Network Uncertainty and Interbank Markets

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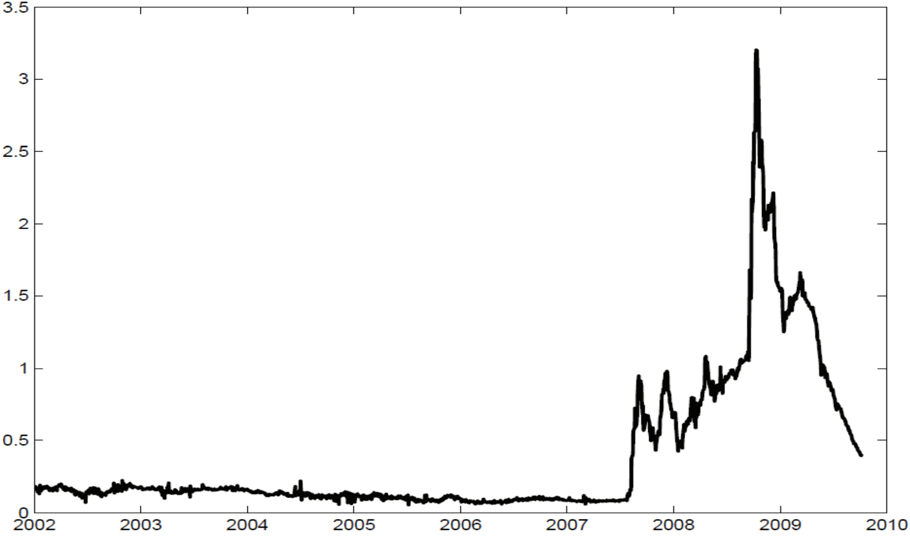
Outline

Stylized Facts from 2007-09 Financial Crisis-I: LIBOR-OIS Spread



"Stylized" Facts from 2007-09 Financial Crisis-II

1. Interbank credit spreads are usually small, and interbank risk does not appear to matter much.
2. As conditions deteriorate, it can matter much more.
   - Post-failures of Lehmann and AIG there were suggestions that uncertainy about who was directly or indirectly connected contributed to higher spreads and "stressed funding markets".
   - Uncertainty about solvency also manifested itself in funding runs. **Example**: Bear Stearns and Lehmann Brothers both experienced runs shortly before failing.
3. Domino Asset-Side Contagion was unimportant.
   - No cascading bank-defaults due to banks' asset-side interconnections during the crisis.
   - Difficult to generate cascading asset-side defaults in plausibly calibrated theoretical models.

Were network effects important for credit risk ?

- Domino-Side contagion was unimportant continued...
  - Glasserman and Young (2015) network effects are second order for understanding default cascades if exposures are not concentrated.
  - Helwege and Zhang (2014) conduct event studies on bank defaults and show little effect on stock prices of firms exposed to the default.
- A different approach to network contagion ala Caballero Simsek and Duffie.
  - When banks are financially weakened, network interconnections become more important as generators of credit risk.
  - When perceived credit risk is high enough it generates funding runs and potentially bank failures.
  - Need a tractable model of how network interconnections affect banks' credit risk.

This paper

- Model how banks probabilities of default are affected by:
  1. The network of interbank exposures.
  2. Uncertainty about the shape of the network.
- Two channels for bank network effects.
  1. Valuation Contagion.
  2. Diversification Effects / Covariance Contagion.
- Most closely related research.
  - Model of Valuation Contagion: Ota (2013)
  - Stylized Models of Uncertainty and Freezes:
    Caballero and Simsek (2014),Milne, Li, and Qiu (2015), Pritsker (2013), Zawadoski (2012)
- This paper's contributions:
  1. Relatively unstylized "general" model of default probabilities.
  2. Diversification/covariance effects are captured.
  3. The effects of uncertainty on spreads are modeled.
  4. Rich framework for analyzing asymmetric information about network structure.

