Financial Linkages, Portfolio Choice and Systemic Risk

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Abstract

Financial linkages smooth the shocks faced by individual components of the system, but they also create a wedge between ownership and decision-making. The classical intuition on the role of pooling risk in raising welfare is valid when ownership is evenly dispersed. However, when the ownership of some agents is concentrated in the hands of a few others, greater integration and diversification can lead to excessive risk taking and volatility and result in lower welfare. We also show that individuals undertake too little (too much) risk relative to the first best if the network is homogeneous (heterogeneous), and study optimal networks.

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1 Introduction

Cross-ownership linkages across corporations and banks is a prominent feature of modern economies. Such linkages have the potential to smooth the shocks and uncertainties faced by individual components of the system. But they also create a wedge between ownership on the one hand and control and decision making on the other hand. We wish to understand how such network affect volatility and welfare, and more generally what are the properties of an ideal ownership network.

To study these questions we develop a model in which a collection of agents, interconnected through financial obligations, make a portfolio choice decision. The network reflects the claims that each agent has on others: so a link from A to B specifies the claims of A on the value of B. Agents have mean-variance preferences. Every agent i can invest his endowment in a risk-free asset (with return r) or in a (distinct) risky asset i (with mean $\mu_i > r$ and variance σ_i^2).[1] The portfolio choices of agents and the network of cross-ownerships together define the distribution of individual payoffs.

We begin by deriving a summary measure that aggregates all direct and indirect claims induced by the financial cross-holdings: we refer to this as *ownership*. Thus every cross-holdings network induces a *ownership* adjacency matrix Γ. In this matrix, the entry γ_{AB} summarizes all the direct and indirect claims that A has on the economic returns of B. The entry γ_{AA} is referred to as *self-ownership* of agent A: it captures the extent to which A bears the wealth effects of his portfolio choice. Our first observation is that optimal investment in a risky asset is inversely related to self-ownership. It follows that, other things being equal, the expected value and variance are higher for agents with greater ownership of low self-ownership agents, as these are the high risk takers.

Equipped with these basic results, we turn to the effects of changes in networks. Networks with low self-ownership induce higher investments in risky assets and, therefore, exhibit a higher mean but also higher volatility in value. This means that, *a priori*, the welfare effects of changes in networks are unclear. Inspired by the literature on finance, we explore changes in networks using the concepts of *integration* and *diversification*. A network S is said to be more integrated than network S' if every link in S is weakly stronger and some are strictly stronger. A network S is more diversified than network S' if every agent in S has a more diversified profile of ownerships. We find that the effects of integration and diversification depend crucially on

[1]For example, an agent may be a bank that can invest in government bonds or finance local entrepreneurs' risky projects.

the topology of the network. Here we discuss integration; similar observations apply to the effects of diversification.

A regular network is one in which all nodes have a similar cross-ownership pattern. We show in such networks an increase in integration leads to higher aggregate utilities. However, greater integration in heterogenous networks – where ownership patterns differ widely across agents – actually lowers aggregate utilities. In symmetric networks, self-ownership is bounded from below, and this, in turn, sets an upper bound on the level of risky investment and, hence, on the costs of volatility.

In heterogenous networks by contrast integration can sharply lower the self-ownership of some agents: this raises their risky investment disproportionately. This, in turn, pushes up volatility for everyone and may lower aggregate utilities; in fact, it may decrease the utility of all agents. We illustrate this argument through an analysis of a class of networks that are empirically salient: the *core-periphery networks.* Core-periphery networks consist of a core and a periphery group of nodes. Every node in the core is connected to all other core nodes. Every peripheral node is connected all nodes in the core. Figure 1 illustrates a core-periphery network. Inter-bank networks have a core-periphery structure, and empirical work suggests that this finding holds for different definitions of financial obligations and different levels of aggregations, see Soramaki et al. (2007), Martinez-Jaramillo et al (2014), Craig and von Peter (2014). Vitali et al. (2011) report that transnational corporations form a giant bow-tie structure and that a large portion of control flows to a small tightly-knit core of financial institutions.

Our theoretical predictions on risk taking and volatility appear to be broadly consistent with recent empirical results on these networks. We show that there may be greater volatility and lower welfare with growing integration in core-periphery networks. Our key result that core banks take more risk than periphery banks is consistent with recent empirical studies on inter-bank networks, e.g.,van Lelyveld and Veld (2012) and Langfield, Liu and Ota (2014).

We then turn to a normative study of networks. Given a fixed network, we derive the socially optimal portfolio of investments. This characterization clarifies the externality generated by financial linkages: an agent focuses exclusively on his own risk exposure, whereas the collective optimum entails a trade-off between expected returns and the sum of own *and* others' variance. The general insight here is that an agent will take too much risk relative to what is collectively desirable when his ownership is concentrated in a few hands. By contrast, investment in risky assets is too low relative to what is collectively desirable when cross-ownerships are widely dispersed.

Finally, we study optimal network design. Deeper and more extensive ties smooth returns, but, by lowering self-ownership, they also raise investments in risky assets. This raises both expected returns and the variance. Our analysis of optimal networks clarifies that deepening the linkages in a symmetric way resolves the tension. Regardless of whether the planner can choose agents' risky investments, the best network is a complete network in which every agent owns exactly $1/n$ of everyone else.

We now locate the paper in the context of the literature. We build on two important strands of research. The first line of work is the research on cross-holdings and linkages (Brioschi, Buzzacchi, and Colombo (1989), Eisenberg, and Noe (2001), Fedenia, Hodder, and Triantis (1994) and the recent work of Elliott, Golub abd Jackson (2014)). The second line of work is the distinction between ownership and control; here, we draw on the long and distinguished tradition that began with the classic work of Berle and Means (1932) and on the more recent work by Fama and Jensen (1983) and Shleifer and Vishnu (1989).[2] To the best of our knowledge, the present paper is the first to study the implications of portfolio choice for financial integration and diversification and the optimal design of networks within a common framework.

An important assumption of our model is that, when studying the implication of the wedge between ownership and control in risk taking, we take the cross-ownership structure, and therefore the exposure to financial markets, as given. So one way to interpret our analysis is that we study how exogenous regulations and rigidities in the investments in stocks of other firms will affect the level of investment of a firm in a risky project that is not directly accessible to other firms. This is in line with a large literature that has focussed on situations in which not all the elements of a firms balance sheet can be chosen. In particular, Rochet (1992) reevaluates the work of Koehn and Santomero (1980) and Kim and Santomero (1988) in a model in which the bank equity capital is fixed, in the short run over which the model spans; this reflects the real distinction in the way equity capital can be altered in the short run relative to other securities.

We next turn to our model of portfolio choice. In a complete markets setting, any uncertainty on returns is washed out and only expected value matters. However, whenever access is restricted or markets are incomplete, risk matters. As stressed by Rochet (1992), "it is hard to believe that a deep understanding of the banking sector can be obtained within the set-up

[2] "The property owner who invests in a modern corporation so far surrenders his wealth to those in control of the corporation that he has exchanged the position of independent owner for one in which he may become merely recipient of the wages of capital" (Berle and Means (1932), page 355).

of complete contingent markets, essentially because of the Modigliani-Miller indeterminacy principle". This motivates a richer model of bank behavior: we build on a prominent strand of the literature that has used the portfolio model of Pyle (1971) and Hart and Jaffee (1974) to study banks. Within this framework, banks are assumed to behave as competitive portfolio managers, taking prices and yields as given and choosing their portfolio (composition of their balance sheets and liabilities) in order to maximize the expected utility of the bank's financial net worth.[3]

In the recent work on contagion in financial networks, attention has focused on the role of the distribution of shocks and the architecture of networks, see e.g., Allen and Gale (2000), Babus (2015), Farboodi (2014), Gottardi and Vega-Redondo (2011), Elliott, Golub and Jackson (2014), Elliott and Hazell (2015), Greenwood, Landier, and Thesmar (2015) and Gai and Kapadia (2010). For a survey, see Cabrales, Gale and Gottardi (2015). The distinguishing feature of our work is that the origin of the shocks – the investments in risky assets – is itself an object of individual decision making. Thus, the focus of our work is, first, on how the network of linkages shapes the level of risk taking by agents and, second, on how it spreads the rewards of the risky choices across different parts of the system. Therefore, our work on the effects of integration and diversification and on optimal network design should be seen as complementary to the existing body of work.[4]

Section 2 introduces the model. Section 3 presents our characterization of risk taking in a network, and Section 4 studies the effects of changes in networks on welfare. Section 5 studies first-best investments and Section 6 examines optimal networks. Section 7 presents our study of a model with correlations across returns of risky assets. Section 8 summarizes the main results and discusses extensions of the model. The proofs of the results are presented in the Appendix.

[3]The portfolio choice banking model has been successfully used to evaluate the effect of capital regulations on risk taking, see e.g., Koehn and Santomero (1980), Kim and Santomero (1988), Keeley and Furlong (1990), Zhou (2013) and Gersbach and Rochet (2012).

[4]In a recent paper, Belhaj and Deroian (2012) study risk taking by agents located within a network. There are two modeling differences: they assume positive correlation in returns to risky assets and bilateral output sharing with no spillovers in ownership. So, with independent assets, there are no network effects in their model. Our focus is on the effects of integration and diversification and the design of optimal networks (with weights on systemic risk). These issues are not addressed in their paper.

2 Model

There are $\mathcal{N} = \{1, ..., n\}$, $n \geq 2$ agents. Agent i has has an endowment $w_i \in \mathbb{R}$ and chooses to allocate it between a safe asset, with return $r > 0$, and a (personal) risky project i, with return z_i. We assume that z_i is normally distributed with mean $\mu_i > r$ and variance σ_i^2. For simplicity, in the basic model, we assume that the n risky projects are uncorrelated. Section 7 discusses the case of assets with correlated returns. Investments by agent i in the risky asset and the safe asset are denoted by $\beta_i \in [0, w_i]$ and $\omega_i - \beta_i$, respectively. Let $\beta = \{\beta_1, ..., \beta_n\}$ denote the profile of investments.

Agents are embedded in a network of cross-holdings; we represent the network as an $n \times n$ matrix S, with $s_{ii} = 0$, $s_{ij} \geq 0$ and $\sum_{j \in \mathcal{N}} s_{ji} < 1$ for all $i \in \mathcal{N}$. We interpret s_{ij} as the claim that agent i has on agent j's economic value V_j.

Let D be a $n \times n$ diagonal matrix, in which the i^{th} diagonal element is $1 - \sum_{j \in \mathcal{N}} s_{ji}$. Define $\Gamma = D[I - S]^{-1}$. Observe that since for every $i \in \mathcal{N}$, $\sum_{j \in \mathcal{N}} s_{ji} < 1$, it follows that we can write $\Gamma = D \sum_{k=0}^{\infty} S^k$. Therefore, the γ_{ij} cell is obtained by summing up all weighted paths from i to j in the cross-holdings network S– i.e., for every $i \neq j$,

$$\gamma_{ij} = [1 - \sum_{j \in \mathcal{N}} s_{ji}] \left[0 + s_{ij} + \sum_k s_{ik} s_{kj} + .. \right].$$

It is then natural to interpret γ_{ij} as i's *ownership* of j. Finally, note that Γ is column-stochastic, $\gamma_{ii} = 1 - \sum_{j \neq i} \gamma_{ji}$. We borrow the formulation of cross-holdings from Brioschi, Buzzacchi, and Colombo (1989), Fedenia, Hodder, and Triantis (1994), and, more recently, Elliott, Golub and Jackson (2014) and Elliott and Hazell (2015). Following Elliott, Golub and Jackson (2014) we interpret this formulation as a linear approximation of an underlying set of contracts linking financial institutions.[5]

Empirical work has highlighted the prominence of a core-periphery structure in financial networks (Bech and Atalay (2010), Afonso and Lagos (2012), McKinsey Global Institute (2014), Van Lelyveld I., and t' Veld (2012)). We present the ownership matrix for this network.

