Endogenous Leverage and Asset Pricing in Double Auctions

Thomas Breuer, Martin Jandačka, Martin Summer, Hans Joachim Vollbrecht

Frontiers of Systemic Risk Forecasting Conference, LSE, November 28, 2013

Introduction

- We study the exchange and pricing of leveraged assets in an agent based model of a continuous double auction.
- Specifically we want to understand how the leverage that can be achieved in the market is determined and how leverage affects the prices of assets.
- There has recently been progress in general equilibrium theory that significantly contributes to a conceptual understanding of theses two questions.
- Like in an economic experiment we study these concepts via continuous double auctions to validate this theory.

The Bigger Picture: Leverage and Systemic Risk

A deeper understanding of the economics of leverage and collateralized lending is key to the understanding of the recent financial crisis and to financial crisis more generally because

- Higher leverage makes agents more sensitive to changes in asset prices.
- Leverage is intimately connected to asset price bubbles.
- Leverage is the key driver of systemic risk. It is at the root of problems related to
  - correlations of exposures
  - counterparty risk and contagion of insolvency
  - funding mechanisms
  - market driven contagion

Literature

- Leverage and Asset Pricing: While there is a traditional literature on collateralized lending like Kiyotaki and Moore (1997), Bernanke, Gertler, Gilchrist (1996) or Holmstrom and Tirole (1997), Shleifer and Vishny (1992), the General Equilibrium theory of leverage due to (Geanakoplos (1997), (2003), (2010), Geanakoplos and Zame (2010), Geanakoplos and Fostel (2008)) made two major advances:
  1. The theory explains how leverage is determined endogenously in equilibrium.
  2. The theory is a pure supply and demand theory with quantity constraints without informational, institutional or behavioral elements.
- Double Auctions: Game theory: Wilson, 1987, Easley and Ledyard 1993, Mertens 2003, Giraud, 2007. Experimental Economics: Smith, 2008, Sunder, 1995, Bossaerts, 2002.

Why Double Auctions?

- Continuous double auctions are competitive trading mechanisms that have been mainly studied in experimental economics.
- Experimentalists believe that the continuous double auction comes close to an environment which abstract equilibrium theories of competitive trading try to describe.
- It is therefore an interesting mechanism to produce evidence on whether the perspective offered by the theory is appropriate.
- It helps to build our intuition and knowledge about the scope and the limitation of the theory.

Findings: How well does the abstract theory predict the outcomes in the double auction?

1. While double auctions for competitive asset trading are known for a long time we show how a double auction can be extended to the case of asset trades involving leverage and possibly default.
2. The double auction confirms the pricing, allocation and leverage prediction of the theory very well in the case where riskless debt is available.
3. The same double auction fails to align with the predictions of theory when riskless debt is not available.
4. One can however find institutional amendments that improve flexibility of exchange in a way that restores the good match between double auction outcomes and theory predictions also in this case.

Findings: When does GE theory of leverage provide an appropriate perspective?

1. Our simulations provide evidence that the theory is an appropriate framework for studying trade in collateralized assets for certain markets like Repo-markets or some securitization markets.
2. The abstraction from institutional details, made by the theory, is not innocuous.
3. In particular the theoretical results about endogeneity of leverage and asset pricing are not robust with respect to these details.

Two Properties of Collateral Equilibrium

There are two key properties of collateral equilibrium emphasized by the theory:

1. In equilibrium all debt instruments available in the market will have a market price but not all instruments will be traded. There is endogenous contract selection by the market. In special cases this contract selection is unique. In this case competitive markets pin down an endogenous level of leverage.
2. In equilibrium assets that can be used as collateral for loans will earn a premium above its fundamental value, the present value of the dividend stream provided by these assets.

