Bank Runs, Prudential Tools and Social Welfare in a Global Game General Equilibrium Model*

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Abstract

I develop a general equilibrium model of bank runs in a global game framework. The model features banking crises triggered by endogenous bank runs. A bank run probability – systemic risk – is increasing in bank leverage and decreasing in bank liquid asset holdings. Bank risk shifting and pecuniary externalities induce excessive leverage and insufficient liquidity, elevating systemic risk in a competitive equilibrium. Addressing this inefficiency calls for prudential tools on both leverage and liquidity. Imposing one tool only causes risk migration: banks respond by taking more risk in another area. I extend the model and discuss bank-specific capital requirements, risk weights, concentration risk, shadow banks and deposit insurance.

Keywords: Bank runs; global games; endogenous financial crisis probability; capital and liquidity requirements; bank-specific capital requirements; concentration risk.

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1 Introduction

Ten years have passed since the global financial crisis. In 2019 the new regulatory framework, Basel III, to prevent the recurrence of such a crisis will be fully implemented. The recovery phase of the banking system is over and we are moving toward the evaluation phase of the financial regulatory reform.

A key to the evaluation phase is to understand how and to what extent new regulations enhance financial system resilience and to assess their social benefits and costs. Doing so is challenging, however, due to the still incomplete financial intermediation theory (Financial Stability Board 2017). What we need is a model that helps us understand more about how the financial system responds to new regulations. In light of the objective of Basel III – building a more resilient financial system to systemic risk – and given its multiple-tool approach, three ingredients are essential for developing such a model: (i) a systemic risk event that triggers a banking crisis, (ii) banking system resilience to such an event, and (iii) externalities that warrant the implementation of multiple tools.

In this paper, I develop a simple model that features these essential ingredients. Specifically, I embed a bank run global game model studied by Rochet and Vives (2004) into a two-period general equilibrium model in the spirit of Christiano and Ikeda (2013, 2016). The model features bank runs as a systemic event, reflecting the historical fact that most of the financial crises have involved bank runs (Gorton 2012 and Reinhart and Rogoff 2009). The probability of bank runs – systemic risk – is endogenously determined as a function of banking system fundamentals such as bank leverage and bank liquid asset holdings. These fundamentals form banking system resilience to systemic risk. Thus, the model provides a link between systemic risk and banking system fundamentals.

To identify externalities and derive policy implications for prudential tools, I first study a benchmark model in which banks hold risky assets only. The model consists of three types of agents: households, banks and fund managers. Households and banks receive an endowment – household income and bank capital, respectively – in the beginning of the first period. Households allocate the income into consumption and bank deposits. Banks offer a deposit contract such that banks pay a promised interest rate as long as they do not default. The contract allows early withdrawals of funds in the beginning of the second period. Banks combine deposits and bank capital to invest in a risky project. If the project return is low enough, the banks, unable to pay the interest rate, default and the depositors incur a loss. Households can avoid such a loss if they successfully withdraw deposits early. Because fund managers have information advantages about the bank asset return, households delegate their early withdrawal decision to fund managers. But early withdrawal is costly for banks as they have to sell some assets at a fire sale price. This costly liquidation causes illiquidity-driven-bank-defaults if a large number of fund managers withdraw funds early. As shown by Rochet and Vives (2004) this structure gives rise to to a global game in which a bank run occurs if the bank asset return is lower than a certain threshold. Both households and banks take into account the bank run probability in choosing how much to lend and borrow, respectively. In the second period, banks distribute the profits to households and households consume everything at hand.

A unique feature of this model is that bank leverage is pinned down without any binding constraints. But for bank runs, banks would increase leverage as long as the expected bank asset return is greater than the interest rate, as in the various financial friction models studied by Christiano and Ikeda (2013). With bank runs, however, a higher leverage increases bank-run-led default probability and thereby decreases the expected profits. Taking a balance between the two, the bank leverage has an interior solution.

To derive welfare implications, I set up a second best problem in which a benevolent regulator chooses leverage subject to bank run risk. The first best outcome should involve no bank run because bank runs occur as a result of coordination failures. However, forcing fund managers not to run is neither possible in the model nor practical in reality. By restricting leverage, the regulator can affect fund managers' decisions on runs and the degree of the banking system vulnerability to runs.

I analytically show that bank leverage is excessive in a competitive equilibrium relative to the outcome of the regulator's problem for two reasons: banks' risk-shifting and pecuniary externalities. First, a key assumption that drives the excessive leverage is that banks cannot commit to their actions that they take after receiving deposits and that households cannot enforce banks to take specific actions. This lack of commitment and enforcement gives rise to a credit market in which only a deposit interest rate works as a market signal. For households, banks offering the same interest rate look perfectly identical ex ante. Households provide funds to take into account the riskiness of banks, rationally expecting that banks will behave to maximize profits given the interest rate. In this sense, unlike Kareken and Wallace (1978), the deposits are fairly priced. But each bank chooses high leverage, knowing that doing so would not affect the interest rate they face. In aggregate, the banking system ends up with excessive leverage and an elevated crisis probability, a phenomenon known as risk shifting in the spirit of Jensen and Meckling (1976).

Second, the model has pecuniary externalities that work through the interest rate as in Christiano and Ikeda (2016). The cost of bank runs depends on the interest rate, which, in turn, is affected by leverage. Banks ignore the externalities because they take prices as given in a competitive equilibrium.

The excessive leverage warrants a prudential instrument on leverage to limit systemic risk. Doing so, however, involves a trade-off by restricting financial intermediation from households to the real sector. Prudential policy has to balance between stabilizing the financial system – decreasing the crisis probability – and promoting the real economy by maintaining banks' capacity to intermediate funds.

Having studied the benchmark model, the paper extends it to incorporate a bank liquidity choice. In this model, banks choose how much liquidity to hold in addition to the amount of loans to a risky project, taking into account that such a choice will affect their bank run probability. In particular, while an increase in liquidity lowers the return on bank assets, it lowers the probability of bank runs because liquidity can be used as a buffer against early withdrawals. Liquidity holdings, required by regulators or not, are usable liquidity in this model and hence the model abstracts away from Goodhart (2008)'s concern on liquidity usability.

The model provides sharp analytical results on interactions between leverage and liquidity requirements. The competitive equilibrium features insufficient liquidity, given leverage, and excessive leverage, given liquidity, relative to the outcome of the regulator's problem. The result implies that with only one instrument put in place, either a leverage or liquidity tool, such an economy is always inefficient. Hence, it calls for implementing prudential tools on both leverage and liquidity to address the inefficiency. It is worth mentioning that the insufficient liquidity stems from banks' risk shifting only while the excessive leverage arises from both risk shifting and pecuniary externalities. This is because a liquidity choice in the asset side does not affect the total amount of borrowing, and hence it does not induce pecuniary externalities.

The model also highlights the general equilibrium effect on the jointly optimal leverage and liquidity requirements. Its numerical example illustrates that a leverage restriction should be tightened more than a liquidity requirement relative to the competitive equilibrium allocation if the supply curve of funds, which is derived from the household problem, is steep. As the supply curve becomes steeper, restricting leverage reduces the interest rate more, yielding an additional benefit of reducing the crisis probability. This result suggests that the jointly optimal requirements can differ significantly depending on the supply side of funds, e.g. a small open economy or a closed economy.

A key insight obtained from the model is risk migration: risk can migrate from one area to another. Imposing one tool only – either leverage or liquidity – induces banks to take more risk in another non-regulated area, attenuating the intended effects of the instrument. This bank behaviour can be regarded as regulatory arbitrage.

The benchmark model is so stylised that it has rich applications for banks' behaviour and other prudential instruments including bank/sector specific capital requirements and caps on concentration risk. Yet, risk migration between two risk areas penetrates the applications. In the model with heterogeneous banks, the two risk areas are leverage of regulated banks and leverage of unregulated 'shadow' banks. This model also allows us to study bank-specific capital requirements and sectoral capital requirements if both types of banks are regulated. In the model with a bank portfolio choice, the two risk areas are leverage and a portfolio choice. Because of risk shifting motives, banks prefer a riskier portfolio than perfectly diversified one. But, unlike Kareken and Wallace (1978), the banks do not necessarily choose the riskiest portfolio because doing so makes the default probability too high, lowering the banks' expected profits. This model allows us to study concentration risk in specific lending.

For a further application, the paper considers a role of deposit insurance. Deposit insurance has been regarded as an institutional milestone for addressing bank runs by retail depositors. In the model, however, bank runs persist as long as deposit insurance is imperfect, which is the case for large depositors and non-banks in practice. The paper shows that imperfect deposit insurance makes excessive leverage even excessive and elevates systemic risk further.

Related literature This paper contributes to the emerging literature on the interaction of multiple prudential tools. As I have emphasized, this paper features endogenous bank-run-led crises in a general equilibrium model. In this regard, the most closely related paper is Kashyap et al. (2017), who *numerically* study a general equilibrium version of Diamond and Dybvig (1983) in a global game framework and argue that no single regulation is sufficient to implement the social optimum. In contrast, this paper, building on a simpler global game bank run model à la Rochet and Vives (2004) and a simpler general equilibrium model à la Christiano and Ikeda (2013, 2016), derives *analytical* results on the sources of inefficiencies and the role of multiple prudential tools.[1]

[1]Because of its simplicity, the global game of Rochet and Vives (2004) is incorporated into the Bank

Vives (2014), using the model of Rochet and Vives (2004), argues that regulations should focus on the balance sheet composition of financial intermediaries.[2] This paper endogenises the balance sheet composition and thereby takes into account the feedback effects of regulations on the composition.

De Nicolò et al. (2012), Covas and Driscoll (2014), Van den Heuvel (2016) and Boissay and Collard (2016) study capital and liquidity regulations in a dynamic general equilibrium model. This paper is static but features endogenous financial crises, which allows us to study a link between a crisis probability and bank fundamentals, especially capital/leverage and liquidity. In a static setting but without endogenous bank runs, Goodhart et al. (2012a, 2012b) also study the role of multiple regulatory tools.

This paper shares similar policy implications with Kara and Ozsoy (2016). They study a model with fire sale externalities and analytically show that both capital and liquidity requirements are essential to achieve constrained efficiency. With only one tool imposed on one risk area, risk migrates to and leakages from other areas. Focusing on similar externalities, Walther (2015) also studies the role of capital and liquidity requirements.

This paper is also related to the huge literature on capital requirements and the emerging literature on liquidity requirements. Recent surveys on the literature on capital requirements include Rochet (2014), Martynova (2015), and Dagher et al. (2016). Gorton and Winton (2003) provides a comprehensive review on the literature. Regarding liquidity requirements, as put by Allen and Gale (2017), 'the literature on liquidity regulation is still at an early stage.' Diamond and Kshayp (2016) and Allen and Gale (2017) review the early-stage literature.

Broadly this paper can be seen as an initial step toward the recent developments in macroeconomic models that incorporate bank runs: Ennis and Keister (2003) and Martin, et al. (2014), both of which build on the idea of Diamond and Dybvig (1983); Gertler and Kiyotaki (2015) and Gertler et al. (2016, 2017), all of which study runs as self-fulfilling rollover crises, following Calvo (1988) and Cole and Kehoe (2011); Angeloni and Faia (2013) who extend Diamond and Rajan (2000, 2001) bank run models.

The rest of the paper is organized as follows. Section 2 presents the benchmark model

of Canada's stress-test model for the banking sector (Fique 2017). Another important global game bank run model is Goldstein and Pauzner (2005). This paper adopts Rochet and Vives (2004) for analytical tractability in the general equilibrium framework.

[2]Papers that have focused on the role of liquidity and liquidity tools only in a global game framework include Konig (2015) and Morris and Shin (2016). Bebchuk and Goldstein (2011) study alternative government responses to an endogenous credit market freeze similar to a bank run considered in this paper.

in which banks choose leverage only. Section 3 conducts welfare analysis and clarifies the sources of inefficiencies. Section 4 extends the model to incorporate bank liquidity and studies roles and interactions of leverage and liquidity requirements. Section 5 presents further extensions of the benchmark model to study bank-specific capital/leverage restrictions, risk weights, risk concentration, deposit insurance and shadow banks. Section 6 concludes by summarising the paper's theoretical predictions for bank behaviour.

2 Model with Leverage

In this section, I present the benchmark model in which banks choose leverage only. The section proceeds by first describing the environment of the model and the behaviour of agents. It then defines an equilibrium and conducts a comparative statics analysis. The derivation of non-trivial equations and the proof of all propositions are provided in the appendix.

2.1 Environment

The model has two periods, $t = 1, 2$. There is one type of goods, which can be used for consumption or investment. The economy is inhabited by three types of agents: households, fund managers and banks. Each type consists of a continuum of agents with measure unity. Banks are owned by households. In period $t = 1$, households and banks receive endowment y and n units of the goods, respectively. Households consume and save in banks for next period consumption. Banks invest the sum of bank capital n and deposits in a risky project. Fund managers, as delegates of households, manage funds by deciding whether to withdraw funds earlier or not. Banks default if they cannot pay the promised interest rate. In period $t = 2$, banks pay the promised rate if they can, and transfer their profits to households, who consume everything at hand.

2.2 Households

For each household preferences are given by the quasi-linear utility,

$$u(c_1) + \mathbb{E}(c_2),$$

where c_t is consumption in period t, $\mathbb{E}(\cdot)$ is an expectation operator, and $u(\cdot)$ is a strictly increasing, strictly concave and twice differentiable function and satisfies $\lim_{c_1 \to 0} u'(c_1) = \infty$. In period $t = 1$ households consume c_1 and make a bank deposit of d, subject to the flow budget constraint, $c_1 + d \leq y$. A contract between households and banks is a deposit contract. Specifically, banks pay an interest rate of vR, where R is the promised fixed interest rate and v is the recovery rate which takes 1 if banks do not default and $v < 1$ if they default. The recovery rate v is given by the ratio of the banks' liquidation value to the debt obligation value of Rd. Households are assumed to delegate the management of deposits to fund managers who have an information advantage. Fund managers can withdraw funds early at a right timing as will be explained in the next section. Households diversify the management of their funds in banks over a continuum of fund managers, so that the realization of v is the same for all households, which allows the model to keep the representative agent framework. In period $t = 2$, households consume c_2, subject to $c_2 \leq vRd + \pi$, where π is bank profits. Both R and v are endogenously determined.

A key assumption is that households cannot enforce banks to choose certain actions after banks take in deposits and that banks cannot commit to any such actions. Under the assumption, knowing that banks make a choice for their own interest, the households rationally form a bank default probability P – systemic risk – and the recovery rate v. Then, solving the household problem yields the upward-sloping supply curve of deposits:

$$R = \frac{u'(y-d)}{1 - P + \mathbb{E}(v|\text{default})P}, \tag{1}$$

where $\mathbb{E}(\cdot|\text{default})$ is an expectation operator conditional on the event of bank defaults. In equilibrium, there is no idiosyncratic bank default; there is, if any, only system-wide bank default i.e. all banks default at the same time. The supply curve (1) implies that households are willing to supply funds d at the interest rate R given the systemic risk P.

2.3 Fund managers

Fund managers have information advantage over households about a stochastic bank asset return R^k on a risky project. In the beginning of period $t = 2$, just after R^k is realized, but before it is known by households, fund manager $i \in (0, 1)$ receives a private noisy signal s_i about R^k, which follows a normal distribution:

$$s_i = R^k + \epsilon_i, \quad \text{with} \quad \epsilon_i \sim N\left(0, \sigma_\epsilon^2\right).$$

Parameter σ_ϵ captures the degree of the noise of the information. While s_i itself is private information, the distribution is public information.

A role of fund managers is to make a decision of withdrawing funds early or not, using their private information. If a fund manager on behalf of a household withdraws early and the bank is solvent at this stage, the fund manager secures R per unit of funds and the household receives R per unit of deposits. But if a fund manager does not withdraw and the bank defaults later, the household receives an interest rate strictly less than R. Only fund managers can provide this professional service of early withdrawal with a right timing.

For analytical tractability in the general equilibrium setting, following Rochet and Vives (2004), I assume that fund managers and households adopt a behavioural rule of this type: fund manager i withdraws early if and only if the perceived probability of bank default, P_i, exceeds some threshold $\gamma \in (0, 1)$:

$$P_i > \gamma. \tag{2}$$

This rule is followed, for example, by an exogenous payoff for fund managers such that they are rewarded if they make the 'right decision' about costly withdrawals. If a net benefit of withdrawing over staying is given by $\Gamma_0 > 0$ when the bank defaults and $-\Gamma_1 < 0$ when the bank survives, maximising the expected payoff yields the behavioural threshold γ, given by $\gamma = \Gamma_1/(\Gamma_0 + \Gamma_1)$. In this case, the payoffs Γ_0 and Γ_1, irrespective of goods or non-goods such as efforts or reputations, are assumed to be infinitesimally small, so that these values can be ignored in the general equilibrium consideration.

As shown by Rochet and Vives (2004), in this environment fund managers employ a threshold strategy such that they withdraw if and only if $s_i < \bar{s}$. The threshold $\bar{s}$ is determined jointly with banks' problem described below.

The behavioural rule (2) is surely the source of inefficiencies that leads to a coordination failure in the form of bank runs. But in this paper, I regard it as an inevitable nature of the financial system that issues short-term debts, and focus on the resilience of the financial system susceptible to runs.

2.4 Banks

In period $t = 1$, banks offer a deposit contract to households and take in a deposit of d. Banks then combine their net worth n and the deposits d and invest in a risky project with a stochastic return R^k, which follows a normal distribution:

$$R^k \sim N\left(\mu, \sigma_k^2\right),$$

It is assumed that the return of the risky project is high enough to satisfy $\mu > R$ and σ_k are such that the probability of the gross return R^k falling below zero is essentially zero.[3] Under the assumption, banks always take in deposits and invest in a risky project. Although there are no firms, this modelling is equivalent to the presence of firms with such a linear technology and with no frictions between banks and firms. Hence, the banks' investment in a risk project should be interpreted as financial intermediation from households to firms.