Example 1 *Ownership in the core-periphery network*

There are n_p peripheral agents and n_c central agents, $n_p + n_c = n$; i_c and i_p refer to the (generic) central and peripheral agent. A link between two central agents has strength $s_{i_c j_c} = s$, and a

[5]For every S, we can obtain a corresponding Γ; however, the converse is not always the case. For sufficient conditions on Γ that guarantee the existence of corresponding S, see Elliot, Golub and Jackson (2014).

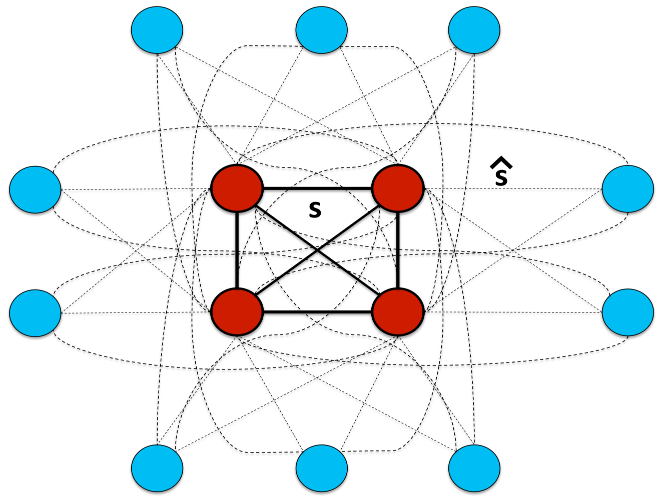

Figure 1: Core-periphery Network, $n_c = 4$, $n_p = 10$.

link between a central and a peripheral agent has strength $s_{i_c i_p} = s_{i_p i_c} = \hat{s}$, and there are no other links. Figure 1 presents such a network.

Computations presented in the Appendix show that the self-ownership of a central node i_c and a peripheral node are, respectively,

$$\begin{aligned}
\gamma_{i_c,i_c} &= \frac{[1-(n_c-1)s-n_p\hat{s}][1-(n_c-2)s-n_cn_p\hat{s}^2+n_p\hat{s}^2]}{(s+1)[1-s(n_c-1)-n_cn_p\hat{s}^2]},\\
\gamma_{i_p,i_p} &= \frac{[1-n_c\hat{s}][1-(n_c-1)s-n_c\hat{s}^2(n_p-1)]}{1-s(n_c-1)-n_cn_p\hat{s}^2}.
\end{aligned}$$

Similarly, the cross-ownerships are given by:

$$\gamma_{i_c,j_p} = \frac{[1-(n_c-1)s-n_p\hat{s}]\hat{s}}{1-s(n_c-1)-n_cn_p\hat{s}^2} \quad \text{and} \quad \gamma_{j_p,i_c} = \frac{[1-n_c\hat{s}]\hat{s}}{1-s(n_c-1)-n_cn_p\hat{s}^2},$$

$$\gamma_{i_c,j_c} = \frac{[1-(n_c-1)s-n_p\hat{s}][s+n_p\hat{s}^2]}{(s+1)(1-s(n_c-1)-n_cn_p\hat{s}^2)} \quad \text{and} \quad \gamma_{i_p,j_p} = \frac{[1-n_c\hat{s}]n_c\hat{s}^2}{1-s(n_c-1)-n_cn_p\hat{s}^2}.$$

We note that the complete network ($n_p = 0$) and the star network ($n_p = n-1$) constitute special cases of the core-periphery network. $\square$

We now define the economic value V_i. For a realization z_i and agent i's investments $(\beta_i, w_i - \beta_i)$, the returns generated by i are given by

$$W_i = \beta_i z_i + (w_i - \beta_i)r. \tag{1}$$

It then follows that the economic value of agent i is

$$V_i = \sum_j \gamma_{ij} W_j. \tag{2}$$

We now turn to the choice problem for agents. We assume that agents seek to maximize a mean-variance utility function:[6]

$$U_i(\beta_i, \beta_{-i}) = E[V_i(\beta)] - \frac{\alpha}{2} Var[V_i(\beta)].$$

Using expressions (1) and (2), we can rewrite expected utility as

$$U_i(\beta_i, \beta_{-i}) = \sum_{j \in \mathcal{N}} \gamma_{ij}[w_j r + \beta_j(\mu_j - r)] - \frac{\alpha}{2} \sum_{j \in \mathcal{N}} \gamma_{ij}^2 \beta_j^2 \sigma_j^2. \tag{3}$$

Let $\beta^* = (\beta_1, .., \beta_n)$ denote the vector of optimal choices.

3 Risk-taking in networks

We begin by characterizing optimal agent investments and then elaborate on the implications for utility and systemic risk.

Agents' utility is given by (3); observe that the cross partial derivatives with respect to investments are zero.[7] So, the optimal investment by agent i may be written as:

$$\beta_i^* = \arg \max_{\beta_i \in [0, w_i]} \gamma_{ii}[w_i r + \beta_i(\mu_i - r)] - \frac{\alpha}{2} \gamma_{ii}^2 \beta_i^2 \sigma_i^2.$$

If agent i has no cross-holdings –i.e., $s_{ij} = s_{ji} = 0$ for all $i \neq j \in \mathcal{N}$– then $\gamma_{ii} = 1$, and, therefore, the optimal investment is

$$\hat{\beta}_i = \frac{\mu_i - r}{\alpha \sigma_i^2}.$$

We shall refer to $\hat{\beta}_i$ as agent i's *autarchy* investment. With this definition in place, we state our characterization result on optimal risk taking.

[6]For a discussion of the foundations of mean-variance utility, see Gollier (2001).

[7]This means that agents' investment choices can be studied independently; this independence sets our paper apart from the literature on network games, which has been recently reviewed by Bramoulle and Kranton (2015) and Jackson and Zenou (2015).

Proposition 1 *The optimal investment of agent i is:*

$$\beta_i^* = \min\left\{w_i, \frac{\hat{\beta}_i}{\gamma_{ii}}\right\}. \tag{4}$$

In an interior solution, expected value and variance for agent i are:

$$E[V_i] = r\sum_{j\in\mathcal{N}}\gamma_{ij}w_j + \sum_{j\in\mathcal{N}}\hat{\beta}_j(\mu_j - r)\frac{\gamma_{ij}}{\gamma_{jj}} \qquad Var[V_i] = \sum_{j\in\mathcal{N}}\hat{\beta}_j^2\sigma_j^2\left(\frac{\gamma_{ij}}{\gamma_{jj}}\right)^2. \tag{5}$$

Note that for sufficiently large w_i the optimal investment if i is interior, i.e., $\beta_i^* = \hat{\beta}_i/\gamma_{ii}$. Hereafter we assume that w_i is large for every i and so optimal investments are interior. Proposition 1 yields a number of insights about the network determinants of risk taking. First, relative to autarky, cross-holdings raise agents' propensity to take risk: agent i's risk-taking investment is negatively related to his self-ownership, as captured by γ_{ii}. Thus, if two agents face similarly risky projects, $\mu_i = \mu_j$ and $\sigma_i^2 = \sigma_j^2$, then agent i invests more than agent j in the risky project if, and only if, $\gamma_{ii} < \gamma_{jj}$. This result follows from the agency problem that cross-holding networks generate: agent i optimizes the mean-variance utility of $\gamma_{ii}W_i$, and not of W_i.

The simplicity of optimal investment policy allows us to develop a relationship between networks and expected returns, volatility and correlations across agents' economic values. An inspection of the expressions $E[V_i]$ and $Var[V_i]$ reveals that agents with higher volatility and higher expected value are those with higher ownership of agents with low self-ownership, as the latter invest more in their risky project. For example, if $\sigma_i^2 = \sigma_j^2$, and $\mu_i = \mu_j$, for all i, j, then the variance of V_i is higher than the variance of V_j if, and only if,

$$\sum_{l\in\mathcal{N}}\left(\frac{\gamma_{il} - \gamma_{jl}}{\gamma_{ll}}\right)^2 > 0.$$

4 Integration and Diversification

Empirical research shows that financial linkages have deepened over the past three decades –e.g., Kose, Prasad, Rogoff and Wei (2006), Lane and Milesi-Ferretti (2003). Motivated by this work, we will study two types of changes in networks: integration and diversification.

For expositional simplicity, in this section, we assume that agents are ex-ante identical–i.e.,

$\mu_i = \mu$, $\sigma_i^2 = \sigma^2$ and $w_i = w$. We start by developing some general results on how changes in networks affect aggregate welfare.

Recall that an agent's investment in autarchy is given by $\hat{\beta} = (\mu - r)/\alpha\sigma^2$. We can write agent utility under optimal investment as

$$U_i(S) = wr \sum_j \gamma_{ij} + \hat{\beta}(\mu - r) \sum_j \left[\frac{\gamma_{ij}}{\gamma_{jj}} - \frac{1}{2} \frac{\gamma_{ij}^2}{\gamma_{jj}^2} \right].$$

Aggregate welfare is the sum of agent utilities:

$$\begin{aligned} W(S) &= \sum_i \left[wr \sum_j \gamma_{ij} + \hat{\beta}(\mu - r) \sum_j \left[\frac{\gamma_{ij}}{\gamma_{jj}} - \frac{1}{2} \frac{\gamma_{ij}^2}{\gamma_{jj}^2} \right] \right] \\ &= wrn + \frac{(\mu - r)^2}{\alpha\sigma^2} \sum_i \sum_j \left[\frac{\gamma_{ij}}{\gamma_{jj}} - \frac{1}{2} \frac{\gamma_{ij}^2}{\gamma_{jj}^2} \right], \end{aligned}$$

where we have used the fact that Γ is column stochastic. We then obtain that $W(S) > W(S')$, if, and only if,

$$\sum_{j \in \mathcal{N}} \left[\frac{1}{\gamma_{jj}(S)} - \frac{1}{\gamma_{jj}(S')} \right] > \frac{1}{2} \sum_j \sum_{i \in \mathcal{N}} \left[\frac{\gamma_{ij}^2(S)}{\gamma_{jj}^2(S)} - \frac{\gamma_{ij}^2(S')}{\gamma_{jj}^2(S')} \right]. \tag{6}$$

The expression $\sum_{j \in \mathcal{N}} 1/\gamma_{jj}$ may be seen as a measure of the aggregate level of risk taking in the network. It is proportional to the aggregate expected returns generated by a network: low self-ownership generates high aggregate expected returns. The term

$$\sum_{j \in \mathcal{N}} \frac{\gamma_{ij}^2(S)}{\gamma_{jj}^2(S)}$$

reflects the costs of aggregate volatility. The inequality expresses the costs and benefits of changes in expected returns vis-a-vis changes in variance in terms of the ownership matrix Γ.[8] We will apply this inequality to compare welfare across different cross-holding networks.

In general, the relation between S and Γ can be quite complicated; to make progress, we

[8] To get a sense of how the network structure affects the two sides of the inequality, consider two scenarios: 1) where j's ownership is evenly distributed $\gamma_{ij} = 1/n$ for all $i \in \mathcal{N}$; and 2) where $\gamma'_{jj} = 1/n$, and the remaining ownership of every j is concentrated in the hands of a single agent 1, $\gamma'_{1j} = (n-1)/n$ and $\gamma'_{ij} = 0$ for all $i \neq 1, j$. Inequality (6) tells us that the left-hand side is 0, but the right-hand side is higher by a factor $n(n-1)$.

focus first on first- and second-order effects of changes in the linkages, and then we investigate comparative statics in core-periphery networks.