Theoretical Key Ideas: Endogenous Leverage

- In a simple two date finance model with uncertainty about the future state of the world a financial contract is a vector $V = (V_1, \ldots, V_S)$, where V_s is called the face value in state s.
- If collateral is added to this model the contract has to specify the face value and a collateral requirement in terms of a real asset or some durable consumption good. Say the real asset pays dividends $A = (A_1, \ldots, A_S)$ then a financial contract is a pair $\mathcal{V} = (V, c)$. This contract pays

$$\textsc{Del}\left((V, c), s\right) = \min\{V_s, A_s c\} \tag{1}$$

- Key idea: Contracts with different face value or different collateral requirement are economically different and have an individual price.

Theoretical Key Ideas: Asset Pricing

- In the standard theory of competitive financial markets the price of a financial contract with face value V is

$$q = E\left[\frac{\pi_s^i}{\pi_0^i} V_s\right] \tag{2}$$

- The price with collateral is however not

$$q = E[(\pi_s^i/\pi_0^i)\mathrm{DEL}((V, c), s)] \tag{3}$$

- because individuals are constrained in their choices today by collateral requirements of the form

$$c^j z_1^i \leq z_0^i \tag{4}$$

Theoretical Key Ideas: Asset Pricing

- Denote the multiplier of the collateral constraint by μ^i then the pricing equations for the collateral asset and the financial promise which finances this asset becomes

$$q_0 = E\left[\frac{\pi^i_s}{\pi^i_0} A^j_s\right] + \frac{1}{\pi^i_0}\mu^i \tag{5}$$

$$q_1 = E\left[\frac{\pi^i_s}{\pi^i_0} \mathrm{DEL}\left((V^j, c^j), s\right)\right] + \frac{1}{\pi^i_0} c^j \mu^i \tag{6}$$

- In general both the asset price and the price of the financial instrument will be distorted: There is a collateral premium and a liquidity wedge.

A Simple Benchmark Example

- Since we study trading of leveraged assets via an agent based double auction we need a particular parametrization. We use a simple example due to Geanakoplos 2010.
- Continuum of consumers $I = [0, 1]$, two states $S = \{U, D\}$. The index i is identified with the probability that the agent assigns to the up state U, so agents with a higher i are more optimistic. Endowments are $\omega^i = (1, 0, 0)$ and $\delta^i = 1$ for all agents.
- Linear preferences given by $u^i(x^i) = x_0^i + i\, x_U^i + (1 - i)\, x_D^i$.
- One real asset $A = (1, 0.2)$ and two debt contracts with $V^1 = (0.2, 0.2)$ and $V^2 = (0.5, 0.5)$.

How Does the Double Auction Work?

- Endowments, assets, financial contracts and preferences are parametrized as in the theoretical benchmark.
- The DA is organized in a random sequence of order generation by the agents and evolves in time steps.
- There is a finite number N_a of agents $i \in [0, 1]$ equidistantly distributed. Agents with higher i are more optimistic.
- In each trading period agents submit limit orders to buy or sell δ_d units of the real asset against cash. Sellers make utility improving random price offers q_0^i from $U[\mathbb{E}^i(A), 1]$. Buyers make a random offer q_0^i from $U[0.2, \mathbb{E}^i(A)]$

How Does the Double Auction Work?

- When cash is exhausted agents can still place orders for loan financed real assets.
- A sell offer is a tuple $(q_0^i, q_1^i; \delta_d)$ with $q_0^i \in U[\mathbb{E}^i(A), 1]$, $q_1^i \in U[0.2, \mathbb{E}^i(A)]$ and j chosen at random from $\{(0.2, 1), (0.5, 1)\}$.
- A buy offer is accordingly a tuple $(q_0^i, q_1^i; \delta_d)$ with $q_0^i \in U[0.2, \mathbb{E}^i(A)]$, $q_1^i \in U[0, V^j]$ and j chosen at random from $\{(0.2, 1), (0.5, 1)\}$.
- Bid and ask orders are ranked and matched against offers with non-negative bid ask spreads. When the budget and collateral constraints are satisfied transactions are executed.
- The double auction stops when an upper limit of trading steps with no further matches has been observed.

