Systemic Risk and Central Clearing Counterparty Design

Andreea Minca
(joint with Hamed Amini and Damir Filipović)

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What this paper is about

- Examine effects of central clearing counterparty (CCP) on a financial network from ex post and ex ante (systemic risk measure) perspective
- Propose CCP design with "hybrid" guarantee fund that is netted against liabilities
- Simple enough for exact analysis of trade off between systemic risk reduction and banks' incentive to join CCP
- Sophisticated enough to capture real world orders of magnitude of capital, guarantee funds, and fees (stylised CDS OTC market data BIS 2010)

Main findings

- Ex post: CCP reduces banks' liquidation and shortfall losses, improves aggregate surplus
- Ex ante: find explicit threshold on CCP capital and guarantee fund for systemic risk reduction
- Design of "hybrid" guarantee fund netted against liabilities is superior to ("pure" guarantee) default fund plus margin fund
  - hybrid implies similar systemic risk
  - hybrid gives much larger banks' incentive compatibility

Outline

1. Financial network
2. Central counterparty clearing
3. Ex post effects of central counterpary clearing
4. Systemic risk and incentive compatibility
5. Simulation study

Outline

Setup

- Two periods $t = 0, 1, 2$
- Values at $t = 1, 2$ are random variables on $(\Omega, \mathcal{F})$
- m interlinked banks $i = 1 \ldots m$

Instruments

Bank i holds

- Cash γ_i: zero return
- External asset (e.g. long-term investment maturing at $t = 2$):
  - fundamental value Q_i at $t = 1, 2$
  - liquidation value $P_i < Q_i$ at $t = 1$
- Interbank liabilities:
  - formation at $t = 0$
  - realization/expiration at $t = 1$: L_{ij}
- No external debt

Example of interbank liabilities: CDS (premiums paid before $t = 0$. At $t = 1$ change in credit spreads or defaults)

Interbank liabilities realize at $t = 1$

- $L_{ij}(\omega)$ cash-amount bank i owes bank j
- $L_i = \sum_{j=1}^{m} L_{ij}$ total nominal liabilities of bank i
- $\sum_{j=1}^{m} L_{ji}$ total nominal receivables from other banks (assets)

Bank i's nominal balance sheet at $t = 1$

- Assets

$$\gamma_i + \sum_{j=1}^m L_{ji} + Q_i$$

- Liabilities

$$L_i + \text{nominal net worth}$$

- Nominal cash balance

$$\gamma_i + \sum_{j=1}^m L_{ji} - L_i$$

Liquidation of external asset at $t = 1$

- If bank i's cash balance is negative,

$$\gamma_i + \sum_{j=1}^{m} L_{ji} < L_i,$$

  it sells external assets at liquidation price $P_i < Q_i$

- Bank i is bankrupt if

$$\underbrace{\gamma_i + \sum_{j=1}^{m} L_{ji} + P_i}_{\text{liquidation value of assets}} < L_i,$$

  and then bank j receives a part of liquidation value of bank i's assets

Interbank liability clearing equilibrium

Interbank liability clearing equilibrium defined as (L^*_{ij}) satisfying

1. Fair allocation:

$$0 \leq L^*_{ij} \leq L_{ij}$$

2. Clearing: $L^*_i = \sum_{j=1}^m L^*_{ij}$ satisfies

$$L^*_i = L_i \wedge \left(\gamma_i + \sum_{j=1}^m L^*_{ji} + P_i\right), \; i = 1 \ldots m$$

Assumption: Let (L^*_{ij}) be an interbank liability clearing equilibrium

Example of interbank clearing equilibrium

Eisenberg and Noe (2001): proportionality rule $\Pi_{ij} = L_{ij}/L_i$ and

$$L^*_{ij} = \Pi_{ij} L^*_i$$

with clearing vector $\boldsymbol{L}^* = (L^*_1, \ldots, L^*_m)$ determined as fixed point

$$\Phi(\boldsymbol{L}^*) = \boldsymbol{L}^*$$

where $\Phi : [0, \boldsymbol{L}] \to [0, \boldsymbol{L}]$ is given by

$$\Phi_i(\ell) = L_i \wedge \left(\gamma_i + \sum_{j=1}^{m} \ell_j \Pi_{ji} + P_i\right), \ i = 1 \ldots m$$

Eisenberg and Noe (2001): If $\gamma_i + P_i > 0$ for all i then there exists a unique interbank clearing equilibrium.

