Information Acquisition and Response in Peer-Effects Networks

C. Matthew Leister Monash University

Conference on Economic Networks and Finance LSE, December 11, 2015

Idiosyncratic values/costs

- Idiosyncratic values/costs
- Strategic position

- Idiosyncratic values/costs
- Strategic position

Dual role of information:

- Idiosyncratic values/costs
- Strategic position

Dual role of information:

1. infer the state of the world,

- Idiosyncratic values/costs
- Strategic position

Dual role of information:

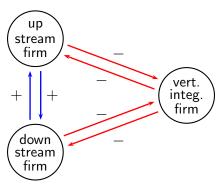
- 1. infer the state of the world,
- in equilibrium, infer the observations and subsequent actions of neighbors.

Peer-effects networks with incomplete information

$$u_i(x_1, \dots, x_N) = \underbrace{\left(a_i + \omega + \sum_{k \neq i} \sigma_{ik} x_k\right)}_{\text{marginal value to } x_i} x_i - \underbrace{\frac{1}{2} \sigma_{ii} x_i^2}_{\text{O.C. to } x_i}$$

$$u_i(x_1,\ldots,x_N) = \left(a_i + \omega + \sum_{k\neq i} \sigma_{ik} x_k\right) x_i - \frac{1}{2} \sigma_{ii} x_i^2$$

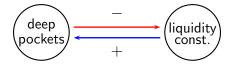
A competitive supply chain



 ω : demand for novel product x_{firm} : production

$$u_i(x_1,\ldots,x_N) = \left(a_i + \omega + \sum_{k\neq i} \sigma_{ik} x_k\right) x_i - \frac{1}{2} \sigma_{ii} x_i^2$$

Traders with heterogeneous funding constraints



 ω : long term asset value x_{trader} : market order/inventory

Basic questions

(1) How does heterogeneity in strategic positioning influence the incentives to acquire information?

Basic questions

- (1) How does heterogeneity in strategic positioning influence the incentives to acquire information?
- (2) Who over and who under acquires information? Who gains to influence others' beliefs?

Positive results

Information response game $\stackrel{EQ}{\longrightarrow}$ value to information.

Equilibrium properties:

- a. game on correlation-adjusted network (second stage),
- b. negative responses (second stage),
- c. multiple information acquisition equilibria (first stage).

Introduction Setup Equilibrium Welfare Conclusion

Welfare results

- Extent of symmetry among pair-wise peer effects drives direction of two inefficiencies:
 - a. informational externalities (network charact.: in-walks),
 - b. strategic value to information acquisition (network charact.: closed-walks).
- 2. Symmetric networks (for e.g.)
 - a. "bunching" for moderate peer effects: equilibrium information asymmetries inefficiently low,
 - b. significant strategic substitutes: acquisition of negative responders *inefficiently low*,
 - c. positive strategic distortion \propto connectedness in network.
- 3. "Antisymmetric" networks: inefficiencies reverse.

Policy implications

Transparency-based policy:

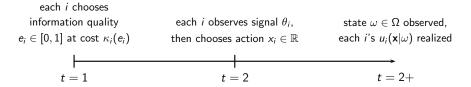
 $targeted\ certification\ of\ information\ investments.$

Introduction Setup Equilibrium Welfare Conclusion

Literature

- Network games with incomplete information:
 Calvó-Armengol & de Martí (2007,2009), Calvó-Armengol, de Martí, Prat (2015), de Martí & Zenou (2015).
- Coordination games with endogenous information:
 - Novshek & Sonnenschein (1983,1988), Vives (1988,2008), Hauk & Hurkens (2007).
 - Morris & Shin (2002), Hellwig & Veldkamp (2009), Myatt & Wallace (2012,2013), Colombo, Femminis, & Pavan (2014).
- Finance:
 - Grossman & Stiglitz (1980), Kyle (1985,1989), Babus & Kondor (2013).