A Summary of Results

When agents can trade a riskless debt instrument, then

1. The double auction converges both to allocations and to prices close to the equilibrium predictions.
2. Leverage emerges endogenously through contract type selection in the market as predicted by the theory of collateral equilibrium.

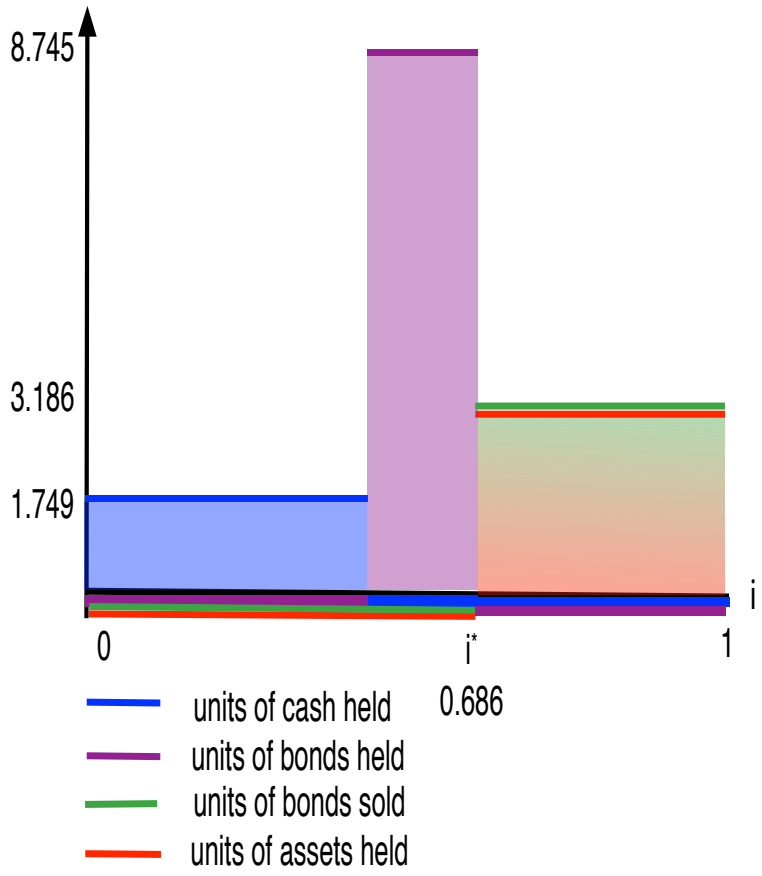
When agents can only use risky debt instruments these predictions fail.

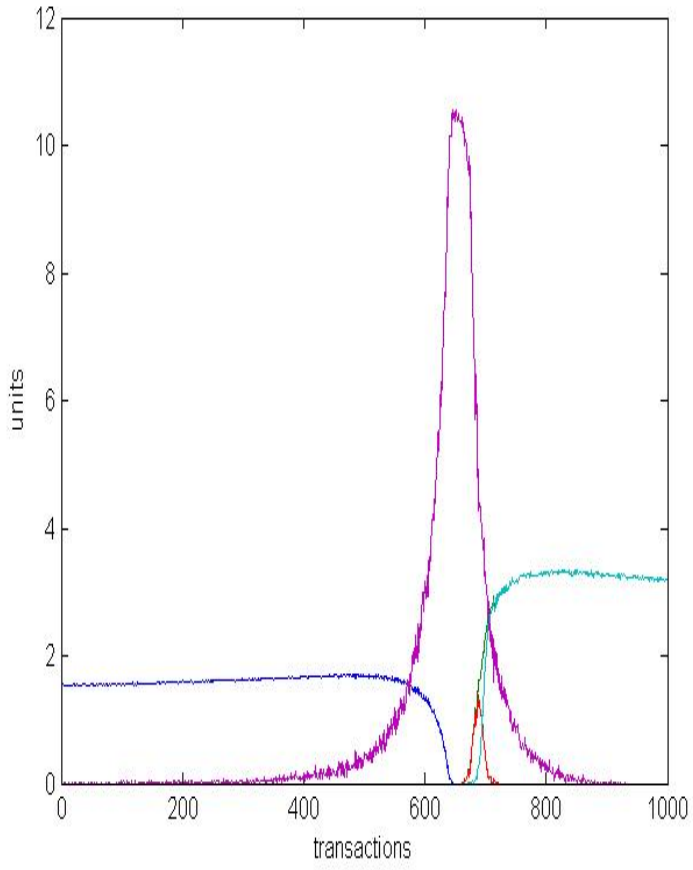
1. While allocations still come close to the equilibrium predictions, prices are now consistently and significantly different from equilibrium predictions.
2. Contract selection works poorly in this case. There are consistent and significant differences to what theory would predict. Compared to the situation with riskless debt collateral is used inefficiently.

