Networks in Production: Asset Pricing Implications

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Third Economic Networks and Finance Conference
London School of Economics

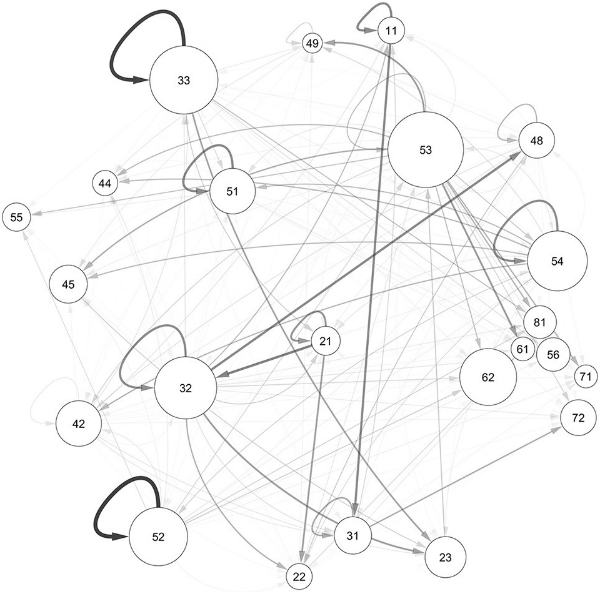
December 2015

Introduction

- Input-output network and technology
- How are changes in the input-output network priced?
- Theory – general equilibrium model

  Network factors: priced sources of risk
- Data – new asset pricing factors

Introduction: input-output network



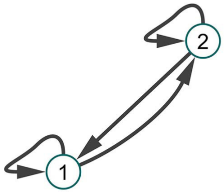
Introduction: concentration and sparsity

- **Concentration** (nodes/circles)

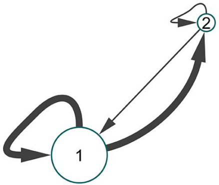
  Large sectors – concentrated network
  Output concentration
  Decreases output

- **Sparsity** (edges/arrows)

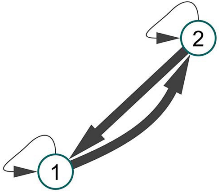
  Few thick arrows – sparse network
  Input specialization
  Increases output



(a) Low Concentration
Low Sparsity



(b) High Concentration
High Sparsity



(c) Low Concentration
High Sparsity

Introduction: how are the network factors priced?

- **Concentration** innovations

  Decrease consumption growth and increase marginal utility

  Negative price of risk

  $\therefore$ more exposure to concentration $\Rightarrow$ lower returns
  Return spread of -4% with similar FF/CAPM alpha

- **Sparsity** innovations

  Increase consumption growth and decrease marginal utility

  Positive price of risk

  $\therefore$ more exposure to sparsity $\Rightarrow$ higher returns
  Return spread of 6% with similar FF/CAPM alpha

Related Papers

- Multisector models, input-output and aggregation:
  - Long and Plosser (1983)
  - Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012)
- Networks and asset pricing:
  - Ahern (2012)
  - Kelly, Lustig, and Van Nieuwerburgh (2012)
- Production-based asset pricing:
  - Papanikolaou (2011)
  - Loualiche (2012)
  - Kung and Schmid (2013)
- Sectoral composition risk:
  - Martin (2013)
  - Cochrane, Longstaff, and Santa-Clara (2008)

Multisector Model

Representative Household

- n goods
- Epstein-Zin recursive preferences

$$U_t = \left[(1-\beta)\, \mathcal{C}_t^{1-\rho} + \beta \left(\mathbb{E}_t \left(U_{t+1}^{1-\gamma} \right) \right)^{\frac{1-\rho}{1-\gamma}} \right]^{\frac{1}{1-\rho}}$$

w/ Cobb-Douglas consumption aggregator: $\mathcal{C}_t = \prod_{i=1}^{n} c_{i,t}^{\alpha_i}$

- Budget constraint

$$\sum_{i=1}^{n} P_{i,t} c_{i,t} + \sum_{i=1}^{n} \varphi_{i,t+1} \left(V_{i,t} - D_{i,t} \right) = \sum_{i=1}^{n} \varphi_{i,t} V_{i,t}$$