Bank i's terminal net worth at $t = 2$

- Fraction of liquidated external asset

$$Z_i = \frac{\left(L_i - \gamma_i - \sum_{j=1}^{m} L_{ji}^*\right)^+}{P_i} \wedge 1$$

- Assets

$$A_i = \gamma_i + \sum_{j=1}^{m} L_{ji}^* + Z_i P_i + (1 - Z_i) Q_i$$

- Net worth

$$C_i = A_i - L_i$$

Bankruptcy characterization

- Shortfall of bank i equals

$$C_i^- = L_i - L_i^*$$

- Bank i is bankrupt if and only if

$$C_i < 0 \quad (\text{or } L_i^* < L_i)$$

- If bank i is bankrupt then all its external assets are liquidated

$$Z_i = 1$$

Aggregate surplus identity

Lemma: The aggregate surplus depends on interbank liabilities only through implied liquidation losses:

$$\sum_{i=1}^{m} C_i^+ = \sum_{i=1}^{m} \gamma_i + \sum_{i=1}^{m} Q_i - \sum_{i=1}^{m} Z_i(Q_i - P_i).$$

→ Forced liquidation of external assets lowers aggregate surplus.

→ Absent external asset, cash gets only redistributed in network. No dead weight losses.

Outline

Central Clearing Counterparty (CCP)

- We label the CCP as $i = 0$
- All liabilities are cleared through the CCP

$\rightarrow$ star shaped network

- Proportionality rule: CCP liabilities have equal seniority

$\rightarrow$ interbank clearing equilibrium is trivial (no fixed point problem)

Capital structure of CCP

- The CCP is endowed with
  - external equity capital γ_0
  - **guarantee fund**

$$\sum_{i=1}^{m} \mathbf{g_i}$$

  where $\mathbf{g_i} \leq \gamma_i$ is received from bank i at time $t = 0$
- Guarantee fund is hybrid of margin fund and default fund:
  - GF payment g_i netted against bank liability (margin fund)
  - GF absorbs shortfall losses of defaulting banks (default fund)
- Banks' shares in the guarantee fund have equal seniority

Liabilities

- Bank i's net exposure to CCP

$$\Lambda_i = \sum_{j=1}^{m} L_{ji} - \sum_{j=1}^{m} L_{ij}$$

- Bank i's nominal liability to the CCP (**netting**)

$$\widehat{L}_{i0} = \left(\Lambda_i^- - \mathbf{g_i}\right)^+$$

- CCP's nominal liability to bank i

$$\widehat{L}_{0i} = (1 - f)\Lambda_i^+$$

→ CCP charges a volume based fee f on bank i's receivables

$$f \times \Lambda_i^+$$

Nominal guarantee fund

- Bank i's nominal share in the guarantee fund:

$$G_i = (\Lambda_i + g_i)^+ - \Lambda_i^+$$

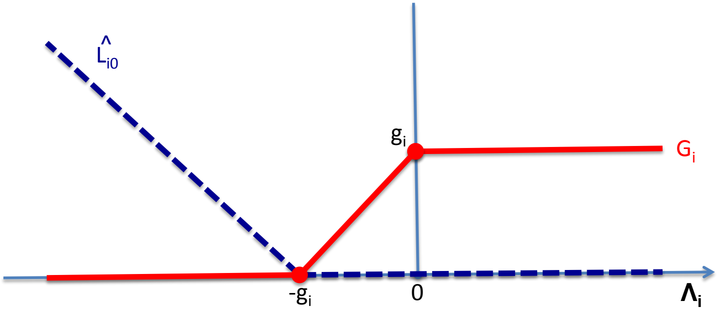


Figure: G_i and $\widehat{L}_{i0}$ as functions of Λ_i

CCP's nominal balance sheet at $t = 1$

Denote $G_{\mathrm{tot}} = \sum_{i=1}^{m} G_i$ total nominal value of guarantee fund

- Assets: $\gamma_0 + \sum_{i=1}^{m} g_i + \sum_{i=1}^{m} \widehat{L}_{i0}$,
- Liabilities: $\widehat{L}_0 + G_{\mathrm{tot}} +$ nominal net worth $\left(\gamma_0 + \sum_{i=1}^{m} f \Lambda_i^{+}\right)$.

