PART II

- Computing default probability in system context
- stochastic, dynamic
- ex-ante

Tasca, P. and Battiston, S., Diversification and Financial Stability. ETH Risk Cent. Work. Pap. Ser. ETH-RC-12-013 (2012).

Model Contributions - Resilience of financial architectures

- balance-sheet approach to default (Eisenberg-Noe 2001)
- systemic defaults: comparative static of different architecture (e.g. density), no ex-ante optimization of links
  - intermediate density is bad: Gai-Kapadia 2010, Gai-Kapadia-Haldane 2011
  - intermediate density is good: Battiston ea. 2012a-b, Roukny ea. 2013

- This paper:
  - incorporates external assets.
  - no cascade of defaults, propagation of distress based on balance-sheet Merton-like approach

Contributions - Risk diversification

- quantification of benefits of risk diversification (Markowitz (1952), Tobin (1958) and Samuelson (1967))
- diversification can have undesired effects, several works and several mechanisms: (Goldstein and Pauzner, 2004; Ibragimov and Walden, 2007; Brock et al., 2009; Wagner, 2009; Ibragimov et al., 2011; Wagner, 2011; Stiglitz, 2010).

This paper:

- No friction, no amplification.
- The mere presence of a future (positive/negative) trend implies tradeoff on optimal density

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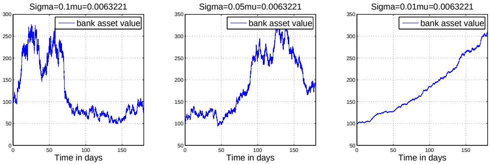
The mechanism at work

Banks diversification in external assets decreases the likelihood of the banking system to escape from the economic trend.



In uptrend periods diversification is likely to be good

- **highlights the positive trend.**
- **increases the time to default**

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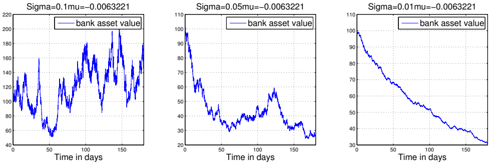

In downtrend periods diversification is likely to be bad

- **highlights the negative trend.**
- **decreases the time to default**

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Contributions - Ex-ante default probability

- Most works on defaults in financial networks are based on **ex-post** approach (Eisenberg-Noe 2001)
- Given a network of interlocked balance sheet and a shock, work out recursively who defaults next and how much are worth the assets of surviving banks.
  - Cifuentes ea. 2005; Elsinger ea. 2006; Gai-Kapadia 2010; Battiston ea. 2012 (DebtRank)

This paper:

- Make steps towards computing **ex-ante** default probability
- Note that in most works based on Merton-approach banks are in isolation
- Ex-ante default probability is useful in all situations where liability structure is affected by events other than default
- Issue deserving attention but only few works: Kubler ea. 2003; Ota 2013

In a nutshell

Model

- combining balance-sheet approach and continuous time stochastic approach
- interbank market + external assets
- dynamics of fragility is driven by external asset prices
- fragility of each bank depends on fragility of all other banks
- default probability of each bank depends on default probability of all other banks
- no optimization on linkages (similar in spirit to Gai ea. 2011)

Results

- optimal portfolio diversification is intermediate, in the presence of uncertain trend
- other exercises are possible

Perspectives

In Tasca-Battiston 2012b, **Market Procylicality**:

- we connect price dynamics to leverage dynamics
- we compute how systemic default probability depends on market impact and bank sensitivity to capital requirements.

In progress, Battiston, Tasca, Stiglitz:

- 2 stage framework, fix-point approach to compute $P_i = f(P_1, ..., P_j, ..., P_N)$
- we find multiple equilibria, sensitivity to parameters
- derivatives exacerbate uncertainty

Balance Sheet approach: m External, n Interbank Assets

- **Economy**: **n** risk-averse **leveraged** banks with **m** external assets and **n** interbank asset
- **Balance-Sheet identity**:

$$\mathbf{a}_i := \sum_k^m \mathbf{Z}_{ik}\mathbf{y}_k + \sum_j^n \mathbf{W}_{ij}\mathbf{l}_j$$

  - $\mathbf{l}_j := \bar{\mathbf{l}}_j/[(1+r_j)]^{-t}$: market value of bank j's debt;
  - $\mathbf{y_k}$: price of external asset-k;
  - $\mathbf{Z}_{n\times m}$: exposures matrix to external assets; z_{ik} relative exposure of i to asset-k
  - $\mathbf{W}_{n\times n}$: exposures matrix to interbank assets; w_{ij} relative exposure of i to bank-j
  - r_j: discount rate for obligor-j's debt.