4.1 Thin networks

We assume that the strength of each link in S is sufficiently small–i.e., the network of cross-holdings S is *thin*. Define $\eta_i^{in} = \sum_{j\in\mathcal{N}} s_{ji}$ and $\eta_i^{out} = \sum_{j\in\mathcal{N}} s_{ij}$ as the in-degree and out-degree of $i \in \mathcal{N}$, respectively. For thin networks, we can then write the terms in Γ as:

$$\gamma_{ii} \simeq 1 - \eta_i^{in} + \sum_l s_{il}s_{li} \text{ and } \gamma_{ij} \simeq s_{ij}(1 - \eta_i^{in}) + \sum_l s_{il}s_{lj}. \tag{7}$$

This, in turn, implies that

$$\frac{\gamma_{ii}}{\gamma_{ij}} \simeq s_{ij} + \sum_l s_{il}s_{lj} \text{ and } \frac{\gamma_{ii}^2}{\gamma_{ij}^2} \simeq s_{ij}^2. \tag{8}$$

With these simplifications in hand, we are ready to state our first result on thin networks.

Proposition 2 *Assume that $\sigma_i^2 = \sigma^2$, $\mu_i = \mu$ and $w_i = w$. There exist $\overline{w} > 0$ and $\bar{s} > 0$ so that if $w > \overline{w}$ and $||S||_{\max} < \bar{s}$ and $||S'||_{\max} < \bar{s}$, then $W(S) > W(S')$ if*

$$\sum_{i\in\mathcal{N}} \left[\eta_i^{out}(S)(1+\eta_i^{in}(S)) - \eta_i^{out}(S')(1+\eta_i^{in}(S'))\right] > \frac{1}{2}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}} \left[s_{ij}^2 - s_{ij}'^2\right]. \tag{9}$$

Note that the reverse implication "only if" holds when the inequality in (9) is not strict (i.e., $\geq$). The inequality in the Proposition follows from substituting the ratios (8) in equation (6), and rearranging terms.

We now formally define integration and diversification in networks. For a vector $s_i = \{s_{i1}, ..., s_{in}\}$, define the variance of s_i as $\sigma_{s_i}^2 = \sum_j (s_{ij} - \eta_i^{out}/(n-1))^2$.

Definition 1 ***Integration*** *We say that S is more integrated than S' if $s_{ij} \geq s'_{ij}$ $\forall i, j \in \mathcal{N}$, and $s_{ij} > s'_{ij}$ for some $i, j \in \mathcal{N}$.*

The definition of integration reflects the idea that links between entities have become stronger.

Definition 2 ***Diversification*** *We say that S' is more diversified than S if $\eta_i^{out}(S') = \eta_i^{out}(S)$ and $\sigma_{s'_i}^2 \leq \sigma_{s_i}^2$ $\forall i \in \mathcal{N}$, and $\sigma_{s'_i}^2 < \sigma_{s_i}^2$ for some $i \in \mathcal{N}$.*

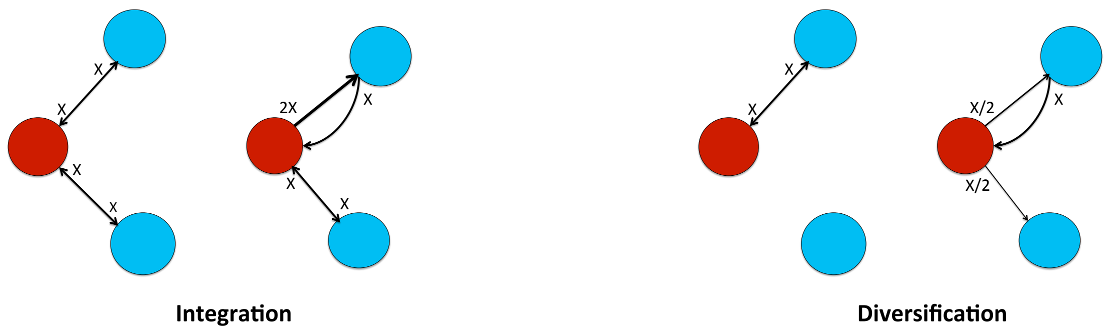

Figure 2: Changes in Networks

The definition of diversification reflects the idea that an existing sum of strength of ties is more evenly spread out. Figure 2 illustrates these definitions. These definitions of integration and diversification capture ideas that are similar to those used in the literature. For example, our notion of integration implies the definition of integration of Golub, Elliott and Jackson (2014). Our definitions are easier to apply in the model we study, but the effects we point out are not specific to these definitions.

Our next result builds on Proposition 2 to draw out the effects of greater integration and diversification on aggregate utility.

Corollary 1 *Assume that $\sigma_i^2 = \sigma^2$, $\mu_i = \mu$ and $w_i = w$. There exist $\overline{w} > 0$ and $\bar{s} > 0$ so that if $w > \overline{w}$ and $||S||_{\max} < \bar{s}$ and $||S'||_{\max} < \bar{s}$, the following holds:*

1. *If S is more integrated than S', then $W(S) > W(S')$.*

2. *If S is more diversified than S', then $W(S) > W(S')$ if*

$$2\sum_i \eta_i^{out}(S')\left[\eta_i^{in}(S') - \eta_i^{in}(S)\right] < \sum_i [\sigma^2_{s'_i} - \sigma^2_{s_i}]. \tag{10}$$

In the second part of the Corollary, the reverse implication "only if" holds when the inequality in (10) is not strict (i.e., $\leq$). An increase in integration lowers self-ownership and pushes up investment in risky assets. Higher investment in risky assets, in turn, raises expected returns and variability in returns. However, the costs of the increased variability are second-order and, in thin networks, are dominated by the benefits of higher expected returns.[9]

We now take up diversification: consider the case of a network in which some agents have high in-degree and some agents have low in-degree. In a thin network, the former have low self-ownership and, therefore, make large risky investments; by contrast, the latter group of agents have a high self-ownership and invest less in the risky asset. An increase in diversification leads to a reallocation away from high in-degree nodes to low in-degree nodes. Since investment in the risky project is proportional to $1/\gamma_{ii}$, high in-degree agents lower their risky investment more than the low in-degree agents raise it. Hence, both aggregate volatility and expected returns decline. Condition (10) in Proposition 1 clarifies the relative magnitude of these changes. In particular, the right-hand side reflects the decrease in the cost of the variance due to diversification; since S is more diversified than S', the right-hand side is always positive. The left-hand side represents the change in the expected aggregate returns due to diversification.

We illustrate the trade-offs involved in diversification with the help of two simple examples.

Example 2 *Diversification, heterogeneous networks and welfare.*

Suppose $n = 4$ and network S is defined as: $s'_{12} = s'_{43} = \epsilon$ and $s'_{13} = s'_{42} = 2\epsilon$, and all other links are zero. Next, define network S as: $s_{12} = s_{43} = s_{13} = s_{42} = 3\epsilon/2$, and all other links are zero. Note that S is more diversified than S', but the in-degree of each agent is the same in S and in S'. Let ϵ be small so that the network is thin. Thus, the left-hand side of condition (10) equals zero, and the right-hand side is positive. It follows that aggregate welfare is higher under the more diversified network S.

Suppose that $n = 3$, and network S' is defined as follows: $s'_{12} = s'_{21} = \epsilon$ and all other links are 0. The network S is defined as $s_{12} = s_{13} = \epsilon/2$, and $s_{21} = s'_{21}$. This is a thin network for sufficiently small ϵ. Note that the left-hand side of condition (10) is equal to ϵ^2, and the right-hand side is equal to $\epsilon^2/2$. Aggregate welfare is lower in the more diversified network S.
□

4.2 Core periphery networks

Recall that in a core periphery network, there are n_p peripheral agents and n_c core agents. We now provide comparative statics results for two extreme form of core-periphery networks,

[9]The effects of integration on agent utilities will vary: if A owns a much larger part of B, then the ownership and, hence, the utility of B will typically go down, while the utility of A will go up.

the complete network, and the star network. In the former, each agent is in the core and the network is symmetric, i.e.g, $n_c = n$; in contrast, in the star there is only one agent in the core, and all other agents are peripheral, i.e., $n_c = 1$ and $n_p = n - 1$.

Proposition 3 *Assume $\sigma_i^2 = \sigma^2$, $\mu_i = \mu$ and that $w_i = w$ is large for all i. Consider that S is a complete network, i.e., $s_{ij} = s$ for all $i \neq j$. When s increases each agent invests more in the risky project and earns a higher utility.*

Recall that the ownership matrix Γ in a complete network is

$$\gamma_{ij} = \frac{s}{s+1} \qquad \text{and} \qquad \gamma_{ii} = 1 - (n-1)\gamma_{ij}.$$

Greater s lowers self-ownership and, from Proposition 1, we know that this means that all agents raise their investment in risky assets. As a consequence, both the expected value $E[V_i]$ and the variance $Var[V_i]$ increase in s. Substituting the ownerships in expression (3) tells us that the expected utility of each agent is increasing in s. Overall, Proposition ?? shows that in symmetric networks deeper integration increases aggregate utilities even in thick networks

We now move to asymmetric networks. In Example 1, we get the star network if we set $n_c = 1$. The self-ownerships of central and peripheral agents are, respectively:

$$\gamma_{i_c i_c} = \frac{1 - n_p \hat{s}}{1 - n_p \hat{s}^2} \qquad \text{and} \qquad \gamma_{i_p i_p} = \frac{[1-\hat{s}][1-\hat{s}^2(n_p - 1)]}{1 - n_p \hat{s}^2}.$$

Proposition 4 *Assume $\sigma_i^2 = \sigma^2$, $\mu_i = \mu$ and that $w_i = w$ is large for all i. Consider that S is a star network and i^* is the center, i.e., $s_{i^*j} = s_{ji^*} = \hat{s} \in [0, 1/(n-1)]$ for all $j \neq i^*$.*

- *The central agent makes larger investments in the risky asset relative to the other agents. Furthermore, an increase in $\hat{s}$ increases the investment in the risky asset of each agent.*
- *There exists $0 < \underline{s} < \bar{s} < 1/n_p$ so that an increase in $\hat{s}$ increases aggregate utilities if $\hat{s} < \underline{s}$ and it decreases aggregate utilities if $\hat{s} > \bar{s}$.*

It is possible to verify that $\gamma_{i_c i_c} < \gamma_{i_p i_p}$ and, from Proposition 1, this implies that the central agent makes larger investments in the risky asset.

We now turn to utilities. For small $\hat{s}$, each agent has high self-ownership, and, therefore, investments in the risky projects are limited. An increase in $\hat{s}$ increases investment in risky assets which leads to an increase in expected returns and in the costs of the variance.

Since investments are small, the costs of the variance is effectively shared, mainly, by the peripheral agents. Overall, aggregate utilities increase. However, when $\hat{s}$ is high, the central player has very little self-ownership and very little ownership of peripheral players. In contrast, the peripheral players have positive and large ownership of the central player. As a consequence, peripheral players absorb the large risky investments that the central player undertakes. Hence, an increase in $\hat{s}$ has a large negative externalities on peripheral agents and aggregate utilities decline.

It is worth emphasizing that the above patterns for the star network are obtained in more general core-periphery networks with several central agents and numerical analysis is available from the authors upon request. Furthermore, we have obtained these results in a setting where the nodes are ex-ante identical in terms of endowment and the links are symmetric. In empirically observed financial networks, the core nodes have larger endowments than the periphery nodes: this will further amplify the negative effects of integration on volatility of the system. On the issue of links, in some empirical contexts, such as international flows, the strength of the link from the periphery to the core has grown: this will further strengthen the decline in self-ownership of core nodes and amplify the effects we identify.[10]

Overall, the results in this section illustrate the powerful effects of network architecture on portfolio choice and welfare. They motivate a normative analysis of networks.

5 Optimal investments and the nature of externalities

This section presents a characterization of first-best investments in networks and then examines the difference between first-best and individually optimal investments. This leads us to study the costs of decentralization across networks.

We suppose that the 'planner' seeks to maximize aggregate utilities:

$$W^P(\beta, S) = \sum_{i \in \mathcal{N}} E[V_i] - \frac{\alpha}{2} \sum_{i \in \mathcal{N}} Var[V_i]. \tag{11}$$

For a given S, the planner chooses investments in risky assets, $\beta^P = \{\beta_1^P, \beta_2^P, .., \beta_n^P\}$, to maximize (11). We obtain:

[10] A variety of financial networks have been empirically studied in recent years; see the introduction for references to this literature.