$V_{i,t}$ cum-dividend price of firm i

$\varphi_{i,t}$ share holding of firm i

$D_{i,t}$ dividend of firm i

$c_{i,t}$ consumption of good i

Firms

- n firms and n goods: firm i produces good i
- i buys inputs $\{y_{i1,t}, \ldots, y_{in,t}\}$ from other firms
- Final output $Y_{i,t}$: combination of inputs
- Maximization problem

$$
\begin{aligned}
D_t \quad &= \max_{\{y_{ij,t}\}_j, I_{i,t}} \quad P_{i,t}Y_{i,t} - \sum_{j=1}^{n} P_{j,t}y_{ij,t} \\
&s.t. \qquad Y_{i,t} = \varepsilon_{i,t} I_{i,t}^{\eta} \\
&\qquad\qquad I_{i,t} = \prod_{j=1}^{n} y_{ij,t}^{w_{ij,t}}
\end{aligned}
$$

$\eta < 1$ diminishing returns

$\varepsilon_{i,t}$ sector specific productivity

$w_{ij,t}$ network weight of firm i on firm j

alt.

Network

$$I_{i,t} = \prod_{j=1}^{n} y_{ij,t}^{w_{ij,t}} \qquad\qquad W_t = \begin{bmatrix} w_{11,t} & \cdots & w_{1n,t} \\ \vdots & \ddots & \vdots \\ w_{n1,t} & \cdots & w_{nn,t} \end{bmatrix}_{n \times n}$$

- Network Weights

  $w_{ij,t}$: fraction i spends on inputs from j

  $w_{ij,t}$: elasticity of $I_{i,t}$ with respect to input j

- Network Properties

$$\sum_{j=1}^{n} w_{ij,t} = 1 \qquad \text{and} \qquad w_{ij,t} \geq 0$$

- W_t: exogenous, stochastic, arbitrary dynamics

Competitive Equilibrium

Definition

A competitive equilibrium consists of spot market prices $(P_{1,t}, \cdots, P_{n,t})$, value of the firms $(V_{1,t}, \cdots, V_{n,t})$, consumption bundle $(c_{1,t}, \cdots, c_{n,t})$, shares holdings $(\varphi_{1,t}, \cdots, \varphi_{n,t})$ and inputs bundles $(y_{ij,t})_{ij}$ such that

1. Given prices, household and firms maximize
2. Markets clear

$$c_{i,t} + \sum_{j=1}^{n} y_{ji,t} = Y_{i,t} \ \forall i, t \quad \text{(goods)}$$

$$\varphi_{i,t} = 1 \ \forall i, t \quad \text{(assets)}$$

Output Shares

- Output share of firm i

$$\delta_{i,t} = \frac{P_{i,t}Y_{i,t}}{\sum_{j=1}^{n} P_{j,t}Y_{j,t}}$$

- In equilibrium

$$\begin{aligned} \delta_{j,t} &= (1-\eta)\alpha_j + \eta \sum_{i=1}^{n} w_{ij,t}\delta_{i,t} \\ &= (1-\eta)\alpha_j + \\ &\quad \eta \sum_{i=1}^{n} \alpha_i w_{ij,t} + \eta^2 \sum_{i=1}^{n}\sum_{k=1}^{n} \alpha_i w_{ik,t} w_{kj,t} + \dots \end{aligned}$$

- Feedback effects: decaying rate η

Theorem

- In equilibrium, consumption growth is given by

$$\frac{1}{1-\eta}\left[(e_{t+1}-e_t)-(1-\eta)(\mathcal{N}_{t+1}^{\mathcal{C}}-\mathcal{N}_t^{\mathcal{C}})+\eta(\mathcal{N}_{t+1}^{\mathcal{S}}-\mathcal{N}_t^{\mathcal{S}})\right]$$

where

$$\begin{aligned} e_t &= \textstyle\sum_{i=1}^{n}\delta_{i,t}\log\varepsilon_{i,t} && \text{(residual TFP)} \\ \mathcal{N}_t^{\mathcal{C}} &= \textstyle\sum_{i=1}^{n}\delta_{i,t}\log\delta_{i,t} && \text{(concentration)} \\ \mathcal{N}_t^{\mathcal{S}} &= \textstyle\sum_{i=1}^{n}\delta_{i,t}\sum_{j=1}^{n}w_{ij_t}\log w_{ij,t} && \text{(sparsity)} \end{aligned}$$

and $\delta_{j,t}$ is the equilibrium output share of firm j

$$\delta_{j,t}=(1-\eta)\alpha_j+\eta\sum_{i=1}^{n}\alpha_i w_{ij,t}+\eta^2\sum_{i=1}^{n}\sum_{k=1}^{n}\alpha_i w_{ik,t}w_{kj,t}+\dots$$

