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EFFICIENCY PROPERTIES OF RATIONAL EXPECTATIONS EQUILIBRIA WITH ASYMMETRIC INFORMATION*

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ABSTRACT

In this paper we provide a characterization of the welfare properties of rational expectations equilibria of economies in which, prior to trading, agents have some information over the realization of uncertainty. We study a model with asymmetrically informed agents, treating symmetric information as a limiting case. Trade takes place in asset markets that may or may not be complete. We show that equilibria are characterized by two forms of inefficiency, *price inefficiency* and *spanning inefficiency*, and that generically both of them are present. Price inefficiency arises whenever equilibrium prices reveal some information. It formalizes and generalizes the so-called Hirshleifer effect, by showing that generically an interim Pareto improvement is possible even conditional on the information that is available to agents in equilibrium; the primary source of the inefficiency is a pecuniary externality. Spanning inefficiency, on the other hand, arises if prices are not fully revealing and markets are incomplete relative to the uncertainty faced by agents in equilibrium. In this case, an ex-post improvement can generically be implemented by providing agents with more information, thus expanding their risk-sharing opportunities and reducing informational asymmetries, even though this additional information restricts the set of allocations that are incentive compatible and individually rational.

Journal of Economic Literature Classification Numbers: D52, D60, D82.

Keywords: Rational Expectations Equilibrium, Asymmetric Information, Incomplete Markets.

1. Introduction

In this paper we provide a characterization of the welfare properties of competitive rational expectations equilibria of economies in which agents have some information prior to trading. Our goal is to understand the precise nature of the inefficiencies that arise in this setting.

Our analysis allows for a range of information structures, from the case where agents are symmetrically informed to situations where they have different degrees of private information; in the latter case equilibrium prices may convey additional information. We evaluate agents' welfare conditionally on different levels of their information (before or after the arrival of individual signals, or of public signals such as prices). Furthermore, we examine whether inefficiencies survive when the set of attainable allocations is restricted by constraints in addition to resource feasibility. These constraints reflect the limitations imposed by the private nature of agents' information, the timing of transactions, and the voluntarity of exchange (incentive compatibility, measurability, and individual rationality), as well as other limitations that arise from the nature of the market mechanism. Accordingly, different notions of welfare and constrained optimality are generated. We are able to precisely pinpoint the sources of inefficiency of an equilibrium allocation by identifying, for each level of information conditional on which welfare is evaluated, a maximal set of constraints on feasible reallocations subject to which a Pareto improvement still exists.

The analysis is developed in the context of a class of two-period exchange economies. In the initial period each agent, after receiving a private signal about the realization of uncertainty, trades in asset markets, which may be incomplete, to reallocate risk in the second period. The structure of uncertainty and information that we consider is such that equilibrium allocations are incentive compatible, whatever the information revealed by prices.[1]

We show that equilibria are characterized by two forms of inefficiency, *price inefficiency* and *spanning inefficiency*, and that generically both of them are present.

Price inefficiency arises when welfare is evaluated conditionally on coarser information than is available to agents in equilibrium (for example, conditional on agents' private signals when some information is revealed by prices, or prior to a public signal in the symmetric information case). In this situation, some uncertainty is resolved before markets open. Given

[1] Equilibrium allocations can then be properly compared to other incentive compatible allocations.

the welfare criterion, agents would like to insure against the realization of this uncertainty, but are unable to do so. Thus an additional form of market incompleteness is endogenously generated by information revelation. A Pareto improvement can then be achieved via transfers across the states that are known before trading begins. This is equivalent to creating new assets that exploit the insurance opportunities precluded by information revelation.

It may be possible to achieve such an improvement by exogenously decreasing agents' information and assigning them the corresponding equilibrium allocation. This was first demonstrated in an example by Hirshleifer (1971). We show here that, generically, we can find a Pareto improving allocation, without a decrease in the information available to agents. An improvement can be generated merely by perturbing prices in each initial state, while preserving their informational content, and choosing portfolios for agents that are budget-feasible at the perturbed prices (thus without "creating" new assets). Furthermore, such an allocation can be chosen to satisfy the same incentive compatibility and individual rationality constraints that apply in equilibrium.