In the beginning of period $t = 2$, R^k is realized. But, before the return $R^k(n + d)$ is finalized, some fund managers may withdraw their funds from banks. In response, the banks have to sell some assets. This early liquidation is costly: early liquidation of one unit of bank asset generates only a fraction $1/(1 + \lambda)$ of R^k, where $\lambda > 0$. The underlying assumption is that in response to early withdrawal requests banks raise funds by selling illiquid assets, which have been invested in the risky project, to households who have a linear but inferior technology than bankers. The technology transforms one unit of bank asset into $1/(1+\lambda)R^k$. With perfect competition and no friction between households and banks, the fire sale price of the early liquidated asset is $1/(1+\lambda)R^k$, where λ captures the degree of the discounting of the fire sale or put simply the cost of early liquidation.

Let x denote the number of fund managers who withdraw funds. Then, to cover the early withdrawal of xRd, banks have to liquidate $(1 + \lambda)xRd/R^k$ units of bank assets.[4] After the liquidation, the banks have $R^k(n+d)-(1+\lambda)xRd$ in hand. If this amount is less than the promised payment under the deposit contract, $(1-x)Rd$, the banks go bankrupt. That is, the banks default if and only if

$$R^k < R\left(1 - \frac{1}{L}\right)(1 + \lambda x), \tag{3}$$

where $L \equiv (n + d)/n$ is bank leverage.

Under the withdrawal strategy for fund manager i, $s_i < \bar{s}$, the number of fund managers who withdraw is given by $x(R^k, \bar{s}) = \Pr(s_i < \bar{s}) = \Pr(\epsilon_i < \bar{s} - R^k) = \Phi((\bar{s} - R^k)/\sigma_\epsilon)$, where

[3] For example, the probability of R^k falling below zero for the normal distribution with the mean return of $\mu = 1.035$ and the standard deviation of $\sigma_k = 0.025$ is smaller than $1e$-300 percent.

[4] To see this, let z denote the quantity of bank assets to be liquidated. Then, z should satisfy $1/(1 + \lambda)R^k z = xRd$, which leads to $z = (1 + \lambda)xRd/R^k$.

$\Phi(\cdot)$ is a standard normal distribution function. Condition (3) implies that the probability of bank default perceived by fund manager i is given by

$$P_i = \Pr\left(R^k < R\left(1-\frac{1}{L}\right)\left[1+\lambda x(R^k,\bar{s})\right]|s_i\right). \tag{4}$$

Conditions (2)-(4) imply that the equilibrium threshold $\bar{s}^*$ is a solution to the following set of equations:

$$\Pr\left(R^k < R^{k*}|\bar{s}^*\right) = \gamma, \tag{5}$$

$$R^{k*} = R\left(1-\frac{1}{L}\right)\left[1+\lambda x(R^{k*},\bar{s}^*)\right]. \tag{6}$$

Rochet and Vives (2004) show these two equations have a unique solution for $\bar{s}^*$ and R^{k*} if the standard deviation of the signal σ_ϵ is small enough. This paper focuses on this case.

Both $\bar{s}^*$ and R^{k*} depend on the interest rate R and leverage L. In particular, an increase in leverage raises $\bar{s}^*$ and R^{k*} so that more fund managers withdraw funds and the probability of bank default increases. Banks take into account this effect in choosing leverage.

Banks are subject to a technical restriction such that leverage should not be too high: $L \leq L_{\max}$. This restriction differs from a prudential tool introduced later. With a high enough $L_{\max}$, the restriction is not binding in equilibrium, but it plays a role of excluding an uninteresting profitable deviation of $L=\infty$ as will be discussed shortly. One interpretation of this restriction is a physical limit $L_{\max} = (y-1)/n$ at which households lend all their funds to banks.

Banks are protected by limited liability. In addition, they cannot commit to their choice of leverage in advance. This lack of commitment and the households' inability to enforce banks to take certain actions imply that banks cannot write a deposit contract that depends on leverage and equivalently the probability of bank default. Hence, the problem of banks is to choose leverage L to maximize the expected profits $\mathbb{E}(\pi)$, taking the interest rate R as given,

$$\max_{\{L\}} \int_{R^{k*}(L)}^{\infty} \left\{R^k L - R\left[1+\lambda x\left(R^k,\bar{s}^*(L)\right)\right](L-1)\right\} ndF(R^k), \tag{7}$$

subject to $L \leq L_{\max}$, where $F(\cdot)$ is a normal distribution function with mean μ and standard deviation σ_k, and $\bar{s}^*(L)$ and $R^{k*}(L)$ are solutions for $\bar{s}^*$ and R^{k*} as a function of L, respectively. In problem (7) the banks ignore the potential feedback effect of leverage on the interest rate: if the banks chose lower leverage, they would become safer and the interest rate they face would be reduced accordingly. This ignorance, arising from the lack of commitment by banks and the lack of enforcement by households, causes risk shifting in the spirit of Jensen and Meckling (1976).

Given that the technical restriction $L \leq L_{\max}$ is non-binding, the first-order condition is:

$$0 = \int_{R^{k*}}^{\infty} (R^k - R) dF(R^k) - R\lambda (L-1) \int_{R^{k*}}^{\infty} \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} dF\left(R^k\right) - R\lambda \int_{R^{k*}}^{\infty} x dF\left(R^k\right). \quad (8)$$

The first-term in the right-hand-side of equation (8) is the expected marginal return by increasing leverage and the remaining terms in the right-hand-side of (8) is the expected marginal costs, which consist of two terms. The first term is the expected marginal liquidation cost. An increase in L raises threshold $\bar{s}^*$ and increases the number of fund managers who withdraw, which leads to an increase in the liquidation cost. The second term is the expected liquidation cost. Equation (8) and assumption $\mu > R$ implies that $\partial \mathbb{E}(\pi)/\partial L|_{L=1} > 0$, so that a unique solution to (8) satisfies the second-order condition as well.[5]

Is there no profitable deviation from the solution to (8)? This is where the technical restriction, $L \leq L_{\max}$, comes into play. For the sake of exposition and analytical tractability, consider a limit equilibrium in which the fund managers' noisy signal becomes perfectly accurate, i.e. $\sigma_\epsilon \to 0$. In the limit equilibrium, $\bar{s}^* = R^{k*}$ and the threshold is given by:

$$R^{k*} = R\left(1 - \frac{1}{L}\right)\left[1 + \lambda(1-\gamma)\right], \quad (9)$$

and the optimality condition of the banks' problem (8) is reduced to:

$$0 = \int_{R^{k*}}^{\infty} (R^k - R) dF(R^k) - \lambda(1-\gamma) f\left(R^{k*}\right)\left[1 + \lambda(1-\gamma)\right] R^2 \frac{L-1}{L^2}, \quad (10)$$

Equation (9) implies that $\lim_{L\to\infty} R^{k*} = R\left[1 + \lambda(1-\gamma)\right]$, so that even in the limit, $L \to \infty$, the default probability is strictly less than unity: $\lim_{L\to\infty} F\left(R^{k*}\right) < 1$. This and condition (10) suggests $\partial \mathbb{E}(\pi)/\partial L > 0$ for a large value of L. Were it not for $L \leq L_{\max}$, there would be a profitable deviation by choosing $L = \infty$. This issue has to do with the fact that the

[5]At $L = 1$, $R^{k*} = 0$ and there is essentially no bank run. Hence, given that the probability of the gross return R^k falling below zero is essentially zero, the final term in the right-hand-side of equation (8) is essentially zero.

domain of the distribution for R^k is unbounded above. Should it exist the upper bound, which is lower than $\mu < R\left[1+\lambda\left(1-\gamma\right)\right]$, for example, as in a uniform distribution, there would be no need for such a technical restriction.[6]

The banks' problem implies that all banks choose the same level of leverage and default, if any, at the same time. Hence, the bank default probability – systemic risk – is given by:

$$P = \Pr(R^k < R^{k*}) = F\left(R^{k*}\right). \tag{11}$$

If banks default, the banks are liquidated and their value is distributed among creditors. Consequently, the recovery rate v in equation (1) is given by:

$$v = \min\left\{1, \max\left\{\frac{R^k}{R}\frac{L}{L-1} - \lambda x(R^k, \bar{s}^*), \frac{1}{1+\lambda}\frac{R^k}{R}\frac{L}{L-1}\right\}\right\}. \tag{12}$$

If the banks have survived, the recovery rate is 1. If they have defaulted but did not have sold all the bank assets, the recovery rate is given by the first term in the max operator in (12). The second term in the max operator corresponds to the recovery rate when the banks have sold all the assets and have defaulted. The banks sell all the assets if the return on bank assets is lower than $\underline{R}^k$, which is defined by:

$$\underline{R}^k = R\left(1-\frac{1}{L}\right)(1+\lambda)x(\underline{R}^k, \bar{s}^*).$$

The threshold $\underline{R}^k$ is clearly lower than the default threshold R^{k*}.

2.5 Equilibrium

A competitive equilibrium for this economy consists of the interest rate R and leverage L that satisfy the supply curve for funds (1), the demand curve for funds (8) and the market clearing condition, $d = (L-1)n$, where R^{k*}, $\bar{s}^*$, v and P in these curves are given by (5), (6), (12) and (11), respectively. With a solution of R and L at hand, household consumption series c_1 and c_2 are obtained from the household budget constraints.

A unique feature of the bank problem that leads to the demand curve (8) is that bank leverage L is uniquely determined as an interior solution without any financial frictions that

[6] A uniform distribution has also a lower bound, which implies that bank run probability can fall to zero if leverage is sufficiently low. But, with a normal distribution bank run probability is always positive. This is a main reason why this paper considers a normal distribution.

directly constrain leverage. In other papers, such frictions include banks' moral hazard of running away with borrowings (Gertler and Kiyotaki 2015), banks' hidden effort as moral hazard (Christiano and Ikeda 2016), asymmetric information and costly state verification (Bernanke et al. 1999), and limited pledgeability (Kiyotaki and Moore 1997). In this model, however, it is bank run risk and the resulting market discipline that help pin down bank leverage. Their effect is captured by the third term of the right-hand-side of equation (8) (equation (10) in the limit case). Too high leverage makes banks' liability vulnerable to bank runs, increases the bank run probability, raises expected liquidation costs and lowers profits. Because of this effect banks refrain from choosing too high leverage and as a result bank leverage has an interior solution.

2.6 Comparative Statics

The competitive equilibrium for this economy depends on parameters such as μ, γ, λ, y and n. The following proposition summarizes how the demand curve (10) and the supply curve (1) for the credit market are affected with respect to a change in these parameter values.

Proposition 1 (Comparative statics)**.** *Consider the credit market described by the demand curve (10) and the supply curve (1). Consider a limit case where $\sigma_\epsilon \to 0$. Assume that bank default probability is not too high, $P \leq 0.5$, and the leverage is not too low, $L > 5/3 > \left(1 - \frac{0.4}{1+\lambda(1-\gamma)}\right)^{-1}$. Then, the following results hold.*

(i) An increase in the mean bank asset return μ shifts the demand curve outward.

(ii) An increase in the liquidation cost λ (or a decrease in the threshold probability γ) shifts the demand curve inward.

(iii) An increase in the household endowment y shifts the supply curve outward.

(iv) An increase in the bank capital n shifts the supply curve inward.

The comparative statics analysis supports a view that credit booms tend to be associated with vulnerability to financial crises (Schularick and Taylor, 2012). In the model a typical credit boom would feature increases in the bank mean return μ, the household endowment y and the bank capital n. On the demand side, a perception of low liquidation costs (low λ) and low threshold probability by fund managers (low γ) could add a further outward shift in the demand curve. On the supply side, if the effect of y dominates the effect of n, the supply curve shift outward, which, combined with an increase in the demand, leads to a rise in the leverage and the crisis probability. A credit boom builds up financial system vulnerability that triggers a banking crisis.

3 Welfare Analysis

In this section, I conduct a welfare analysis on the benchmark model presented in Section 2. The results are twofold. First, leverage is excessive in the competitive equilibrium relative to that chosen by a benevolent regulator, so that restraining leverage improves welfare. Second, the sources of the inefficiency are bank risk shifting and pecuniary externalities.

3.1 Regulator's Problem

What is an optimal point to which the competitive equilibrium allocation is compared? The first best should feature no bank run. But, in this paper, a bank run is regarded as an inevitable feature of the financial system. Hence, I take a regulator perspective and set up a benevolent regulator's problem in which the regulator chooses leverage L to maximize social welfare subject to bank run risk and the supply curve for funds (1). In other words, in place of banks the regulator chooses leverage, but unlike banks the regulator maximizes social welfare and takes into account the general equilibrium effect of the choice of L on R. The social welfare, SW, is given by the expected households' utility, $SW = u(c_1) + \mathbb{E}(c_2)$, because banks are owned by the households.

The regulator's problem is explicitly written as:

$$\begin{aligned} \max_{\{L\}} \ & u\left(y-(L-1)\,n\right)+\mathbb{E}(R^k)Ln \\ & -\lambda\left\{\int_{\underline{R}^k}^{\infty}\left[x(R^k,\bar{s}^*(L,R(L)))\right]R(L)(L-1)ndF(R^k)+\int_{-\infty}^{\underline{R}^k}\frac{R^kL}{1+\lambda}dF(R^k)\right\}n. \end{aligned} \tag{13}$$

subject to $L \leq L_{\max}$, where $R(L)$ is implicitly given by the supply curve (1) and threshold $\bar{s}^*$ is written as a function of R as well as L to take into account the effect of R on the threshold. The regulator balances the expected benefit of financial intermediation, which is given by the first row of the regulator's objective (13), and the expected loss due to the fire sale of bank assets, which is given by the second row of the regulator's objective (13).

The loss is governed by the parameter, $\lambda > 0$, that captures the cost of early liquidation.

The first-order condition of the regulator's problem is given by:

$$\begin{aligned}0 = &- R\left[1 - P + \mathbb{E}(v|\text{default})P\right] + \mathbb{E}(R^k) - \lambda R \int_{\underline{R}^k}^{\infty} x dF - \lambda \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF \\ &- \lambda R(L-1)R \int_{\underline{R}^k}^{\infty} \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF - \lambda(L-1)\int_{\underline{R}^k}^{\infty} \left(\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} R + x\right)\frac{\partial R}{\partial L} dF, \end{aligned} \tag{14}$$

where the supply curve (1) was used to substitute out for $u'(y-(L-1)n)$. Condition (14) distinguishes from the banks' optimality condition (8) in two respects. First, the planner takes into account all possible states including bank run states, but the banks focus only on non-default states due to limited liability. Second, the planner internalizes the impact of leverage L on the interest rate R, which is captured by the final term in (14), while the banks do not as they take R as given.

3.2 Roles of leverage restrictions

Now we are in a position to study whether the competitive equilibrium has excessive leverage. If the slope of the social welfare evaluated at the competitive equilibrium allocation, $\partial SW/\partial L|_{\text{CE}}$, is negative, the leverage is excessive: restricting the leverage improves welfare. Because the competitive equilibrium solves the banks' optimal condition, it has to be $\partial \mathbb{E}(\pi)/\partial L|_{\text{CE}} = 0$. Then, $\partial SW/\partial L|_{\text{CE}}$ is written and expanded as:

$$\begin{aligned}\left.\frac{\partial SW}{\partial L}\right|_{\text{CE}} =& \left.\frac{\partial SW}{\partial L}\right|_{\text{CE}} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial L}\right|_{\text{CE}} \\ =& -\frac{1}{L-1}\left[\int_{\underline{R}^k}^{R^{k*}} R^k dF + \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF\right] \\ &- \lambda(L-1)\left[\int_{\underline{R}^k}^{R^{k*}} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF + \int_{\underline{R}^k}^{\infty}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L} dF\right], \end{aligned} \tag{15}$$

where $\partial x/\partial \bar{s}^* = \Phi'((\bar{s}^* - R^k)/\sigma_\epsilon)(1/\sigma_\epsilon) > 0$.[7] The first term on the right-hand-side of (15) is negative under the assumption that the probability of R^k falling below zero is essentially zero. As shown in the appendix, an increase in leverage L raises the threshold $\bar{s}^*$ and an increase in the interest rate R raises the threshold: $\partial \bar{s}^*/\partial L > 0$ and $\partial \bar{s}^*/\partial R > 0$. In addition, under a plausible condition the supply curve (1) is upward-sloping, i.e.

[7] Bank capital n is abstracted away from these conditions because they are all proportional to n. The same applies hereafter in calculating similar optimality conditions.

$\partial R/\partial L > 0$. In this case, equation (15) implies $\partial SW/\partial L|_{\text{CE}} < 0$, which leads to the following proposition.

Proposition 2 (Excessive leverage)**.** *Assume that the supply curve (1) is upward sloping. Then, in a competitive equilibrium, bank leverage is excessive from a social welfare viewpoint. Restraining leverage can improve social welfare.*

A corollary of Proposition 2 is that the probability of bank runs – the systemic risk – is too high in a competitive equilibrium. The excessive leverage implies the high threshold R^{k*}, which, in turn, implies the elevated systemic risk.

The excessive leverage and the resulting elevated systemic risk in the competitive equilibrium provides a rational for policymakers to introduce prudential policy to improve welfare. The second best allocation, which solves the benevolent regulator's problem, can be achieved, for example, by imposing a leverage restriction $\bar{L}$ on banks such that $L \leq \bar{L} = L^*$, where L^* is a solution to equation (14). Similarly, the second best is achieved by imposing restrictions on a capital ratio, $n/(n+d)$, such that it is no less than $1/L^*$.