Liability clearing equilibrium

- Fraction of external assets liquidated $(\widehat{L}_{i0} \times \widehat{L}_{0i} = 0)$

$$\widehat{Z}_i = \frac{\left(\gamma_i - g_i - \widehat{L}_{i0}\right)^-}{P_i} \wedge 1$$

- Clearing payment of bank i to CCP

$$\widehat{L}_i^* = \widehat{L}_{i0} \wedge (\gamma_i - g_i + P_i)$$

- Value of CCP's total assets become

$$\widehat{A}_0 = \gamma_0 + \sum_{i=1}^m g_i + \sum_{i=1}^m \widehat{L}_i^*$$

- Clearing payment of CCP

$$\widehat{L}_0^* = \widehat{L}_0 \wedge \widehat{A}_0$$

- Bank i receives (proportionality rule)

$$\widehat{L}_{0i}^* = \frac{\widehat{L}_{0i}}{\widehat{L}_0} \times \widehat{L}_0^*$$

Liquidation of the guarantee fund at $t = 2$

- Guarantee fund = first layer, prior to nominal net worth

$$G^*_{\mathrm{tot}} = G_{\mathrm{tot}} \wedge \left(\widehat{A}_0 - \widehat{L}^*_0 - \gamma_0 - \sum_{i=1}^{m} f \Lambda_i^+ \right)^+$$

- Bank i receives (proportionality rule)

$$G^*_i = \frac{G_i}{G_{\mathrm{tot}}} \times G^*_{\mathrm{tot}}$$

Terminal net worth

- CCP

$$\widehat{C}_0 = \widehat{A}_0 - \widehat{L}_0 - G^*_{\mathrm{tot}}$$

- Bank i's assets

$$\widehat{A}_i = \gamma_i + \widehat{Z}_i P_i + (1 - \widehat{Z}_i) Q_i + \frac{\widehat{L}_{0i}}{\widehat{L}_0} \times \widehat{L}^*_0 + G^*_i - g_i$$

- Bank i's net worth

$$\widehat{C}_i = \widehat{A}_i - \widehat{L}_{i0}$$

- Shortfall of CCP and banks becomes

$$\widehat{C}^-_i = \widehat{L}_i - \widehat{L}^*_i$$

Aggregate surplus identity with CCP

Lemma: The aggregate surplus with CCP depends on clearing mechanism only through implied liquidation losses:

$$\sum_{i=0}^{m} \widehat{C}_i^+ = \sum_{i=0}^{m} \gamma_i + \sum_{i=1}^{m} Q_i - \sum_{i=1}^{m} \widehat{Z}_i(Q_i - P_i).$$

Outline

Independence from fee and guarantee fund policy

Write $\boldsymbol{g} = (g_1, \ldots, g_m)$.

Lemma:

- Number of liquidated assets $\widehat{Z}_i$ does not depend on $(f, \boldsymbol{g})$
- Shortfall of bank i does not depend on $(f, \boldsymbol{g})$

$$\widehat{C}_i^- = (\Lambda_i + P_i + \gamma_i)^-$$

- Aggregate surplus dos not depend on $(f, \boldsymbol{g})$

Scope

- Compare financial network with and without CCP
- **Convention:** For comparison we set

$$C_0 = \gamma_0$$

CCP ex post effects

Theorem:

The CCP reduces

- liquidation losses $\widehat{Z}_i \leq Z_i$
- bank shortfalls (bankruptcy cost) $\widehat{C}_i^- \leq C_i^-$

The CCP improves

- aggregate terminal bank net worth $\sum_{i=1}^m \widehat{C}_i \geq \sum_{i=1}^m C_i$
- aggregate surplus

$$\sum_{i=0}^m \widehat{C}_i^+ = \sum_{i=0}^m C_i^+ + \underbrace{(Q_i - P_i) \sum_{i=1}^m (Z_i - \widehat{Z}_i)}_{\geq 0}$$

The CCP imposes shortfall risk $\widehat{C}_0^- \geq 0$

CCP impact on banks' net worth decomposition

Theorem: Difference in net worth of bank i is decomposed in

$$\widehat{C}_i - C_i = T_1 + T_2 + T_3$$

corresponding to

- counterparty default:

$$T_1 = -\frac{\Lambda_i^+}{\sum_{i=1}^m \Lambda_i^+} \widehat{C}_0^- + \sum_{j=1}^m (L_{ji} - L_{ji}^*)$$