Thus our result substantially generalizes Hirshleifer's example, by showing that the inefficiency is generic and that it is essentially due to a pecuniary externality. It clearly illustrates that, in economies in which some uncertainty is resolved before markets open, prices have a role beyond the usual one of being indices of scarcity—since agents are subject to multiple budget constraints, one for each initial state, prices affect their insurance opportunities. There are then clear similarities with the generic inefficiency results established for incomplete market economies (for example, Geanakoplos and Polemarchakis (1986)). The logic of the argument is, however, rather different, as the desired change in prices cannot be induced here by a reallocation prior to the realization of uncertainty, since there are no markets open at that time (as in Example 1 of Hart (1975); our result can also be seen as a generalization of this example).

Spanning inefficiency, on the other hand, arises if markets are incomplete relative to the uncertainty faced by agents in equilibrium, and prices are not fully revealing (or, in the case of symmetric information, it is possible to release public information prior to trade). In this case an improvement can be obtained even with respect to the most stringent welfare criterion wherein utilities are evaluated conditionally on the pooled information of all agents. This improvement is achieved by increasing the information available to agents, thereby relaxing the measurability constraints imposed by their limited information, and thus expanding the set of state-contingent payoffs that they can realize by trading the existing assets. We show that a Pareto improvement can generically be obtained even if we restrict attention to marginal changes in information, the additional information used by the allocation rule is disclosed to agents, and we require the allocation to satisfy the tighter incentive compatibility and individual rationality constraints associated with this increased level of information.

This result generalizes the validity of the Blackwell effect (after Blackwell (1951)), concerning the positive value of information in single-agent decision problems, where an increase in information expands the feasible choice set. It shows that such an effect is typically present even in market situations where an improvement in risk-sharing opportunities through an increase in information is gained at the cost of tighter individual rationality and incentive constraints.

When there is asymmetric information in equilibrium, less informed agents trade at a disadvantage and hence are less willing to trade. In this situation, an increase in information may improve welfare by reducing the informational asymmetry among agents. We refer to this as the adverse selection effect. It is not possible in general to disentangle the adverse selection effect from the Blackwell effect, since the two effects arise from the same set of constraints, *i.e.* measurability. Nevertheless, it is possible to construct examples of partially revealing equilibria in which spanning inefficiency is due solely to adverse selection (see Example 4).

The value of information in competitive economies has been widely investigated in the literature. Since Hirshleifer's (1971) contribution, the usual approach is to compare the ex-ante welfare of equilibria associated with different levels of information. In economies with symmetric information the level of information is treated as an exogenous parameter (see, for example, Green (1981), Hakansson *et al* (1982), Milne and Shefrin (1987), and Schlee (2000)); on the other hand, with asymmetric information the analysis is limited to economies which have a continuum of equilibria with different degrees of revelation (Krebs (1999), Citanna and Villanacci (2000)). While these papers show that an increase in agents' information may lead to either an improvement or reduction of welfare in equilibrium, there are various factors at play (as our results show) and one cannot identify the reasons for the change in welfare. For instance, a fully revealing equilibrium may Pareto dominate a partially revealing one because of the pecuniary effect of a change in prices or because the measurability constraint is relaxed; it is difficult to interpret such a result solely in terms of the (positive) value of information. Furthermore, the welfare results obtained with this approach usually do not have generic validity.[2]

A significant exception to this line of research is Laffont (1985), who carries out a more conventional welfare analysis, comparing equilibrium allocations to other incentive compatible allocations. In particular, he provides an example of a fully revealing equilibrium that is interim inefficient, and a partially revealing equilibrium that is ex-post inefficient. Our analysis considerably generalizes these examples, by showing that they hold generically and under stronger conditions on the set of admissible reallocations. It also allows us to precisely identify the source of inefficiency in such examples.

The paper is organized as follows. Section 2 describes the economy. Competitive equilibria are defined in Section 3. In Section 4, the various welfare notions are introduced and motivated, and the inefficiency results are informally presented and discussed. They are then formally stated in Sections 5 and 6. Proofs are collected in the Appendix.

2. The Economy

We consider a large economy in which there is both aggregate and idiosyncratic uncertainty. By aggregate uncertainty we mean uncertainty of the "common value" type that affects several or all agents, while idiosyncratic uncertainty is of the "private value" type, being specific to a single agent. Agents have private information about both sources of uncertainty, but their information regarding the aggregate uncertainty is nonexclusive. The structure of uncertainty and information is motivated by the need to ensure that competitive rational expectations equilibria are incentive compatible.