Timeline of the game



Model primitives: second stage (t = 2)

• Each *i* chooses $x_i \in \mathbb{R}$, yielding *i*'s payoffs (t = 2):

$$u_i(\mathbf{x}|\omega) = \left(\omega + \rho \sum_{k \neq i} \sigma_{ik} x_k\right) x_i - \frac{1}{2} x_i^2,$$

where $\omega \in \Omega \subseteq \mathbb{R}$, $\sigma_{ij} \in \mathbb{R}$ for each i, j, and $\rho \in [0, 1]$,

Model primitives: second stage (t = 2)

• Each *i* chooses $x_i \in \mathbb{R}$, yielding *i*'s payoffs (t = 2):

$$u_i(\mathbf{x}|\omega) = \left(\omega + \rho \sum_{k \neq i} \sigma_{ik} x_k\right) x_i - \frac{1}{2} x_i^2,$$

where $\omega \in \Omega \subseteq \mathbb{R}$, $\sigma_{ij} \in \mathbb{R}$ for each i, j, and $\rho \in [0, 1]$,

• *i* observes signal $\theta_i \in \Theta \subseteq \mathbb{R}$ of quality $e_i \in [0,1]$,

Model primitives: second stage (t = 2)

• Each *i* chooses $x_i \in \mathbb{R}$, yielding *i*'s payoffs (t = 2):

$$u_i(\mathbf{x}|\omega) = \left(\omega + \rho \sum_{k \neq i} \sigma_{ik} x_k\right) x_i - \frac{1}{2} x_i^2,$$

where $\omega \in \Omega \subseteq \mathbb{R}$, $\sigma_{ij} \in \mathbb{R}$ for each i, j, and $\rho \in [0, 1]$,

- *i* observes signal $\theta_i \in \Theta \subseteq \mathbb{R}$ of quality $e_i \in [0,1]$,
 - Pure strategy: $X_i:\Theta\times[0,1]\to\mathbb{R}$.

ntroduction Setup Equilibrium Welfare Conclusion

Model primitives: second stage (t = 2)

• Each *i* chooses $x_i \in \mathbb{R}$, yielding *i*'s payoffs (t = 2):

$$u_i(\mathbf{x}|\omega) = \left(\omega + \rho \sum_{k \neq i} \sigma_{ik} x_k\right) x_i - \frac{1}{2} x_i^2,$$

where $\omega \in \Omega \subseteq \mathbb{R}$, $\sigma_{ij} \in \mathbb{R}$ for each i, j, and $\rho \in [0, 1]$,

- *i* observes signal $\theta_i \in \Theta \subseteq \mathbb{R}$ of quality $e_i \in [0,1]$,
 - Pure strategy: $X_i:\Theta\times[0,1]\to\mathbb{R}$.

Assumption 1

 $(\mathbf{I} - [s_{ij}\sigma_{ij}])^{-1}$ is well defined for every $\mathbf{s} \in [0,1]^{N(N-1)}$.

Model primitives: first stage (t = 1)

- Each i = 1, ..., N privately invests in information quality $e_i \in [0, 1]$.
- *i*'s cost of information quality $\kappa_i(\cdot) \in C^2$ satisfies $\kappa_i(0), \kappa_i'(0) = 0$, with non-decreasing $\kappa_i''(e_i) \geq 0$.

Assumption 2

For $v_0 > 0$, there exists an unique $e_i^{\dagger} \in (0,1)$ solving $v_0 e_i^{\dagger} = \kappa_i'(e_i^{\dagger})$.

Model primitives: first stage (t = 1)

- Each i = 1, ..., N privately invests in information quality $e_i \in [0, 1]$.
- *i*'s cost of information quality $\kappa_i(\cdot) \in C^2$ satisfies $\kappa_i(0), \kappa_i'(0) = 0$, with non-decreasing $\kappa_i''(e_i) \geq 0$.

Assumption 2

For $v_0>0$, there exists an unique $e_i^\dagger\in(0,1)$ solving $v_0e_i^\dagger=\kappa_i'(e_i^\dagger)$.

All conditions satisfied for normal state and signals case.

Model primitives: beliefs and expectations

• Belief: $\mu_i(\mathbf{e}_{-i})$, density function over $\mathbf{e}_{-i} \in [0,1]^{N-1}$.

Model primitives: beliefs and expectations

- Belief: $\mu_i(\mathbf{e}_{-i})$, density function over $\mathbf{e}_{-i} \in [0,1]^{N-1}$.
- Consistency: $\mu_i(\mathbf{e}_{-i}) = 1$ for t = 1 for given \mathbf{e}_{-i} , with $\mu_i(\mathbf{e}'_{-i}) = 0$ otherwise.