- liquidation loss:

$$T_2 = (Z_i - \widehat{Z}_i)(Q_i - P_i) \geq 0$$

- fees and losses in guarantee fund:

$$T_3 = -f\Lambda_i^+ - \frac{G_i}{G_{\mathrm{tot}}} (G_{\mathrm{tot}} - G_{\mathrm{tot}}^*) \leq 0$$

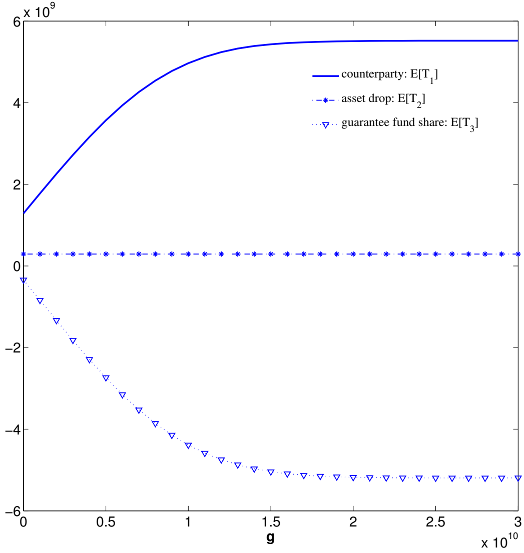


Figure: Expected differences in stand-alone risk components with and without

Outline

Systemic risk measure

- Write $\boldsymbol{C} = (C_0, \ldots, C_m)$ and $\widehat{\boldsymbol{C}} = (\widehat{C}_0, \ldots, \widehat{C}_m)$
- Generic coherent risk measure $\rho(X)$
- Aggregation function, $\alpha \in [1/2, 1]$,

$$A_\alpha(\boldsymbol{C}) = \underbrace{\alpha \sum_{i=0}^m C_i^-}_{\text{bankruptcy cost}} - \underbrace{(1-\alpha) \sum_{i=0}^m C_i^+}_{\text{tax benefits}}$$

- Systemic risk measure (Chen, Iyengar, and Moallemi 2013)

$$\mathcal{R}(\boldsymbol{C}) = \rho\left(A_\alpha(\boldsymbol{C})\right)$$

Impact on aggregation function

Lemma:

$$A_\alpha(\widehat{\boldsymbol{C}}) - A_\alpha(\boldsymbol{C}) = \alpha \widehat{C}_0^- - \Delta_\alpha$$

where

$$\Delta_\alpha = \alpha \sum_{i=1}^m \left(C_i^- - \widehat{C}_i^- \right) + (1-\alpha)(Q-P) \sum_{i=1}^m \left(Z_i - \widehat{Z}_i \right)$$

is nonnegative, $\Delta_\alpha \geq 0$, and does not depend on $(f, \boldsymbol{g})$. Hence

$$\mathcal{R}(\widehat{\boldsymbol{C}}) - \mathcal{R}(\boldsymbol{C}) = \rho\left(A_\alpha(\widehat{\boldsymbol{C}})\right) - \rho\left(A_\alpha(\boldsymbol{C})\right) \leq \rho\left(A_\alpha(\widehat{\boldsymbol{C}}) - A_\alpha(\boldsymbol{C})\right)$$
$$\leq \alpha\rho\left(\widehat{C}_0^-\right) + \rho(-\Delta_\alpha)$$

with equlity if $\rho(X) = \mathbb{E}[X]$.

Systemic risk reduction

Theorem: The CCP reduces systemic risk, $\mathcal{R}(\widehat{\boldsymbol{C}}) < \mathcal{R}(\boldsymbol{C})$, if[1]

$$\underbrace{\alpha\rho\left(\widehat{C}_0^-\right)}_{\text{shortfall risk of CCP}} < \underbrace{-\rho\left(-\Delta_\alpha\right)}_{\text{risk-adjusted value of } \Delta_\alpha}$$

where

$$\Delta_\alpha = \alpha \underbrace{\sum_{i=1}^m \left(C_i^- - \widehat{C}_i^-\right)}_{\text{cost of intermediation}} + (1-\alpha) \underbrace{\sum_{i=1}^m \left(Z_i - \widehat{Z}_i\right)\left(Q_i - P_i\right)}_{\text{mitigation on liquidation losses}} \geq 0$$

does not depend on $(f, \boldsymbol{g})$.