3.3 Sources of inefficiencies

What are the sources of inefficiencies that give rise to excessive leverage? Equation (15) is suggestive, but it is not clear exactly what causes the excessive leverage. To address this question, I consider the same problem but without bank risk shifting. In this economy, the household optimality condition (1) stays the same, but what changes is the banks' behaviour. Now the banks can commit to their choice of leverage so that they provide a deposit contract that specifies leverage L as well as the interest rate R. The banks choose a pair of leverage and the interest rate, $\{L, R\}$, to maximize the same expected profits (7) subject to the technical constraint $L \leq L_{\max}$ and the households' participation constraint:

$$R[1 - F(R^{k*})] + \int_{\underline{R}^k}^{R^{k*}} \left[R^k \frac{L}{L-1} - R\lambda x(R^k, \bar{s}^*) \right] dF + \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} \frac{L}{L-1} dF \geq R^e, \quad (16)$$

for some return R^e, where R^{k*}, $\bar{s}^*$ and $\underline{R}^k$ are all a function of L and R. The left-hand-side of (16) corresponds to the expected return received by households, $R[1 - P + \mathbb{E}(v|\text{default})]$. As long as condition (16) holds, which promises the expected return of R^e, households are willing to supply funds irrespective of a pair of L and R. In equilibrium, the constraint (16) is binding and $R^e = u'(y - (L-1)n)$.

The binding constraint (16) disciplines the banks' behaviour as an increase in leverage and a resulting increase in bank riskiness raises the interest rate. Indeed, the binding constraint (16) implicitly defines R as a function of L as $R = R_B(L)$, where $\partial R_B/\partial L > 0$.[8] Because of this feedback effect, the banks choose lower leverage than that in the benchmark model presented in Section 2. The optimality condition of the banks' problem is delegated to the appendix.

Leverage is still excessive in a competitive equilibrium even in the economy without bank risk shifting, but the degree of excessiveness is mitigated. Let CE' denote such a competitive equilibrium. The slope of the social welfare evaluated at the competitive equilibrium is given by

$$\left.\frac{\partial SW}{\partial L}\right|_{\mathrm{CE}'} = \lambda(L-1)\left[\int_{\underline{R}^k}^{\infty}\left(\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}R + x\right)dF\right]u''(c_1) \in \left(\left.\frac{\partial SW}{\partial L}\frac{1}{n}\right|_{\mathrm{CE}}, 0\right). \tag{17}$$

This equation shows that in this model the only source of inefficiencies is pecuniary externalities that work through the interest rate R, which is captured by the second derivative of the utility function, $u''(c_1)$. This result is summarized in the following proposition.

Proposition 3 (Excessive leverage in the model without bank risk shifting)**.** *Consider the benchmark model without bank risk shifting in which the supply curve (1) is upward sloping. In a competitive equilibrium, bank leverage is excessive because of pecuniary externalities that work through the interest rate.*

Propositions 2 and 3 reveal that the sources of inefficiencies in the benchmark model are twofold: bank risk shifting and pecuniary externalities. Regarding bank risk shifting, in the benchmark model, banks compete for attracting deposits by using the interest rate only. Even if a bank attempts to become safer by lowering leverage, the bank cannot lower the interest rate because it would lose depositors. Hence, such an attempt cannot be a profitable deviation from the equilibrium. Instead, in the model without bank risk shifting, the market works through bank riskiness as well as the interest rate. In this case, banks take into account the effect of leverage on the interest rate and consequently the leverage becomes lower than in the benchmark model. Regarding pecuniary externalities, an increase in bank leverage raises the interest rate as a general equilibrium effect and increases the costs associated with bank asset fire sales, $\lambda R x(L-1)$. This effect is ignored

[8] A condition for $\partial R_B/\partial L > 0$ is the same as that for $\partial R/\partial L > 0$, which is assumed to hold. A relationship between $\partial R/\partial L$ and $\partial R_B/\partial L$ is such that $\partial R/\partial L - \partial R_B/\partial L \propto -u''(c_1) > 0$. Hence, the slope of $R(L)$ is sleeper than that of $R_B(L)$.

by bankers who take the interest rates, R or R^e, as given in the economy with or without bank risk shifting, respectively.

4 Leverage and Liquidity

In this section, I extend the benchmark model presented in Section 2 to incorporate liquid assets in a bank balance sheet. This section first presents the extended model and analytical results on social welfare and the sources of inefficiencies. It then proceeds to numerical analyses on the roles and interactions of liquidity and leverage requirements regarding social welfare and systemic risk.

4.1 Model with Leverage and Liquidity

In this model, a bank balance sheet consists of safe liquidity as well as risky lending. Specifically, banks have an access to a safe technology with gross return unity. Assets invested in a safe technology are called liquidity, which is drawn at any time without any costs.

In period $t = 1$, banks allocate the sum of their net worth n and the deposit d to liquidity M and lending $n + d - M$. In response to fund managers' early withdrawal claim of xRd, banks use liquidity first because doing so is not costly, and they sell the assets invested in a risky project to households at a fire sale price if the amount of liquidity is not enough to cover the amount of the claim: $xRd > M$. In this case, the banks sell $(1+\lambda)(xRd-M)/R^k$ units of bank assets. If the banks' revenue, $R^k(n + d - M) - (1 + \lambda)(xRd - M)$, cannot cover the promised payment to the depositors who have not withdrawn early, $(1 - x)Rd$, the banks go bankrupt. Instead, if the banks can cover the early withdrawal request by using liquidity, i.e. $xRd < M$, they do not liquidate any risky assets and they are subject to only a fundamental default. Hence, banks default if and only if

$$R^k < \frac{R - m}{\frac{L}{L-1} - m} \left(1 + \lambda \frac{\max\{xR - m, 0\}}{R - m} \right), \tag{18}$$

where $m \equiv M/d$ is a liquidity-deposit ratio (hereafter a liquidity ratio or liquidity for short) and $L \equiv (n+d)/n$ is leverage. This condition is reduced to condition (3) in the case of bank leverage only if $m = 0$. Condition (18) implies that thresholds $\bar{s}^*$ and R^{k*} are determined by equation (5) and

$$R^{k*} = \frac{R-m}{\frac{L}{L-1}-m}\left(1+\lambda\frac{x(R^{k*},\bar{s}^*)R-m}{R-m}\right), \tag{19}$$

where $x(R^{k*},\bar{s}^*) = \Phi((\bar{s}^* - R^{k*})/\sigma_\epsilon)$. At the thresholds of R^{k*} and $\bar{s}^*$, the amount of early withdrawals exceeds the bank liquidity, i.e. $x(R^{k*},\bar{s}^*)R - m > 0$. Otherwise, the banks would not default for R^k close to but smaller than R^{k*}. Equation (19) is the extension of equation (6) to incorporate a bank liquidity choice.

The banks' problem is to maximize the expected profits $\mathbb{E}(\pi)$ by choosing leverage and liquidity,

$$\max_{\{L,m\}} \int_{R^{k*}(L,m)}^{\infty} \Big\{ R^k L - (R^k - 1)(L-1)m \\ - \left[R + \lambda \max\left\{x\left(R^k, \bar{s}^*(L,m)\right)R - m, 0\right\}\right](L-1)\Big\} n dF(R^k),$$

subject to $L \leq L_{\max}$ and $0 \leq m \leq L/(L-1)$, where the thresholds $\bar{s}^*(L,m)$ and $R^{k*}(L,m)$ are a solution to equations (5) and (19), written as a function of L and m. A marginal increase in the liquidity ratio m is associated with the opportunity cost of $(R^k-1)(L-1)$, but it reduces the likelihood of fire sales and its expected costs $\lambda\max\{xR-m,0\}(L-1)$. High enough liquidity, e.g. $m = L/(L-1)$, insulates banks from bank runs and makes them perfectly bank-run-proof, but $R > 1$ is assumed so that such a choice cannot be a solution to the problem.[9]

For solving the banks' problem, let $\bar{R}^k$ define a threshold for R^k such that bank liquidity just covers the amount of early withdrawal, i.e. $x(\bar{R}^k, \bar{s}^*)R = m$. Solving for $\bar{R}^k$ yields:

$$\bar{R}^k = \bar{s}^* - \sigma_\epsilon \Phi^{-1}\left(\frac{m}{R}\right).$$

Now the first-order conditions of the banks' problem, which characterize an interior solution

[9] If leverage is too low, the gross interest rate can fall below unity, violating the assumption of $R > 1$. One way to address this problem is to assume that the gross return of liquidity is lower than unity. Another way is to assume that the gross return of liquidity depends on R and is given by $R - s$, where $s > 0$ is a liquidity premium. In this case the presence of R would become another source of a pecuniary externality. To make the model as simple as possible, the return of liquidity is assumed to be unity.

for leverage L and liquidity m, are given by:

$$0 = \int_{R^{k*}}^{\infty} [R^k - (R^k - 1)m - R] dF(R^k)$$
$$- \int_{R^{k*}}^{\bar{R}^k} \left[\lambda(Rx - m) + (L-1)\lambda R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} \right] dF(R^k), \tag{20}$$
$$0 = - \int_{R^{k*}}^{\infty} (R^k - 1) dF(R^k) + \lambda \int_{R^{k*}}^{\bar{R}^k} \left(1 - R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial m} \right) dF(R^k). \tag{21}$$

Equation (20) corresponds to $0 = \partial \mathbb{E}(\pi)/\partial L$, which is reduced to the optimal condition in the benchmark model (10) when $m = 0$. Equation (20) implies that the fire sale cost due to a marginal increase in leverage appears only when liquidity cannot cover the amount of early withdrawals, i.e. when $R^k < \bar{R}^k$. Similar to the benchmark model, the marginal impact of raising the threshold $\bar{s}^*$ on x, the number of fund managers who withdraw funds early, is positive and the marginal impact of leverage L on the threshold $\bar{s}^*$ is positive: $\partial x/\partial \bar{s}^* > 0$ and $\partial \bar{s}^*/\partial L > 0$.

Equation (21) corresponds to $0 = \partial \mathbb{E}(\pi)/\partial m$. The first term in the right-hand-side of equation (21) is the opportunity cost of holing liquidity, i.e. the net expected return on the risky project the banks would have earned if they had not held liquidity but invested in the risky project. The second term in the right-hand-side of equation (21) is the marginal benefit of holding liquidity and by lowering the number of fund managers who withdraw early, x. As shown in the appendix, an increase in liquidity lowers the threshold $\bar{s}^*$, i.e. $\partial \bar{s}^*/\partial m < 0$ if and only if the interest rate is not high enough to satisfy:

$$R < \frac{1+\lambda}{1+\lambda x} \frac{L}{L-1}. \tag{22}$$

Under condition (22) an increase in liquidity m reduces the thresholds $\bar{s}^*$ and R^{k*} and lowers the bank run probability $F(R^{k*})$ and thereby increases the resilience of the financial system. Instead, if condition (22) does not hold, the interest cost on the bank liability is so high that a decrease in the expected revenue due to an increase in liquidity holding causes the banks to be more vulnerable to bank runs, raising the threshold R^{k*} and the bank run probability $F(R^{k*})$. Hereafter condition (22) is imposed on this model.

Given a unique solution to the first-order condition with respect to liquidity, (21), the banks' liquidity holding is positive if and only if $\partial \mathbb{E}(\pi)/\partial m|_{m=0} > 0$:

$$\int_{R^{k*}}^{\infty} \left[-(R^k - 1) + \lambda \left(1 - R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial m} \right) \right] dF(R^k) > 0.$$

Hence, given a unique solution to (21), the sufficient condition for $m > 0$ is:

$$\mathbb{E}(R^k | \text{no default}) < 1 + \lambda. \tag{23}$$

That is, banks hold liquidity if the expected return of the risky loan conditional on no default is not so high, satisfying condition (23). In other words, the banks hold low return safe assets when the opportunity cost of doing so is not high.

It is worth noting that the sufficient condition for positive liquidity (23) does not apply to a limit equilibrium in which $\sigma_\epsilon \to 0$. In this case $\bar{R}^k \to R^{k*}$ for $m > 0$ and as a result the second term of the right-hand-side of condition (21) vanishes. Hence, in the limit equilibrium, banks do not hold liquidity. This is intuitive. In the limit equilibrium, it is either all fund managers withdraw or no one withdraws. As long as the amount of liquidity is not enough to prevent banks from defaulting due to runs by all fund managers, a marginal increase in liquidity has no impact on this nature and thereby generates no marginal benefit. On the other hand, if there is a region of fire sales with no default, i.e. $\bar{R}^k - \bar{R}^* > 0$, as in the case of $\sigma_\epsilon > 0$, building additional liquidity yields benefits from reducing the costs of fire sales. Hence, the noisy information, $\sigma_\epsilon > 0$, is essential in analysing the model with leverage and liquidity.

The supply side of funds – the household problem – is the same as in the benchmark model except for the recovery rate v. Assuming that banks can satisfy early withdrawal requests, a fraction x of fund managers who withdraw early receive R per unit of deposit. When banks default, a remaining fraction, $1 - x$, of fund managers divide banks' return $[R^k(n+d-M) - \lambda(xRd - M)]$ equally and receive $[R^k(n+d-M) - \lambda(xRd - M)]/[(1-x)d]$ per unit of deposit. Because households diversify over fund managers, households receive a weighted sum of the returns when banks default: $R^k(L/(L-1) - m) + m - \lambda(Rx - m)$. This recovery rate assumes that banks do not sell all the risky assets. The recovery rate when the banks sell all the risky assets is given by $(R^k/(1+\lambda))(L/(L-1) - m) + m$.

Consequently, the recovery rate is given by

$$v = \min\left\{1, \max\left\{R^k\left(\frac{L}{L-1} - m\right) + m - \lambda(Rx - m), \frac{R^k}{1+\lambda}\left(\frac{L}{L-1} - m\right) + m\right\}\right\}. \tag{24}$$

This expression also applies to a case when banks default because they cannot satisfy the request of early withdrawals. The recovery rate is increasing in liquidity m when banks do not sell all the assets and $R^k < 1+\lambda$, which holds under the assumption of (23). As in the benchmark model presented in Section 2, it is useful to define a threshold $\underline{R}^k$ under which banks sell all the risky assets:

$$\underline{R}^k = (1+\lambda)\frac{Rx(\underline{R}^k, \bar{s}^*) - m}{\frac{L}{L-1} - m}.$$

From equation (19) it is clear that $\underline{R}^k < R^{k*}$.

A competitive equilibrium for this economy consists of the interest rate R, leverage L and liquidity m that satisfy the supply curve for funds (1), the demand curve for funds (20), the optimality condition for liquidity (21) and the market clearing condition, $d = (L-1)n$, where R^{k*}, $\bar{s}^*$, P and v in these equations are given by (5), (19), (11) and (24), respectively.

4.2 Roles of Liquidity and Leverage Requirements

In a competitive equilibrium is liquidity insufficient from a social welfare viewpoint? Does leverage continue to be excessive? To address these questions, as in Section 3 I set up a benevolent regulator's problem, where the regulator chooses leverage L and liquidity m to maximize social welfare:

$$\begin{aligned}\max_{\{L,m\}} \quad u(y-(L-1)n) + &\left\{\int_{\bar{R}^k}^{\infty} \left[R^k L - (R^k-1)(L-1)m\right] dF\right.\\ &+ \int_{\underline{R}^k}^{\bar{R}^k} \left[R^k L - (R^k-1)(L-1)m - \lambda(xR-m)(L-1)\right] dF\\ &\left.+ \int_{-\infty}^{\underline{R}^k} \left[\frac{R^k}{1+\lambda}L - \left(\frac{R^k}{1+\lambda} - 1\right)(L-1)m\right]\right\} n,\end{aligned}$$

subject to $L \leq L_{\max}$ and $0 \leq m \leq L/(L-1)$, where $R = R(L,m)$ is given by the supply curve (1) and $\bar{s}^* = \bar{s}^*(L,m,R)$ is given by a solution to equations (5) and (19). The interest rate depends on liquidity in addition to leverage because the interest rate depends on the recovery rate, which is affected by liquidity.

The first-order condition of the regulator's problem with respect to liquidity yields

$$
\begin{aligned}
0 = &-\int_{\underline{R}^k}^{\infty}(R^k-1)dF - \int_{-\infty}^{\underline{R}^k}\left(\frac{R^k}{1+\lambda}-1\right)dF \\
&+\lambda\int_{\underline{R}^k}^{\bar{R}^k}\left[1-R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m}-\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}+x\right)\frac{\partial R}{\partial m}\right]dF. \qquad (25)
\end{aligned}
$$

In contrast to equation (21) that characterizes the banks' privately optimal choice of liquidity, equation (25) takes into account the opportunity cost of holding liquidity and the benefits of mitigating fire sales in default states. Furthermore, it considers the effect of liquidity on the interest rate. As shown in the appendix, if condition (23) holds and the supply curve (1) is upward sloping, an increase in liquidity lowers the interest rate, $\partial R/\partial m < 0$, by decreasing the default probability and increasing the recovery rate.[10]

The regulator's optimality condition, evaluated at the level of liquidity $m = m^*$ implied by the privately optimal choice (21), is given by:

$$
\begin{aligned}
\left.\frac{\partial SW}{\partial m}\right|_{m=m^*} &= \left.\frac{\partial SW}{\partial m}\right|_{m=m^*} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial m}\right|_{m=m^*} \\
&= \int_{\underline{R}^k}^{R^{k*}}(1-R^k)dF + \int_{-\infty}^{\underline{R}^k}\left(1-\frac{R^k}{1+\lambda}\right)dF \\
&+\lambda\int_{-\infty}^{R^{k*}}\left(1-R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m}\right)dF - \lambda\int_{-\infty}^{\bar{R}^k}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}+x\right)\frac{\partial R}{\partial m}dF.
\end{aligned}
$$

The curvature of the social welfare evaluated at m^* consists of four integral terms. The sign of the last two terms is positive if the supply curve (1) is upward sloping and conditions (22) and (23) are satisfied. Hence, if the first two terms are positive as well, $\partial SW/\partial m|_{m=m^*} > 0$ follows. This leads to the following proposition.