The Main Ideas in the Paper

1. When banks are interconnected their probabilities of default are jointly determined, and difficult to jointly solve for.
2. There are reasonable models of credit risk for unconnected entities.
3. Solution: Solve for credit risk for unconnected entities and perturb these models to allow for connections.
    - When banks are not connected, they hold risk-free assets instead of interbank assets.
    - Their liabilites are treated as if only to those outside the banking system.
4. The above analysis measures credit-risk for known connections. To solve for credit risk more generally, integrate against the distribution of connections.

Main Results Preview

- Using a Merton-style baseline model for defaults, with a time horizon of T default probabilities when banks are interconnected are approximated for dates 0 and 1. Manipulating leads to a formula for how default probs when connected evolves between dates 0 and 1.

$$\begin{aligned} &PD_c(1,\omega_b) \approx PD_c(0,\omega_b) + [I-\psi]^{-1}[PD_{nc}(1,\omega_b) - PD_{nc}(0,\omega_b)] \\ &+ [I-\psi]^{-1}[PD_\sigma(1,\omega_b) - PD_\sigma(0,\omega_b)] + [I-\psi]^{-1}[PD_\mu(1,\omega_b) - PD_\mu(0,\omega_b)] \\ &+ [I-\psi]^{-1}\eta(R_b - \iota R_f) \\ &= PD_c(0) + \text{Valn Contag} + \text{Cov Contag} + 2 \text{ Approx Adjustments} \end{aligned} \tag{1}$$

- $[I-\psi]^{-1}$ is network multiplier term.
- ψ is fragility-weighted matrix of interbank portfolio weights.
- $\psi = \gamma(1)\omega_b LGD$
- $\gamma(1)$ is diagonal matrix of banks' financial fragilities at date 1.

The setting

- M banks $m = 1, \ldots M$. Time Periods $0, 1, \ldots, T$.
- Date 0: Banks choose asset and liability portfolios.
- Date T: Assets and Liabs Mature. Banks default if $A_T < L_T$.
- N non-interbank risky assets. $R_t \sim$ i.i.d.$\mathcal{N}(\mu, \Sigma_R)$.
- Risk-free assets with per period gross return R_f.
- Banks liabilibities (interbank assets) promise gross yield Y_m.
- Asset portfolios have mean per-period return $\mu_{a,m}$ and variance $\sigma^2_{a,m}$
- Date 0: Banks PDs are approximated.
- Date 1: R_1 is learned. Banks $PD's|R_1$ are approximated.

Pricing Interbank Deposits

1. Deposits are modeled as risky bullet-bonds that pay face value 1 at maturity.
2. The required net and gross returns on deposits are r_b and R_b.
3. The date t price of a deposit in bank m is

$$P_{m,d}(t) = \text{Exp}[-r_b(T - t) - LGD_m PD_m(t)]$$

4. The gross return on deposits in bank m between dates 0 and 1 depends on the innovation in m's probability of default.

$$\begin{aligned} \tilde{R}_{b,m}(1) &= \text{Exp}[r_b - LGD_m(PD_m(1) - PD_m(0))] \\ &\approx R_b + LGD_m[PD_m(0) - PD_m(1)] \end{aligned}$$

5. The value of bank $\hat{m}$'s assets at date 1 is $A_{\hat{m}}(0)R_{a,\hat{m}}(1)$. If bank $\hat{m}$ has interbank assets, then the value of its assets will depend on innovations in the probabilities of default of other banks.

Baseline Approach to Modeling Banks PDs

- Notation:
  - $A(t)$ = Assets at t, $L(t)$ = Liabilities at t. $NW(t) = A(t) - L(t)$.
  - $C(t)$ = Capital Ratio at t = $NW(t)/A(t)$
  - $PD_m(t)$ = Prob that bank m defaults at T given date t info.
  - $X_m(t)$ are additional covariates that affect default risk.

$$PD_m(t) = \frac{1}{1 + \text{Exp}\left(\alpha + \beta \frac{C_m(t)+(\mu_m - y_m)(T-t)}{\sigma_{a,m}\sqrt{T-t}} + \theta' X_m(t)\right)}$$

- Blends Merton and Jarrow-style models of default risk.
- If bank m holds interbank assets then his PD depends on theirs.
- When banks hold each others assets PD's are jointly determined through valuation contagion.
- Also $\sigma_{a,m}$ depends on interbank linkages for each bank.