Model primitives: beliefs and expectations

- Belief: $\mu_i(\mathbf{e}_{-i})$, density function over $\mathbf{e}_{-i} \in [0,1]^{N-1}$.
- Consistency: $\mu_i(\mathbf{e}_{-i}) = 1$ for t = 1 for given \mathbf{e}_{-i} , with $\mu_i(\mathbf{e}'_{-i}) = 0$ otherwise.
- E1.

$$\mathbb{E}_{i} [\omega] = \mathbb{E}_{i} [\theta_{i}] = 0,$$

$$v_{0} := \mathbb{E}_{i} [\omega^{2}] = \mathbb{E}_{i} [\theta_{i}^{2} | e_{i}],$$

E2.

$$\mathbb{E}_i\left[\omega|\theta_i,e_i\right]=e_i\theta_i,$$

E3.

$$\mathbb{E}_{i}\left[\theta_{i}|\theta_{i},e_{i},e_{j}\right]=e_{i}e_{j}\theta_{i},$$

for each $e_i \in [0,1]$.

Equilibrium facts



1. Multiple IAE e^* may exist even with a unique IRE β^* for each e.

Equilibrium facts



- 1. Multiple IAE e^* may exist even with a unique IRE β^* for each e.
- 2. Significant strategic substitutes: can have $\beta_i^* < 0$.

Equilibrium facts



- 1. Multiple IAE e^* may exist even with a unique IRE β^* for each e.
- 2. Significant strategic substitutes: can have $\beta_i^* < 0$.
- 3. Significant peer effects required for 1. or 2. to obtain.

ntroduction Setup **Equilibrium** Welfare Conclusion

Equilibrium facts



- 1. Multiple IAE e^* may exist even with a unique IRE β^* for each e.
- 2. Significant strategic substitutes: can have $\beta_i^* < 0$.
- 3. Significant peer effects required for 1. or 2. to obtain.

Proposition

Under Assumptions 1 and 2, there exists a $\bar{\rho} > 0$ such that for $\rho \in [0, \bar{\rho})$, a unique IAE \mathbf{e}^* with $\beta_i^* > 0$ for all i obtains.

Welfare

Welfare

For any e, giving X*:

$$\nu_{i}(\mathbf{X}^{*}|\mathbf{e}) := \mathbb{E}_{i}[u_{i}(\mathbf{X}^{*}|\theta_{i}, e_{i}, \mu_{i}^{*})|e_{i}, \mu_{i}^{*}] - \kappa_{i}(e_{i})$$

$$\vdots$$

$$= \frac{1}{2}\nu_{0}\beta_{i}^{*2} - \kappa_{i}(e_{i}).$$

Define the utilitarian problem:

$$\max_{\mathbf{e} \in [0,1]^N} \sum_k \nu_k \left(\mathbf{X}^* | \mathbf{e} \right).$$

Define the utilitarian problem:

$$\max_{\mathbf{e} \in [0,1]^N} \sum_{k} \nu_k \left(\mathbf{X}^* | \mathbf{e} \right).$$

$$\bullet \frac{\partial}{\partial e_{i}} \sum_{k} \nu_{k} (\mathbf{X}^{*}|\mathbf{e}) \\
= \frac{\partial \nu_{i} (\mathbf{X}^{*}|\mathbf{e})}{\partial e_{i}} \Big|_{\beta_{k}^{*}, k \neq i} + \sum_{k \neq i} \frac{\partial \nu_{i} (\mathbf{X}^{*}|\mathbf{e})}{\partial \beta_{k}^{*}} \frac{\partial \beta_{k}^{*}}{\partial e_{i}} + \sum_{k \neq i} \frac{\partial \nu_{k} (\mathbf{X}^{*}|\mathbf{e})}{\partial \beta_{k}^{*}} \frac{\partial \beta_{k}^{*}}{\partial e_{i}}.$$

Define the utilitarian problem:

$$\max_{\mathbf{e} \in [0,1]^N} \sum_{k} \nu_k \left(\mathbf{X}^* | \mathbf{e} \right).$$

$$\bullet \frac{\partial}{\partial e_{i}} \sum_{k} \nu_{k} (\mathbf{X}^{*} | \mathbf{e})$$

$$= \underbrace{\left(v_{0} \frac{\beta_{i}^{*2}}{e_{i}} - \kappa'(e_{i})\right)}_{= 0 \text{ in IAE } \mathbf{e}^{*} \text{ f.o.c.}}_{= 0 \text{ in public acquisition eq. } \mathbf{e}^{pb} \text{ f.o.c.}$$

= 0 in planner's solution e^{pl} f.o.c.