Proposition 4 (Insufficient liquidity)**.** *Assume that the supply curve (1) is upward sloping and conditions (22) and (23) hold. Assume further that the threshold R^{k*} is low enough to satisfy $\int_{-\infty}^{R^{k*}}(1-R^k)dF > 0$. Then, for given leverage, banks' liquid asset holdings are insufficient from a social welfare viewpoint. Raising liquidity can improve welfare.*

Proposition 4 does not require that leverage is at the competitive equilibrium level. Indeed, Proposition 4 holds for an arbitrary level of L. Hence, Proposition 4 implies that

[10] This negative relationship is consistent with the empirical finding by Miller and Sowerbutts (2018) for the major US banks.

bank liquidity is insufficient not only in a competitive equilibrium but also in an equilibrium with $m > 0$ where leverage is restrained e.g. by a prudential tool on leverage. This result suggests that there is room for imposing a liquidity tool to improve social welfare even if a leverage restriction is already put in place.

Turning to welfare implications for leverage in this extended model, the first-order condition of the regulator's problem with respect to leverage is given by:

$$
\begin{aligned}
0 = \frac{\partial SW}{\partial L} =& - R\left[1 - P + P\mathbb{E}(v|\text{default})\right] + \int_{\underline{R}^k}^{\infty} \left[R^k - (R^k - 1)m\right] dF \\
& - \lambda \int_{\underline{R}^k}^{\bar{R}^k} \left[(xR - m) + R(L-1)\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} + (L-1)\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L}\right] dF \\
& + \int_{-\infty}^{\underline{R}^k} \left[\frac{R^k}{1+\lambda} - \left(\frac{R^k}{1+\lambda} - 1\right) m\right] dF.
\end{aligned}
$$

Similar to the choice of liquidity, the regulator's optimality condition, evaluated at the banks' privately optimal choice $L = L^*$ implied by condition (20), is given by:

$$
\begin{aligned}
\left.\frac{\partial SW}{\partial L}\right|_{L=L^*} =& \left.\frac{\partial SW}{\partial L}\right|_{L=L^*} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial L}\right|_{L=L^*} \\
=& -\frac{1}{L-1}\left[\int_{\underline{R}^k}^{R^{k*}} R^k dF + \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF\right] \\
& -\lambda(L-1)\left[\int_{\underline{R}^k}^{R^{k*}} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF + \int_{\underline{R}^k}^{\bar{R}^k} \left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L} dF\right].
\end{aligned}
$$

The sign of $\partial SW/\partial L|_{L=L^*}$ is negative if the supply curve of funds (1) is upward sloping, i.e. $\partial R/\partial L > 0$. This leads to the following proposition.

Proposition 5 (Excessive leverage)**.** *Assume that the supply curve (1) is upward sloping. Then, for given liquidity, bank leverage is excessive from a social welfare viewpoint. For given liquidity, restraining leverage can improve welfare.*

Similar to Propositions 4 that shows insufficient liquidity, Proposition 5 holds for any level of bank liquidity. Even if liquidity is at some regulated level, banks choose excessive leverage relative to the constrained optimal level. Hence, Propositions 4 and 5 warrant imposing both leverage and liquidity requirements.

The sources of inefficiencies that give rise to excessive leverage and insufficient liquidity are the same as those in the benchmark model, i.e. bank risk shifting and pecuniary externalities that work through the interest rate. However, the choice of liquidity is free from the pecuniary externalities as the composition of bank assets does not affect the marginal utility in period 1. Hence, without risk shifting the banks' liquidity choice would coincide with the solution to the regulator's problem. This result is formalized in the following proposition.

Proposition 6 (Optimal liquidity but excessive leverage without bank risk shifting)**.** *Consider a version of the extended model in which banks have no risk shifting motives and the supply curve (1) is upward sloping. In a competitive equilibrium, liquidity is at the level that would be chosen by the benevolent regulator, but leverage is excessive because of pecuniary externalities that work through the interest rate.*

Proposition 6 highlights that in this model the externalities arising from bank risk shifting are essential for obtaining the result of insufficient liquidity in a competitive equilibrium.

4.3 Parameterization

The previous section analytically showed that leverage is excessive *given* liquidity and liquidity is insufficient *given* leverage in a competitive equilibrium. However, it is not clear whether they are so *jointly*. Also, how banks respond to changes in leverage and liquidity requirements and key parameter values are yet to be known. Addressing these questions requires numerical analyses. To this end, this section parameterizes the extended model presented in Section 4.1.

Parameter values are set so that the extended model generates key endogenous variables not far from those observed for major US banks. Yet, it should be noted that the model aims to capture a financial system as a whole which generates short-term liabilities vulnerable to runs. After all, the model is so stylized that numerical analyses are intended to show the model's qualitative implications rather than quantitative implications.

Parameters σ_ϵ, γ and λ and y are set to hit the following target values in a competitive equilibrium: the leverage of $L = 15$, the liquidity ratio of $m = 0.05$, the crisis probability of $P = 0.05$ and the borrowing interest rate of $R = 1.02$. For the six largest US banks, over the period of 2008–2017 the leverage, measured by the ratio of total assets to Core Equity Tier 1 capital, is 13.4 on average and the liquidity ratio, measured by the ratio of liquid assets to total liabilities, is 0.037 on average (Miller and Sowerbutts, 2018).[11] The target

[11]The liquid assets are the sum of cash, withdrawable reserves and US treasury securities. The ratio

values for L and m are not far from these observations. The target value of $P = 0.05$ is consistent with the historical fact that suggests that in any given country, banking crises occur on average once in every 20 to 25 years, i.e. the average annual probability of a crisis is 4–5 percent (Basel Committee on Banking Supervision 2010 (BCBS hereafter)). The bank capital n is set so that the consumption in period 1 is close to the consumption in period 2. The resulting parameter values are $\sigma_\epsilon = 8.68/10000$, $\gamma = 0.66$, $\lambda = 0.17$, $y = 1.63$ and $n = 0.055$.

The mean return on bank lending is set to $\mu = 1.035$ so that the after-taxed return on equity at the mean return when there is no fire sales is about 15 percent, which is higher than those observed in the post-crisis period of 2008–17, but it is in line with the pre-crisis period of 2000–07.[12] The standard deviation of the return on bank lending is set to $\sigma_k = 0.025$ so that there exists an equilibrium that satisfy the target values discussed above.[13] Admittedly the return is highly volatile, but such volatility is required for the equilibrium to have the target level of a 5 percent crisis probability.

Finally, the functional form of the period-1 utility is assumed to be $u(c_1) = (c_1^{1-\alpha})/(1-\alpha)$ and two values $\alpha = 0.01$ and 0.1 are considered. A smaller value of α means that the utility function becomes close to be linear and the degree of the pecuniary externalities identified in the model gets smaller. Although the two values are small relative to an often-assumed case of log utility ($\alpha = 1$), these values are enough to show contrasting implications on prudential tools, highlighting a general equilibrium effect through the curvature of $u(\cdot)$.

4.4 Leverage and Liquidity Requirements

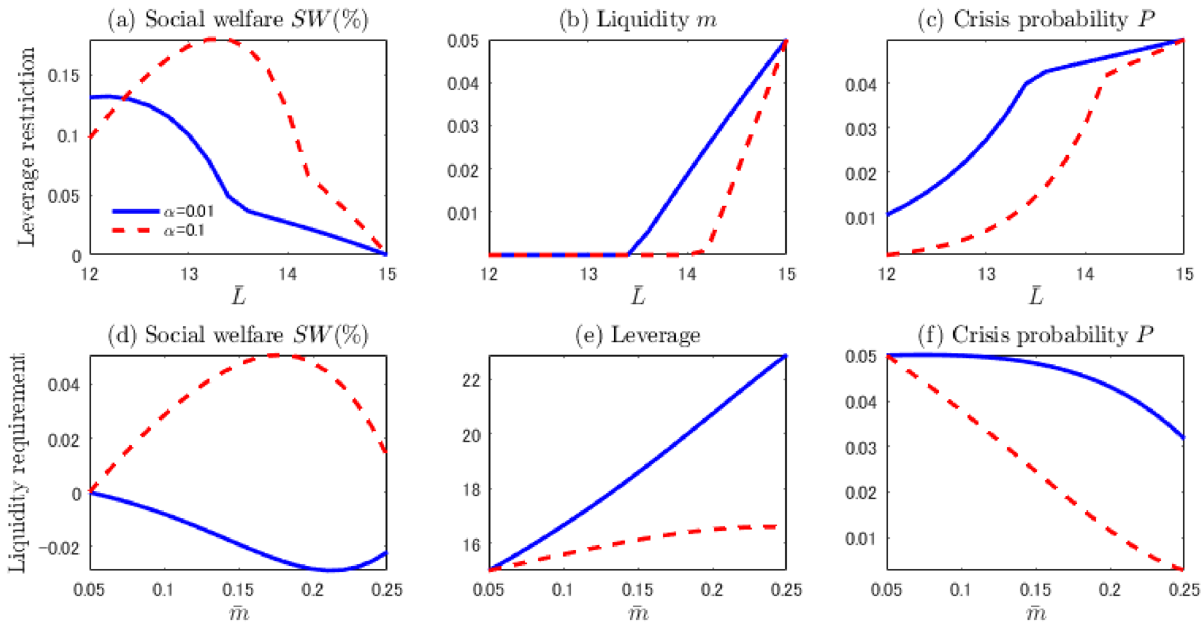
To understand the joint impact of leverage and liquidity requirements, first, I consider cases of one tool only for leverage and liquidity, respectively, which is followed by an analysis on the joint effects of the two tools.

of liquid assets to total assets, reported by Miller and Sowerbutts (2018), is transformed into the ratio of liquid assets to total liabilities using leverage. The data are available from 2008 because two banks in the sample were purely investment banks until 2008 and their data source – the Federal Reserve's Financial Reports (form FRY-9C) – was not available before 2008 for these two banks.

[12]The after-taxed return on equity at the mean return when there is no fire sales is given by $(1-\tau)[\bar{R}^k L - (\mu-1)(L-1)m - R(L-1) - 1]$, where τ is the tax rate. In the calculation, the tax rate is assumed to be 30 percent.

[13]If σ_k is set too low, there is no parameter value for e.g. $\gamma \in (0, 1)$ that supports the equilibrium with the target values.

Figure 1: Impacts of leverage (upper panels) and liquidity (lower panels) requirements



Note: In Panels (a) and (d), social welfare is measured as a percentage deviation from the level of social welfare at the competitive equilibrium without any restrictions.

4.4.1 Leverage restriction only

Consider the extended model presented in Section 4.1 in which only a restriction on leverage is put in place. This situation is reminiscent of the periods under the Basel I and II in which liquidity requirements were absent. Panels (a)-(c) of Figure 1 show the impacts of the leverage restriction, $L \leq \bar{L}$, on social welfare, liquidity and the crisis probability, respectively, for the economies with $\alpha = 0.01$ (blue solid line) and 0.1 (red dashed line). Without any restriction the leverage is $L = 15$. As the leverage restriction is tightened from $L = 15$ to lower values, the social welfare is improved (Panel (a)) and the crisis probability is reduced (Panel (c)). However, the banks respond by reducing liquidity holdings (Panel (b)). Hence, imposing a leverage tool only induces banks to migrate risk from leverage to liquidity.

The degree of the risk migration is greater for the economy with $\alpha = 0.1$ than that with $\alpha = 0.01$. This is because a tightening in leverage limits the amount of deposits and lowers the interest rate R, which further reduces the crisis probability. This general equilibrium effect is stronger for the economy with a greater curvature of the period-1 utility function. Banks reduce liquidity holdings more to counter a decrease in profits caused by a tight restriction on leverage when the curvature of the utility function is greater. The economy with $\alpha = 0.01$ has a smaller general equilibrium effect of the leverage restriction on the interest than the economy with $\alpha = 0.1$ and thereby it calls for a tighter leverage restriction around L=12 to achieve the highest level of social welfare by means of the leverage restriction only (Panel (a)).

4.4.2 Liquidity requirement only

Next, consider a situation in which only a liquidity requirement, $m \geq \bar{m}$, is put in place. Panels (d)-(f) of Figure 1 show the impacts of the liquidity tool on social welfare, leverage and the crisis probability, respectively, for the economies with $\alpha = 0.01$ and $\alpha = 0.1$. As the liquidity requirement is tightened, the crisis probability is contained for both economies (Panel (f)). However, while the social welfare is improved for the economy with $\alpha = 0.1$, it is deteriorated for the economy with $\alpha = 0.01$ (Panel (d)). This difference is driven by the divergent responses of leverage (Panel (e)). For the economy with a lower curvature of the utility function, the effect of increasing leverage on the interest rate is smaller, so that the banks respond by increasing leverage to a tightened liquidity requirement much more than in the economy with a higher curvature of the utility function. This negative effect dominates the benefit of increasing bank liquidity holdings, and as a result, imposing the liquidity requirement worsens welfare rather than improves it. This numerical example is still consistent with Proposition 4, which shows that imposing a liquidity requirement improves welfare *given* leverage. In this example, doing so worsens welfare, because leverage is not fixed; the banks respond by increasing leverage. This risk migration is a culprit of the welfare deterioration as a result of imposing the liquidity requirement only for the economy with $\alpha = 0.01$.

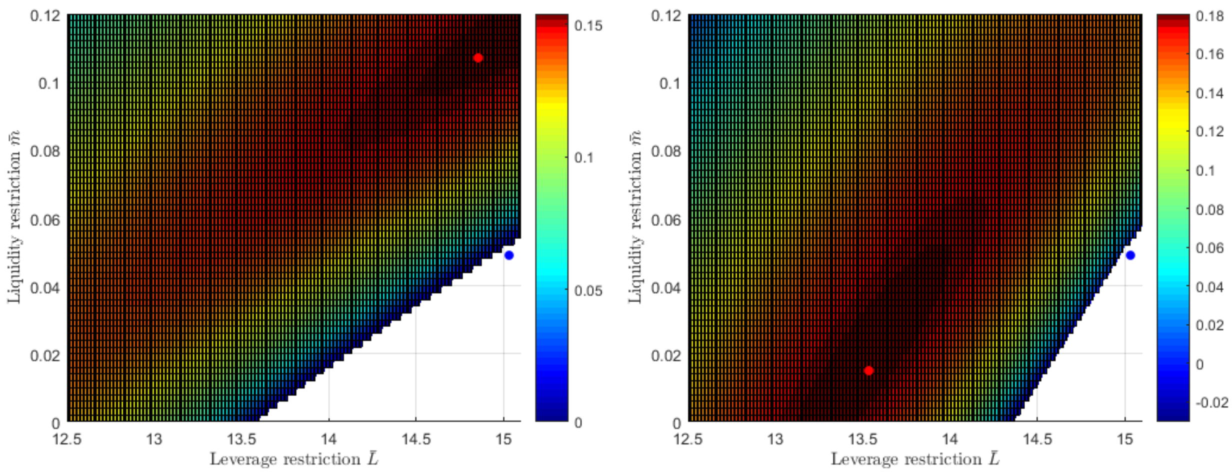
4.4.3 Coordination of leverage and liquidity tools

The previous analysis on one tool only highlights need for joint restrictions on leverage and liquidity to address risk migration from one area to another. Then, what is an optimal policy coordination between leverage and liquidity tools? How does the optimal coordination differ from the cases of one tool only?

Figure 2 addresses these questions by plotting social welfare as a function of the two requirements for the model with $\alpha = 0.01$ (Panel (a)) and that with $\alpha = 0.1$ (Panel (b)). Let subscript $_{BR}$ and $_{CE}$ denote a solution to the benevolent regulator's problem and the

Figure 2: Impacts of leverage and liquidity requirements on social welfare

(a) $\alpha = 0.01$ (b) $\alpha = 0.1$



Note: Social welfare is measured as a percentage deviation from that of the competitive equilibrium. A red circle corresponds to a solution to the constrained planner problem and a blue circle corresponds to the competitive equilibrium.

competitive equilibrium, respectively. First, the optimal coordination $\{L_{BR}, m_{BR}\}$ depends crucially on the curvature of the period-1 utility function, i.e. the general equilibrium effect of leverage on the interest rate. Relative to the competitive equilibrium, the solution to the regulator's problem features tightened leverage and tightened liquidity, i.e. $L_{BR} < L_{CE}$ and $m_{BR} > m_{CE}$, in the case of $\alpha = 0.01$ (Panel (a)). But the solution features tightened leverage and loosened liquidity, i.e. $L_{BR} < L_{CE}$ and $m_{BR} < m_{CE}$, in the case of $\alpha = 0.1$ (Panel (b)). In this case, the general equilibrium effect of the leverage restriction on the crisis probability, through its effect on the interest rate, is so great that lowering leverage is more effective than increasing liquidity to address the inefficiency. It is worth noting that even though the optimal level of liquidity is lower than that in the competitive equilibrium, the liquidity requirement is still binding. Without the requirement, the banks would chosen a lower level of liquidity as shown in Figure 1(b). In the case of $\alpha = 0.01$, the general equilibrium effect is small and hence tightening both leverage and liquidity becomes optimal.

Second, relative to the cases of a leverage tool only, the optimal coordination between leverage and liquidity requirements calls for milder requirements on leverage. On the one hand, in the case of the leverage tool only, the optimal level of leverage that achieves the highest possible welfare is 12 and 13.2 for $\alpha = 0.01$ and 0.1, respectively. On the other hand, the optimal coordination requires the leverage of 14.9 and 13.5, respectively. Hence, with a liquidity requirement in place, a less strict leverage restriction is needed to achieve the highest possible social welfare than in the case of a leverage tool only.