Prices and Allocations with Riskless Debt

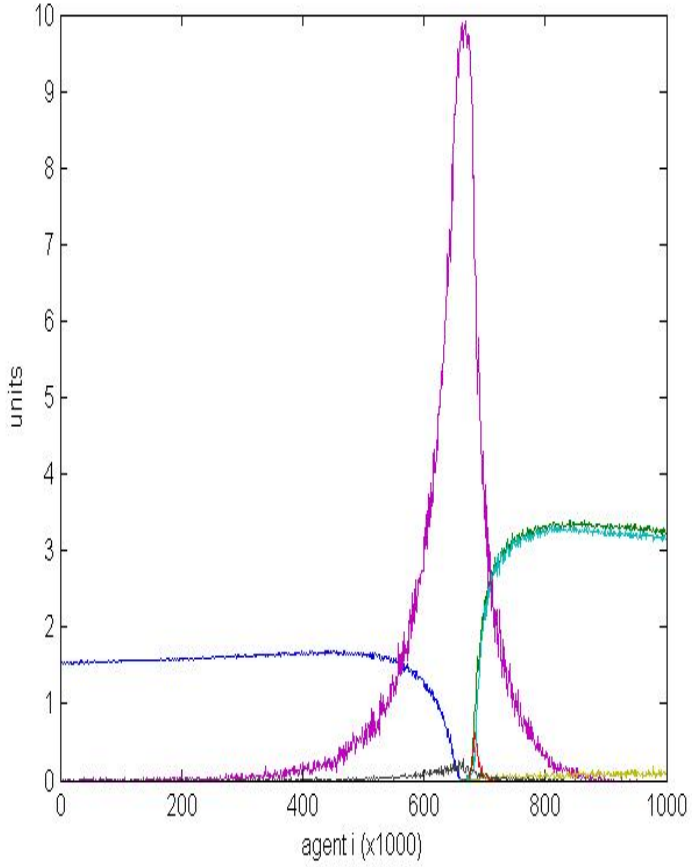
prices		allocation		
asset price q_0	bond price q_1	marg. agent i^*	cash held by pessimists	assets held by optimists
collateral equilibrium				
0.749	0.200	0.686	1.749	3.186
end of double auction				
0.745	0.198	0.686	1.616	3.145
(0.006)	(0.002)	(0.003)	(0.002)	(0.021)