Define the utilitarian problem:

$$\max_{\mathbf{e} \in [0,1]^N} \sum_{k} \nu_k \left(\mathbf{X}^* | \mathbf{e} \right).$$

$$\bullet \frac{\partial}{\partial e_{i}} \sum_{k} \nu_{k} \left(\mathbf{X}^{*} | \mathbf{e} \right) \\
= \left(v_{0} \frac{\beta_{i}^{*2}}{e_{i}} - \kappa'(e_{i}) \right) + \underbrace{v_{0} \beta_{i}^{*} \sum_{k \neq i} e_{i} \rho \sigma_{ik} e_{k} \frac{\partial}{\partial e_{i}} \beta_{k}^{*}}_{\left(\text{marginal}\right) \text{ strategic value}} \left(\underbrace{v_{0} \sum_{k \neq i} \beta_{k}^{*} \frac{\partial}{\partial e_{i}} \beta_{k}^{*}}_{\left(\text{marginal}\right) \text{ externalities}} \right)$$

Define the utilitarian problem:

$$\max_{\mathbf{e} \in [0,1]^N} \sum_k \nu_k \left(\mathbf{X}^* | \mathbf{e} \right).$$

$$\bullet \frac{\partial}{\partial e_{i}} \sum_{k} \nu_{k} (\mathbf{X}^{*} | \mathbf{e}) \\
= \left(v_{0} \frac{\beta_{i}^{*2}}{e_{i}} - \kappa'(e_{i}) \right) + \underbrace{v_{0} \beta_{i}^{*} \sum_{k \neq i} e_{i} \rho \sigma_{ik} e_{k} \frac{\partial}{\partial e_{i}} \beta_{k}^{*}}_{\xi_{i}^{*} (\mathbf{X}^{*} | \mathbf{e})} + \underbrace{v_{0} \sum_{k \neq i} \beta_{k}^{*} \frac{\partial}{\partial e_{i}} \beta_{k}^{*}}_{\xi_{i}^{*} (\mathbf{X}^{*} | \mathbf{e})}.$$

Define the utilitarian problem:

$$\max_{\mathbf{e} \in [0,1]^N} \sum_{k} \nu_k \left(\mathbf{X}^* | \mathbf{e} \right).$$

$$\bullet \frac{\partial}{\partial e_{i}} \sum_{k} \nu_{k} (\mathbf{X}^{*} | \mathbf{e}) \\
= \left(\nu_{0} \frac{\beta_{i}^{*2}}{e_{i}} - \kappa'(e_{i}) \right) + \underbrace{\nu_{0} \beta_{i}^{*} \sum_{k \neq i} e_{i} \rho \sigma_{ik} e_{k} \frac{\partial}{\partial e_{i}} \beta_{k}^{*} + \nu_{0} \sum_{k \neq i} \beta_{k}^{*} \frac{\partial}{\partial e_{i}} \beta_{k}^{*}}_{\text{(marginal) public-value}}$$

Theorem (marginal inefficiencies)

For information qualities ${f e}$, consistent beliefs ${m \mu}$ and IRE ${f X}^*$:

$$\xi_{i}^{st}(\mathbf{e}, \mathbf{X}^{*}) = 2v_{0} \frac{\beta_{i}^{*2}}{e_{i}^{*}} \mathbf{1}_{i}' \mathbf{I}_{e} \Sigma \mathbf{I}_{e} (\mathbf{I} - \mathbf{I}_{e} \Sigma \mathbf{I}_{e})^{-1} \mathbf{I}_{e} \Sigma \mathbf{I}_{e} \mathbf{1}_{i},$$

$$\xi_{i}^{ex}(\mathbf{e}, \mathbf{X}^{*}) = 2v_{0} \frac{\beta_{i}^{*}}{e_{i}^{*}} (\beta^{*} - \beta_{i}^{*} \mathbf{1}_{i})' \mathbf{I}_{e} \Sigma \mathbf{I}_{e} (\mathbf{I} - \mathbf{I}_{e} \Sigma \mathbf{I}_{e})^{-1} \mathbf{1}_{i}.$$

Theorem (marginal inefficiencies)