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Leverage

- **Leverage** meant as: face-value of debt/market value of asset

$$\phi_i := \bar{\mathbf{l}}_i./\mathbf{a}_i = \bar{\mathbf{l}}_i / \left(\sum_j^m \mathbf{Z}_{ik}\mathbf{y}_k + \sum_j^n \mathbf{W}_{ij}\bar{\mathbf{l}}_j/[(1+r_j)]^{-t} \right)$$

Objective: study process of ϕ_i to determine probability of default: $P_i = P\{\phi_i > 1\}$.

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- In principle: r_j function of default probability of bank j, which is what we want to determine...
- In on-going work: we explore endogenous and simultaneous determination of vector of default probability P
- Here: first order approximation: $r_j = r_j(\phi_j) \approx r_f + \beta\phi_j$

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Leverage Dynamics

$$\phi_i := \bar{\mathbf{l}}_i./\left(\sum_k^m \mathbf{Z}_{ik}\mathbf{y}_k + \sum_j^n \mathbf{W}_{ij}\frac{\bar{\mathbf{l}}_j}{(1+r_f+\beta\phi_j)}\right)$$

- External assets: $B_k(t)$ standard Brownian, i.i.d. returns

$$dy_k(t) = \mu_k dt + \sigma_k dB_k(t) \quad \forall k = 1, ..., m$$

Exercises:

- Compute (numerically or analytically) **ex-ante** default probability and **ex-ante** time to default
- How does P depends on network structure? (e.g. density of interbank, density of external)

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Mean Field Analysis

- Assume individual behaves as the mean (yet there is interaction!):
  $\bar{l}_i = \bar{l} \quad \forall i = 1, \ldots n$; $\phi_i = \phi \quad \forall i = 1, \ldots n$; y external asset portfolio.
- Fragility is the solution of a quadratic equation:

$$\phi = \frac{\bar{l}}{y + l} = \frac{\bar{l}}{y + \frac{\bar{l}}{1 + r_f + \beta\phi}}$$

$$\phi = \frac{1}{2\beta y}\left[\bar{l}(\beta - 1) - Ry + \left(4\beta\bar{l}Ry + \left(\bar{l}(1 - \beta) + Ry\right)^2\right)^{1/2}\right]$$

  where $R = 1 + r_f$.
- Fragility depends on portfolio value y

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Dynamics of external assets cash-flow

- Each external asset:

$$dy_k(t) = \mu_k dt + \sigma_k dB_k(t) \quad \forall k = 1, ..., m$$

  - $B_k(t)$ is standard Brownian;
  - i.i.d. returns and $d(B_k, B_l) = \rho_{kl}$.
- By the M-F approx., $y = \frac{1}{m} \sum_j^m y_k$. Then,

$$dy = \mu_y \, dt + \sigma_y \, dB$$

where

  - $\mu_y := \frac{1}{m} \sum_{k=1}^m \mu_k$;
  - $\sigma_y := \sqrt{\frac{\sigma^2}{m} + \frac{m-1}{m} \bar{\rho} \sigma^2}$

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Default Probability

Proposition

- **Default Probability:** $\mathbb{P}(\phi = b_\phi)$.
  Probability that fragility ϕ, *with* $\phi_0 \in (0 \leq a_\phi, b_\phi \leq 1)$ *exits through* b_ϕ*;*
- **Default Probability:** $\mathbb{P}(y = a_y)$.
  - *explicit form*

$$\mathbb{P}(y = a_y) := \frac{\left(\int_{y_0}^{b_y} dy\psi(y)\right)}{\left(\int_{a_y}^{b_y} \mathrm{d}y\psi(y)\right)}; \quad \psi(x) = \exp\left(\int_0^x -\frac{2\mu_y}{\sigma_y^2}\,\mathrm{d}y\right)$$

  - *closed form solution*

$$\mathbb{P}(y = a_y) = \left(\exp\left(-\frac{2\mu_y y_0}{\sigma_y^2}\right) - \exp\left(-\frac{2\mu_y b}{\sigma_y^2}\right)\right) / \left(\exp\left(-\frac{2\mu_y a}{\sigma_y^2}\right) - \exp\left(-\frac{2\mu_y b}{\sigma_y^2}\right.\right.$$

Let define

- $p := \mathbb{P}(\mu_y \leq 0)$, probability of having a positive trend
- $q := \mathbb{P}(y = a_y \mid \mu_y < 0)$, probability of having a default in the case of a negative trend;
- $g := \mathbb{P}(y = a_y \mid \mu_y > 0)$, probability of having a default in the case of a positive trend;.