A similar implication holds for liquidity in the case of $\alpha = 0.1$: a liquidity tool only requires the liquidity ratio of around 0.18, while the optimal coordination calls for the liquidity ratio of only 0.016. However, this result does not hold for $\alpha = 0.01$ because tightening a liquidity requirement worsens welfare as discussed in Section 4.4.2.

4.5 Comparative Statics Analysis

4.5.1 Comparative statics: competitive equilibrium

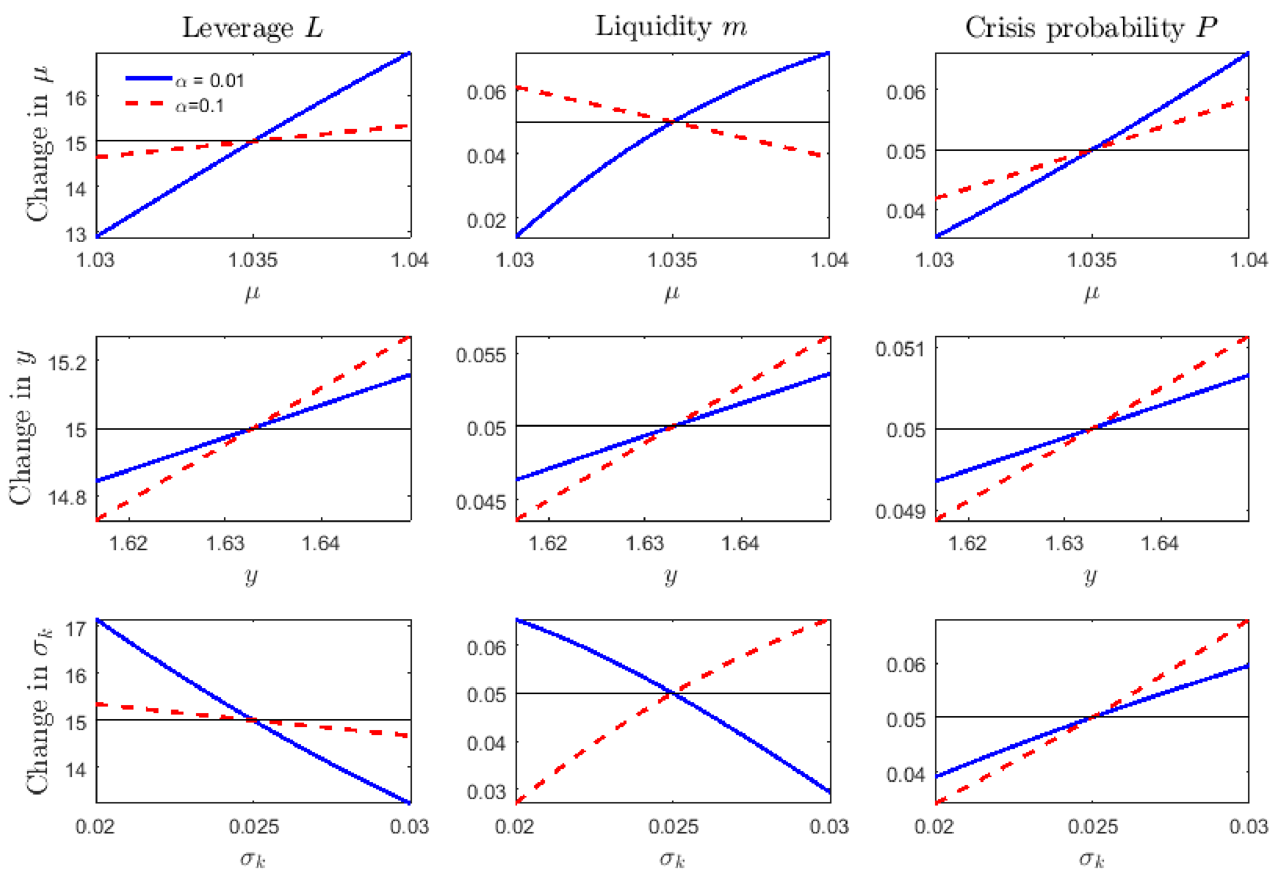
Having studied the welfare implications of the model, in this section, I study how the model economy without any restrictions responds to changes in key parameter values. Figure 3 plots how leverage, liquidity and the crisis probability change in response to changes in the mean return on bank assets μ, the household income y and the standard deviation of the bank asset return σ_k for the models with $\alpha = 0.01$ and 0.1, respectively

Figure 3 reveals three findings. First, similar to Proposition 1 for the baseline model with a bank leverage choice only, both leverage and the crisis probability increase as the mean return μ and the household income y increase. This result holds for both cases of $\alpha = 0.01$ and 0.1.

Second, in response to an increase in the standard deviation – uncertainty – of the bank asset return σ_k, banks lower leverage but the crisis probability increases for both cases of $\alpha = 0.01$ and 0.1. Although leverage is an important determinant of the crisis probability, the crisis probability increases when banks are deleveraging.

Third, in response to changes in the mean bank asset return μ and the uncertainty of the bank asset return σ_{R^k}, in the case of $\alpha = 0.01$, leverage and liquidity move in the opposite direction in terms of contributions to the crisis probability, i.e. they are strategic substitutes. But in the case of $\alpha = 0.1$ leverage and liquidity move in the same direction. i.e. they are strategic complements. Specifically, when the mean return rises, the banks respond by increasing leverage in both cases of $\alpha = 0.01$ and 0.1, but they behave differently in a liquidity choice: they increase liquidity in the case of $\alpha = 0.01$ while they decrease liquidity in the case of $\alpha = 0.1$. This difference has to do with the general equilibrium effect of leverage on the interest rate and on the crisis probability. When the curvature of the period-1 utility is relative flat, e.g. $\alpha = 0.01$, a higher leverage is less associated with a

Figure 3: Comparative statics of the competitive equilibrium

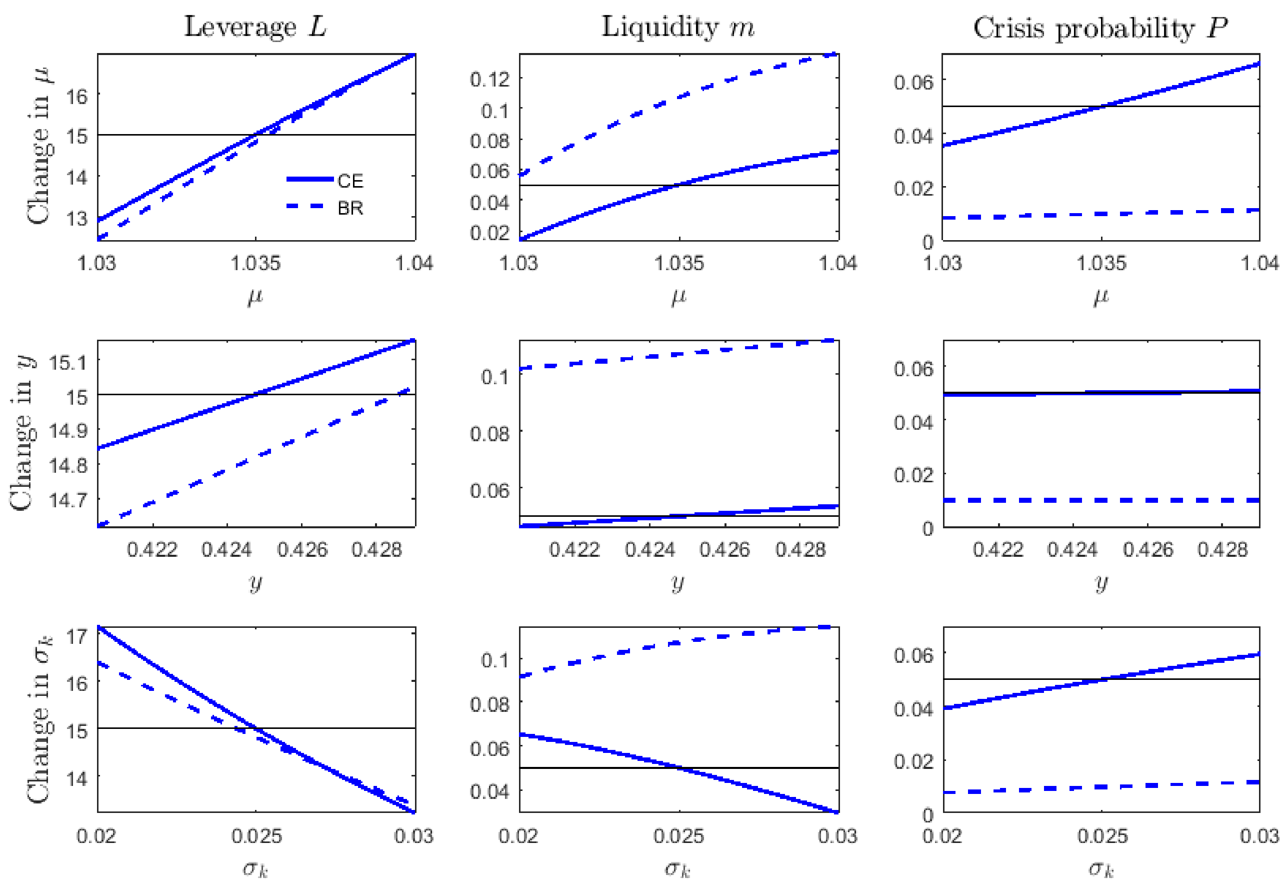


rise in the interest rate than otherwise would be a case. Hence, the banks find it profitable to increase leverage and limit the associated increase in the crisis probability by increasing liquidity holdings. If, instead, the general equilibrium effect is strong, e.g. $\alpha = 0.1$, the banks restrain an increase in leverage but reduce liquidity to raise the bank asset return. A similar mechanism works for the case of a change in the uncertainty of the bank asset return.

4.5.2 Comparative statics: constrained optimal allocation

How does the constrained optimal allocation – a solution to the benevolent regulator's problem – change in response to changes in key parameter values? Figure 4 plots the constrained optimal allocation for leverage, liquidity and the crisis probability in the case of $\alpha = 0.01$ in response to changes in the mean return on bank assets μ, the household income y and the uncertainty of bank asset returns σ_k. The case of $\alpha = 0.1$ is omitted as its implications are similar.

Figure 4: Comparative statics of the constrained optimal allocation



Note: CE (blue solid line) and BR (blue dashed line) represent the competitive equilibrium and the solution to the benevolent regulator's problem, respectively.

Figure 4 reveals two findings. First, the constrained optimal levels of leverage and liquidity change in response to changes in the parameter values. In most cases the constrained optimal levels change in parallel with changes in the competitive equilibrium allocation. For example, both the constrained optimal level and the competitive equilibrium level of leverage increase as the mean return on bank assets rises. However, this is not always a case: the two levels can move in the opposite direction. For example, in response to an increase in the uncertainty of bank asset returns the constrained optimal level of liquidity increases while its counterpart in the competitive equilibrium decreases (bottom medium panel of Figure 4). These observations suggest that the optimal prudential policy, which aims to achieve the constrained optimal allocation, differs in a non-trivial manner depending on parameter values that characterize the banking system and the economy.

Second, the constrained optimal level of the crisis probability is relatively stable around

1 percent, irrespective of changes in the parameter values. This makes a contrast with volatile changes in the constrained optimal levels of leverage and liquidity. Also, the stable crisis probability implies that the degree of the crisis probability curbed by the optimal prudential policy – a difference between P_{CE} and P_{BR} – becomes greater as P_{CE} increases. This is evident in response to increases in the mean return on bank assets (top right panel of Figure 4) and the uncertainty of bank asset returns (bottom right panel of Figure 4). The stable crisis probability in the constrained optimal allocation implies that if the crisis (default) probability is observable, setting a target level of the crisis probability and letting banks to behave freely as long as the probability is no higher than the target level, rather than imposing multiple tools, may be a robust way to improve welfare in various economies with a different banking system.

5 Extensions

The benchmark model presented in Section 2 is so stylised that it can be extended in various ways. In this section, I provide some extensions that are used to discuss bank-specific capital requirements, risk weights and deposit insurance. The extensions bring some implications for shadow banking and concentration risk. Unless mentioned, the same parameter values set in Section 4.3 are used in this section. Main implications are unaffected by the discussed values of the curvature of the utility function, and hence $\alpha = 0.1$ is used in this section.

5.1 Model with Heterogeneous Banks

5.1.1 Overview of the model

I extend the benchmark model to incorporate two types of banks, indexed by $j \in \{1, 2\}$. For simplicity, the two types of banks differ only in the riskiness of lending. The type-j banks specialise in lending to sector j and cannot lend to the other sector. Lending to sector j yields the same expected return μ, but the riskiness differs between the two sectors: $R_j^k \sim N(\mu, \sigma_j^2)$ with $\sigma_1 \neq \sigma_2$. The remaining part of the model is essentially the same as in the benchmark model.

The equilibrium for this economy is characterized by the following four equations with four unknowns $\{R_j, L_j\}_{j=1}^2$: for $j = 1, 2$,

$$
\begin{aligned}
R_j =& \frac{u'(y - (L_1 - 1)n - (L_2 - 1)n)}{1 - P_j + \mathbb{E}(v_j|\text{default})P_j}, \\
0 =& \int_{R_j^{k*}}^{\infty} (R^k - R)dF_j - R_j\lambda (L_j - 1) \int_{R_j^{k*}}^{\infty} \frac{\partial x_j}{\partial \bar{s}_j^*} \frac{\partial \bar{s}_j^*}{\partial L_j} dF_j - R_j\lambda \int_{R_j^{k*}}^{\infty} x_j dF_j,
\end{aligned}
$$

where $P_j = F_j(R_j^{k*})$ is the default probability for the type-j banks, $F_j(\cdot)$ is the cumulative normal distribution function with mean μ and standard deviation σ_j. The thresholds R_j^{k*} and $\bar{s}_j^*$ are given by equations (5) and (6) and the recovery rate v_j is given by (12) with a modification to add subscript j.

For a numerical illustration, the type-2 banks are assumed to be riskier than the type-1 banks. Specifically, the standard deviation of the type-2 bank asset return is 1.5 times as high as that of the type-1 banks. The bank net worth is set to a half of the level set in Section 4.3 for each type of banks so that the aggregate bank net worth remains the same.

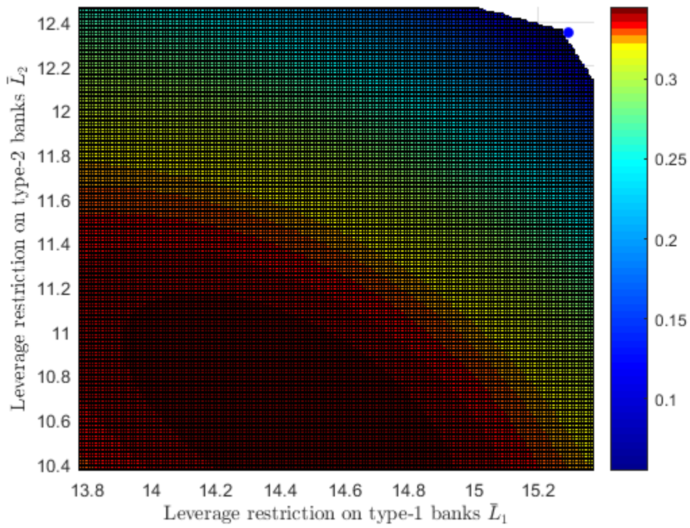
In the competitive equilibrium, the type-2 banks have a lower leverage but a higher default probability than do the type-1 banks, reflecting the higher riskiness of the bank asset return. The leverage and the default probability are $L_1 = 15.3$ and $P_1 = 0.074$ for the type-1 banks and $L_2 = 12.4$ and $P_2 = 0.099$ for the type-2 banks. Hence, in this model, a low leverage mirrors the riskiness of the banks and does not necessarily signals the safety of the banks.

5.1.2 Heterogeneous capital requirements and risk weights

A heterogeneity in bank riskiness calls for bank-specific leverage/capital requirement. Figure 5 shows the joint effects of bank-specific leverage restrictions on social welfare. Limiting leverage for both types of banks improves welfare and the optimum is attained around $\bar{L}_1 = \bar{L}_1^* \equiv 14.5$ and $\bar{L}_2 = \bar{L}_2^* \equiv 10.6$. Reflecting the heterogeneous riskiness of bank assets, the leverage restriction imposed on banks differs between the two types of banks.