For information qualities ${f e}$, consistent beliefs ${m \mu}$ and IRE ${f X}^*$:

$$\xi_{i}^{st}(\mathbf{e}, \mathbf{X}^{*}) = 2v_{0} \frac{\beta_{i}^{*2}}{\mathbf{e}_{i}^{*}} \mathbf{1}_{i}' \mathbf{I}_{\mathbf{e}} \Sigma \mathbf{I}_{\mathbf{e}} (\mathbf{I} - \mathbf{I}_{\mathbf{e}} \Sigma \mathbf{I}_{\mathbf{e}})^{-1} \mathbf{I}_{\mathbf{e}} \Sigma \mathbf{I}_{\mathbf{e}} \mathbf{1}_{i},$$

$$\xi_{i}^{ex}(\mathbf{e}, \mathbf{X}^{*}) = 2v_{0} \frac{\beta_{i}^{*}}{\mathbf{e}_{i}^{*}} (\beta^{*} - \beta_{i}^{*} \mathbf{1}_{i})' \mathbf{I}_{\mathbf{e}} \Sigma \mathbf{I}_{\mathbf{e}} (\mathbf{I} - \mathbf{I}_{\mathbf{e}} \Sigma \mathbf{I}_{\mathbf{e}})^{-1} \mathbf{1}_{i}.$$

$$\xi_i^{st}\left(\mathbf{e},\mathbf{X}^*\right) \propto \mathbf{1}_i'\left(\sum_{j=1}^{\infty}([e_ie_j
ho\sigma_{ij}]_{i\neq j})^{\tau}\right)\mathbf{1}_i$$
:

summation of closed walks on $[e_i e_j \rho \sigma_{ij}]_{i \neq j}$ beginning and ending on i.

Theorem (marginal inefficiencies)

For information qualities ${f e}$, consistent beliefs ${m \mu}$ and IRE ${f X}^*$:

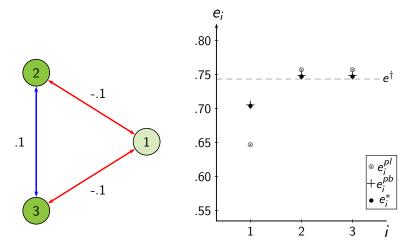
$$\xi_{i}^{st}(\mathbf{e}, \mathbf{X}^{*}) = 2v_{0} \frac{\beta_{i}^{*2}}{e_{i}^{*}} \mathbf{1}_{i}' \mathbf{I}_{e} \Sigma \mathbf{I}_{e} (\mathbf{I} - \mathbf{I}_{e} \Sigma \mathbf{I}_{e})^{-1} \mathbf{I}_{e} \Sigma \mathbf{I}_{e} \mathbf{1}_{i},$$

$$\xi_{i}^{ex}(\mathbf{e}, \mathbf{X}^{*}) = 2v_{0} \frac{\beta_{i}^{*}}{e_{i}^{*}} (\beta^{*} - \beta_{i}^{*} \mathbf{1}_{i})' \mathbf{I}_{e} \Sigma \mathbf{I}_{e} (\mathbf{I} - \mathbf{I}_{e} \Sigma \mathbf{I}_{e})^{-1} \mathbf{1}_{i}.$$

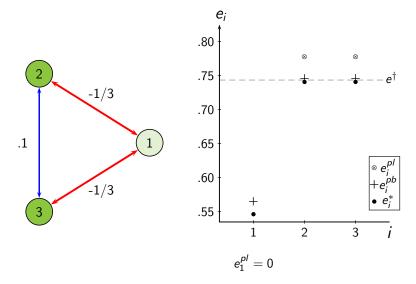
$$\xi_i^{\mathsf{ex}}\left(\mathbf{e},\mathbf{X}^*\right) \propto \left(\boldsymbol{\beta}^* - \beta_i^* \mathbf{1}_i\right)' \left(\sum_{\tau=1}^{\infty} ([e_i e_j \rho \sigma_{ij}]_{i \neq j})^{\tau}\right) \mathbf{1}_i$$
:

summation of walks on $[e_i e_j \rho \sigma_{ij}]_{i \neq j}$ beginning with j and ending on i, weighted by β_i and aggregate over $j \neq i$.