There are two periods and a single physical consumption good. The economy is populated by finitely many types of agents with a continuum of each type. A typical agent is indexed by (h, τ), where $h \in \mathcal{H}$ (with $\#\mathcal{H} = H$, H finite) denotes his type, and $\tau \in [0, 1]$ endowed with Lebesgue measure. The aggregate uncertainty is described by the random variables $\tilde{s}$ and $\tilde{t}$, taking values in the finite sets $\mathcal{S}$ and $\mathcal{T}$ respectively (with $\#\mathcal{S} = S$ and $\#\mathcal{T} = T$). At date 0, agent (h, τ) observes a signal $\tilde{s}^{h\tau}$ taking values in the finite set $\mathcal{S}^h$ (with $\#\mathcal{S}^h = S^h$). The agent's type h is publicly observable, while the realization of

[2] Both Krebs (1999) and Citanna and Villanacci (2000) consider models in which each economy, as parameterized by agents' endowments, has a continuum of equilibria. They show that for a generic subset of economies there exists at least one equilibrium in the continuum (in Citanna and Villanacci, this is an equilibrium where information is redundant) which can be improved upon ex-ante. Our inefficiency results, on the other hand, apply to all equilibria of economies in a generic set, and hold for a stronger welfare criterion (interim or ex-post).

the signal $\tilde{s}^{h\tau}$ is private information. The endowment of an agent of type h is given by $\omega^h : \mathcal{S}^h \times \mathcal{S} \times \mathcal{T} \to (0, \infty)$. Thus, within the same type, agents' endowments differ only insofar as they have different private signals. We assume that $\overline{S} := \sum_h S^h \geq 2$.

We take the variables $\tilde{s}$ and $\tilde{t}$ to be independent; as will become clear shortly, this entails no loss of generality. The signals $\{\tilde{s}^{h\tau}\}$ are independent of $\tilde{t}$ but may be correlated with $\tilde{s}$. We assume that $\tilde{t}$ has full support, as does the joint distribution of $(\tilde{s}^{h\tau}, \tilde{s})$, for every h.[3] We denote a generic element of the sets $\mathcal{S}$, $\mathcal{T}$, and $\mathcal{S}^h$ by s, t, and s^h, respectively. We also assume the following (π denotes probabilities):

Assumption 1.

(i) $\pi(\tilde{s}^{h\tau} = s^h) = \pi(\tilde{s}^{h\tau'} = s^h) \qquad \forall h \in \mathcal{H},\ s^h \in \mathcal{S}^h$, and $\tau, \tau' \in [0, 1]$.

(ii) $\pi(\tilde{s}^{h\tau} = s^h,\ \tilde{s}^{h'\tau'} = s^{h'} \mid s) = \pi(\tilde{s}^{h\tau} = s^h \mid s)\, \pi(\tilde{s}^{h'\tau'} = s^{h'} \mid s)$
$\forall (h', \tau') \neq (h, \tau),\ s^h \in \mathcal{S}^h,\ s^{h'} \in S^{h'},\ s \in \mathcal{S}.$

(iii) $\forall s, s' \in \mathcal{S}\, (s \neq s'),\ \exists\, s^h \in \mathcal{S}^h$ (for some h) s.t. $\pi\{\tilde{s}^{h\tau} = s^h \mid s\} \neq \pi\{\tilde{s}^{h\tau} = s^h \mid s'\}$.

In other words, for any given type h the private signals of the agents have the same distribution. Also, conditional on $\tilde{s}$, agents' private signals are independent across (h, τ).[4] Informally, we may think of the signal $\tilde{s}^{h\tau}$ as containing a systematic or common value component given by the information it reveals about $\tilde{s}$, and a residual idiosyncratic component $(\tilde{s}^{h\tau} \mid \tilde{s} = s)$. Assumptions 1(i) and 1(ii) imply that, for every type h, the idiosyncratic components are *i.i.d.* across τ. Finally, 1(iii) implies that $\tilde{s}$ can be inferred by observing $\tilde{s}^{h\tau}$ for every (h, τ), except possibly for a set of agents of Lebesgue measure zero. Assumption 1(iii) is in fact just a nontriviality condition that allows us to interpret $\tilde{s}$ as the component of the aggregate uncertainty that can be inferred from the private information of agents.