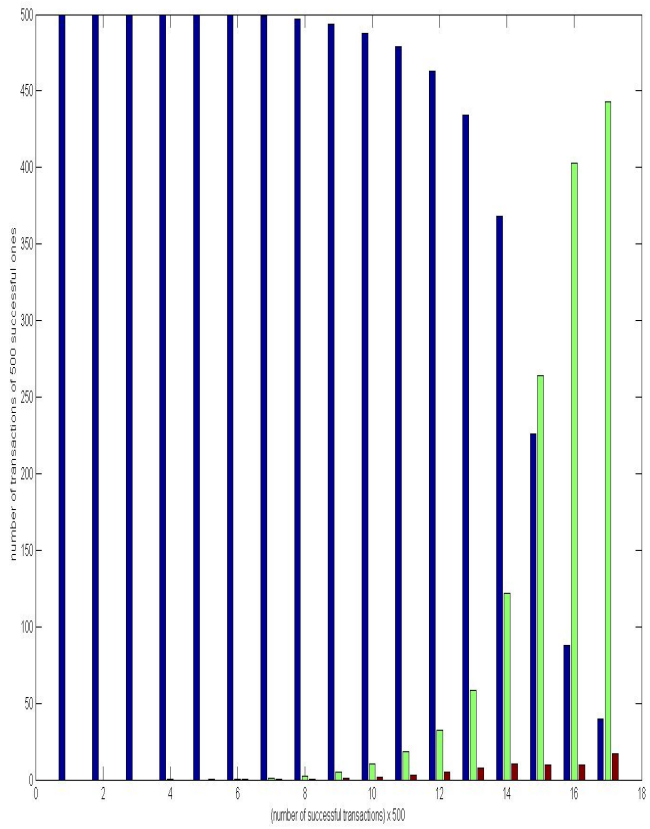
Prices and Allocations with Riskless Debt





Prices and Allocations with Riskless Debt





Prices and Allocations with Risky Debt Only

bond type	equilibrium prices		equilibrium allocation				
	collateral equilibrium						
	asset price	bond price	marginal agent	marginal agent	cash held by each	bonds given by	assets held by each
V^j	q_0	q_1	i_1^*	i_2^*	pessimist	medium	optimist
0.5	0.716	0.375	0.583	0.801	1.716	4.578	5.032
	outcome double auction						
0.5	0.850	0.446	0.601	0.726	1.588	3.217	3.331

Prices and Allocations with Risky Debt Only

Face Values		bonds issued Type 1	Type 2	free assests
Market institution: No swaps private clearing of collateral				
0.2	0.5	91.9%	6.3%	1.8%
0.3	0.5	67.8%	8.3%	23.9%
Market institution: Swaps central clearing of collateral				
0.2	0.5	93.3%	5.6%	1.1%
0.3	0.5	91.1%	8.2%	0.7%