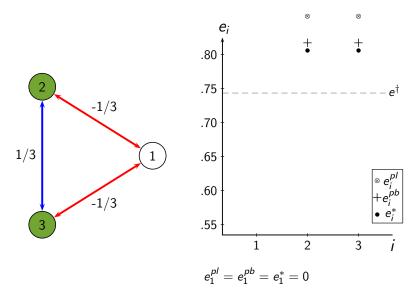
Example: three-player symmetric network, common κ



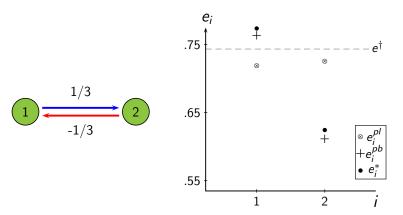
Example: three-player symmetric network, common κ



Example: three-player symmetric network, common κ



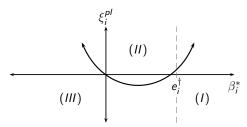
Example: two-player antisymmetric network, common κ



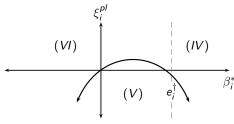
Welfare and policy design

Welfare and the neutral player

symmetric networks



antisymmetric networks





• N = 8 traders comprise non-trivial share of market.

- N = 8 traders comprise non-trivial share of market.
- x_i : i's inventory/market order (e.g. Kyle (1985)); $\bar{x} := \sum_{i=1}^8 x_i$.

- N = 8 traders comprise non-trivial share of market.
- x_i : i's inventory/market order (e.g. Kyle (1985)); $\bar{x} := \sum_{i=1}^8 x_i$.
- t=2 market price $\phi(\bar{x})=A+B\bar{x}$, B>0.

- N = 8 traders comprise non-trivial share of market.
- x_i : i's inventory/market order (e.g. Kyle (1985)); $\bar{x} := \sum_{i=1}^8 x_i$.
- t=2 market price $\phi(\bar{x})=A+B\bar{x}$, B>0.
- ullet ω : risky asset's long term value.

- N = 8 traders comprise non-trivial share of market.
- x_i : i's inventory/market order (e.g. Kyle (1985)); $\bar{x} := \sum_{i=1}^{8} x_i$.
- t = 2 market price $\phi(\bar{x}) = A + B\bar{x}$, B > 0.
- ullet ω : risky asset's long term value.
- t = 2 payoffs:

$$u_i(\mathbf{x}|\omega) = (\omega + p_i\phi(\bar{x}))x_i - x_i^2$$

$$= \left(\omega + p_iA + p_iB\sum_{k\neq i}x_k\right)x_i - (1 - p_iB)x_i^2.$$

Liquidity flush market:

• $p_i < 0$ for each unconstrained i.

Liquidity flush market:

- $p_i < 0$ for each unconstrained i.
- Market *crowding* in information acquisition.

Market efficiency in liquidity crises

Liquidity flush market:

- $p_i < 0$ for each unconstrained i.
- Market crowding in information acquisition.
- Traders set $e_i^*, \beta_i^* < e^{\dagger}$ (region (II)): *over*-acquire; over exertion in informationally inefficient markets.

Market efficiency in liquidity crises

Liquidity crises:

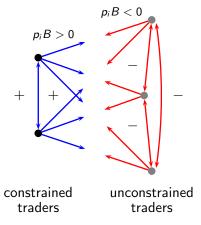
- Liquidity spirals à la Brunnermeier and Pedersen (2009)
 - ightarrow upward sloping demand.

Market efficiency in liquidity crises

Liquidity crises:

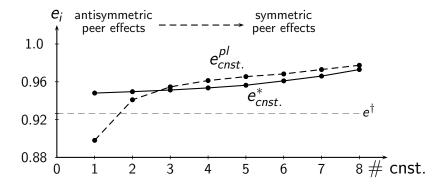
- Liquidity spirals à la Brunnermeier and Pedersen (2009)
 → upward sloping demand.
- $p_i > 0$ for liquidity-constrained trader i.

Market structure:



Liquidity crisis paradigm shift:

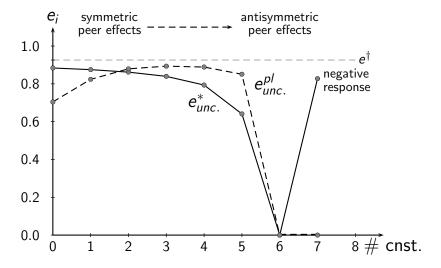
- Constrained traders set $e_i^*, \beta_i^* > e^{\dagger}$
 - 1. Flush market: antisymmetric relationships \rightarrow over-acquire.
 - 2. Crisis: symmetric relationships \rightarrow *under*-acquire.