A single capital/leverage restriction can achieve the same outcome if it is complemented by risk weights. This is so-called risk-weighted-based capital requirement. A risk weight is normalised at 100 percent for the type-1 bank loans and 100ω percent for the type-2 bank loans and a risk-weighted-based capital requirement is normalised at $1/\bar{L}_1^*$. By construction, the capital ratio (or leverage) is restrained at the optimal level for the type-1 banks. To achieve the optimal level for the type-2 banks i.e. $n/(n + d_2) = 1/\bar{L}_2^*$, the risk weight ω has

Figure 5: Impacts of bank-specific leverage restrictions on social welfare



Note: Social welfare is measured as a percentage deviation from that of the competitive equilibrium. A blue circle at the upper right corner indicates the competitive equilibrium.

to be such that $n/(n + \omega d_2) = 1/\bar{L}_1^*$. Solving the equations for ω yields

$$\omega = \omega^* \equiv \frac{\bar{L}_1^* - 1}{\bar{L}_2^* - 1} > 1.$$

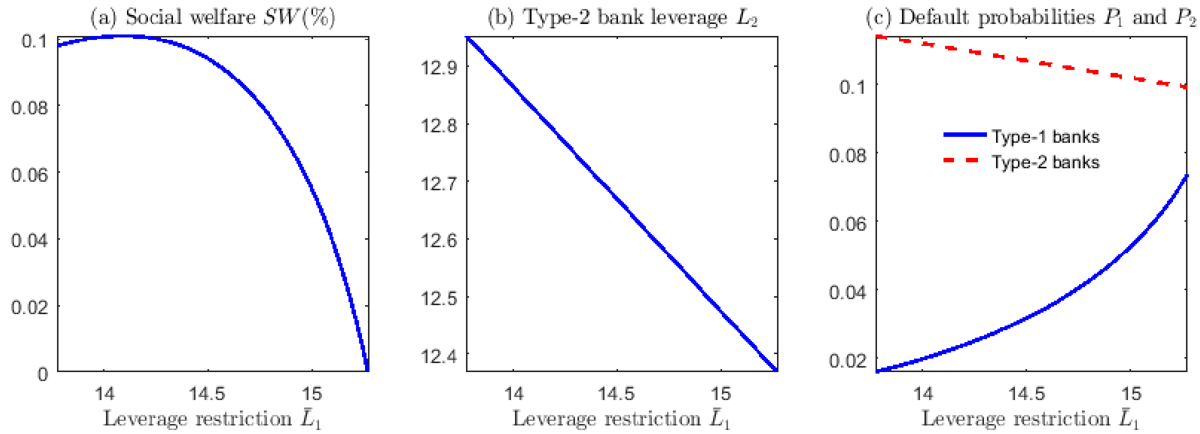
The optimal risk weight for the type-2 bank loans is more than 100 percent, reflecting their high riskiness.

5.1.3 Shadow banks

Shadow banks, by definition, lie outside the reach of banking regulations. In the model with heterogeneous banks, the type-2 banks, which specialise in riskier loans, can be interpreted as shadow banks if they are free from regulations, while the type-1 banks, which specialise in less risky loans, can be seen as traditional banks if they are regulated.

With restrictions imposed only on the traditional banks, the traditional banks become safer, but the shadow banks become riskier. Figure 6 plots the impacts of a leverage restriction on the type-1 banks only on social welfare, the type-2 bank leverage and the default probabilities. As the leverage restriction is tightened, the type-2 bank leverage increases and so does the type-2 bank default probability. The social welfare is improved for somewhat, but its achievable level of around 0.1 percent is far below the optimum of

Figure 6: Impacts of a leverage restriction on the type-1 banks only



Note: Social welfare is measured as a percentage deviation from that of the competitive equilibrium.

above 0.3 percent when both types of banks are regulated.

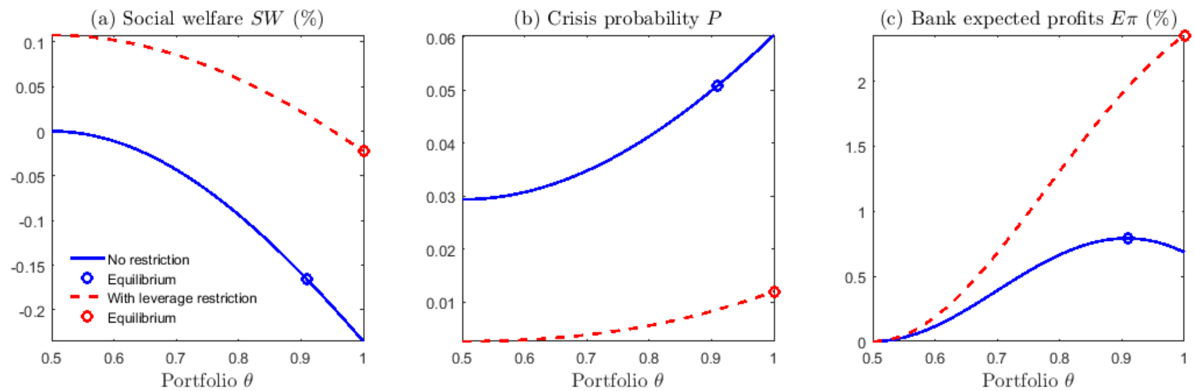
5.2 Model with a Portfolio Choice

Banks may choose a less-diversified and riskier portfolio than the socially desirable one when they have risk shifting motives. To formalise this idea, I extend the baseline model presented in Section 5.1 to incorporate a portfolio of loans. Specifically, banks make loans to two sectors, indexed by $j \in \{1, 2\}$. The returns of the two sectors follow a joint normal distribution, $\boldsymbol{R}^k \sim N(\boldsymbol{\mu}, \Sigma)$, where $\boldsymbol{R}^k \equiv [R_1^k, R_2^k]'$ is a vector of returns of the two sectors. In addition to leverage banks choose a portfolio of loans, $\boldsymbol{\theta} \equiv [\theta, 1-\theta]'$, where $\theta \in [0, 1]$ is a fraction of total loans invested in sector $j = 1$. The return of the bank asset portfolio is then given by $R^k(\theta) \equiv \boldsymbol{\theta}'\boldsymbol{R}^k$, which follows $N(\mu(\theta), \sigma_k(\theta)^2)$, where $\mu(\theta) \equiv \boldsymbol{\theta}'\boldsymbol{\mu}$ is the mean return and $\sigma_k(\theta) \equiv (\boldsymbol{\theta}'\Sigma\boldsymbol{\theta})^{\frac{1}{2}}$ is the standard deviation of the portfolio. Each fund manager i observes a bank portfolio as well as leverage and receives independent signals $s_{ij} = R_j^k + \epsilon_{ij}$ with $\epsilon_{ij} \sim N(0, \sigma_{\epsilon_j}^2)$ for $j = 1, 2$. Given a bank portfolio, this extended model works essentially the same way as in the benchmark model. Fund manager i withdraws deposits early if and only if $\boldsymbol{\theta}'\boldsymbol{s}_i$ is less than the threshold $\bar{s}^*(L, \theta)$, where $\boldsymbol{s}_i \equiv [s_{i1}, s_{i2}]'$ is a vector of noisy signals. A difference is that now the threshold depends on the bank asset portfolio θ as well as the leverage L.

To illustrate concentration risk, I assume that the two sectors are identical.[14] The only difference from the benchmark model is that banks can reduce their loan risk by diversifying over loans to the two sectors. Specifically, banks are able to minimize the risk of their loan

[14] BCBS (2014) points out the concentration risk as potential risk.

Figure 7: Bank portfolio choice and concentration risk

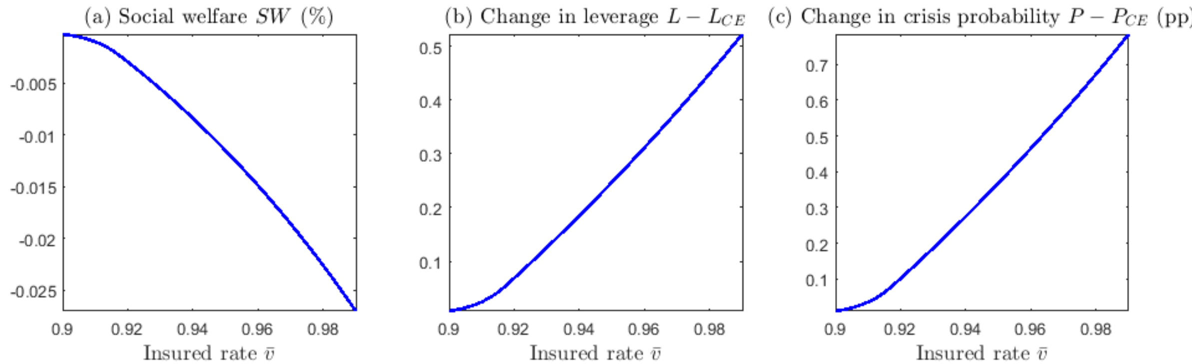


Note: Social welfare is measured as a percentage deviation from that in the economy with no restriction and with $\theta = 0.5$. Bank expected profits are measured as a percentage deviation from those with $\theta = 0.5$ for each curve.

portfolio by setting $\theta = 0.5$. Not surprisingly, the smallest portfolio risk achieves the highest social welfare, as shown by the blue line in Figure 7(a). However, banks do not choose such a portfolio but instead select the riskier and more concentrated portfolio of around $\theta = 0.9$ to maximise the profits (Figure 7(c)). As a result, the crisis probability rises to 5 percent from 3 percent, a level which would be realised if the banks chosen the perfectly diversified portfolio (Figure 7(b)).

The model and its numerical example highlights need for addressing concentration risk with a unique prudential instrument. Imposing a leverage restriction can improve welfare, but as in the model with liquidity and leverage and the model with heterogeneous banks, doing so causes risk to leakage from a non-regulated area, which is a portfolio choice in this model. For example, if a regulator imposes the leverage restriction, $\bar{L} = L_{CE} - 1$, that is tighter by 1 than what banks would choose without any restriction, the banks respond by concentrating completely in sector-1 lending, i.e., by setting $\theta = 1$, as shown in the red dashed line in Figure 7(c). As a result, the crisis probability becomes materially higher and the social welfare gets significantly lower than what would be achievable if the banks chose the perfectly diversified portfolio of $\theta = 0.5$. Hence, a unique prudential tool that restricts risk concentration in one type of lending is required to address the risk leakages from the leverage area to the portfolio area.

Figure 8: Impacts of imperfect deposit insurance



Note: Social welfare is measured as a percentage deviation from that of the competitive equilibrium with no deposit insurance. L_{CE} and P_{CE} denote the leverage and the crisis probability in the competitive equilibrium with no deposit insurance.

5.3 Model with Deposit Insurance

Perfect deposit insurance, which ensures the recovery rate of unity, $v = 1$, will eliminate bank runs in theory, but such an insurance is hardly put in place in practice. Typically, the coverage of bank deposit insurance is limited and there is no insurance on money-like short-term debt. In short, deposit insurance is imperfect in practice.

Imperfect deposit insurance falls short of eliminating bank runs. As long as households and fund managers follow the behavioural rule (2), bank runs can still occur. A key modelling assumption is that the fund managers' incentive to run, summarized by parameter γ in (2), is unaffected by the presence of deposit insurance.

To explore the impact of deposit insurance on financial stability and social welfare, the benchmark model is extended to incorporate imperfect deposit insurance that protect households from incurring losses more than $100(1-\bar{v})$ percent of the promised interest rate R. Hence, $\bar{v}$ forms the floor of the recovery rate. The government finances $(\bar{v} - v)R$ per unit of funds, imposing lump-sum taxes on households in period $t = 2$. Then, the supply curve of funds (1) is modified to:

$$R = \frac{u'(y - (L-1)n)}{1 - P + \mathbb{E}(\max\{v, \bar{v}\}|\text{default})P}. \tag{26}$$

Equation (26) implies that an increase in the insurance rate $\bar{v}$ shifts the supply curve outward and makes excessive leverage even more excessive and worsens the crisis probability.

Figure 8 confirms this prediction. As the coverage rate of the deposit insurance rises, the leverage becomes more excessive (Panel (b)), the crisis probability increases further

(Panel (c)), and as a result, the social welfare deteriorates (Panel (a)).

6 Conclusion

This paper has developed a model of endogenous bank runs in a global game general equilibrium framework. The benchmark model presented in Section 2 has highlighted banks' risk shifting and pecuniary externalities as a source of inefficiencies that give rise to an inefficiently high financial crisis probability. The paper has extended the benchmark model and studied the role of imposing multiple prudential tools in addressing the inefficiency: leverage and liquidity tools in Section 4; bank-specific (or sector-specific) capital requirements in Section 5.1; a leverage restriction and a exposure limit on certain-type of lending in Section 5.2. These tools are closely related with and motivated by the actual regulations and reforms put in place after the global financial crisis (BCBS 2011, 2013, 2014). The benchmark model, upon which the extended models are built and used to study these tools, hence provides a unified basic framework for studying banking crises, banks' behaviour and prudential policy tools.

The models studied in the paper make several empirical predictions. Their common theme is that risk can migrate from one area to others. And this is a main reason why multiple restrictions are required to address the issue. In the case of capital/leverage and liquidity discussed in Section 4, a tightening in capital requirements causes banks to reduce the holdings of liquid assets. In the case of traditional and shadow banks discussed in Section 5.1, a tightening in capital requirements on traditional banks induce shadow banks to grow and make them riskier. In the case of capital requirements and exposure limits discussed in Section 5.2, a tightening in capital requirements leads banks to choose a riskier asset portfolio by increasing exposure to a certain sector.

The paper has highlighted risk migration between two different risk spaces, e.g. capital/leverage and liquidity, for simplicity and clarity. In practice there would be risk migration among more than two areas, e.g. capital/leverage, liquidity and portfolios, under the name of 'balance sheet optimisation.' The paper abstracts away from a heterogeneity in bank liabilities, but this can be another area of risk migration. Analysing risk migration in all possible areas would be extremely difficult, if not impossible. Yet, the models presented in this paper have allowed us to disentangle the impacts of one or two prudential tools on two risk spaces, a crisis probability and social welfare. In the case of leverage and liquidity tools, the model has also shed light on the general equilibrium effect through the interest rate on the constrained optimal allocation.

The models presented in this paper have considered various prudential tools on risk spaces, but they still lack a important dimension: time. Adding a time dimension is essential for considering time-varying tools, e.g. countercyclical capital buffers, and also for highlighting other potential source of externalities. Having kept this potential extension in mind, I have constructed the benchmark model so that it would be easily incorporated into a dynamic general equilibrium model. I plan to tackle on this in a future work.

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Appendix

Derivation of equation (9). As shown in Section 2 the threshold R^{k*} is a solution to equations (5) and (6). These equations are written explicitly as:

$$\Phi\left(\sqrt{\frac{1}{\sigma_k^2}+\frac{1}{\sigma_\epsilon^2}}R^{k*}-\frac{\frac{1}{\sigma_k^2}\mu+\frac{1}{\sigma_\epsilon^2}\bar{s}^*}{\sqrt{\frac{1}{\sigma_k^2}+\frac{1}{\sigma_\epsilon^2}}}\right)=\gamma, \tag{27}$$

$$R^{k*}=R\left(1-\frac{1}{L}\right)\left[1+\lambda\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\right], \tag{28}$$

where $\Phi(\cdot)$ is the standard normal distribution function. Equation (27) implies that $\lim_{\sigma_\epsilon\to 0}\Phi((R^{k*}-\bar{s}^*)/\sigma_\epsilon)=\gamma$. Therefore, $\lim_{\sigma_\epsilon\to 0}\Phi((\bar{s}^*-R^{k*})/\sigma_\epsilon)=1-\gamma$. Substituting this result into equation (28) leads to equation (9).

Derivation of equation (10). Equation (10) is the limiting case of equation (8) where $\sigma_\epsilon\to 0$. First, we derive an expression for $\partial\bar{s}^*(L)/\partial L$ in equation (8). Totally differentiating equations (27) and (28) yields

$$dR^{k*}=\frac{1}{\frac{\sigma_\epsilon^2}{\sigma_k^2}+1}d\bar{s}^*,$$

$$dR^{k*}=\frac{R}{L^2}\left[1+\lambda\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\right]dL+R\left(1-\frac{1}{L}\right)\lambda\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}(d\bar{s}^*-dR^{k*})$$

Combining these equations yields

$$\frac{d\bar{s}^*}{dL}=\frac{\left(\sigma_k^2+\sigma_\epsilon^2\right)\frac{R}{L^2}\left[1+\lambda\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\right]}{\sigma_k^2-\left(1-\frac{1}{L}\right)\lambda\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\sigma_\epsilon},$$

where $\phi(\cdot)$ is the standard normal pdf. Note that $\lim_{\sigma_\epsilon\to 0}\phi((\bar{s}^*-R^{k*})/\sigma_\epsilon)=\phi(\lim_{\sigma_\epsilon\to 0}(\bar{s}^*-R^{k*})/\sigma_\epsilon)=\phi(\Phi^{-1}(1-\gamma))$. Then, in the limit, $d\bar{s}^*/dL$ is given by

$$\lim_{\sigma_\epsilon\to 0}\frac{d\bar{s}^*}{dL}=\frac{R}{L^2}\left[1+\lambda(1-\gamma)\right].$$

Next, consider $\int_{R^{k*}}^{\infty}[\partial x(R^k,\bar{s}^*)/\partial\bar{s}^*]dF(R^k)$ in equation (8), where $F(\cdot)$ is the normal distribution function with mean μ and variance σ_k^2. It is explicitly written as

$$\begin{aligned}\int_{R^{k*}}^{\infty}\frac{\partial x(R^k,\bar{s}^*)}{\partial\bar{s}^*}dF(R^k)&=\int_{R^{k*}}^{\infty}\phi\left(\frac{\bar{s}^*-R^k}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}dF(R^k)\\&=\int_{R^{k*}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\bar{s}^*-R^k}{\sigma_\epsilon}\right)^2}\frac{1}{\sigma_\epsilon}\frac{1}{\sqrt{2\pi}\sigma_k}e^{-\frac{1}{2}\left(\frac{R^k-\mu}{\sigma_k}\right)^2}dR^k.\end{aligned}$$