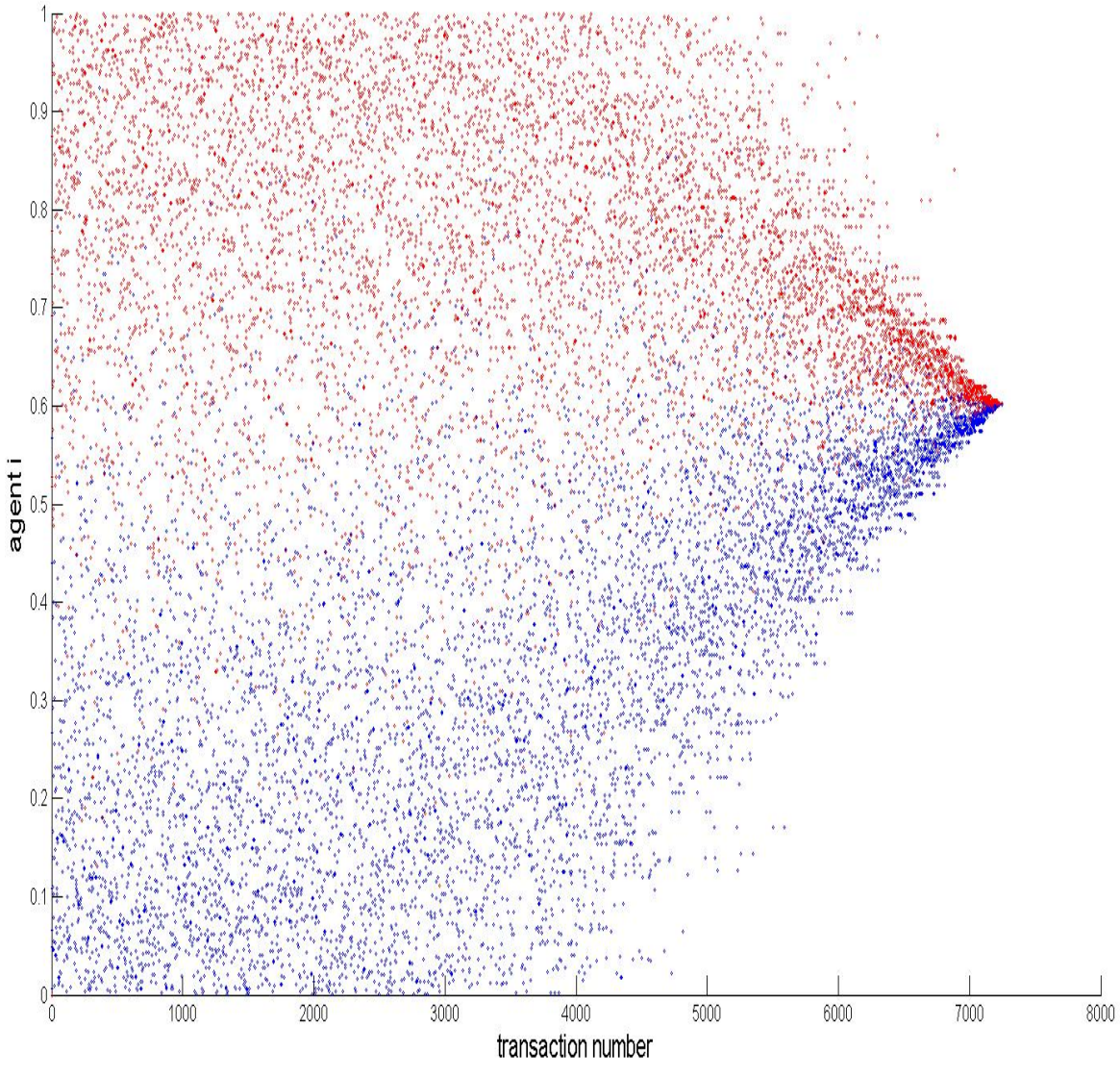
Why does the Double Auction (not) work?

Two crucial features built into the double auction The explanation of this convergence behavior comes from two crucial features built into the mechanism of the double auction:

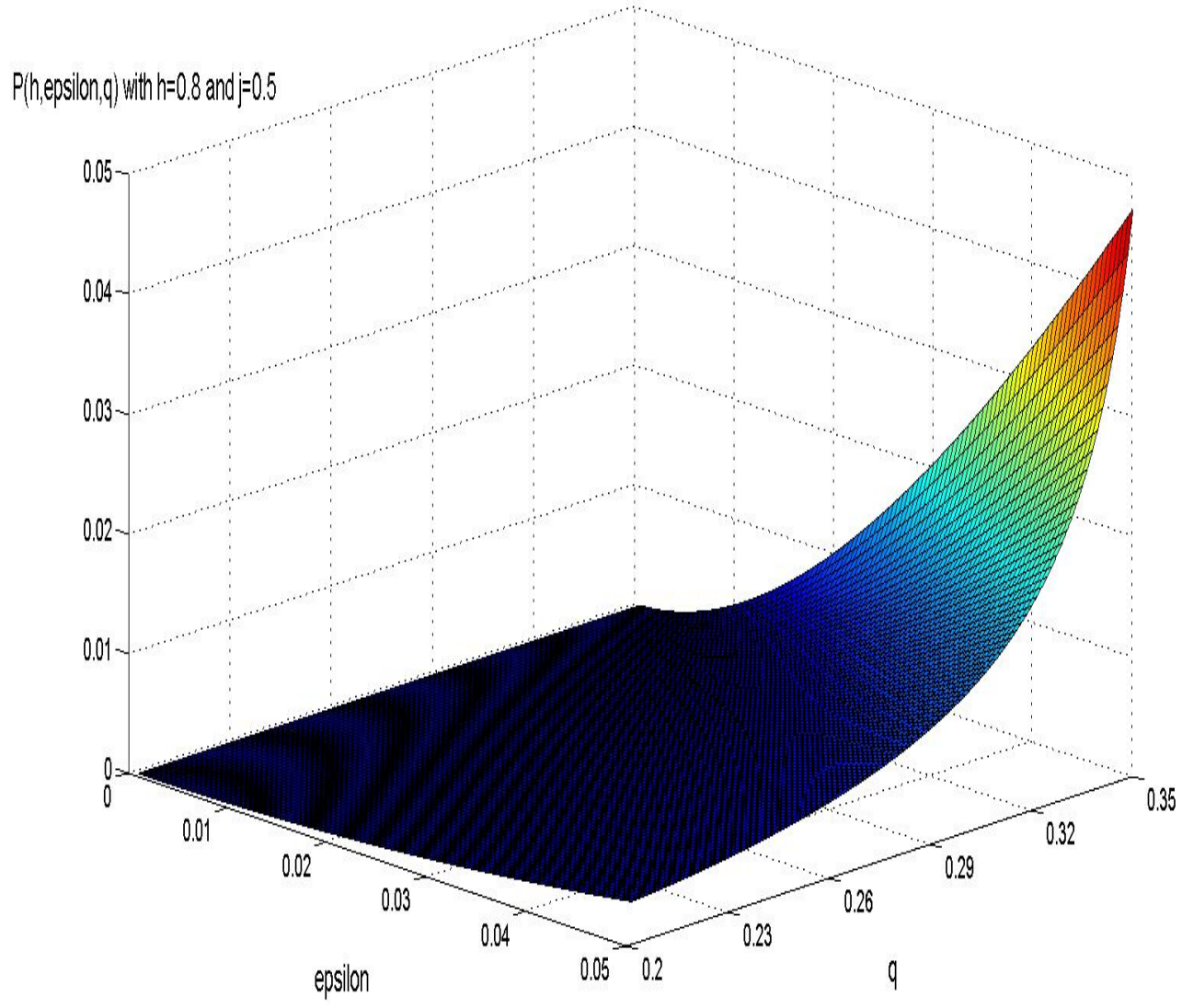
1. The asset always flows in the direction from the lower i to the higher i.
2. The closer the i of asker and bidder around marginal agent i^*, the smaller is the likelihood of a successful match.

When there is only one marginal agent the reservation price of i^* is exactly the equilibrium price. This is the case when riskless debt is available.

Why does the Double Auction (not) work? Convergence



Why does the Double Auction (not) work? Matching



Adding Markets and Change Clearing of Collateral

We now change two things in the double auction

1. We enlarge trading opportunities. Real assets and bonds could up to now only be exchanged indirectly via cash. We now allow direct swaps of assets and bonds without going through cash first.
2. We now allow central clearing of collateral: If an agent has sold a bond and has committed the necessary collateral, he can free this collateral again by buying a bind from somebody else.

Institutional Variation improves Convergence

Face Values		bonds issued Type 1	Type 2	free assests
Market institution: No swaps private clearing of collateral				
0.2	0.5	91.9%	6.3%	1.8%
0.3	0.5	67.8%	8.3%	23.9%
Market institution: Swaps central clearing of collateral				
0.2	0.5	93.3%	5.6%	1.1%
0.3	0.5	91.1%	8.2%	0.7%

New institution: Prices and Allocation

bond types		final prices			price fit	utility gap
		Asset	Bond 1	Bond 2		
Market institution: No swaps private clearing of collateral						
0.2	0.5	0.754	0.198	0.409	0.352%	-0.168%
Market institution: Swaps central clearing of collateral						
0.2	0.5	0.757	0.200^{+}	0.411	-2.390%	-0.223%

Conclusions

- GE theory provides an appropriate conceptual perspective on the exchange and pricing of leveraged assets under certain circumstances.
- The theory provides good predictions in a DA environment provided the institution allows a very flexible and frictionless build up and unwinding of leveraged positions.
- The key mechanisms in pricing and the endogenous build up of leverage seem to be at work in the agent based Double Auction.