Proposition 5 *The optimal investment of the social planner in risky project $i = 1, ..., n$ is given by*

$$\beta_i^P = \min\left[w_i, \frac{1}{\sum_{j\in\mathcal{N}} \gamma_{ji}^2}\hat{\beta}_i\right]. \tag{12}$$

In order to understand the externalities created by the network of holdings, we compare the marginal utility of increasing β_i for agent i, with the marginal utility of the utilitarian planner. We have:

$$\begin{aligned}
\frac{\partial U_i}{\partial \beta_i} &= (\mu_i - r)\gamma_{ii} - \alpha\sigma_i^2\beta_i\gamma_{ii}^2, \\
\frac{\partial W(S)}{\partial \beta_i} &= (\mu_i - r) - \alpha\sigma_i^2\beta_i\sum_{j\in\mathcal{N}}\gamma_{ji}^2.
\end{aligned}$$

The agent ignores the impact of his risky investment on the aggregate expected returns *and* also on the sum of the agent's variance. In particular, an agent underestimates the impact of his investment on the aggregated expected value by $(1 - \gamma_{ii})$, and on the sum of variances by $\sum_{j\neq i}\gamma_{ji}^2$. Note that $\sum_{j\neq i}\gamma_{ji}^2$ is higher when the ownership of agent i is concentrated in a few other agents. This yields the following general insight: when the cross-holding network of agent i is highly concentrated, agent i's investment in risky assets is too high relative to what is collectively optimal. The converse is true if agent i's cross-holdings are widely dispersed.

Corollary 2 *Assume that w_i is large for all $i \in \mathcal{N}$. Agent i over-invests as compared to the planner, $\beta_i > \beta_i^P$ if, and only if,*

$$\gamma_{ii} < \sum_{j\in\mathcal{N}}\gamma_{ji}^2.$$

We now consider how the network affects the cost of decentralization. Given a network S, the cost of decentralization is defined as

$$K(S) = W(\beta^P, S) - W(\beta^*, S).$$

Using Proposition 1 and Proposition 5, we obtain

$$K(S) = \frac{(\mu - r)^2}{\alpha\sigma^2}\left[\sum_j\left(\frac{1}{\sum_l \gamma_{lj}^2} - \frac{1}{\gamma_{jj}}\right) - \frac{1}{2}\sum_i\sum_j\left(\gamma_{ij}^2\frac{1}{(\sum_l\gamma_{lj}^2)^2} - \frac{1}{\gamma_{jj}^2}\right)\right].$$

We would like to order networks in terms of this cost of decentralization. While it is difficult to obtain a result when comparing arbitrary networks, we are able to make progress if we restrict attention to thin networks.

Proposition 6 *Assume that $\sigma_i^2 = \sigma^2$, $\mu_i = \mu$ and $w_i = w$ for all $i \in \mathcal{N}$. Suppose that S and S' are both thin networks. There exist $\overline{w} > 0$ and $\bar{s} > 0$ so that if $w > \overline{w}$ and $||S||_{\max} < \bar{s}$ and $||S'||_{\max} < \bar{s}$, the cost of decentralization is higher under S than under S' if, and only if,*

$$\sum_j \eta_j^{in}(S)^2 > \sum_j \eta_j^{in}(S')^2. \tag{13}$$

Note that if S is more integrated than S', then $\eta_i^{in}(S) \geq \eta_i^{in}(S')$ for all $i \in \mathcal{N}$, and the inequality is strict for some i, which implies that condition (13) holds. That is, the cost of decentralization is higher in more-integrated networks. Intuitively, by increasing integration, agents' self-ownership decreases, and, therefore, the agency problem is stronger.

On the other hand, take two network S and S' for which the sum of in-degrees across agents is constant. Then, condition (13) tells us that the cost of decentralization is higher in networks in which in-degrees are concentrated on a few nodes, as in the core-periphery network. In these networks, it follows from Corollary 2, that the few agents with a large in-degree over-invest in the risky asset, creating far too much variability among the connected agents.

6 Optimal Network Design

This section considers the nature of the optimal network. It is useful to separately develop both a first-best and a second-best analysis. In the first-best analysis, the planner designs the network S to maximize objective (11) and dictates collectively optimal investments according to (12). In the second-best analysis, the planner designs the network S to maximize objective (11) but takes into account that, for a given S, agents choose investments according to (4). The following result summarizes our analysis.

Proposition 7 *Assume that w_i is large for all $i \in \mathcal{N}$. The first-best network design and the second-best network design is the complete network with maximum link strength $s_{ij} = 1/(n-1)$ for all $i \neq j$.*

To solve the first-best and second-best design problem, we first derive the optimal Γ, and then we derive the network S that induces the optimal Γ. We start by establishing that homogeneous networks – where links and weights are spread evenly across nodes - dominate heterogeneous networks. This is because agents are risk-averse, and concentrated and unequal ownership exacerbates the costs of variance. This leads to a preference for homogeneous networks: networks where, for every i, $\gamma_{ji} = \gamma_{j'i}$ for all $j, j' \neq i$.

In the first-best, within homogeneous networks, stronger links are better, as they allow for greater smoothing of shocks, and this is welfare-improving due to agents' risk aversion. In the second-best design problem, within homogeneous network, the designer can replicate the first best outcome by setting the same network as in the first-best design. In fact, when $s_{ij} = 1/(n-1)$ for all $i \neq j$, then $\gamma_{ij} = 1/n$ for all i, j, and therefore equilibrium investment coincides with socially optimal investments.

7 Correlations

We relax the assumption that the returns of projects are uncorrelated. We show existence of an equilibrium; we provide sufficient conditions for uniqueness and for existence of an interior equilibrium. Finally, we provide an example in the extreme case in which projects are positive perfectly correlated; this example shows that in asymmetric networks some individuals over-invest in risk taking; in this sense our insights in the basic model carry over to a setting with correlations.

Recall that each project z_i is normally distributed with mean μ_i and variance σ_i^2 and therefore $z = \{z_1, ..., z_n\}$ is a multivariate normal distribution. Let Ω be the covariance matrix. Under the assumption that z is a non-degenerate multivariate normal distribution, it follows that Ω is positive definite. Note that

$$U_i(\beta_i, \beta_{-i}) = \sum_{j \in \mathcal{N}} \gamma_{ij}(wr + \beta_j(\mu_i - r)) - \frac{\alpha}{2} \sum_{j \in \mathcal{N}} \sum_{j' \in \mathcal{N}} \gamma_{ij}\beta_j\gamma_{ij'}\beta_{j'}\sigma^2_{jj'}, \tag{14}$$

and the sign of $\partial^2 U_i/(\partial\beta_i\partial\beta_j)$ is the same as the sign of $-\sigma_{ij}$; that is, investments in risky asset i and j are strategic substitutes (resp. strategic complement) whenever the returns from the two projects are positively correlated (negatively correlated).

Let $\circ$ be the Hadamard product. Let also $\mathbf{b}$ be a n dimensional vector where the i-th element is $(\mu_i - r)$.

Proposition 8 *There always exists an equilibrium and the equilibrium is unique if* $\sum_j s_{ij} < 1/2$ *for all* $i \in \mathcal{N}$. *Furthermore, there exists a* $\bar{w} > 0$ *and a* $\bar{s} > 0$ *so that if* $w > \bar{w}$ *and* $||S||_{max} < \bar{s}$ *the unique equilibrium is interior and takes the following form* $\beta = \{\beta_1, ..., \beta_n\}$*:*

$$\beta = \frac{1}{\alpha}[\Gamma \circ \Omega]^{-1}\mathbf{b}.$$

Existence and uniqueness follows by verifying the sufficient conditions developed by Rosen (1965). The analysis becomes easier when we take the case of projects that are positively perfectly correlated. This environment is equivalent to assume that there is only one risky project and all individuals can invest in such project.[11] Recall that $\eta_i^{in} = \sum_j s_{ji}$

Proposition 9 *Assume projects are perfectly positive correlated, that* $\mu_i = \mu$ *for all* $i \in \mathcal{N}$ *and that* w *is large. An interior equilibrium exists if, and only if,* $\frac{1}{1-\eta_i^{in}} - \sum_j \frac{s_{ij}}{1-\eta_j^{in}} > 0$ *for all* i. *In an interior equilibrium*

$$\beta_i \quad = \quad \frac{\mu - r}{\sigma^2 \alpha}\left[\frac{1}{1-\eta_i^{in}} - \sum_j \frac{s_{ij}}{1-\eta_j^{in}}\right], \tag{15}$$

Note that in an interior equilibrium each individual is exposed to the same amount of risky investment in the sense that for each individual i and j it must be the case that $\sum_l \gamma_{il}\beta_l = \sum_l \gamma_{jl}\beta_l$. Furthermore, this amount is the same as the one that an individual will choose in isolation, i.e., $\sum_l \gamma_{il}\beta_l = \sum_l \gamma_{jl}\beta_l = \frac{(\mu-r)^2}{\alpha\sigma^2}$. This fact, together with the fact that Γ is column stochastic, implies that the sum of risky investment across individuals equals the sum of investment in the risky asset across individuals in the case where the network is empty.

From the explicit characterization of Proposition 9, it is easy to provide the following comparative statics:

Proposition 10 *Assume projects are perfectly positive correlated. In an interior equilibrium: 1.) a change in the network increases the utility of individual* i *if and only if it increases his total ownership* $\sum_j \gamma_{ij}$*; 2.) a change in the network has not impact on aggregate utilities; 3.) equilibrium investments are socially efficient*

[11] In fact, the insights we provide in this section will also carry over to an environment where there are n assets, whose returns are i.i.d, and each individual can invest in each of these assets. In the equilibrium of these models, each individual i will choose a total investment in risky assets, say β_i, and then spread such investment equally across the n assets, i.e., individual i invests $\beta_{is} = \beta_i/n$ on each asset s. It is easy to show that the equilibrium investment β_i in this model with n assets is the same as the one that we derived here for one asset.

We conclude by showing that, when the networks are heterogenous, the equilibrium is not interior, and this leads to over-investment in risky assets.

Example 3 *Two-individuals case and over-investment.*

Note first that, with two agents, S is characterised by s_{12} and s_{21}, and it is immediate to derive: $\gamma_{ii} = (1 - s_{ji})/(1 - s_{ij}s_{ji})$ and $\gamma_{ji} = 1 - \gamma_{ii}$, for all $i = 1, 2$ and $j \neq i$. Second, we provide a full characterisation of equilibrium. From Proposition 9 we know that an interior equilibrium exists if, and only if, $\frac{1}{1-\eta_i^{in}} - \sum_j \frac{s_{ij}}{1-\eta_j^{in}} > 0$ for all i, which, in this example, reads as $\gamma_{ii} > 1/2$ for $i = 1, 2$. We now characterise non-interior equilibrium. It is immediate to see that the marginal returns to agent i are strictly positive at $\beta_i = \beta_j = 0$; therefore $\beta_1 = \beta_2 = 0$ cannot be equilibrium. Consider the case where $\beta_i = 0$ and $\beta_j > 0$. Given $\beta_i = 0$, the FOC for j leads to

$$\beta_j = \frac{\mu - r}{\alpha\sigma^2}\frac{1 - s_{ij}s_{ji}}{1 - s_{ij}},$$

and since $\beta_i = 0$, it has to be the case that the marginal utility of i at $\beta_i = 0$ and $\beta_j = \frac{\mu-r}{\alpha\sigma^2}\frac{1-s_{ij}s_{ji}}{1-s_{ij}}$ is non-positive, which holds, if, and only if, $\mu - r - \alpha\gamma_{ii}\beta_j\sigma^2 \leq 0$, if, and only if, $\gamma_{jj} < 1/2$. Combining these results we have that the equilibrium is unique and that there are three regions, which are the same as the one depicted in figure 3. In Region 1, $\gamma_{11} > 1/2$ and $\gamma_{22} > 1/2$, both agents invest positively in the risky asset and their investment is specified in Proposition 9. In Region 2 (resp. Region 3), where $\gamma_{11} < 1/2$ and $\gamma_{22} > 1/2$ (resp. $\gamma_{11} > 1/2$ and $\gamma_{22} < 1/2$), agent 2 (resp. agent 1) does not invest in the risky project and agent 1 (resp. agent 2) invests $\beta_1 = \frac{\mu-r}{\alpha\sigma^2}\frac{1-s_{21}s_{12}}{1-s_{21}}$ (resp. $\beta_2 = \frac{\mu-r}{\alpha\sigma^2}\frac{1-s_{21}s_{12}}{1-s_{12}}$).