Approximating Banks PDs when Interconnected

1. Bank m's portfolio weights:

$$\omega_m = (\omega_{m,R}, \omega_{m,Rf}, \omega_{m,ib})$$

2. Parameterizd with parameter ν.

$$\omega_m(\nu) = (\omega_{m,R}, \omega_{m,Rf} + (1-\nu)\sum \omega_{m,ib}, \nu\omega_{m,ib})$$

3. When $\nu = 1$ banks portfolio weights are as in 1.
4. When $\nu = 0$ banks are not interconnected.
5. The effects of interconnection on PD are approximated via a first-order Taylor series in ν centered at $\nu = 0$ and evaluated at $\nu = 1$.
   - The Taylor series produces equations for $PD_{c,m}(1)$, banks probability of default when they are interconnected.
   - The Taylor series in ν affects the return on interbank assets between dates 0 and 1, it alters $\mu_{a,m}$, and $\sigma_{a,m}$.

Modeling how linkages affect $\sigma_{a,m}$

1. $\sigma_{a,m}(\nu)$ is given by:

$$\sigma_{a,m}(\nu) = \sqrt{(\omega'_{m,R}, \nu\omega'_{m,b}) \begin{pmatrix} \Sigma_R & \Sigma_{R,b} \\ \Sigma'_{R,b} & \Sigma_b \end{pmatrix} (\omega'_{m,R}, \nu\omega'_{m,b})'} \quad (2)$$

2. When $\nu = 0$ $\sigma_{a,m}$ is the standard deviation of m' non-interbank assets.
3. The effect on $\sigma_{a,m}$ from linking with other banks when not connected depends on $\Sigma_{b,R}$, the Covariance of returns on interbank and non-interbank risky assets:

$$\left.\frac{\partial \sigma_{a,m}}{\partial \nu}\right|_{\nu=0} = \frac{\omega'_{m,R}\Sigma_{R,b}\omega_{m,b}}{\sigma_{a,m}\ |_{\nu=0}}. \quad (3)$$

Approximating $Cov(R, R_b)$

- Approximating this covariance is very challenging. It is a work in progress.
- The return on interbank assets between dates 0 and 1 conditional on ω_b and R (which is suppressed in the notation) is:

$$\tilde{R}_b(1) = R_b - LGD[PD_c(1, \omega_b) - PD_c(0, \omega_b)] \quad (4)$$

- Ideally to compute this covariance one would draw from the joint distribution of R, ω_b and numerically compute the covariance using the formulas for $PD_c(1, \omega_b) - PD_c(0, \omega_b)$. This is not quite possible because $PD_c(1, \omega_b)$ is a function of Σ_{bR}, creating a difficult fixed point problem.
- A second best solution is approximate Σ_{bR} with simulation using all terms in $PD_c(1, \omega_b) - PD_c(0, \omega_b)$ that do not involve Σ_{bR}
- A third best solution is linearize parts of $PD_c(1, \omega_b) - PD_c(0, \omega_b)$ in terms of R and compute Σ_{bR} using the approximation.

Additional notes on the approximations for $PD_c(1)$ and $PD_c(0)$

- Network multiplier effects appear in the approximation for $PD_c(1)$ because of $PD_c(1)$ depends on returns on interbank assets between dates 0 and 1 which depends on $PD_c(1)$. This is a valuation contagion effect.
- $PD_c(0)$ also appears in the approximation for $PD_c(1)$ because interbank returns depend on $PD_c(1) - PD_c(0)$.
- There are no valuation contagion effects at date 0. So, there are fewer network effects at date 0 than at date 1.