The terms in the power of e are arranged as

$$
\begin{aligned}
& -\frac{1}{2}\left(\frac{\bar{s}^*-R^k}{\sigma_\epsilon}\right)^2-\frac{1}{2}\left(\frac{R^k-\mu}{\sigma_k}\right)^2 \\
=& -\frac{1}{2}\left[\frac{\bar{s}^{*2}-2\bar{s}^*R^k+R^{k2}}{\sigma_\epsilon^2}+\frac{R^{k2}-2R^k\mu+\mu^2}{\sigma_{R^k}^2}\right] \\
=& -\frac{1}{2}\left[\left(\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}\right)R^{k2}-2\left(\frac{\bar{s}^*}{\sigma_\epsilon^2}+\frac{\mu}{\sigma_k^2}\right)R^k+\frac{\bar{s}^*}{\sigma_\epsilon^2}+\frac{\mu^2}{\sigma_k^2}\right] \\
=& -\frac{1}{2}\left(\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}\right)\left[R^{k2}-2\frac{\frac{\bar{s}^*}{\sigma_\epsilon^2}+\frac{\mu}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}R^k+\frac{\frac{\bar{s}^*}{\sigma_\epsilon^2}+\frac{\mu^2}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}\right] \\
=& -\frac{1}{2}\left(\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}\right)\left[\left(R^k-\frac{\frac{\bar{s}^*}{\sigma_\epsilon^2}+\frac{\mu}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}\right)^2-\left(\frac{\frac{\bar{s}^*}{\sigma_\epsilon^2}+\frac{\mu}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}\right)^2+\frac{\frac{\bar{s}^*}{\sigma_\epsilon^2}+\frac{\mu^2}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}\right] \\
=& -\frac{1}{2}\left(\frac{R^k-\frac{\frac{\bar{s}^*}{\sigma_\epsilon^2}+\frac{\mu}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}}{\sqrt{\frac{\sigma_\epsilon^2\sigma_k^2}{\sigma_\epsilon^2+\sigma_k^2}}}\right)^2+\frac{1}{2}\left[\frac{\left(\frac{\bar{s}^*}{\sigma_\epsilon^2}+\frac{\mu}{\sigma_k^2}\right)^2}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}-\frac{\bar{s}^*}{\sigma_\epsilon^2}-\frac{\mu^2}{\sigma_k^2}\right].
\end{aligned}
$$

Then, $\int_{R^{k*}}^{\infty}[\partial x(R^k,\bar{s}^*)/\partial\bar{s}^*]dF(R^k)$ is written as

$$
\int_{R^{k*}}^{\infty}\frac{\partial x(R^k,\bar{s}^*)}{\partial\bar{s}^*}dF(R^k)=\left(\int_{z^*}^{\infty}\phi(z)dz\right)\frac{1}{\sqrt{2\pi}}\sqrt{\frac{1}{\sigma_\epsilon^2+\sigma_{R^k}^2}}\exp\left\{\frac{1}{2}\left[\frac{\left(\frac{\bar{s}^*}{\sigma_\epsilon^2}+\frac{\mu}{\sigma_k^2}\right)^2}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}-\frac{\bar{s}^*}{\sigma_\epsilon^2}-\frac{\mu^2}{\sigma_k^2}\right]\right\},
$$

where

$$
z^*=\frac{R^{k*}-\frac{\frac{\bar{s}^*}{\sigma_\epsilon^2}+\frac{\mu}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}}{\sqrt{\frac{\sigma_\epsilon^2\sigma_k^2}{\sigma_\epsilon^2+\sigma_k^2}}}
$$

Note that $\lim_{\sigma_\epsilon\to 0}=\Phi^{-1}(\gamma)$ and

$$
\lim_{\sigma_\epsilon\to 0}\frac{1}{2}\left[\frac{\left(\frac{\bar{s}^*}{\sigma_\epsilon^2}+\frac{\mu}{\sigma_k^2}\right)^2}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}-\frac{\bar{s}^*}{\sigma_\epsilon^2}-\frac{\mu^2}{\sigma_k^2}\right]=-\frac{1}{2}\left(\frac{\bar{s}^*-\mu}{\sigma_k}\right)^2.
$$

Therefore, the limit of $\int_{R^{k*}}^{\infty}[\partial x(R^k,\bar{s}^*)/\partial\bar{s}^*]dF(R^k)$ is given by

$$
\lim_{\sigma_\epsilon\to 0}\int_{R^{k*}}^{\infty}\frac{\partial x(R^k,\bar{s}^*)}{\partial\bar{s}^*}dF(R^k)=(1-\gamma)f(\bar{s}^*),
$$

where $f(\cdot)$ is the pdf of the normal distribution with mean μ and variance σ_k^2.

Finally, the term, $\int_{R^{k*}}^{\infty}x(R^k,\bar{s}^*(L))dF(R^k)$, in equation (8) goes to zero as $\sigma_\epsilon\to 0$. Therefore, in the limit of $\sigma_\epsilon \to 0$, equation (8) is reduced to equation (10).

Proof of Proposition 1.

(i) The first-order condition of the banks' problem in the limit equilibrium (10) is written as $0 = \partial \mathbb{E}(\pi)/\partial L$, where

$$\begin{aligned}\frac{\partial \mathbb{E}(\pi)}{\partial L} &= \int_{\frac{R^{k*}-\mu}{\sigma_k}}^{\infty} (\mu + \sigma_k z)\, d\Phi(z) \\ &- \left\{ \left[1 - \Phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right)\right] R + \lambda(1-\gamma)\left[1+\lambda(1-\gamma)\right]\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right) R^2 \frac{L-1}{L^2} \right\}.\end{aligned}$$

A marginal change in this derivative with respect to a marginal increase in μ is given by

$$\begin{aligned}\frac{\partial^2 \mathbb{E}(\pi)}{\partial L \partial \mu} &= 1 - \Phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right) + \left[R^{k*} - R\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right)\right]\frac{1}{\sigma_k} \\ &+ \frac{\lambda(1-\gamma)\left[1+\lambda(1-\gamma)\right]}{\sigma_k}\phi'\left(\frac{R^{k*}-\mu}{\sigma_k}\right) R^2 \frac{L-1}{L^2}.\end{aligned}$$

Because $\max_z \phi(z) < 0.4$, the assumptions of this proposition imply $R^{k*} > R\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right)$ and $\phi'(\cdot) > 0$, and thereby the sign of the above derivative is positive: $\partial^2 \mathbb{E}(\pi)/(\partial L \partial \mu) > 0$. Given that the solution L is an optimal solution, the $\partial \mathbb{E}(\pi)/\partial L$ curve is downward sloping. Then, $\partial^2 \mathbb{E}(\pi)/(\partial L \partial \mu) > 0$ implies that the $\partial \mathbb{E}(\pi)/\partial L$ curve shifts upward, implying that the optimal L increases. Hence, the demand curve shifts outward.

(ii) A marginal change of $\partial \mathbb{E}(\pi)/\partial L$ with respect to a marginal increase in λ is given by

$$\begin{aligned}\frac{\partial^2 \mathbb{E}(\pi)}{\partial L \partial \lambda} &= -\left[R^{k*} - R\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right)\right]\frac{1}{\sigma_k}\frac{\partial R^{k*}}{\partial \lambda} \\ &\quad -\frac{\lambda(1-\gamma)\left[1+\lambda(1-\gamma)\right]}{\sigma_k}\phi'\left(\frac{R^{k*}-\bar{R}^k}{\sigma_k}\right) R^2 \frac{L-1}{L^2}\frac{\partial R^{k*}}{\partial \lambda} \\ &\quad -(1-\gamma)[1+2\lambda(1-\gamma)]\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right) R^2 \frac{L-1}{L^2},\end{aligned}$$

where $\partial R^{k*}/\partial \lambda = R(1-1/L)(1-\gamma) > 0$. Hence, $\partial^2 \mathbb{E}(\pi)/(\partial L \partial \lambda) < 0$, which implies that an increase in λ shifts the demand curve inward. Similarly, a marginal change of $\partial \mathbb{E}(\pi)/\partial L$ with respect to a marginal increase in γ is given by

$$\begin{aligned}\frac{\partial^2 \mathbb{E}(\pi)}{\partial L \partial \gamma} &= -\left[R^{k*} - R\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right)\right]\frac{1}{\sigma_k}\frac{\partial R^{k*}}{\partial \gamma} \\ &\quad -\frac{\lambda(1-\gamma)\left[1+\lambda(1-\gamma)\right]}{\sigma_k}\phi'\left(\frac{R^{k*}-\mu}{\sigma_k}\right) R^2 \frac{L-1}{L^2}\frac{\partial R^{k*}}{\partial \gamma} \\ &\quad +\lambda[1+2\lambda(1-\gamma)]\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right) R^2 \frac{L-1}{L^2}\end{aligned}$$

where $\partial R^{k*}/\partial\gamma = -R(1-1/L)\lambda < 0$. Hence, $\partial^2\mathbb{E}(\pi)/(\partial L\partial\gamma) > 0$, which implies that a decrease in γ shifts the demand curve inward.

(iii) The supply curve (1) is written as

$$R = \frac{u'\left(y-(L-1)\,n\right)}{1-P+\mathbb{E}(v|\text{default})P}.$$

From this it is clear that an increase in y shifts the supply curve outward.

(iv) Similarly, the supply curve implies that an increase n shits the curve inward.

Derivation of equation (15). The first-order condition of the regulator's problem is $\partial\text{SW}/\partial L = 0$, where

$$\begin{aligned}\frac{\partial\text{SW}}{\partial L} =& -R\left[1-P+\mathbb{E}(v|\text{default})P\right]+\mathbb{E}(R^k)-\lambda R\int_{\underline{R}^k}^{\infty}xdF-\lambda\int_{-\infty}^{\underline{R}^k}\frac{R^k}{1+\lambda}dF\\ &-\lambda(L-1)\int_{\underline{R}^k}^{\infty}R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial L}dF-\lambda(L-1)\int_{\underline{R}^k}^{\infty}R\left(\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial R}+x\right)\frac{\partial R}{\partial L}dF,\end{aligned}$$

The first-order condition of the bank's problem is $\partial\mathbb{E}(\pi)/\partial L = 0$, where

$$\frac{\partial\mathbb{E}(\pi)}{\partial L} = \int_{R^{k*}}^{\infty}(R^k-R)dF-\lambda\left(L-1\right)\int_{R^{k*}}^{\infty}R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial L}dF-R\lambda\int_{R^{k*}}^{\infty}xdF.$$

Then, $\partial\text{SW}/\partial L$ evaluated at the competitive equilibrium is given by

$$\begin{aligned}\left.\frac{\partial\text{SW}}{\partial L}\right|_{\text{CE}} =& \left.\frac{\partial\text{SW}}{\partial L}\right|_{\text{CE}}-\left.\frac{\partial\mathbb{E}(\pi)}{\partial L}\right|_{\text{CE}}\\ =& \int_{\underline{R}^k}^{R^{k*}}R^kdF+\frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k}R^kdF-R\mathbb{E}(v|\text{default})P-\lambda R\int_{\underline{R}^k}^{R^{k*}}xdF\\ &-\lambda(L-1)\left[\int_{\underline{R}^k}^{R^*}R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial L}dF+\int_{\underline{R}^k}^{\infty}\left(R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial R}+x\right)\frac{\partial R}{\partial L}\right]\end{aligned}$$

Because the recovery rate v is given by equation (12), $R\mathbb{E}(v|\text{default})P$ is given by:

$$R\mathbb{E}(v|\text{default})P = \int_{\underline{R}^k}^{R^{k*}}\left(R^k\frac{L}{L-1}-\lambda Rx\right)dF+\frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k}R^k\frac{L}{L-1}dF.$$

Then, the first-order condition of the regulator's problem, evaluated at the competitive equilibrium, is written as:

$$\begin{aligned}\left.\frac{\partial\text{SW}}{\partial L}\right|_{\text{CE}} \propto& -\frac{1}{L-1}\left[\int_{\underline{R}^k}^{R^{k*}}R^kdF+\frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k}R^kdF\right]\\ &-\lambda(L-1)\left[\int_{\underline{R}^k}^{R^*}R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial L}dF+\int_{\underline{R}^k}^{\infty}\left(R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial R}+x\right)\frac{\partial R}{\partial L}\right].\end{aligned}$$

This completes the derivation of (15).

Derivation of $\partial\bar{s}^*/\partial L$ and $\partial\bar{s}^*/\partial R$ in the benchmark model. Totally differentiating equations (27) and (28) with respect to R, $\bar{s}^*$ and R^{k*} yields:

$$\begin{aligned}
dR^{k*} &= \frac{\sigma_k^2}{\sigma_\epsilon^2+\sigma_k^2}d\bar{s}^*,\\
dR^{k*} &= \left(1-\frac{1}{L}\right)(1+\lambda x)dR + R\left(1-\frac{1}{L}\right)\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}(d\bar{s}^*-dR^{k*}).
\end{aligned}$$

Also, totally differentiating equation (28) with respect to L, $\bar{s}^*$ and R^{k*} yields:

$$dR^{k*} = \frac{R}{L^2}(1+\lambda x)dL + R\left(1-\frac{1}{L}\right)\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}(d\bar{s}^*-dR^{k*}).$$

Rearranging these equations leads to:

$$\begin{aligned}
\frac{\partial\bar{s}^*}{\partial R} &= \frac{\left(1+\frac{\sigma_\epsilon^2}{\sigma_k^2}\right)\left(1-\frac{1}{L}\right)(1+\lambda x)}{1-\frac{\sigma_\epsilon}{\sigma_k^2}R\left(1-\frac{1}{L}\right)\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)} > 0,\\
\frac{\partial\bar{s}^*}{\partial L} &= \frac{\left(1+\frac{\sigma_\epsilon^2}{\sigma_k^2}\right)\frac{R}{L^2}(1+\lambda x)}{1-\frac{\sigma_\epsilon}{\sigma_k^2}R\left(1-\frac{1}{L}\right)\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)} > 0.
\end{aligned}$$

The sign of these derivatives is positive because the denominator, which is identical between the two, is positive for the threshold $\bar{s}^*$ to uniquely exist.

The slope of the supply curve (1). The supply curve (1) is written in terms of leverage as:

$$R(1-P) + \int_{\underline{R}^k}^{R^{k*}}\left(R^k\frac{L}{L-1}-R\lambda x\right)dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k}R^k\frac{L}{L-1}dF = u'(y-(L-1)n),$$

Totally differentiating the equation with respect to R and L yields

$$\begin{aligned}
&\left\{1-P-\lambda\int_{\underline{R}^k}^{R^{k*}}\left[R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial R}+x\right]dF\right\}dR\\
=&\left\{\int_{\underline{R}^k}^{R^{k*}}\left[\frac{R^k}{(L-1)^2}+R\lambda\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial L}\right]dF + \frac{1}{1+\lambda}\int_{-\infty}^{R^{k*}}\frac{R^k}{(L-1)^2}dF - u''(c_1)n\right\}dL.
\end{aligned}$$

Then, the slope of the supply curve is given by:

$$\frac{dR}{dL} = \frac{\int_{\underline{R}^k}^{R^{k*}}\left[\frac{R^k}{(L-1)^2}+R\lambda\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial L}\right]dF + \frac{1}{1+\lambda}\int_{-\infty}^{R^{k*}}\frac{R^k}{(L-1)^2}dF - u''(c_1)n}{1-P-\lambda\int_{\underline{R}^k}^{R^{k*}}\left[R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial R}+x\right]dF}. \tag{29}$$

The numerator of (29) is positive. Hence, the slope of the supply curve is positive if and only if the denominator is positive.

Banks' problem without bank risk shifting motives. Banks choose leverage L to maximize

$$\int_{R^{k*}}^{\infty} \left\{ R^k L - R\left[1 + \lambda x(R^k, \bar{s}^*(L,R))\right](L-1) \right\} n dF(R^k)$$

subject to the technical constraint $L \leq L_{\max}$ and the households' participation constraint (16), which is reproduced here for convenience:

$$R[1 - F(R^{k*})] + \int_{\underline{R}^k}^{R^{k*}} \left[R^k \frac{L}{L-1} - R\lambda x(R^k, \bar{s}^*) \right] dF + \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} \frac{L}{L-1} dF \geq R^e.$$

As in the benchmark problem, we restrict our attention to a case in which the technical constraint is non-binding. It is obvious that the households' participation constraint is binding. The binding constraint implicitly defines the interest rate as a function of leverage: $R = R_B(L)$. The slope of this curve is derived in a similar manner as in the supply curve and given by

$$\frac{dR_B}{dL} = \frac{\int_{\underline{R}^k}^{R^{k*}} \left[\frac{R^k}{(L-1)^2} + R\lambda \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} \right] dF + \frac{1}{1+\lambda} \int_{-\infty}^{R^{k*}} \frac{R^k}{(L-1)^2} dF}{1 - P - \lambda \int_{\underline{R}^k}^{R^{k*}} \left[R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} + x \right] dF} \tag{30}$$

Compared to the slope of the supply curve, (29), the only difference in the slope of R_B is the absence of $-u''(c_1)n$ in the numerator.