Third, we provide a full characterisation of the social optimum. If the optimum is interior, then we know from part 3 of Proposition 10 that $\hat{\beta}_i$ equals expression (15), which, in this example reads

$$\hat{\beta}_i = \frac{\mu - r}{\sigma^2\alpha}\left[\frac{1 - 2s_{ij} + s_{ij}s_{ji}}{(1 - s_{ij})(1 - s_{ji})}\right] \tag{16}$$

and, it is easy to verify that, $\hat{\beta}_1 > 0$ and $\hat{\beta}_2 > 0$ if and only if: $\gamma_{11} > 1/2$ and $\gamma_{22} > 1/2$. The social welfare that is generated is $2wr + \frac{(\mu-r)^2}{\sigma^2\alpha}$. Consider now that $\hat{\beta}_i = 0$ and $\hat{\beta}_j > 0$. Then the FOC for j must hold which leads to

$$\hat{\beta}_j = \frac{\mu - r}{\sigma^2\alpha}\frac{1}{\gamma_{ij}^2 + \gamma_{jj}^2}$$

and the social welfare generated is $2wr + \frac{(\mu-r)^2}{\sigma^2\alpha}\frac{1}{2[\gamma_{ij}^2+\gamma_{jj}^2]}$.

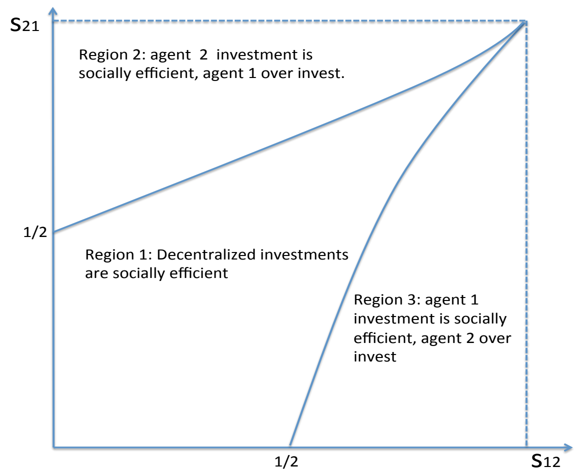

Figure 3: Equilibrium and first best investments.

It is easy to see that $2[\gamma_{ij}^2 + \gamma_{jj}^2] \geq 1$, and the inequality is strict whenever $\gamma_{jj} \neq 1/2$, which implies that the social welfare when $\hat{\beta}_1$ and $\hat{\beta}_2$ are both positive is higher than when one of them is 0. So, whenever $\gamma_{22} > 1/2$ and $\gamma_{11} > 1/2$ the optimal solution is interior. Next, note that the welfare associate to $\hat{\beta}_1 = 0$ and $\hat{\beta}_2 > 0$ is higher than the welfare associated to $\hat{\beta}_1 > 0$ and $\hat{\beta}_2 = 0$ if and only if $\gamma_{12}^2 + \gamma_{22}^2 < \gamma_{21}^2 + \gamma_{11}^2$, which is satisfied if, and only if, $\gamma_{22} < 1/2$ and $\gamma_{11} > 1/2$. Finally, by comparing the optimal investment β_j when $\gamma_{jj} > 1/2$ and $\gamma_{ii} < 1/2$, with the equilibrium investment, it is easy to check that individual j over invests relative to the social planner.

Finally, the comparison between equilibrium and social optimum is summarised in Figure 3. It shows that when the cross-holding network is asymmetric then we have over-investment of some of the agents, which is similar to the case of independent projects.

8 Conclusion and remarks on model

We have developed a model in which the network of financial obligations mediates agents' risk taking behavior. The framework allows us to discuss the costs and benefits of greater integration and greater diversification and how they depend on the underlying network's characteristics.

In the basic model, we have taken the view that ownership does not translate into control in a straightforward way. For expositional purposes, we assume that ownership and control are completely separate. We now discuss two different ways of bringing ownership more in line with decision rights. An online Appendix contains supplementary material that covers technical results.

Suppose that γ_{ij} signifies that agent i has control over γ_{ij} fraction of agent j's initial endowment w_j. One way of interpreting this control is to say that agent i can invest $\gamma_{ij}w_j$ in the risk-free asset or in the risky project i. In this interpretation, $\gamma_{ij}w_j$ is a transfer from j to i that occurs before shocks are realized. Therefore, Γ redefines the agents' initial endowments. Since, under the mean-variance preferences, initial endowments do not influence risk taking (unless the solution is corner), the network plays no important role.

In an alternative scenario, suppose that ownership conveys control, but the control is 'local': agent i can invest $w\gamma_{ij}$ in the risk-free asset and in the risky project of agent j. The choice of agent i is, then, a vector of investments $\beta_i = \{\beta_{i1}, ..., \beta_{in}\}$, where β_{ij} is the investment in risky project j of endowment $w_{ij} = \gamma_{ij}w_j$, and $\beta_{ij} \in [0, \gamma_{ij}w_j]$. It is possible to show that, in this case, individually optimal investment levels are independent of the network, and agents' choices mimic those of a central planner with mean-variance preferences over aggregate returns $V = \sum_i V_i$.

These two examples illustrate that whenever ownership gives control in a "frictionless" way, the role of the network in shaping risk taking is uninteresting.

9 Appendix

Derivation of Γ matrix for core-periphery matrix: We first derive the Γ matrix for a core-periphery matrix, S. In a core-periphery network there are n_p peripheral individuals and n_c central individuals, $n_p + n_c = n$; i_c is a (generic) central individual and i_p is a peripheral individual. A link between two central individuals is $s_{i_c j_c} = s$ and a link between a central and a peripheral individual $s_{i_c i_p} = s_{i_p i_c} = \hat{s}$, and there are no other links.

Denote by $k^t(i_c, i_c)$ the element $[S^t]_{ii}$ where i is a core player, $k^t(i_p, i_p)$ the element $[S^t]_{ii}$ where i is a peripheral player, $k^t(i_c, j_c)$ the element $[S^t]_{ij}$ where i and j, $i \neq j$ are core players, $k^t(i_p, j_p)$ the element $[S^t]_{ij}$ where i and j, $i \neq j$ are peripheral players, $k^t(i_c, j_p)$ the element $[S^t]_{ij}$ where i is a core player and j is a peripheral player. It is easy to verify that for every

$t \geq 1$ we have

$$\begin{pmatrix} k^t(i_c,i_c) \\ k^t(i_c,j_c) \\ k^t(i_c,j_p) \end{pmatrix} = \begin{pmatrix} 0 & (n_c-1)s & n_p\hat{s} \\ s & (n_c-2)s & n_p\hat{s} \\ \hat{s} & (n_c-1)\hat{s} & 0 \end{pmatrix} \begin{pmatrix} k^{t-1}(i_c,i_c) \\ k^{t-1}(i_c,j_c) \\ k^{t-1}(i_c,j_p) \end{pmatrix}$$

and $k^t(i_p,i_p) = k^t(i_p,j_p) = n_c\hat{s}k^t(i_c,j_p)$, where $k^0(i_c,j_c) = 0$, $k^0(i_c,j_p) = 0$, $k^0(i_c,i_c) = 1$. This is a homogenous system of difference equation with initial conditions $k^1(i_c,j_c) = s$, $k^1(i_c,j_p) = \hat{s}$, $k^1(i_c,i_c) = 0$. So, to solve it suffices to derive the eigenvalues of the matrix of coefficients and the respective eigenvectors. To derive eigenvalues note that

$$\begin{vmatrix} -\lambda & (n_c-1)s & n_p\hat{s} \\ s & (n_c-2)s-\lambda & n_p\hat{s} \\ \hat{s} & (n_c-1)\hat{s} & -\lambda \end{vmatrix} = \begin{vmatrix} -\lambda-s & 0 & 0 \\ s & (n_c-1)s-\lambda & n_p\hat{s} \\ \hat{s} & n_c\hat{s} & -\lambda \end{vmatrix} = (-\lambda-s)\begin{vmatrix} (n_c-1)s-\lambda & n_p\hat{s} \\ n_c\hat{s} & -\lambda \end{vmatrix} = 0$$

if and only if $\lambda = -s$ or $\lambda^2 - \lambda(n_c-1)s - n_pn_c\hat{s}^2 = 0$. Call $\lambda_1 = -s$, and $\lambda_2 > \lambda_3$ the two solutions to the quadratic equation. Let the eigenvector associated to λ_i be denoted by $v_i = [x_i, y_i, z_i]$. Simple calculation implies that $v_1 = [x_1, -x_1/(n_c-1), 0]$ and $v_2 = [x_2, x_2, n_c\hat{s}x_2/\lambda_2]$, $v_3 = [x_3, x_3, n_c\hat{s}x_3/\lambda_3]$. Recalling that

$$\begin{aligned} k^t(i_c,i_c) &= c_1x_1\lambda_1^t + c_2x_2\lambda_2^t + c_3x_3\lambda_3^t, \\ k^t(i_c,j_c) &= c_1y_1\lambda_1^t + c_2y_2\lambda_2^t + c_3y_3\lambda_3^t, \\ k^t(i_c,j_p) &= c_1z_1\lambda_1^t + c_2z_2\lambda_2^t + c_3z_3\lambda_3^t, \end{aligned}$$

and using the derived eigenvalues and eigenvectors we obtain

$$\begin{aligned} k^t(i_c,i_c) &= c_1x_1(-s)^t + c_2x_2\lambda_2^t + c_3x_3\lambda_3^t, \\ k^t(i_c,j_c) &= -c_1x_1\frac{1}{n_c-1}(-s)^t + c_2x_2\lambda_2^t + c_3x_3\lambda_3^t, \\ k^t(i_c,j_p) &= c_2n_c\hat{s}x_2\lambda_2^{t-1} + c_3n_c\hat{s}x_3\lambda_3^{t-1}. \end{aligned}$$

Imposing the initial conditions, we obtain $c_1x_1 = (n_c-1)/n_c$, $c_2x_2 = \frac{1}{n_c}\left[\frac{(n_c-1)s-\lambda_3}{\lambda_2-\lambda_3}\right]$ and

$c_3 x_3 = \frac{1}{n_c}\left[\frac{[\lambda_2-(n_c-1)s}{\lambda_2-\lambda_3}\right]$. And so, after some algebra,

$$\begin{aligned} k^t(i_c,i_c) &= \frac{n_c-1}{n_c}(-1)^t s^t + \frac{1}{(\lambda_2-\lambda_3)n_c}\left[(n_c-1)s(\lambda_2^t-\lambda_3^t)-\lambda_2\lambda_3(\lambda_2^{t-1}-\lambda_3^{t-1})\right], \\ k^t(i_c,j_c) &= -\frac{1}{n_c}(-1)^t s^t + \frac{1}{(\lambda_2-\lambda_3)n_c}\left[(n_c-1)s(\lambda_2^t-\lambda_3^t)-\lambda_2\lambda_3(\lambda_2^{t-1}-\lambda_3^{t-1})\right], \\ k^t(i_c,j_p) &= \frac{n_c\hat{s}}{(\lambda_2-\lambda_3)n_c}\left[(n_c-1)s(\lambda_2^{t-1}-\lambda_3^{t-1})-\lambda_2\lambda_3(\lambda_2^{t-2}-\lambda_3^{t-2})\right]. \end{aligned}$$

We can now derive the matrix Γ. Note that

$$\sum_{t=1}^{\infty} k^t(i_c,i_c) = \frac{(n_c-1)s^2+n_c n_p \hat{s}^2 s + n_p\hat{s}^2}{(s+1)[1-s(n_c-1)-n_c n_p \hat{s}^2]},$$

and since $\gamma_{i_c,i_c} = (1-d_c)[1+\sum_{t=1}^{\infty} k^t(i_c,i_c)]$ we have

$$\gamma_{i_c,i_c} = \frac{[1-(n_c-1)s-n_p\hat{s}][1-(n_c-2)s-n_c n_p\hat{s}^2+n_p\hat{s}^2]}{(s+1)[1-s(n_c-1)-n_c n_p\hat{s}^2]}.$$

We can repeat the same steps for the other cases and straight algebra leads to following expressions that we have reports in example 1. Furthermore, if we set $n_p = 0$ we get the Γ for the complete network with $n_c = n$ nodes. If we set $n_p = n-1$, we get the Γ for the star network.