Incorporating Uncertainty About ω_b

- All of the analysis above treats the matrix of interbank portfolio weights ω_b as known.
- A more realistic case is ω_b is unknown to the public and banks only know their own first-order interbank exposures.
- Suppose ω_b is unknown, but public beliefs about ω_b conditional on information I_1 are for simplicity such that

$$\mathrm{Vec}(\omega_b) \sim \mathcal{N}[\mu(I_1), \Sigma_{\omega_b}(I_1)]$$

- I_1 includes:
  1. Noisy information on each banks total interbank assets and liabilities.
  2. Knowledge of bank regulations that limit banks exposure concentrations.
  3. Knowledge of stylized facts from public literature on banks interbank exposures.

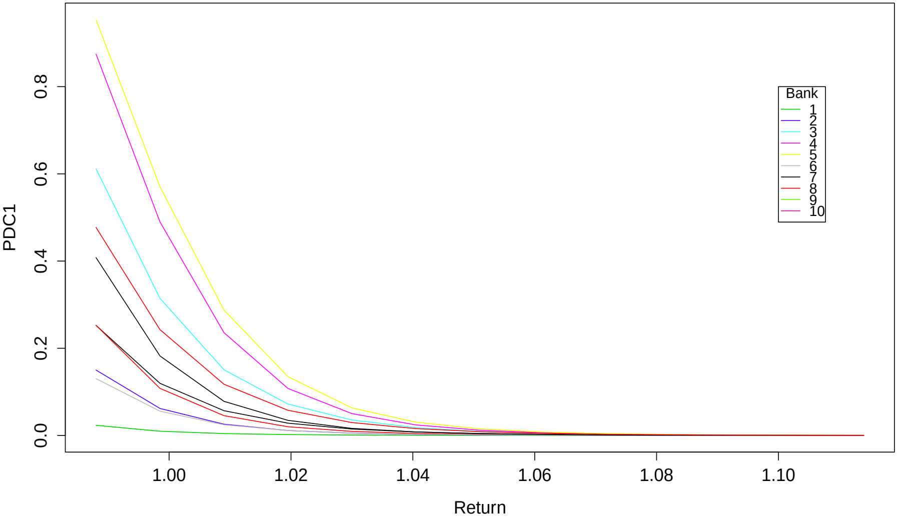
Simulation Example

- There are 5 large banks, 5 small banks.
- Core Assumptions:
  1. No bank can have a greater than 1-day exposure to another bank above 3% of its total assets. (Based on a regulation for national U.S. banks).
  2. Small banks do not receive interbank loans (Based on empirical banking literature).
  3. $Vec(\omega_{ib}) \sim \mathcal{N}(\mu_{\omega_{ib}}, \Sigma_{\omega_{ib}})$
  4. Among large banks, i's deposits with j: $\omega_{i,j} \sim N(.015, .0075^2)$
  5. For small banks lending to large banks: i's deposits with j: $\omega_{i,j} \sim N(.015, .0075^2)$
  6. Total interbank assets and liabilities of each bank are publicly reported.
- Other assumptions:
  1. Risk-free + interbank assets are 20% of the asset portfolio.
  2. Retail and interbank deposits have the same yield.
  3. Banks capital ratios are 8% at date 0.
  4. Banks capital ratios are a little low given risk of the assets.