[1] if and only if for $\rho(X) = \mathbb{E}[X]$

Acceptable equity, fee, and guarantee fund policies

- CCP and banks are risk neutral
- Utility function = expected surplus $\mathbb{E}\left[C_i^+\right]$
- Policy $(\gamma_0, f, \boldsymbol{g})$ is incentive compatible if

$$\mathbb{E}\left[\widehat{C}_i^{\,+}\right] \geq \mathbb{E}\left[C_i^+\right] \quad \forall i = 0 \ldots m.$$

- Policy $(\gamma_0, f, \boldsymbol{g})$ is acceptable if incentive compatible and

$$\mathcal{R}(\widehat{\boldsymbol{C}}) \leq \mathcal{R}(\boldsymbol{C})$$

Symmetric case

Assumption: $\gamma_i \equiv \gamma$, $g_i \equiv g$, and

$$(Q_i, P_i, \{L_{ij}\}_{j=1\ldots m}, \{L_{ji}\}_{j=1\ldots m}), \quad i = 1 \ldots m$$

is exchangeable.

Theorem:

- Policy $(\gamma_0, f, \boldsymbol{g})$ incentive compatible if and only if

$$\gamma_0 \leq \mathbb{E}\left[\widehat{C_0}^+\right] \leq \gamma_0 + \sum_{i=1}^{m} \mathbb{E}\left[\left(Z_i - \widehat{Z}_i\right)(Q_i - P_i)\right]$$

- Existence theorem for acceptable policies
- Every acceptable policy is Pareto optimal

Outline

Parameters

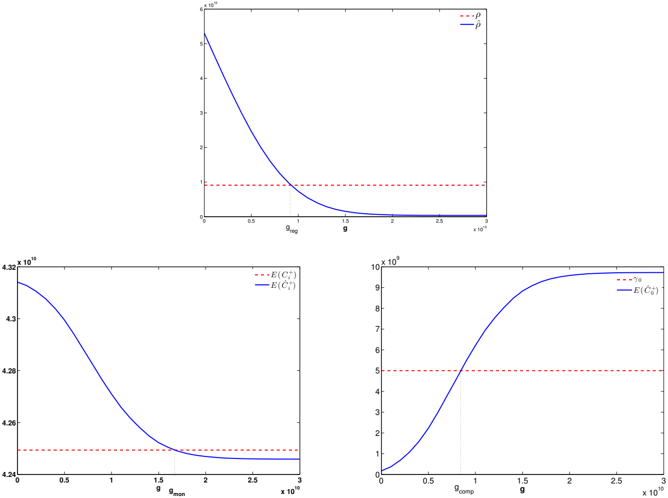
- Symmetric CDS inter dealer network based on BIS 2010 data
- gross market value $W = \$1tn$
- $m = 14$ banks
- $\gamma_i = \gamma = \$10bn$
- $Q_i = Q = \$11bn$, $P_i = Q_i/2$
- CCP: $\gamma_0 = \$5bn$, fee $f = 2\%$ ($\approx 1bp$ of notional)
- Systemic risk measure $\mathcal{R}(\boldsymbol{C}) = \mathbb{E}\left[A_{0.9}(\boldsymbol{C})\right]$
- Model:

$$W = \sum_{i \neq j} \mathbb{E}\left[|X_{ij}|\right], \quad X_{ij} \text{ i.i.d. } N(0, \sigma)$$

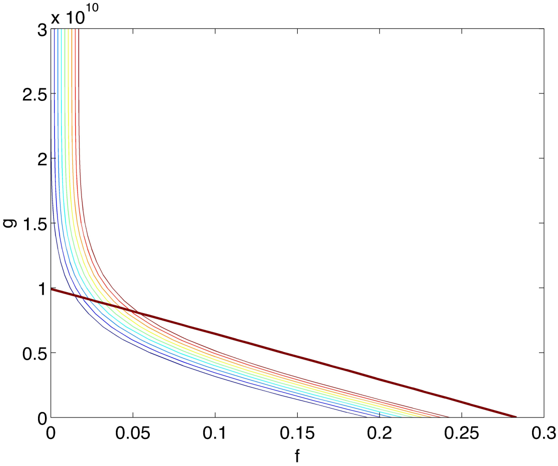
$$L_{ij} = \left(|X_{ij}| - |X_{ji}|\right)^+$$