Proof of Proposition 1: Suppose that the solution is interior. As the objective function is concave, the first-order condition is sufficient. Taking derivatives in (3) with respect to β_i and setting it equal to 0, immediately yields the required expression for optimal investments. Substituting the optimal investments in the expressions for the expected value and variance yields the expressions in the statement of the result. ■

Proof of Proposition 2. We start with the derivation of the second-order approximation of Γ for thin networks. Define the indicator function $\delta_{ij} = 1$, if $i = j$ and $\delta_{ij} = 0$, otherwise. First, note that:

$$\begin{aligned}\gamma_{ij} &= (1-\sum_{p\neq i} s_{pi})\left(\delta_{ij}+s_{ij}+\sum_{p\neq i,j} s_{ip}s_{pj}+\sum_{p\neq i, q\neq j, p\neq q} s_{ip}s_{pq}s_{qj}+...\right)\\ &\simeq \delta_{ij}+s_{ij}+\sum_p s_{ip}s_{pj}-\sum_p s_{pi}(\delta_{ij}+s_{ij}),\end{aligned}$$

which yields to

$$\gamma_{ii}\simeq 1-\eta_i^{in}+\sum_p s_{ip}s_{pi} \text{ and } \gamma_{ij}\simeq s_{ij}-s_{ij}\eta_i^{in}+\sum_p s_{ip}s_{pj}.$$

We can then write:

$$\begin{aligned}\frac{\gamma_{ij}}{\gamma_{jj}} &\simeq \frac{s_{ij}+\sum_p s_{ip}s_{pj}-s_{ij}\sum_p s_{pj}}{1-\left(\sum_p s_{pj}-\sum_p s_{jp}s_{pj}\right)}\\ &\simeq \left(s_{ij}+\sum_p s_{ip}s_{pj}-s_{ij}\sum_p s_{pj}\right)\left(1+\sum_p s_{pj}-\sum_p s_{jp}s_{pj}+\left(\sum_p s_{pj}\right)\left(\sum_h s_{hj}\right)\right)\\ &\simeq s_{ij}+\sum_p s_{ip}s_{pj}-s_{ij}\sum_p s_{pj}+s_{ij}\sum_p s_{pj}=s_{ij}+\sum_p s_{ip}s_{pj},\end{aligned}$$

and, similarly,

$$\frac{\gamma_{ij}^2}{\gamma_{jj}^2}\simeq s_{ij}^2.$$

Therefore

$$\frac{\gamma_{ij}}{\gamma_{jj}}-\frac{1}{2}\frac{\gamma_{ij}^2}{\gamma_{jj}^2}\simeq s_{ij}+\sum_p s_{ip}s_{pj}-\frac{1}{2}s_{ij}^2$$

Using expression (6), we obtain that in thin networks, $\sum_i U_i(S)>\sum_i U_i(S')$ if

$$\sum_i\sum_j\left[s_{ij}+\sum_p s_{ip}s_{pj}-\frac{1}{2}s_{ij}^2\right]>\sum_i\sum_j\left[s'_{ij}+\sum_p s'_{ip}s'_{pj}-\frac{1}{2}s'^2_{ij},\right]$$

and using the definition of η_i^{in} and η_i^{out}, this condition can be rewritten as condition (10) in the Proposition. The "only if" part also follows. ■

Proof of Corollary 1: If S is more integrated than S' then $\eta_i^{out}(S)\geq\eta_i^{out}(S')$ and the inequality is strict for some i. This implies that moving from S' to S there is a positive first order effect in aggregate utilities. Therefore, for $\bar{s}$ small enough, aggregate utility is higher in S and than S'. Next, the proof of the second part of the Corollary follows by Proposition 2 after noticing that $\eta_i^{out}(S) = \eta_i^{out}(S')$ for all $i \in \mathcal{N}$ and using the definition of $\sigma^2_{s_i}$. ■

Proof of Proposition 3. Setting $n_p = 0$ and calling $n_c = n$, we get the complete network, with link strength $s \leq 1/(n-1)$. The element of Γ are therefore $\gamma_{ij} = s/(s+1)$ and $\gamma_{ii} = 1-(n-1)\gamma_{ij}$. Individual investment is negatively related to γ_{ii}, which is clearly decreasing in s, for $s \in [0, 1/(n-1)]$. Next, observe that

$$E[V_i] = wr \sum_{j \in \mathcal{N}} \gamma_{ij} + \frac{(\mu - r)^2}{\alpha\sigma^2} \sum_{j \in \mathcal{N}} \frac{\gamma_{ij}}{\gamma_{jj}} = wr + \frac{(\mu - r)^2}{\alpha\sigma^2} \left[\frac{1+s}{1-(n-2)s}\right].$$

It is straightforward to see that $E[V_i]$ is increasing in s. Similar computation shows that

$$Var[V_i] = \frac{(\mu - r)^2}{\alpha^2\sigma^2} \left[1 + \frac{(n-1)s^2}{[1-(n-2)s]^2}\right],$$

and it is immediate to see that it is increasing in s. Next, the expected utility of i reads as

$$U_i = E[V_i] - \frac{\alpha}{2} Var[V_i] = wr + \frac{(\mu - r)^2}{2\alpha\sigma^2} \left[1 + \frac{s[2 - s(2n-3)]}{[1-(n-2)s]^2}(n-1)\right],$$

and

$$\frac{\partial U_i}{\partial s} = \frac{(n-1)(\mu - r)^2}{\alpha\sigma^2} \frac{[1 - s(n-1)]}{[1-(n-2)s]^3} > 0,$$

where the last inequality follows by noticing that, by assumption, $s(n-1) < 1$. Finally, it is easy to check the result on the covariance. ■

Proof of Proposition 4. Obtain Γ for the star network by setting $n_c = 1$ and $n_p = n - 1$. Part 1 follows by inspection of the net ownership expressions derived for the star network. We now prove part 2. Aggregate utilities in a star network is

$$W(S) = -\frac{1}{2} + \frac{1}{\gamma_{i_c i_c}} + \frac{1}{\gamma_{i_p i_p}} - \frac{n_p}{2} \left[\left(\frac{\gamma_{i_c i_p}}{\gamma_{i_p i_p}}\right)^2 + \left(\frac{\gamma_{i_p i_c}}{\gamma_{i_c i_c}}\right)^2 + (n-2)\left(\frac{\gamma_{i_p j_p}}{\gamma_{i_p i_p}}\right)^2\right]$$

and

$$\begin{aligned}\frac{\partial W(s)}{\partial \hat{s}} &= \frac{\partial(1/\gamma_{i_c i_c})}{\partial \hat{s}} + \frac{\partial(1/\gamma_{i_p i_p})}{\partial \hat{s}} - n_p \left[\frac{\gamma_{i_c i_p}}{\gamma_{i_p i_p}} \frac{\partial(\gamma_{i_c i_p}/\gamma_{i_p i_p})}{\partial \hat{s}}\right] - \\ &- (n-1)\left[\frac{\gamma_{i_p i_c}}{\gamma_{i_c i_c}} \frac{\partial(\gamma_{i_p i_c}/\gamma_{i_c i_c})}{\partial \hat{s}} + (n-2)\frac{\gamma_{i_p j_p}}{\gamma_{i_p i_p}} \frac{\partial(\gamma_{i_p j_p}/\gamma_{i_p i_p})}{\partial \hat{s}}\right]\end{aligned}$$

It is easy to verify that $\frac{\partial W(s)}{\partial \hat{s}}$ is continuous in $\hat{s} \in [0, 1/n_p]$. Furthermore, when $\hat{s}$ goes to 0, then $\frac{\partial(1/\gamma_{i_c i_c})}{\partial \hat{s}}$ goes to n_p and all the other terms goes to 0 and therefore $\frac{\partial W(s)}{\partial \hat{s}}$ goes to n_p. When $\hat{s}$ goes to $1/n_p$ the terms $\frac{\partial(1/\gamma_{i_p i_p})}{\partial \hat{s}}$, $(n-2)\frac{\gamma_{i_p j_p}}{\gamma_{i_p i_p}} \frac{\partial(\gamma_{i_p j_p}/\gamma_{i_p i_p})}{\partial \hat{s}}$, $\frac{\gamma_{i_c i_p}}{\gamma_{i_p i_p}} \frac{\partial(\gamma_{i_c i_p}/\gamma_{i_p i_p})}{\partial \hat{s}}$ converges to a final number. However the terms $\frac{\partial(1/\gamma_{i_c i_c})}{\partial \hat{s}}$ and $\frac{\gamma_{i_p i_c}}{\gamma_{i_c i_c}} \frac{\partial(\gamma_{i_p i_c}/\gamma_{i_c i_c})}{\partial \hat{s}}$ both converge to $+\infty$. Hence

$$\begin{aligned}\text{sign} \lim_{\hat{s} \to 1/n_p} \frac{\partial W(s)}{\partial \hat{s}} &= = \lim_{\hat{s} \to 1/n_p} \frac{\partial(1/\gamma_{i_c i_c})}{\partial \hat{s}} - n_p \frac{\gamma_{i_p i_c}}{\gamma_{i_c i_c}} \frac{\partial(\gamma_{i_p i_c}/\gamma_{i_c i_c})}{\partial \hat{s}} \\ &= \lim_{\hat{s} \to 1/n_p} \left[\frac{n_p[1-2s+n_p s^2]}{[1-n_p \hat{s}]^2} - \frac{(1-s)n_p s}{1-n_p s} \frac{[1-2s+n_p s^2]}{[1-n_p s]^2}\right] \\ &= \lim_{\hat{s} \to 1/n_p} \frac{[1-2s+n_p s^2] n_p}{[1-n_p s]^2} \left[1 - \frac{(1-s)s}{1-n_p s}\right] < 0\end{aligned}$$

■

Proof of Proposition 5: Rewriting the objective function of the planner (11) we obtain that

$$W(S) = r \sum_{i \in \mathcal{N}} w_i + \sum_{i \in \mathcal{N}} \beta_i (\mu_i - r) - \frac{\alpha}{2} \sum_{i \in \mathcal{N}} \beta_i \sigma_i^2 A_i, \tag{17}$$

where $A_i \equiv \sum_{j \in \mathcal{N}} \gamma_{ji}^2$. Suppose the optimum is interior. Then, under the assumption that projects are independent, we obtain that for every $i \in \mathcal{N}$, the first order condition is

$$(\mu_i - r) - \sigma_i^2 \beta_i \alpha A_i = 0. \tag{18}$$

We obtain that the optimal level of investment of the social planner is, for every i,

$$\beta_i^P = \min\left[w_i, \hat{\beta}_i \frac{1}{A_i}\right]. \tag{19}$$

■

Proof of Proposition 6. Recall that, given a network S, the cost of decentralization is

$$K(S)=\frac{(\mu-r)^2}{\alpha\sigma^2}\left[\sum_{j\in\mathcal{N}}\left(\frac{1}{\sum_{l\in\mathcal{N}}\gamma_{lj}^2}-\frac{1}{\gamma_{jj}}\right)-\frac{1}{2}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}}\gamma_{ij}^2\left(\frac{1}{(\sum_{l\in\mathcal{N}}\gamma_{lj}^2)^2}-\frac{1}{\gamma_{jj}^2}\right)\right].$$