Network Concentration

$$\mathcal{N}_t^{\mathcal{C}} = \sum_{i=1}^{n} \delta_{i,t} \log \delta_{i,t}$$

- Sectoral Output Concentration
  - Min if $\delta_{j,t} = \frac{1}{n}$ (equal shares)
  - Max if $\delta_{s,t} = 1$ and $\delta_{j,t} = 0 \ \forall j \neq s$ (concentrated shares)
- Good news for consumption? No
  - Decreases consumption
  - Production relies on fewer sectors: diminishing returns

Network Sparsity

$$\mathcal{N}_t^{\mathcal{S}} = \sum_i \delta_{i,t} \underbrace{\sum_j w_{ij,t} \log w_{ij,t}}_{\equiv \mathcal{N}_{i,t}^{\mathcal{S}}}$$

- High $\mathcal{N}_{i,t}^{\mathcal{S}} \implies$ row i with few high entries (thick arrows)
- High $\mathcal{N}_t^{\mathcal{S}} \implies$ sparse network

$$W_t = \begin{bmatrix} w_{11,t} & \cdots 0 \cdots & w_{1n,t} \\ \vdots & \ddots & \vdots \\ w_{n1,t} & \cdots 0 \cdots & w_{nn,t} \end{bmatrix}_{n\times n}$$

- Dispersion of marginal product and output elasticities
- Gains from input specialization

Example: why does sparsity increase consumption?

- Firm i has \$ k to buy inputs, what is the optimal output?
- $\varepsilon_j = 1$, $P_j = 1$ for every $j = 1, \dots, n$

Scenario 1: high sparsity

- $w_{ij} = 1$ for some j and $w_{is} = 0$ for every $s \neq j$
- $y_{ij} = k$ for some j and $y_{is} = 0$ for every $s \neq j$

$$Y_i = k^\eta$$

Scenario 2: low sparsity

- $w_{ij} = \frac{1}{n}$
- $y_{ij} = \frac{k}{n}$

$$Y_i = \frac{k^\eta}{n^\eta}$$

Why Does Sparsity Increase Consumption?

- (Partial eq.) If i spends $\$k$, then

$$y_{ij,t} = w_{ij,t}\frac{k}{P_{j,t}} \implies Y_{i,t} = \frac{\varepsilon_{i,t}\left(\prod_j w_{ij,t}^{w_{ij,t}}\right)^{\eta}}{\left(\prod_j P_{j,t}^{w_{ij,t}}\right)^{\eta}} k^{\eta}$$

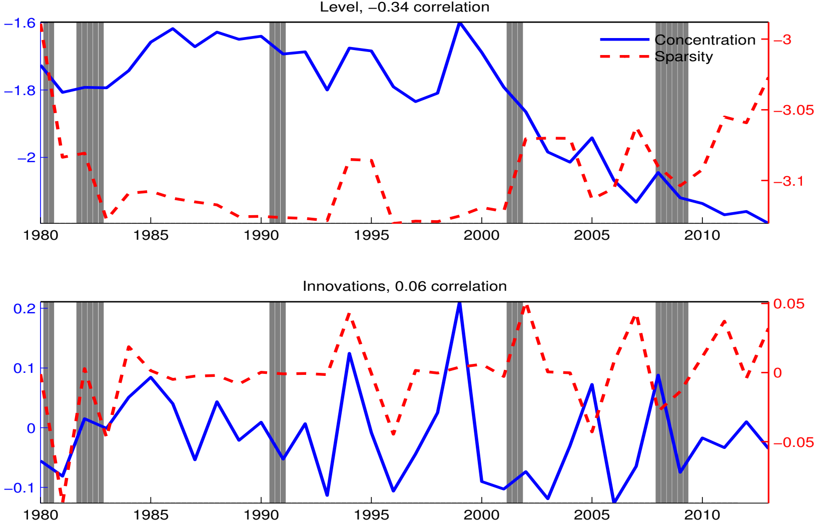
  - **substitution of inputs**: input specialization
  - **changes in marginal cost**: different input bundle
- (General eq.) **Sparsity increases output**

$$\Delta \log \sum_i P_{i,t+1} Y_{i,t+1} = \frac{\eta}{1-\eta} \Delta \sum_i \delta_{i,t+1} \log \prod_j w_{ij,t+1}^{w_{ij,t+1}}$$

  - keeping network concentration constant

Data

Network Factors



Constructing Beta-Sorted Portfolios

1. CRSP monthly data: form annual returns for each stock
2. For each stock, regress excess returns on the factors' innovations over a 15 year window:

$$r^i_{t+1} - r^f_t = \alpha^i + \beta^i_{\mathcal{N}^{\mathcal{S}}} \Delta\mathcal{N}^{\mathcal{S}}_{t+1} + \beta^i_{\mathcal{N}^{\mathcal{C}}} \Delta\mathcal{N}^{\mathcal{C}}_{t+1} + Controls + \xi^i_t$$