Systemic risk, banks' and CCP utility as functions of g

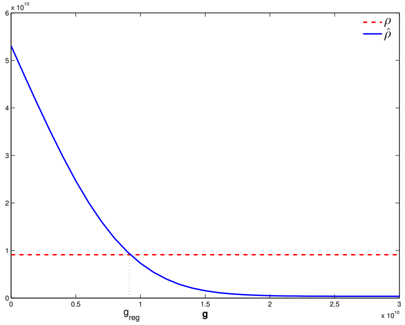
$\exists$ acceptable and incentive compatible policies: $g_{\mathrm{reg}}, g_{\mathrm{comp}} < g_{\mathrm{mon}}$

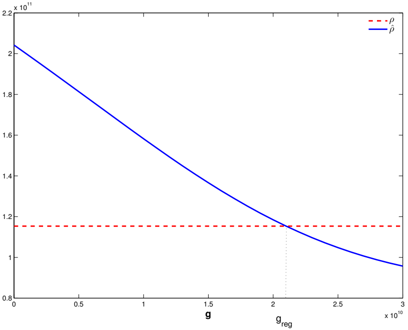


Incentive compatible utility indifference curves and systemic risk zero line in (f, g)



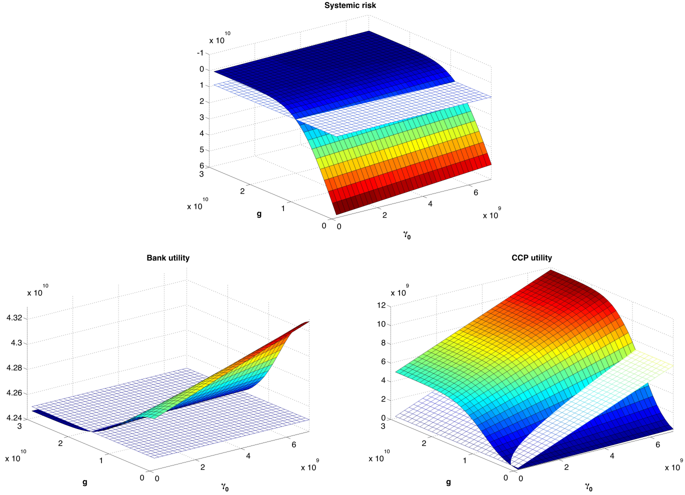
Systemic risk as functions of g for $m = 14$ vs. 10 banks





g_{reg} doubles: concentration risk matters!

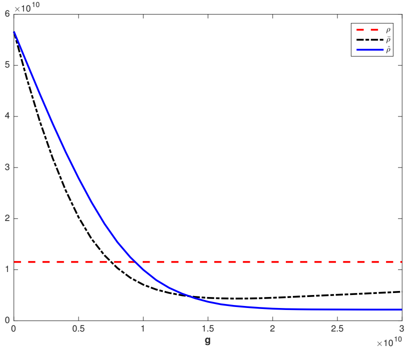
Systemic risk, banks' and CCP utility as functions of g, γ_0

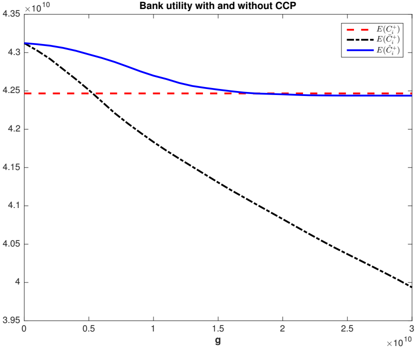


Hybrid vs. pure (default) guarantee fund

Pure guarantee fund: not netted agains liabilities, $\overline{L}_{i0} = \Lambda_i^-$.

Assets remaining with bank i, $\gamma_i - g_i + P_i$, form margin fund.





Systemic risk improvement is limited, while banks have no incentive compatibility: $g_{\mathrm{mon}} < g_{\mathrm{reg}}$.

Conclusion

- General financial network setup with and without CCP
- CCP improves aggregate surplus due to lower liquidation losses
- CCP reduces banks' bankruptcy cost
- CCP introduces tail risk, and may increase systemic risk
- Find exact condition for systemic risk reduction
- Simulation study illustrates range of acceptable CCP equity, fee, and guarantee fund policies
- Hybrid guarantee fund design greatly improves banks incentives to join CCP