Discussion of
"The Macroeconomic Impact of Microeconomic Shocks"
David Baqaee and Emmanuel Farhi

Alireza Tahbaz-Salehi
Columbia Business School

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What is Hulten's Theorem?

- In an efficient economy, the macro impact of a shock to industry i depends on i's sales as a share of aggregate output, up to a first-order approximation.

- Corollary: Firm size distribution is a sufficient statistic for how micro shocks shape macroeconomic outcomes.
- As long as one is concerned with macro outcomes, one can ignore
  - details of firm-to-firm linkages
  - complementarities in production
  - reallocation of primary factors across industries

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- Though mathematically true, the result sounds somewhat unintuitive:
  - Shutting down electricity or the transportation system can have impacts above and beyond each industry's sales as a share of GDP.

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Where Does Hulten's Theorem Come from?

- Consider an economy in which the FWT holds:

$$
\begin{aligned}
C(A_1, \ldots, A_n) = \max \quad & \mathcal{C}(c_1, \ldots, c_n) \\
\text{s.t.} \quad & y_i = A_i f_i(x_{i1}, \ldots, x_{in}, l_i, L_i) \\
& y_i = c_i + \sum_{j=1}^{n} x_{ji}, \quad \sum_{j=1}^{n} l_j = \bar{l}, \quad L_i = \bar{L}_i.
\end{aligned}
$$

- By the envelope theorem: $\frac{\partial C}{\partial A_i} = p_i f_i(x_{i1}, \ldots, x_{in}, l_i, L_i)$.
- Which leads to Hulten's:

$$\frac{\partial \log C}{\partial \log A_i} = \frac{p_i y_i}{C} := \lambda_i \qquad \text{Domar weight of industry } i$$

- Natural (but very much ignored) question: how good is this approximation?

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A Differential Identity

- For any function $C(A_1, \ldots, A_n)$, let,

$$\nabla C = \sum_{i=1}^{n} \frac{\partial \log C}{\partial \log A_i}$$

and define the elasticities

$$1/\rho_{ij} = -\frac{\partial \log(C_i/C_j)}{\partial \log A_i}.$$

- Differential identity:

$$\frac{\partial^2 \log C}{(\partial \log A_i)^2} = \frac{\partial \log C}{\partial \log A_i} \left(\frac{1}{\nabla C} \sum_{j \neq i} (1 - 1/\rho_{ij}) \frac{\partial \log C}{\partial \log A_j} + \frac{\partial \log \nabla C}{\partial \log A_i} \right).$$

Beyond Hulten's Theorem

- As a result of Hulten's, these mechanical objects are economically meaningful in an efficient economy.
- Input-output multiplier:

$$\xi := \nabla C = \sum_{i=1}^{n} \frac{\partial \log C}{\partial \log A_i} = \frac{\text{Gross Output}}{\text{GDP}}$$

- Elasticities:

$$1 - 1/\rho_{ij} = \frac{\partial \log(\lambda_i/\lambda_j)}{\partial \log A_i}.$$

- Hence,

$$\frac{\partial^2 \log C}{(\partial \log A_i)^2} = \frac{\lambda_i}{\xi} \sum_{j \neq i} \lambda_j \frac{\partial \log(\lambda_i/\lambda_j)}{\partial \log A_i} + \lambda_i \frac{\partial \log \xi}{\partial \log A_i}$$

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The Micro Origins of Macro Outcomes

- Second-order approximation:

$$\frac{\partial^2 \log C}{(\partial \log A_i)^2} = \frac{\lambda_i}{\xi} \sum_{j \neq i} \lambda_j \frac{\partial \log(\lambda_i / \lambda_j)}{\partial \log A_i} + \lambda_i \frac{\partial \log \xi}{\partial \log A_i}$$

- Key observations:
  (1) When firm-level shocks are not small, the domar weights may no longer be sufficient statistics for measuring the macro impact of the micro shocks.
  (2) Second-order macro effects depend on first-order “micro effects”.

First-Order Micro Effects in a Structural Model

- Suppose all firms have Cobb-Douglas production technologies, whereas the representative consumer has a CES utility:

$$u(c_1, \ldots, c_n) = \left(\sum_{j=1}^{n} \beta_j c_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

- Input-output matrix: $\mathcal{A}_{ij} = p_i x_{ij} / p_i y_i$.
- Leontief inverse: $\mathcal{L} = (I - \mathcal{A})^{-1}$.

- First-order micro effect:

$$\frac{\partial \lambda_j}{\partial \log A_i} = (\sigma - 1) \left(\sum_{k=1}^{n} \beta_k \ell_{ki} \ell_{kj} - \left(\sum_{k=1}^{n} \beta_k \ell_{ki} \right) \left(\sum_{k=1}^{n} \beta_k \ell_{kj} \right) \right)$$

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Second-Order Macro Effects

- Second-order macro effects are *identical to* first-order micro effects.

$$\frac{\partial^2 \log C}{\partial (\log A_i)^2} = \frac{\partial \lambda_i}{\partial \log A_i} = (\sigma - 1)\left(\sum_{k=1}^{n} \beta_k \ell_{ki}^2 - \left(\sum_{k=1}^{n} \beta_k \ell_{ki}\right)^2\right)$$

- The second-order effects depend on the dispersion of how various goods rely on firm i as a (direct or indirect) supplier: a higher dispersion means a larger second-order term.
- Intuition: Substitutability can only matter when there is differential exposure to the shock.

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Operationalizing the Characterization?

- Hulten, even though imprecise, provides a result in terms of quantities that can be measured.
- Is there an equivalent for the second-order effects?
- Or does one have to rely on a structural model?

A User's Manual?

- The paper mostly concerned with the limitations of relying on Hulten's and makes a convincing case by focusing on the second-order terms.
- But the same criticism applies to the second-order approximation as well, at least quantitatively (even if one thinks higher-order terms are not structurally meaningful).
- In the presence of large shocks, no guarantee that second-order terms are what matter.

- Two alternative take-aways:
  (1) Non-linearities are important and one has to rely on the full non-linear model (as is done in the paper's quantitative section)
  (2) The second-order approximation (ξ & ρ_{ij}) is in and of itself useful.

A User's Manual?

- Possible solution: Upper bound on the size of the approximation error as a function of the shocks and structural elasticities using Taylor's Theorem.

- Clearly, the approximation error is highly network and elasticity dependent. But even rough bounds (say, based on the smallest/largest elasticities) would be useful.

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Summary

- Important contribution, clarifying the role of non-linearities, input-output linkages, and reallocation of factors in translating micro shocks to macro outcomes.

- Clarified a disconnect in my understanding: how come first-order micro effects depend on the elasticities but not the macro effects?

- Would be nice to have a thorough discussion of how the characterizations can be operationalized empirically/quantitatively.