Market efficiency in liquidity crises

Liquidity crisis paradigm shift:

- Constrained traders set $e_i^*, \beta_i^* > e^{\dagger}$
 - 1. Flush market: antisymmetric relationships \rightarrow over-acquire.
 - 2. Crisis: symmetric relationships \rightarrow *under*-acquire.
- Unconstrained traders set $e_i^*, \beta_i^* < e^{\dagger}$
 - 1. Flush market: symmetric relationships \rightarrow over-acquire.
 - 2. Crisis: antisymmetric relationships \rightarrow *under*-acquire.
 - 3. Extreme crisis: few unconstrained traders set $e_i^*, \beta_i^* < 0$.



Policy suggestion in liquidity crises

 Constrained traders impose symmetric, positive informational externalities on each other: under acquire, with positive strategic values...

Policy suggestion in liquidity crises

 Constrained traders impose symmetric, positive informational externalities on each other: under acquire, with positive strategic values...

Couple stress-tests with certification of information investments of constrained traders.

Conclusions

1. Introduce problem of costly information acquisition into new context: general network of peer effects.

- 2. Symmetric networks:
 - a. Equilibrium information inefficiently symmetric.
 - b. Players moving against their information do so too little.
 - c. Strategic values to information are positive.
- 3. Direction of welfare and strategic motives determined by network "position" and extent of symmetry in relationships: direction of inefficiencies reverse in antisymmetric networks.
- 4. Information externalities and "position": β_i^* w.r.t. e_i^{\dagger} and origin, Strategic values and "position": connectedness.

Conclusions II

- 1. Liquidity crisis *paradigm shift*: over acquisition of information in liquid markets, under acquisition in constrained markets.
- 2. Unconstrained "shorters" in crisis: inefficient.
- 3. Transparency-based policy intervention: stress test *with* information investment certification.

Equilibrium characterization

Equilibrium characterization

Theorem (t = 2 information-response equilibrium (IRE))

Under Assumption 1, for any ${\bf e}$ and consistent μ there exists a unique linear IRE of the form:

$$\mathbf{X}^* = \left[X_i^*(\theta_i | e_i) \right] = \left[\beta_i^* \theta_i \right],$$

where each β_i^* solves $\beta_i^* = e_i + \sum_{k \neq i} e_i e_k \rho \sigma_{ik} \beta_k^*$:

$$eta^* := (\mathbf{I} - [e_i e_j
ho \sigma_{ij}]_{i \neq j})^{-1} \mathbf{e}$$

$$= \sum_{\tau=0}^{\infty} ([e_i e_j
ho \sigma_{ij}]_{i \neq j})^{\tau} \mathbf{e}.$$

Equilibrium characterization

Theorem (t = 2 information-response equilibrium (IRE))

Under Assumption 1, for any e and consistent μ there exists a unique linear IRE of the form:

$$\mathbf{X}^* = [X_i^*(\theta_i|e_i)] = [\beta_i^*\theta_i],$$

where each β_i^* solves $\beta_i^* = e_i + \sum_{k \neq i} e_i e_k \rho \sigma_{ik} \beta_k^*$:

$$\beta^* := (\mathbf{I} - [e_i e_j \rho \sigma_{ij}]_{i \neq j})^{-1} \mathbf{e}$$
$$= \sum_{\tau=0}^{\infty} ([e_i e_j \rho \sigma_{ij}]_{i \neq j})^{\tau} \mathbf{e}.$$

 β_i^* : i's "informational centrality" (weighted Bonacich centrality).

Equilibrium characterization



Theorem (t = 1 information-acquisition equilibrium (IAE))

Under Assumption 1, for IRE \mathbf{X}^* and consistent beliefs $\boldsymbol{\mu}$ there exists a (generically unique*) IAE \mathbf{e}^* . For any such IAE, and \forall i with $e_i^* \in (0,1)$:

$$v_0 \frac{\beta_i^{*2}}{e_i^*} = \kappa_i'(e_i^*).$$

Equilibrium characterization



Theorem (t = 1 information-acquisition equilibrium (IAE))

Under Assumption 1, for IRE \mathbf{X}^* and consistent beliefs $\boldsymbol{\mu}$ there exists a (generically unique*) IAE \mathbf{e}^* . For any such IAE, and \forall i with $\mathbf{e}_i^* \in (0,1)$:

$$v_0\frac{\beta_i^{*2}}{e_i^*}=\kappa_i'(e_i^*).$$