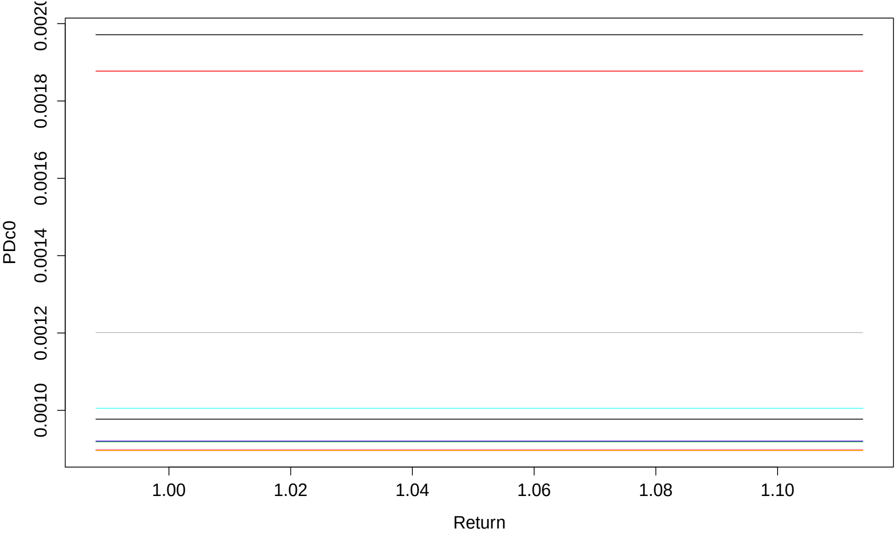
Simulation Methodology

1. Generate Banks interbank assets one-time.
2. Compute total assets and liabilities of each bank.
3. Compute posterior distribution of interbank asset holdings conditional on the public info.
4. Draw from the posterior distribution of ω_b $Nrep = 10,000$ times.
5. For each draw $\tilde{\omega}_b$ and compute $PD_c(1)(\tilde{omega}_b)$.
6. $\widehat{PD}_c(1) = \frac{1}{Nrep}\sum_{rep=1}^{Nrep} PD_{c,rep}(1)$

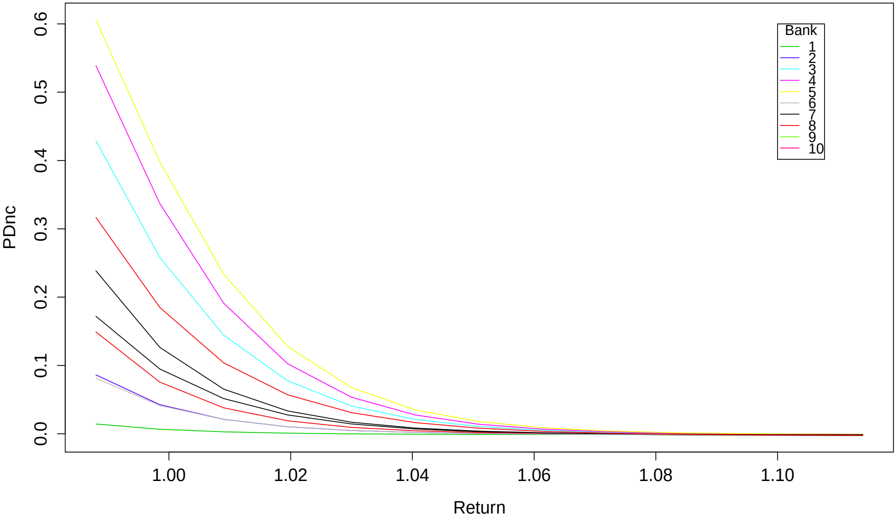
Banks Date 1 Default Probabilities as a Function of R(1)



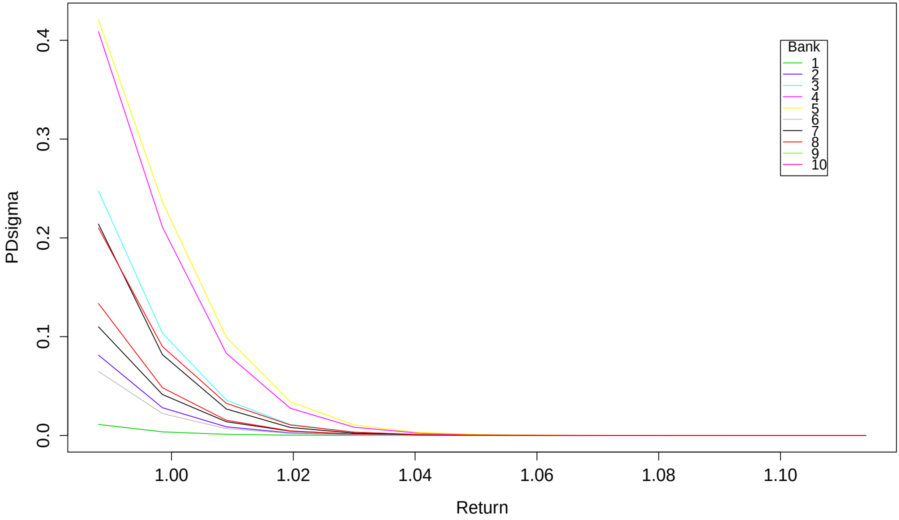
Banks Date 0 Default Probabilities as a Function of R(1)