Proposition (3)

The difference btw default probability in the positive and negative trend tends to one for large portfolio size m:

$$\exists \quad m^* > 1 \mid (q - g) > 1 - \epsilon \quad \forall m > m^*$$

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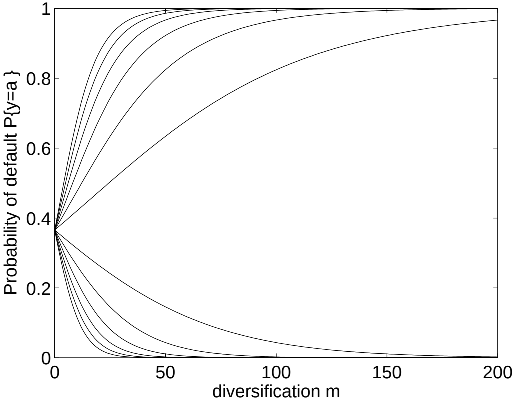

Figure: The lower curves shows the variation of $\mathbb{P}(y = a_y)$ when $\mu_y > 0$ for increasing degree of diversification m. The upper curves shows the variation of $\mathbb{P}(y = a_y)$ when $\mu_y < 0$ for increasing degree of diversification m.

Definition (**Bank's Maximization Problem**)

banks are mean-variance decision makers, such that the utility function $\mathbb{E}U(\Pi_m)$ may be written as a smooth function $V\left(\mathbb{E}(\Pi_m), \sigma^2(\Pi_m)\right)$ of the mean $\mathbb{E}(\Pi_m)$ and the variance $\sigma^2(\Pi_m)$ of Π_m or

$$V\left(\mathbb{E}(\Pi_m), \sigma^2(\Pi_m)\right) := \mathbb{E}U(\Pi_m) = \mathbb{E}(\Pi_m) - \frac{\lambda\sigma^2(\Pi_m)}{2}$$

Then,

$$m* = m \text{ s.t.: } \max_m \mathbb{E}U(\Pi_m) \tag{1}$$

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Expected Profit:

$$\mathbb{E}(\Pi_m) := p\left[q\pi^- + (1-q)\pi^+\right] + (1-p)\left[g\pi^- + (1-g)\pi^+\right] \quad (2)$$

Variance:

$$\sigma^2(\Pi_m) := p\left[q\left(\pi^- - \mathbb{E}(\Pi_m)\right)^2 + (1-q)\left(\pi^+ - \mathbb{E}(\Pi_m)\right)^2\right] + (1-p)\left[g\left(\pi^- - \mathbb{E}(\Pi_m)\right)^2 + (1-g)\left(\pi^+ - \mathbb{E}(\Pi_m)\right)^2\right] \quad (3)$$

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Corollary (Optimal Level of Diversification m^*)

Let the event down-turn occurs with probability p, and the event up-turn occurs with probability $1-p$ where $p \in \Omega_P := [0,1]$. Then, there exists $\Omega_{P^*} \subset \Omega_P$ s.t., for a given $p^* \in \Omega_{P^*}$,

$$m^* = \left[(g^{-1}; q^{-1}) \circ f^{-1}\right](p^*) \Rightarrow \exists \quad \mathbb{E}U(\Pi_{m^*}) \geq \mathbb{E}U(\Pi_m) \quad \forall m \gtrless m^*$$

with $f := 1/\left(1 + \frac{q(m^*)}{g(m^*)}\left(\frac{\partial q(m^*)}{\partial g(m^*)}\right)\right)$.

For a fixed absolute value of $|\mu_y|$ we assign a probability p to the event down-turn and $1-p$ to the up-turn. Then, for certain values of p^*, there exist a specific optimal level of diversification m^* which is a function of p^* and maximizes $\mathbb{E}U(\Pi_m)$.

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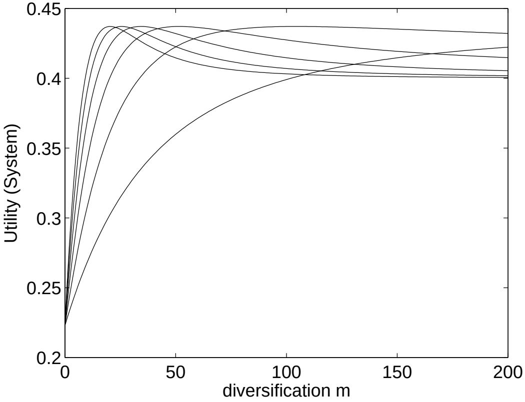

Figure: $\mathbb{E}U(\Pi_m)$ exhibits a maximum w.r.t. m for different levels of $p = 0.4$. Parameters: $\sigma^2 = 0.35$, $r_f = 0.03$, $\pi^- = \pi^+ = 1, \bar{\rho} = 0.1$, $m \in [0, 200]$, $|\mu_y| = 0.005, 0.01, 0.015, 0.020, 0.025, 0.030$.