Substituting $R = R_B(L)$ into the banks' objective function, the first-order condition with respect to L is written as

$$\begin{aligned} 0 = \frac{\partial \mathbb{E}(\pi)}{\partial L} &= \int_{R^{k*}}^{\infty} R^k dF - (1-P)R - \lambda R (L-1) \int_{R^{k*}}^{\infty} \frac{\partial x}{\partial \bar{s}^*} \left(\frac{\partial \bar{s}^*}{\partial L} + \frac{\partial \bar{s}^*}{\partial R} \frac{\partial R_B}{\partial L} \right) dF \\ &\quad - \lambda R \int_{R^{k*}}^{\infty} x dF - \frac{\partial R_B}{\partial L} (L-1) \int_{R^{k*}}^{\infty} (1 + \lambda x) dF. \end{aligned}$$

Derivation of equation (17). The slope of the social welfare function is given by equation (14), which is reproduced here for convenience:

$$\begin{aligned} \frac{\partial SW}{\partial L} &= -R\left[1 - P + \mathbb{E}(v|\text{default})P\right] + \mathbb{E}(R^k) - \lambda R \int_{\underline{R}^k}^{\infty} x dF - \lambda \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF \\ &\quad - \lambda R(L-1)R \int_{\underline{R}^k}^{\infty} \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} dF - \lambda (L-1) \int_{\underline{R}^k}^{\infty} \left(\frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} R + x \right) \frac{\partial R}{\partial L} dF, \end{aligned}$$

where constant proportional term n is omitted for simplifying notations. Let CE' denote the competitive equilibrium without bank risk shifting motives. Then, the slope of the social welfare evaluated at this competitive equilibrium is given by:

$$
\begin{aligned}
\left.\frac{\partial SW}{\partial L}\right|_{\mathrm{CE}'} =& \left.\frac{\partial SW}{\partial L}\right|_{\mathrm{CE}'} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial L}\right|_{\mathrm{CE}'} \\
=& -\frac{1}{L-1}\left[\int_{\underline{R}^k}^{R^{k*}} R^k dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k} R^k dF\right] \\
& -\lambda(L-1)\left[\int_{\underline{R}^k}^{R^*} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}dF + \int_{\underline{R}^k}^{\infty}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}+x\right)\frac{\partial R}{\partial L}dF\right] \\
& + \frac{\partial R_B}{\partial L}\left[\int_{R^{k*}}^{\infty}\left(\lambda\frac{\partial \bar{s}^*}{\partial R}+1+\lambda x+\lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}\right)dF\right](L-1) \\
=& -\int_{\underline{R}^k}^{R^{k*}}\left[\frac{R^k}{L-1}+R\lambda(L-1)\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right]dF + \frac{1}{1+\lambda}\int_{-\infty}^{R^{k*}}\frac{R^k}{L-1}dF \\
& + \frac{\partial R_B}{\partial L}\left[1-P-\lambda\int_{\underline{R}^k}^{R^{k*}}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}+x\right)dF\right](L-1) \\
& -\lambda(L-1)\Delta R\int_{\underline{R}^k}^{R^{k*}}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}+x\right)dF
\end{aligned}
$$

where $\Delta R \equiv \partial R/\partial L - \partial R_B/\partial L \propto -u'' > 0$. Substituting $\partial R_B/\partial L$ out using (30) yields:

$$
\left.\frac{\partial SW}{\partial L}\right|_{\mathrm{CE}'} = \lambda(L-1)\left[\int_{\underline{R}^k}^{R^{k*}}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}+x\right)dF\right]u''(c_1) < 0.
$$

Derivation of $\partial\bar{s}^*/\partial L$, $\partial\bar{s}^*/\partial m$, $\partial\bar{s}^*/\partial R$ and condition (22) in Section 4. In the model of leverage and liquidity, the thresholds $\bar{s}^*$ and R^{k*} are characterized by equations (5) and (19). Equation (19) is written as:

$$
R^{k*} = \frac{R-m}{\frac{L}{L-1}-m}\left[1+\lambda\frac{\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)R-m}{R-m}\right], \tag{31}
$$

Totally differentiating equations (5) with respect to R^{k*} and $\bar{s}^*$ yields

$$
dR^{k*} = \frac{1}{\frac{\sigma_\epsilon^2}{\sigma_k^2}+1}d\bar{s}^*.
$$

Totally differentiating equation (31) with respect to R^{k*}, $\bar{s}^*$ and L yields

$$
dR^{k*} = \frac{1}{[L-m(L-1)]^2}\left(1+\lambda\frac{xR-m}{R-m}\right)dL + \frac{\lambda R x'}{\frac{L}{L-1}-m}\frac{d\bar{s}^*-dR^{k*}}{\sigma_\epsilon},
$$

where $x' \equiv \phi((\bar{s}^* - R^{k*})/\sigma_\epsilon)$. Then, $d\bar{s}^*/dL$ is given by

$$\frac{d\bar{s}^*}{dL} = \frac{\frac{\sigma_\epsilon^2/\sigma_k^2+1}{[L-m(L-1)]^2}\left(1+\lambda\frac{xR-m}{R-m}\right)}{1-\frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R x'}{\frac{L}{L-1}-m}} > 0.$$

Note that the denominator is positive for the model to have a unique solution for $\bar{s}^*$ and R^{k*}. Next, totally differentiating equation (31) with respect to R^{k*}, $\bar{s}^*$ and m yields

$$dR^{k*} = \frac{-(1+\lambda)\frac{L}{L-1}+(1+\lambda x)R}{[L/(L-1)-m]^2}dm + \frac{\lambda R x'}{\frac{L}{L-1}-m}\frac{d\bar{s}^* - dR^{k*}}{\sigma_\epsilon},$$

Then, $d\bar{s}^*/dm$ is given by

$$\frac{d\bar{s}^*}{dm} = \frac{\frac{\sigma_\epsilon^2/\sigma_k^2+1}{[L/(L-1)-m]^2}\left[-(1+\lambda)\frac{L}{L-1}+(1+\lambda x)R\right]}{1-\frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R x'}{\frac{L}{L-1}-m}}.$$

Hence, $d\bar{s}^*/dm < 0$ if the interest rate is low enough to satisfy condition (22):

$$R < \frac{1+\lambda}{1+\lambda x}\frac{L}{L-1}.$$

Finally, totally differentiating equation (31) with respect to R^{k*}, $\bar{s}^*$ and R yields

$$dR^{k*} = \frac{1}{\frac{L}{L-1}-m}(1+\lambda x)dR + \frac{\lambda R x'}{\frac{L}{L-1}-m}\left(d\bar{s}^* - dR^{k*}\right).$$

Then, $d\bar{s}^*/dR$ is given by

$$\frac{d\bar{s}^*}{dR} = \frac{\frac{\sigma_\epsilon^2/\sigma_k^2+1}{L/(L-1)-m}(1+\lambda x)}{1-\frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R x'}{\frac{L}{L-1}-m}} > 0.$$

Derivation of $\partial R/\partial L$ and $\partial R/\partial m$ in Section 4. Using the recovery rate in the model with liquidity, the supply curve of funds (1) is written as

$$\begin{aligned} u'(y-(L-1)n) = R(1-P) + &\int_{\underline{R}^k}^{R^{k*}} \left[R^k\left(\frac{L}{L-1}-m\right)+m-\lambda(Rx-m)\right]dF \\ &+ \int_{-\infty}^{\underline{R}^k}\left[\frac{R^k}{1+\lambda}\left(\frac{L}{L-1}-m\right)+m\right]dF. \end{aligned}$$

Totally differentiating this equation with respect to L and R yields

$$\begin{aligned} -u''(c_1)ndL = &\left[1-P-\lambda\left(\int_{\underline{R}^k}^{R^{k*}} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}+x\right)dF\right]dR \\ &-\left[\int_{\underline{R}^k}^{R^{k*}}\left(\frac{R^k}{(L-1)^2}+\lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right)dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k}\frac{R^k}{(L-1)^2}dF\right]dL \end{aligned}$$

Similarly, totally differentiating it with respect to R and m yields

$$
\begin{aligned}
0 = &\left[1 - P - \lambda\left(\int_{\underline{R}^k}^{R^{k*}} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right) dF\right] dR \\
&+ \left[\int_{\underline{R}^k}^{R^{k*}} \left(-R^k + 1 + \lambda - \lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m}\right) dF + \int_{-\infty}^{\underline{R}^k} \left(-\frac{R^k}{1+\lambda} + 1\right) dF\right] dm.
\end{aligned}
$$

Hence, $\partial R/\partial L$ and $\partial R/dm$ are given by

$$
\begin{aligned}
\frac{\partial R}{\partial L} =& \frac{\int_{\underline{R}^k}^{R^{k*}} \left(\frac{R^k}{(L-1)^2} + \lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right) dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k} \frac{R^k}{(L-1)^2} dF - u''(c_1)n}{1 - P - \lambda\int_{\underline{R}^k}^{R^{k*}} \left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right) dF}, \\
\frac{\partial R}{\partial m} =& \frac{\int_{\underline{R}^k}^{R^{k*}} \left[\lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m} - (1 + \lambda - R^k)\right] dF - \int_{-\infty}^{\underline{R}^k} \left(1 - \frac{R^k}{1+\lambda}\right) dF}{1 - P - \lambda\int_{\underline{R}^k}^{R^{k*}} \left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right) dF}
\end{aligned}
$$

The numerator of the equation for $\partial R/\partial L$ is positive. Hence, the slope of the supply curve is positive, i.e. $\partial R/\partial L$, if and only if

$$
1 - P - \lambda\int_{-\infty}^{R^{k*}} \left[x + R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}\right] dF(R^k) > 0.
$$

The numerator of the equation for $\partial R/\partial m$ is negative under the assumptions of (22) and (23). If the slope of the supply curve is positive, the slope of the interest rate curve with respect to liquidity is negative, i.e. $\partial R/\partial m < 0$.

Proof of Proposition 5. As provided in Section 4, the first-order condition of the regulator's problem with respect to leverage is given by:

$$
\begin{aligned}
0 = \frac{\partial SW}{\partial L} =& - R\left[1 - P + P\mathbb{E}(v|\text{default})\right] + \int_{\underline{R}^k}^{\infty} \left[R^k - (R^k - 1)m\right] dF \\
&- \lambda\int_{\underline{R}^k}^{\bar{R}^k} \left[(xR - m) + R(L-1)\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} + (L-1)\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L}\right] dF \\
&+ \int_{-\infty}^{\underline{R}^k} \left[\frac{R^k}{1+\lambda} - \left(\frac{R^k}{1+\lambda} - 1\right) m\right] dF.
\end{aligned}
$$

where $RP\mathbb{E}(v|\text{default})$ is given by:

$$
\begin{aligned}
RP\mathbb{E}(v|\text{default}) =& \int_{\underline{R}^k}^{R^{k*}} \left[R^k\left(\frac{L}{L-1} - m\right) + m - \lambda(Rx - m)\right] dF \\
&+ \int_{-\infty}^{\underline{R}^k} \left[\frac{1}{1+\lambda}R^k\left(\frac{L}{L-1} - m\right) + m\right] dF.
\end{aligned}
$$

On the other hand, as provided in the main text, the first-order condition of the banks' problem with respect to leverage is given by

$$0=\frac{\partial\mathbb{E}(\pi)}{\partial L}=\int_{R^{k*}}^{\infty}[R^k-(R^k-1)m-R]dF-\lambda\int_{R^{k*}}^{\bar{R}^k}\left[(xR-m)+R(L-1)\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial L}\right]dF.$$

Hence, the slope of the social welfare, evaluated at the banks' privately optimal choice of leverage $L=L^*$, is given by:

$$\begin{aligned}\left.\frac{\partial SW}{\partial L}\right|_{L=L^*}=&-\frac{1}{L-1}\left[\int_{\underline{R}^k}^{R^{k*}}R^kdF+\int_{-\infty}^{\underline{R}^k}\frac{R^k}{1+\lambda}dF\right]\\&-\lambda(L-1)\left[\int_{\underline{R}^k}^{R^{k*}}R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial L}dF+\int_{\underline{R}^k}^{\bar{R}^k}\left(R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial R}+x\right)\frac{\partial R}{\partial L}dF\right].\end{aligned}$$

Hence, under the assumption of the upward-sloping supply curve, $\partial R/\partial L>0$, the sign of $\partial\text{SW}/\partial L|_{L=L^*}$ is negative. This completes the proof of Proposition 5.

Proof of Proposition 6. When banks do not have risk shifting motives, they maximize profits subject to the households' participation constraint, $R[1-P+\mathbb{E}(v|\text{default})]\geq R^e$ for some R^e, and the technical constraint $L\leq L_{\max}$. Given R^e, the households' participation constraint implicitly defines the interest rate as a function of leverage and liquidity, $R=R_B(L,m)$. In particular, the derivatives with respect to L and m respectively are given by

$$\begin{aligned}\frac{\partial R_B}{\partial L}=&\frac{\int_{\underline{R}^k}^{R^{k*}}\left(\frac{R^k}{(L-1)^2}+\lambda R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial L}\right)dF+\frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k}\frac{R^k}{(L-1)^2}dF}{1-P-\lambda\int_{\underline{R}^k}^{R^{k*}}\left(R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial R}+x\right)dF},\\\frac{\partial R_B}{\partial m}=&\frac{\partial R}{\partial m}=\frac{\int_{\underline{R}^k}^{R^{k*}}\left[\lambda R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial m}-(1+\lambda-R^k)\right]dF-\int_{-\infty}^{\underline{R}^k}\left(1-\frac{R^k}{1+\lambda}\right)dF}{1-P-\lambda\int_{\underline{R}^k}^{R^{k*}}\left(R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial R}+x\right)dF}\end{aligned}$$

Taking into account $R=R_B(L,m)$, the first-order condition of the banks' problem with respect to m is given by

$$\begin{aligned}0=&-\int_{R^{k*}}^{\infty}(R^k-1)dF(R^k)+\lambda\int_{R^{k*}}^{\bar{R}^k}\left(1-R\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial m}\right)dF(R^k)\\&-(1-P)\frac{\partial R_B}{\partial m}-\lambda\frac{\partial R_B}{\partial m}\int_{R^{k*}}^{\bar{R}^k}\left(x+R\frac{\partial x}{\partial\bar{s}^*}\frac{\bar{s}^*}{\partial R}\right)dF(R^k).\end{aligned}$$

The last two terms in the right-hand-side of the equation correspond to those related to the effect of liquidity on the interest rate. Because the sign of these terms are positive, banks which have no risk shifting motives have higher liquidity holding than otherwise would be the case. Evaluating the first-order condition of the regulator's problem with respect to liquidity at the competitive equilibrium level of liquidity $m = m^*$ yields:

$$\begin{aligned}\left.\frac{\partial SW}{\partial m}\right|_{m=m^*} &= \frac{\partial R_B}{\partial m}\left[1 - P - \lambda\int_{\underline{R}^k}^{R^{k*}}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m} + x\right)dF\right] \\ &- \left\{\int_{\underline{R}^k}^{R^{k*}}\left[\lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m} - (1 + \lambda - R^k)\right]dF - \int_{-\infty}^{\underline{R}^k}\left(1 - \frac{R^k}{1+\lambda}\right)dF\right\} \\ &= 0.\end{aligned}$$

The final equality was derived by using the expression for $\partial R_B/\partial m$. The first-order condition of the regulator's problem, evaluated at the competitive equilibrium level of leverage, can be derived similarly to the benchmark model. This completes the proof of Proposition 6.

Calibration: the extended model with bank leverage and liquidity. The unit of time is annual. The calibration strategy is to set target values for endogenous variables L, m, R and P and pin down parameter values for σ_ϵ, γ, λ and y jointly. The four parameters, σ_ϵ, γ, λ and y, are set as follows. The probability of bank default is given by $P = \Phi((R^{k*} - \mu)/\sigma_k)$, so that the threshold R^{k*} is given by $R^{k*} = \mu + \sigma_k\Phi^{-1}(P)$. Condition (5) is arranged as

$$\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon} = \frac{\sigma_\epsilon}{\sigma_k^2}(R^{k*} - \mu) - \sqrt{1 + \frac{\sigma_\epsilon^2}{\sigma_k^2}}\Phi^{-1}(\gamma).$$

Also, condition (19) is arranged as

$$\lambda = \left[R^{k*}\frac{\frac{L}{L-1} - m}{R - m} - 1\right]\frac{R - m}{\Phi\left(\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon}\right)R - m} \equiv \lambda(\sigma_\epsilon, \gamma),$$

where the equation for $(\bar{s}^* - R^{k*})/\sigma_\epsilon$ was used in deriving the final equivalence. The first-order conditions (20) and (21) are written as

$$\begin{aligned}0 &= \int_{R^{k*}}^{\infty}[R^k - (R^k - 1)m - R]dF(R^k) - \int_{R^{k*}}^{\bar{R}^k}\left[\lambda(Rx - m) + (L-1)\lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right]dF(R^k), \\ 0 &= -\int_{R^{k*}}^{\infty}(R^k - 1)dF(R^k) + \lambda\int_{R^{k*}}^{\bar{R}^k}\left(1 - R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m}\right)dF(R^k),\end{aligned}$$

where

$$
\begin{aligned}
\mu =& \bar{s}^* - \sigma_\epsilon \Phi^{-1}\left(\frac{m}{R}\right), \\
\frac{\partial x}{\partial \bar{s}^*} =& \phi\left(\frac{\bar{s}^* - R^k}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}, \\
\frac{\partial \bar{s}^*}{\partial L} =& \frac{\frac{\sigma_\epsilon^2/\sigma_k^2+1}{[L-m(L-1)]^2}\left(1+\lambda\frac{xR-m}{R-m}\right)}{1-\frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R}{\frac{L}{L-1}-m}\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)}, \\
\frac{\partial \bar{s}^*}{\partial m} =& \frac{\frac{\sigma_\epsilon^2/\sigma_k^2+1}{[L/(L-1)-m]^2}\left[-(1+\lambda)\frac{L}{L-1}+(1+\lambda x)R\right]}{1-\frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R}{\frac{L}{L-1}-m}\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)}.
\end{aligned}
$$

These two equations are solved for σ_ϵ and γ. In solving the simultaneous equations, σ_ϵ and γ have to satisfy conditions (22) and (23). Also, these parameters have to be such that the denominator of $\partial\bar{s}^*/\partial L$ is positive. With σ_ϵ and γ at hand, parameter λ is determined. Finally, y is set to satisfy equation (1), i.e.,

$$
y = (L-1)\,n + \frac{1}{[R\,(1-P+\mathbb{E}(v|\text{default})P)]^\alpha},
$$

where $\mathbb{E}(v|\text{default})P$ is given by

$$
R\mathbb{E}(v|\text{default})P = \int_{\underline{R}^k}^{R^{k*}}\left[R^k\left(\frac{L}{L-1}-m\right)+m-\lambda(Rx-m)\right]dF + \int_{-\infty}^{\underline{R}^k}\left[\frac{1}{1+\lambda}R^k\left(\frac{L}{L-1}-m\right)+m\right]dF.
$$