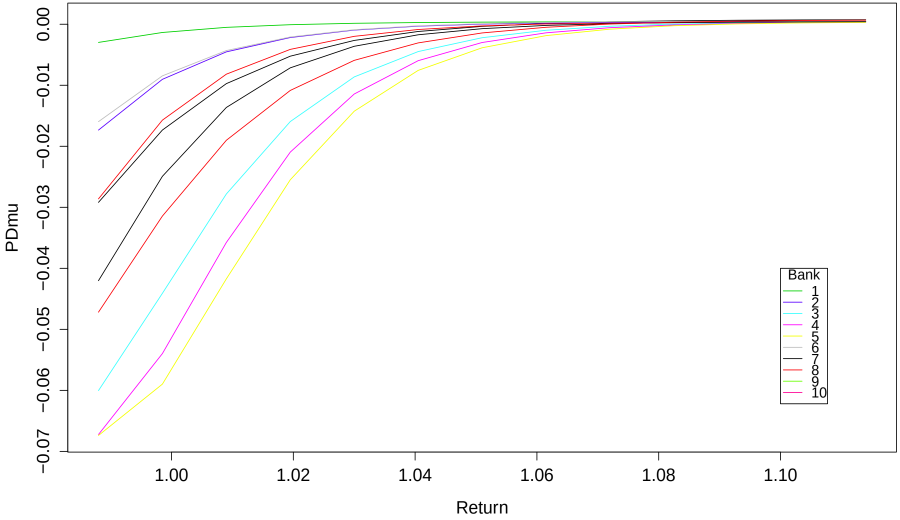
Valuation Contagion Term



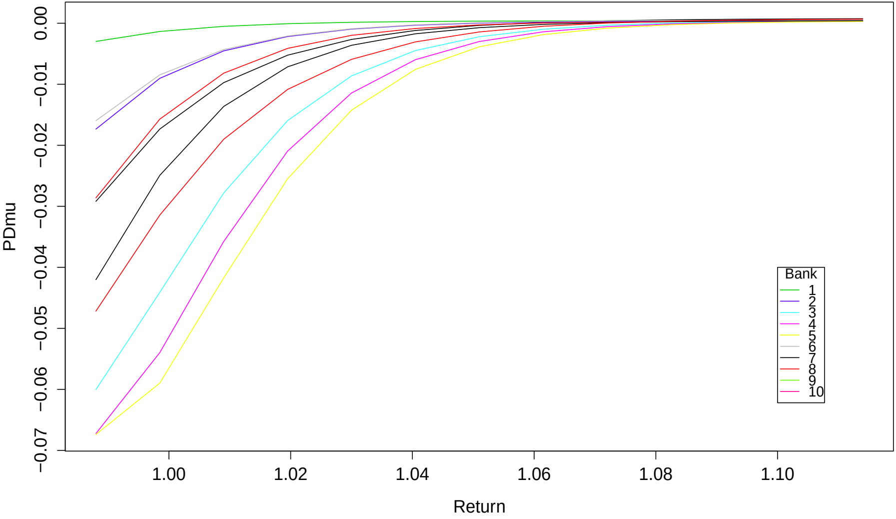
Covariance Contagion Term



Adjustment for Mean Returns Beyond Date 1



Rb - Rf Adjustment Term



Size of network effects.

- Recall banks are undercapitalized.
- I look at two cases. One in which returns are good.
- Another when they are bad. The “bad” multipliers are shockingly large.