First, using the approximation of γ_{ij} we derive

$$\begin{aligned}\sum_{h=1}^{n}\gamma_{hj}^2 &\simeq \sum_{h\neq j}^{n}\left(s_{hj}+\sum_p s_{hp}s_{pj}-s_{hj}\eta_j^{in}\right)^2+\left(1-\eta_j^{in}+\sum_p s_{jp}s_{pj}\right)^2\\ &\simeq 1-2\eta_j^{in}+2\sum_p s_{jp}s_{pj}+\left(\eta_j^{in}\right)^2+\left(\sum_h s_{hj}^2\right).\end{aligned}$$

Therefore

$$\begin{aligned}\frac{1}{\sum_{h=1}^{n}\gamma_{hj}^2} &\simeq 1+2\eta_j^{in}-2\sum_p s_{jp}s_{pj}-\left(\eta_j^{in}\right)^2-\left(\sum_h s_{hj}^2\right)+4\left(\eta_j^{in}\right)^2\\ &= 1+2\eta_j^{in}-2\sum_p s_{jp}s_{pj}-\left(\sum_h s_{hj}^2\right)+3\left(\eta_j^{in}\right)^2,\end{aligned}$$

and

$$\frac{1}{\gamma_{jj}}\simeq 1+\eta_j^{in}-\sum_p s_{jp}s_{pj}+\left(\eta_j^{in}\right)^2.$$

We obtain:

$$\frac{1}{\sum_{h=1}^{n}\gamma_{hj}^2}-\frac{1}{\gamma_{jj}}\simeq \eta_j^{in}-\sum_p s_{jp}s_{pj}+2\left(\eta_j^{in}\right)^2-\left(\sum_h s_{hj}^2\right).$$

On the other hand

$$\begin{aligned}\frac{1}{\left(\sum_{p=1}^{n}\gamma_{pj}^2\right)^2}-\frac{1}{\gamma_{jj}^2} &\simeq \left[1+2\eta_j^{in}-2\sum_p s_{jp}s_{pj}-\left(\sum_h s_{hj}^2\right)+3\left(\eta_j^{in}\right)^2\right]^2\\ &\quad-\left[1+\eta_j^{in}-\sum_{p\neq j} s_{jp}s_{pj}+\left(\eta_j^{in}\right)^2\right]^2\\ &\simeq 2\eta_j^{in}-2\sum_p s_{jp}s_{pj}+7\left(\eta_j^{in}\right)^2-2\left(\sum_h s_{hj}^2\right).\end{aligned}$$

Then

$$\sum_i \sum_j \gamma_{ij}^2 \left(\frac{1}{\left(\sum_{p=1}^n \gamma_{pj}^2\right)^2} - \frac{1}{\gamma_{jj}^2} \right) \simeq \sum_i \sum_{j \neq i} \left(s_{ij} + \sum_p s_{ip} s_{pj} - s_{ij} \eta_j^{in} \right)^2 \left(\frac{1}{\left(\sum_p \gamma_{pj}^2\right)^2} - \frac{1}{\gamma_{jj}^2} \right),$$

and after some algebra we obtain

$$\sum_i \sum_j \gamma_{ij}^2 \left(\frac{1}{\left(\sum_{p=1}^n \gamma_{pj}^2\right)^2} - \frac{1}{\gamma_{jj}^2} \right) \simeq \sum_i \left[2\eta_i^{in} - 2 \sum_p s_{ip} s_{pi} + 3(\eta_i^{in})^2 - 2 \left(\sum_h s_{hi}^2 \right) \right].$$

Putting together these expressions we get an expression for the cost of decentralization:

$$\begin{aligned} \frac{K(S)}{\frac{(\mu - r)^2}{\alpha \sigma^2}} &\simeq \sum_j \left[\eta_j^{in} - \sum_p s_{jp} s_{pj} + 2\left(\eta_j^{in}\right)^2 - \left(\sum_h s_{hj}^2 \right) \right] - \\ &- \frac{1}{2} \sum_j \left[2\eta_j^{in} - 2 \sum_p s_{jp} s_{pj} + 3\left(\eta_j^{in}\right)^2 - 2 \left(\sum_h s_{hj}^2 \right) \right] \\ &= \frac{1}{2} \sum_j \left(\eta_j^{in}(S) \right)^2. \end{aligned}$$

It is now straightforward to complete the proof of Proposition 6. ■

Proof of Proposition 7:

First-best design problem. We start by considering the first best design problem. Substituting in expression (17) the centralised solution $\beta^P = \{\beta_1^P, ..., \beta_n^P\}$, we obtain that

$$W(S, \beta^P) = r \sum_{i \in \mathcal{N}} w_i + \frac{1}{2} \sum_{i \in \mathcal{N}} \hat{\beta}_i (\mu_i - r) \frac{1}{A_i}$$

Recall that $A_i = \sum_{j \in \mathcal{N}} \gamma_{ji}^2$ and therefore A_i only depends on $\{\gamma_{1i}, ..., \gamma_{ni}\}$. Moreover, if we fix i, the expresson

$$\hat{\beta}_i (\mu_i - r) \frac{1}{A_i}$$

is declining in A_i. Next note that if, for some i, $\gamma_{li} > \gamma_{ki}$ for some $l \neq i$ and $k \neq i$, then, we can always find a small enough $\epsilon > 0$ so that, by making the local change $\gamma'_{li} = \gamma_{li} - \epsilon$ and $\gamma'_{ki} = \gamma_{ki} + \epsilon$, we strictly decrease A_i, without altering A_j for all $j \neq i$. Hence, such a local change strictly increases welfare. This implies that at the optimum $\gamma_{li} = \gamma_{ki}$ for all $l, k \neq i$. Set $\gamma_{li} = \gamma_{ki} = \gamma$; hence,$\gamma_{ii} = 1 - (n-1)\gamma$. Then, W is maximized when A_i is minimized, or, equivalently, γ minimizes

$$(n-1)\gamma^2 + [1 - \gamma(n-1)]^2$$

which implies that $\gamma = 1/n$. Note that Γ such that $\gamma_{ij} = 1/n$ for all i and for all j is obtained when S is complete and $s_{ij} = 1/(n-1)$ for all i and for all $j =\neq i$.

Second-best design problem. Note that by setting $s_{ij} = 1/(n-1)$ for all $i \neq j$, we obtain that $\gamma_{ij} = 1/n$ for all i, j and that, as a consequence β^* coincides with the socially optimal choice. Hence, the planner can replicate the firs best outcome just by setting $s_{ij} = 1/(n-1)$ for all $i \neq j$. ■

Proof Proposition 8. Recall that

$$U_i(\beta_i, \beta_{-i}) = \sum_{j \in \mathcal{N}} \gamma_{ij}(wr + \beta_j(\mu_i - r)) - \frac{\alpha}{2} \sum_{j \in \mathcal{N}} \sum_{j' \in \mathcal{N}} \gamma_{ij}\beta_j\gamma_{ij'}\beta_{j'}\sigma^2_{jj'}, \tag{20}$$

and note that $U_i(\beta_i, \beta_{-i})$ is continuous in (β_i, β_{-i}) and it is concave in β_i. Moreover, the strategy space is from a convex and bounded support. Hence, existence follows from Rosen 1965.

The sufficient condition for uniqueness also follows from Rosen 1965. For some positive vector r, let $g(\beta, r)$ be a vector where element i is $r_i \frac{\partial U_i}{\partial \beta_i}$. Let $G(\beta, r)$ be the Jacobian of $g(\beta, r)$. Rosen (1965) shows that a sufficient condition for uniqueness is that there exists a positive vector r such that for every β and β' the following holds

$$(\beta - \beta')^T g(\beta', r) + (\beta' - \beta)^T g(\beta, r) > 0.$$

Moreover, a sufficient condition for the above condition to hold is that there exists a positive vector r such that the symmetric matrix $G(\beta, r) + G(\beta, r)^T$ is negative definite. In our case, by fixing r to be the unit vector, we have that

$$G(\beta, 1) + G(\beta, 1)^T = -\alpha[\Gamma + \Gamma^T] \circ \Omega$$

So, it is sufficient to show that $[\Gamma + \Gamma^T] \circ \Omega$ is positive definite. It is well known that the Hadamard product of two positive definite matrix is also a positive definite matrix. Since Ω is positive definite, it is sufficient to show that $[\Gamma + \Gamma^T]$ is positive definite. Since the sum of positive definite matrix is a positive definite matrix, it is sufficient to show that Γ is positive definite. The condition that $\sum_j s_{ij} < 1/2$, implies that Γ is a strictly diagonally dominant, and therefore positive definite.

The characterization of an interior equilibrium follows by taking the FOCs. It remains to show that there exists a $\bar{s} > 0$ so that if $||S||_{max} < \bar{s}$ the equilibrium is interior. For this note that taking the derivative of $U_i(\beta_i, \beta_{-i})$ with respect to β_i we have

$$\frac{\partial U_i(\beta_i, \beta_{-i})}{\partial \beta_i} = \gamma_{ii}\left[(\mu_i - r) - \alpha \sum_j \gamma_{ij}\beta_j\sigma_{ji}^2\right]. \tag{21}$$

If the equilibrium is non-interior, then there exists a i with $\beta_i = 0$ which implies that

$$(\mu_i - r) - \alpha \sum_{j \neq i} \gamma_{ij}\beta_j\sigma_{ji}^2 \leq 0,$$

but as we take $||S||_{\max}$ smaller and smaller we have that $\sum_{j\neq i} \gamma_{ij}\beta_j\sigma_{ji}^2$ becomes as small as we wish and therefore we get a contradiction (we can do that because we can make each γ_{ij} small enough for each $i \neq j$ and because β_j is bounded above by w).

∎

Proof of Proposition 9 The characterization of equilibrium behavior follows immediately from Proposition 8 by setting $\sigma_{ij}^2 = \sigma^2$ for all i, j. It is immediate to check that in an interior equilibrium $\sum_i \beta_i = n\frac{\mu - r}{\alpha\sigma^2}$. Furthermore, the condition for interior equilibrium follows from inspection of expression (15). We next derive the expression for the equilibrium expected utility of player i. Recall that $E[V_i] = wr\sum_j \gamma_{ij} + (\mu - r)\sum_j \gamma_{ij}\beta_j$; in an interior equilibrium we have that, for every i, $\mu - r - \alpha\sigma^2 \sum_j \gamma_{ij}\beta_j = 0$, and, therefore,

$$E[V_i] \quad = \quad wr\sum_j \gamma_{ij} + \frac{(\mu - r)^2}{\alpha\sigma^2}.$$

Similarly, $Var[V_i] = \sigma^2\left[\sum_j \gamma_{ij}\beta_j\right]^2$, and using the equilibrium conditions we have that:

$$Var[V_i] \quad = \quad \frac{(\mu - r)^2}{\alpha^2\sigma^2}.$$

The equilibrium expected utility of i is therefore

$$\begin{aligned}E[U_i] &= wr\sum_j \gamma_{ij} + \frac{(\mu-r)^2}{\alpha\sigma^2} - \frac{\alpha}{2}\frac{(\mu-r)^2}{\alpha^2\sigma^2}\\ &= wr\sum_j \gamma_{ij} + \frac{1}{2}\frac{(\mu-r)^2}{\alpha\sigma^2}.\end{aligned}$$

This concludes the proof of Proposition 9. ■

Proof of proposition 10. Part 1 follows by noticing that the equilibrium expected utility of i is

$$\begin{aligned}U_i(\beta^*, S) &= wr\sum_j \gamma_{ij} + \frac{(\mu-r)^2}{\alpha\sigma^2} - \frac{\alpha}{2}\frac{(\mu-r)^2}{\alpha^2\sigma^2}\\ &= wr\sum_j \gamma_{ij} + \frac{1}{2}\frac{(\mu-r)^2}{\alpha\sigma^2}.\end{aligned}$$

Part 2 follows by noticing that $\sum_i U_i(\beta^*, S)$ is independent of S.