Private Incentives Vs. Social Welfare

- For policy makers: **negative externalities** might be generated by losses occurred in the bad state of the world.
- Regulator can include **social costs** (e.g., unemployment) that might emerge due to the losses suffered by the financial system.
- Then for the regulator standpoint of view, the total loss to be accounted in bad states, is a monotonically increasing cost function of the amount of losses, e.g. at first order:

$$f(k, \pi^-) := k\pi^- \quad \text{with } k > 1$$

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Treating the policy-maker as an expected utility maximizer, it has an objective function $\mathbb{E}U_R(\Pi_m)$ which is differently expressed w.r.t. $\mathbb{E}U(\Pi_m)$. Explicitly, eq.(14) and (15) become

Expected Profit Regulator:

$$\mathbb{E}_R(\Pi_m) = p\left[qk\pi^- + (1-q)\pi^+\right] + (1-p)\left[g\pi^- + (1-g)\pi^+\right]$$

Variance Regulator:

$$\sigma_R^2(\Pi_m) = p\left[q\left(k\pi^- - \mathbb{E}(\Pi_m)\right)^2 + (1-q)\left(\pi^+ - \mathbb{E}(\Pi_m)\right)^2\right] + (1-p)\left[g\left(\pi^- - \mathbb{E}(\Pi_m)\right)^2 + (1-g)\left(\pi^+ - \mathbb{E}(\Pi_m)\right)^2\right]$$

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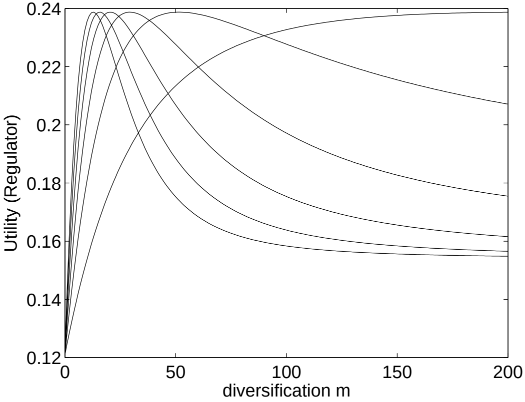
Corollary

Individual banks' incentives favor a financial network that is over-diversified in external assets w.r.t. to the level of diversification that is socially desirable

$$m^* \geq m^R$$



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Comments

- Mean-field approach is functional to have analytical results
- An agent-based version allows to study numerically
    - heterogeneous structures
    - effect of balance-sheet management strategy (see Tasca-Battiston 2012b)
    - feedback loops price-leverage (it is interesting to look at non-equilibrium, complementary to Cifuentes ea. 2005)
- Ex-ante default probability is useful in all situations where liability structure is affected by events other than default

Conclusions

1. **Contribution to the debate on**
   1. robust-yet-fragile properties of the financial system
   2. financial networks architecture and systemic risk
   3. ex-ante default probability
2. **Model layout**
   1. interconnected banks invest inf external asssets/projects
   2. External assets may generate positive or negative cash-flows
3. **Results**
   1. DP/Exp. Utiltiy increases (decrease) with diversification in case of down-turn (up-turn)
   2. Banks'incentives favor a financial network that is over-diversified w.r.t. the diversification that is socially desirable

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Recent Papers

- Debtrank: [Battiston, Puliga, Kaushik, Tasca, Caldarelli, DebtRank: Too-central-to-fail? (2012) *Sci. Rep.* 2:541]
- Complex derivatives [Battiston, Caldarelli, Georg, May, Stiglitz, *Nat. Phys.*, 2013]
- CDS and network reconstruction [Kaushik R., Battiston S., 2013 PLoS-ONE, forth], [Puliga M., Kaushik R., Battiston S., Caldarelli G., 2013 in progress]
- Estimation of systemic risk in networks from partial information: [Musmeci, Puliga, Gabrielli, Battiston, Caldarelli, JOSS 2013, forth.]
- Controllability in e-mid [Delpini, Battiston, Riccaboni, Pammolli, Gabbi, Caldarelli, *Sci. Rep.*, 2013, forth.]
- Controllability in TARGET2 [Galbiati, Delpini, Battiston, (2013) *Nat Phys*]

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Related International Activities

- **FOC** (Forecasting Financial Crises)
- MULTIPLEX (Theory of networks of networks)
- GSDP Global Systems and Policies
- GSS (Global Systems Science)
- SIMPOL (Financial Systems Simulation and Policy Modeling)
- INET - Systemic Risk Task Force: WG Fin. Nets