Table 3: Inverse(I - psi) with Return Plus One Standard Deveation

1.0000004	0.0003696	0.0004169	0.0004585	0.0003366	0.0000121	0.0000122	0.0000122	0.0000122	0.0000122
0.0001555	1.0000003	0.0002358	0.0002594	0.0001940	0.0000064	0.0000065	0.0000065	0.0000065	0.0000065
0.0000080	0.0000150	1.0000000	0.0000210	0.0000128	0.0000008	0.0000008	0.0000008	0.0000008	0.0000008
0.0000840	0.0001172	0.0001321	1.0000002	0.0001077	0.0000038	0.0000038	0.0000038	0.0000038	0.0000038
0.0008197	0.0010548	0.0011402	0.0012393	1.0000007	0.0000255	0.0000257	0.0000256	0.0000256	0.0000256
0.0009851	0.0011290	0.0011824	0.0012485	0.0010922	1.0000001	0.0000321	0.0000321	0.0000321	0.0000321
0.0012901	0.0014699	0.0015456	0.0016165	0.0014110	0.0000404	1.0000001	0.0000404	0.0000404	0.0000404
0.0000781	0.0000978	0.0001069	0.0001140	0.0000927	0.0000043	0.0000043	1.0000000	0.0000043	0.0000043
0.0002754	0.0003235	0.0003471	0.0003633	0.0003094	0.0000111	0.0000112	0.0000112	1.0000000	0.0000112
0.0006029	0.0007029	0.0007494	0.0008002	0.0006721	0.0000232	0.0000233	0.0000232	0.0000232	1.0000000

Table 4: Inverse(I - psi) with Return Minus One Standard Deveation

1.003330	0.032506	0.037551	0.040499	0.028402	0.001074	0.001085	0.001084	0.001080	0.001080
0.025111	1.004242	0.038560	0.041588	0.029636	0.001042	0.001052	0.001051	0.001048	0.001048
0.001291	0.002261	1.000377	0.003123	0.001860	0.000115	0.000116	0.000116	0.000116	0.000116
0.014023	0.019230	0.022195	1.003110	0.016928	0.000629	0.000635	0.000634	0.000632	0.000632
0.068779	0.088105	0.098155	0.104815	1.006640	0.002244	0.002266	0.002264	0.002255	0.002255
0.074904	0.086877	0.094249	0.098238	0.079883	1.000390	0.002545	0.002548	0.002544	0.002543
0.078153	0.090184	0.098165	0.101507	0.082385	0.002559	1.000407	0.002568	0.002564	0.002563
0.010853	0.013582	0.015266	0.016048	0.012266	0.000583	0.000584	1.000063	0.000584	0.000584
0.037953	0.044960	0.049774	0.051486	0.040939	0.001549	0.001553	0.001554	1.000204	0.001552
0.044998	0.052964	0.058322	0.061358	0.048225	0.001760	0.001764	0.001766	0.001763	1.000240

Summary and Conclusions

- Presented a relatively tractable framework for approximating how networks affect banks PDs.
- The framework assesses how networks affect perceived PDs when the network connections are or are not publicly known.
  1. Network connections affect default risk via valuation effects and effects on the variance of banks portfolio returns.
  2. When fragility is low networks don't matter much.
  3. When fragility is high, networks have a larger multiplier effect and matter more.
- Potential application: Choosing stress-scenarios.
  1. Banks financial distress is related to their PDs.
  2. This framework helps capture how networks contribute to distress.
- Additional extensions could look at the role of information in PD risk assessemnt.

Qualifications / Challenges

1. The model relies on a benchmark model of *PDs* when firms are not connected. Such models may not exist. Or, it may be tricky to estimate or calibrate them.
2. The Merton model's simplifying assumption that defaults occur only at a particular date has undesirable properties.
3. The model assumes banks noninterbank asset holdings are publicly known. More realistically, the model should allow for uncertainty about both sets of asset holdings.
4. Parts of interbank networks are sparse. To simulate such networks from a distribution, Gaussianity is a poor choice. It would be better to simulate from a different distribution that still reflects known information about the shape of the network.