   - $\beta^i_{\mathcal{N}^{\mathcal{S}}}$ and $\beta^i_{\mathcal{N}^{\mathcal{C}}}$: exposure of stock i to factors' innovations
   - Sample: stocks with network data
   - Controls: factors in level and orthogonalized TFP

3. Form portfolios sorted by $\beta^i_{\mathcal{N}^{\mathcal{S}}}$ and $\beta^i_{\mathcal{N}^{\mathcal{C}}}$ terciles
4. Compute subsequent year's return for the sorted portfolio
5. Verify return spread

Sorted Portfolios

Table: One Way Sorted Portfolios

	(1)	(2)	(3)	(3)-(1)	t-stat
Panel A: Sparsity					
Avg. Exc. Returns (%)	5.24	8.61	11.25	6.01	2.26
α_{CAPM}	−3.15	2.29	4.78	7.92	3.11
α_{FF}	−3.21	1.47	3.84	7.04	2.91
Volatility (%)	17.60	13.78	15.13	11.60	–
Book/Market	0.76	0.67	0.70	–	–
Avg. Market Value ($bn)	1.53	2.18	1.23		
Panel B: Concentration					
	(1)	(2)	(3)	(3)-(1)	t-stat
Avg. Exc. Returns (%)	10.23	8.51	6.19	−4.04	−2.19
α_{CAPM}	2.62	2.43	−1.60	−4.21	−2.26
α_{FF}	2.00	1.64	−2.00	−4.01	−2.12
Volatility (%)	16.18	13.60	16.27	8.05	–
Book/Market	0.74	0.69	0.70	–	–
Avg. Market Value ($bn)	0.91	2.03	2.00		

more: ret

Why do sectors have different network betas?

- Dividend growth:

$$D_{i,t} = (1-\eta)\delta_{i,t} z_t \implies \Delta d_{i,t+1} = \Delta \log \delta_{i,t+1} + \Delta \log z_{t+1}.$$

- Cross-sectional heterogeneity: changes in output shares
- Concentration beta

  Network centrality / size

- Sparsity beta

$$\mathcal{N}_t^{\mathcal{S}} \equiv \sum_{i=1}^{n} \delta_{i,t} \sum_{j=1}^{n} w_{ij_t} \log w_{ij,t} = \sum_{j=1}^{n} \underbrace{\sum_{i=1}^{n} \delta_{i,t} w_{ij_t} \log w_{ij,t}}_{\text{out-sparsity of sector } j}$$

Concluding Remarks

- New production-based asset pricing factors
  - Network sparsity and concentration
- Sources of aggregate risk
- Sparsity-beta sorted portfolios

  6% return spread per year on avg
- Concentration-beta sorted portfolios

  -4% return spread per year on avg
- Spreads not explained by CAPM or Fama French factors
- Calibrated model replicates return spreads

Annex

Firms

Maximization problem

$$
\begin{aligned}
D_t &= \max_{\{y_{ij,t}\}_j, I_{i,t}} \quad P_{i,t} Y_{i,t} - \sum_{j=1}^{n} P_{j,t} y_{ij,t} \\
& \qquad s.t. \qquad Y_{i,t} = \varepsilon_{i,t} I_{i,t}^{\eta} L_{i,t}{}^{1-\eta} \\
& \qquad\qquad\qquad I_{i,t} = \prod_{j=1}^{n} y_{ij,t}^{w_{ij,t}}
\end{aligned}
$$

- $\eta < 1$ diminishing returns
- $\varepsilon_{i,t}$ sector specific productivity
- $w_{ij,t}$ network weight of firm i on firm j
- $L_{i,t} = 1$

back

Robustness: sorted portfolios

Table: Return Spreads

	Sparsity-beta sort		Concentration-beta sort	
	(3)-(1)	t-stat	(3)-(1)	t-stat
Benchmark	6.01	2.26	−4.04	−2.19
No level control	4.47	1.90	−3.50	−1.55
All CRSP stocks	5.78	2.17	−3.83	−2.13
Out of Sample	0.31	0.14	−3.25	−1.61
R. TFP Cons.	6.03	2.09	−3.42	−1.64
No TFP	5.49	1.92	−4.89	−2.51
16-year window	5.51	1.92	−5.35	−2.73
17-year window	4.91	1.46	−6.00	−2.52
18-year window	4.57	1.22	−5.15	−2.19
19-year window	8.54	2.02	−5.93	−2.45
20-year window	6.48	1.73	−3.60	−1.63

back