Finally, we prove part 3. First we assume that the socially optimal is interior and show that it coincides with the equilibrium behavior. We then show that the social welfare is concave in β. Under perfect positive correlation, we can write the social welfare as

$$W(\beta_i, \beta_{-i}) = nwr + \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}} \gamma_{ij}\beta_j(\mu - r) - \frac{\alpha\sigma^2}{2}\sum_{i\in\mathcal{N}}\left[\sum_{j\in\mathcal{N}}\gamma_{ij}\beta_j\right]^2.$$

Taking the derivative with respect to β_l we have

$$\begin{aligned}\frac{dW}{d\beta_l} &= \sum_{i\in\mathcal{N}}\left[(\mu-r)\gamma_{il} - \alpha\sigma^2\gamma_{il}\sum_{j\in\mathcal{N}}\gamma_{ij}\beta_j\right]\\ &= \mu - r - \alpha\sigma^2\sum_{i\in\mathcal{N}}\gamma_{il}\sum_{j\in\mathcal{N}}\gamma_{ij}\beta_j,\end{aligned}$$

and given the assumption that the optimum is interior, we have that for every l it must to hold

$$\sum_{i\in\mathcal{N}}\gamma_{il}\sum_{j\in\mathcal{N}}\gamma_{ij}\beta_j = \frac{\mu - r}{\alpha\sigma^2}.$$

Note that the equilibrium solution derived in Proposition 9 also solves the above problem.

Indeed, the equilibrium solution has the property that, for every i, $\sum_j \gamma_{ij}\beta_j = \frac{\mu - r}{\alpha\sigma^2}$. It is also immediate to see that there is a unique solution to the above linear system.

We now show that the social welfare is concave in β. To see this note that the Hessian is $-\sigma^2\alpha\Gamma^T\Gamma$, and therefore we need to show that $\Gamma^T\Gamma$ is positive definite. This is true because

$$x^T\Gamma^T = \{x_1\gamma_{11}, .., x_n\gamma_{1n}; ....; x_1\gamma_{n1}, ..., x_n\gamma_{nn}\} \text{ and } \Gamma x = (x^T\Gamma^T)^T$$

and so

$$x^T\Gamma^T\Gamma x = [x_1\gamma_{11} + .. + x_n\gamma_{1n}]^2 + ... + [x_1\gamma_{n1}, ..., x_n\gamma_{nn}]^2 > 0,$$

where the strict inequality follows because $x_i \neq 0$ for some i and because for each i, $\gamma_{ij} > 0$ for some j. ■

For Online Pubblication: Ownership and Control

We discuss the case in which ownership leads to control in decision making. For simplicity we assume that $w_i = w$, $\mu_i = \mu$ and $\sigma_i^2 = \sigma^2$ for all $i \in \mathcal{N}$. We assume that γ_{ij} signifies that individual i has control over a percentage γ_{ij} of j's initial endowment w. That is, individual i unilaterally decides the investment of $\gamma_{ij}w$. We propose two natural scenario and study the consequences for optimal risk taking.

First Scenario. Assume that individual i can invest $\gamma_{ij}w$ in the risk-free asset or in risky project i. In this case, $\gamma_{ij}w$ is a transfer from j to i that occurs before shocks' realization. Hence, Γ simply re-defines initial endowment of individuals: if we start from a situation where $w_i = w$ for all i, then Γ leads that a new distribution $\hat{w} = \{\hat{w}_1, ..., \hat{w}_n\}$ of endowment, where $\hat{w}_i = \sum_j \gamma_{ij}w$. Since, under mean-variance preferences, initial endowment doest not affect the portfolio choice of an individual (as long as the solution is interior), the network S plays no major role in the analysis.

Second Scenario. Suppose that individual i can invest $\gamma_{ij}w$ in the risk free asset or in risky project j. This is a model where γ_{ij} conveys control to i over $\gamma_{ij}w$, but the control is local, in the sense that individual i can only invest $\gamma_{ij}w$ in risky project j. In this case, individual i chooses $\beta_i = \{\beta_{i1}, ..., \beta_{in}\}$, where β_{ij} is the investment in risky project j of endowment $w_{ij} = \gamma_{ij}w$. Of course, $\beta_{ij} \in [0, \gamma_{ij}w]$. It is immediate that individual i's optimal investment is

$$\beta_{ij} = \max \in \left\{ \gamma_{ij} w, \frac{(\mu - r)}{\alpha \sigma_j^2} \right\}.$$

For a given Γ, takes w sufficiently high so that β_{ij} is interior for all ij. This is always possible. Then we can calculate sum of utilities

$$U_i = wr \sum_j \gamma_{ij} + \frac{1}{2} \frac{(\mu - r)^2}{\alpha} \sum_j \frac{1}{\sigma_j^2},$$

and therefore

$$\sum_i U_i = n \left[wr + \frac{1}{2} \frac{(\mu - r)^2}{\alpha} \sum_j \frac{1}{\sigma_j^2} \right].$$

We now observe that this outcome is equivalent to the outcome choosen by a planner with mean variance utility with regard to aggregate output. Indeed,

$$V_i = \sum_j \gamma_{ij} wr + \sum_j \gamma_{ij} \beta_{ij} (z_j - r),$$

and

$$V = nwr + \sum_i \sum_j \gamma_{ij} \beta_{ij} (z_j - r).$$

the optimal investment plan of the planner that maximizes $E[V] - \frac{\alpha}{2} Var[V]$ is then the same as the decentralized solution derived above.

10 References

1. Acemoglu, D., A. Ozdaglar, and A. Tahbaz-Salehi (2015), Systemic Risk and Stability in Financial Networks. *American Economic Review*, 105, 2, 564-608.

2. Acharya, V.V., Pedersen, L.H., T., and Richardson, M. (2010), Measuring systemic risk, NYU Wprking paper.

3. Afonso, G. and R. Lagos (2012), Trade dynamics in the market for federal funds. *Federal Reserve of New York Staff Report.*

4. Allen, F. and D. Gale (2000), Financial contagion, *Journal of Political Economy*, 108, 1, 1-33.

5. Arlotto, A., and M. Scarsini (2009), Hessian orders and multinomial distributions, *Journal of multivariate analysis*, 100, 2324-30.

6. Babus, A. (2015), The formation of financial networks. *Rand Journal of Economics*, forthcoming.

7. Bech, M. L. and E. Atalay (2010), The topology of the federal funds market. *Physica A: Statistical Mechanics and its Applications*, 389, 22, 5223-5246.

8. Belhaj, M. and F. Deroian (2012), Risk taking under heterogenous revenue sharing, *Journal od Development Economics*, 98, 192-202.

9. Berle, A. and G. Means (1932), *The Modern Corporation and Private Property*, Transaction Publishers, New York.

10. Bisias, D., M. Flood, A. W. Lo and S.tavros Valavanis, (2012), A Survey of Systemic Risk Analytics, Working-paper, US, Department of Treasury, Office of Financial Research.

11. Bramoulle and Kranton (2015), Games played on networks, to appear in Y. Bramoulle, A. Galeotti and B. Rogers (eds) *Oxford Handbook on Economics of Networks.*

12. Brioschi, F., L. Buzzacchi, and M. M. Colombo (1989), Risk Capital Financing and the Separation of Ownership and Control in Business Groups, *Journal of Banking and Finance*, 13, 747-772.

13. Cabrales, A., D. Gale, P. Gottardi (2015), Financial Contagion in Networks, in *The Oxford Handbook in the Economics of Networks*, edited by Y. Bramoulle, A. Galeotti and B. Rogers.

14. Cabrales, A., P. Gottardi, and F. Vega-Redondo (2013), Risk-sharing and Contagion in Networks, *Working Paper, EUI.*

15. Cifuentes, R., G. Ferrucci, and H. S. Shin (2005), Liquidity risk and contagion, *Journal of the European Economic Association*, 3, 556-566.

16. Craig, B. and G. von Peter (2014), Interbank tiering and money center banks, *Journal of Financial Intermediation*, 23, 3, 322-347.

17. Eisenberg, L. and T. H. Noe (2001), Systemic Risk in Financial Systems, *Management Science*, 47, 236-249.

18. Elliott, M., B. Golub, and M. O. Jackson (2014), Financial Networks and Contagion, *American Economic Review*, 104, 10, 3115-3153.

19. Elliott, M. and J. Hazell (2015), Endogenous Financial Networks: Efficient Modularity and Why Shareholders Prevent It, Working Paper SSRN-id2658249.

20. Farboodi, M (2014), Intermediation and voluntary exposure to counter-party risk, *Working Paper*, University of Chicago.

21. Fedenia, M., J. E. Hodder, and A. J. Triantis (1994), Cross-Holdings: Estimation Issues, Biases, and Distortions, *The Review of Financial Studies*, 7, 61-96.

22. Gai, P. and S. Kapadia (2010), Contagion in financial networks, *Proceedings of the Royal Society A*, 466, 2401-2423.

23. Gollier, C. (2001), *The Economics of Risk and Time*, MIT Press.

24. Greenwood, R. A. Landier, and D. Thesmar (2015), Vulnerable Banks, *Journal of Financial Economics*, forthcoming.

25. Hart, O.D. and D.M. Jaffee (1974), On the application of portfolio theory to depository financial intermediaries, *Review of Economic Studies*, 41, 129-147.

26. Jackson, M.O. and Y. Zenou (2015), Games on Networks, in P. Young and S. Zamir (eds), *Handbook of Game Theory Vol. 4*, Amsterdam: Elsevier Publisher.

27. Kim, D. and A.M. Santomero (1988), Risk in banking and capital regula- tion, *Journal of Finance*, 43. 1219-1233.

28. Koehn, H. and A.M. Santomero (1980), Regulation of bank capital and portfolio risk, *Journal of Finance* 35, 1235-1244.

29. Kose, M. A., E. Prasad, K. Rogoff and S-J Wei (2006), Financial globalization: a reappraisal, *NBER Working Paper 12484*

30. Lane, P. and G M Milesi-Ferretti (2003) International Financial Integration, *IMF Staff Papers*, Volume 50, Special Issue.

31. Meyer, M.A. and B. Strulovici (2012), Increasing interdependence of multivariate distributions, *Journal of Economic Theory*, 147, 1460-89.

32. Meyer, M.A. and B. Strulovici (2013), The super modular stochastic ordering, Work in paper.

33. Van Lelyveld I., and D. in t' Veld (2012), Finding the core: Network structure in interbank markets. *DNB Working Paper No. 348.*

34. Martinez-Jaramillo, S., B. Alexandrova-Kabadjova, B. Bravo-Benitez, J. P. Solorzano-Margain (2014), An empirical study of the Mexican banking systems network and its implications for systemic risk, *Journal of Economic Dynamics and Control*, 40, 242-265.

35. Pyle. D.. 1971. On the theory of financial intermediation, *Journal of Finance*, 26, 734-747.

36. Soramaki, K., M. L. Bech, J. Arnold, R. J. Glass, and W. E. Beyeler (2007), The topology of interbank payment flows, *Physica A*, 379, 317-333.

37. van Lelyveld, I. and D. in 't Veld (2012), Finding the core: Network structure in interbank markets, *DNB Working Paper No. 348.*

38. Vitali, S., J. B. Glattfelder, and S. Battiston (2011), The network of global corporate control, mimeo, ETH Zurich