Agents of type h have a von Neumann-Morgenstern utility function $u^h : (0, \infty) \to I\!R$. To ensure that demand functions are smooth, we make the following assumption:e

Asset markets, in which $J \geq 2$ securities are traded, open at date zero, after agents have observed their private signals. Asset payoffs are contingent on the aggregate state (s,t) only,[5] and are denoted by $r : \mathcal{S} \times \mathcal{T} \to I\!R^J$. Thus a portfolio $y \in I\!R^J$ results in payoff $r(s,t) \cdot y$ in state (s,t). There is no consumption (or endowments) at date 0. At date 1, all uncertainty is resolved, assets pay off, and agents consume. Since we have a single-good economy, portfolios uniquely determine consumption. The only motives for trade in this economy are risk-sharing, and possibly speculation on the basis of private information.

We assume that there is an asset, say asset J, whose payoff vector for any s, over the state space $\mathcal{T}$, is nonnegative and nonzero, *i.e.* for every $s \in \mathcal{S}$, $r_J(s,t) \geq 0$, for all $t \in \mathcal{T}$, and $r_J(s,t) > 0$, for some $t \in \mathcal{T}$. Together with the monotonicity assumption on utility functions, this ensures that the equilibrium price of asset J is positive. It also guarantees that budget constraints are satisfied with equality. Finally, we denote by R_s the asset payoff matrix conditional on state s, *i.e.*

$$R_s := \begin{pmatrix} \vdots \\ r(s,t)^\top \\ \vdots \end{pmatrix}_{t \in \mathcal{T}}$$

where $^\top$ denotes "transpose." By default all vectors are column vectors, unless transposed.

We parameterize economies by agents' endowments

$$\omega := [\omega^h(s^h, s, t)]_{h \in \mathcal{H}, s^h \in \mathcal{S}^h, s \in \mathcal{S}, t \in \mathcal{T}} \quad \in I\!R^{\overline{S}ST}_{++}.$$

By "generically" we mean "for an open subset of $I\!R^{\overline{S}ST}_{++}$ of full Lebesgue measure."

To summarize the information structure: endowments and asset payoffs are uncertain, and this uncertainty is described by the random variables $\{\tilde{s}^{h\tau}\}_{h \in \mathcal{H}, \tau \in [0,1]}$, $\tilde{s}$, and $\tilde{t}$. The idiosyncratic component of $\tilde{s}^{h\tau}$ affects only the endowment of agent (h,τ). The random variables $\tilde{s}$ and $\tilde{t}$ describe the common aggregate uncertainty that affects the endowments of all agents and asset payoffs. Furthermore, $\tilde{s}$ can be perfectly inferred from the pooled information of agents, $\{\tilde{s}^{h\tau}\}_{h,\tau}$, while $\tilde{t}$ captures any residual uncertainty. Thus, even if $\tilde{s}$ is fully revealed in equilibrium, a motive for trade is still present, given by insurance against

[5] At the time of trade, any remaining uncertainty that an agent faces concerning his future endowment is completely captured by $\tilde{s}$ and $\tilde{t}$. We can then restrict attention to contracts that depend only on aggregate uncertainty, as in the usual rational expectations framework *á la* Radner (1979) in which there is no idiosyncratic uncertainty.

the residual uncertainty $\tilde{t}$.[6]

Our informational assumptions generalize those of Laffont (1985).[7] Agents in our economy are informationally small in the following sense: an individual's private signal is informative about $\tilde{s}$, but $\tilde{s}$ can always be fully inferred from the pooled information of the other agents. Thus the agent's private information about the aggregate uncertainty is nonexclusive in the sense of Postlewaite and Schmeidler (1986) (see also Gul and Postlewaite (1992)). This ensures that equilibrium allocations are incentive compatible with respect to common value information. Information about the idiosyncratic uncertainty, on the other hand, is exclusive. Incentive compatibility with regard to this information follows from the negligibility of an individual agent in a continuum economy. Moreover, the presence of an idiosyncratic component in agents' private signals, ensures that, generically, their trading behavior varies nontrivially with their signals, and hence equilibrium allocations are strictly (Bayesian-Nash) implementable.[8]

Our description of private information is fairly general, and allows us to consider various standard cases in a unified framework. In particular, we can have two types U and I, who are respectively completely uninformed and (almost) perfectly informed. This case arises if $\mathcal{H} = \{U, I\}$, $\mathcal{S}^U$

refer to an allocation simply by specifying portfolios, since portfolios uniquely determine consumption: $c^h = \omega^h + r \cdot y^h$. Using the law of large numbers, the aggregate portfolio of agents of type h in state s is

$$\int_{[0,1]} y^h(\tilde{s}^{h\tau}, s) d\tau = \sum_{s^h \in \mathcal{S}^h} \pi(s^h \mid s)\, y^h(s^h, s).$$

A price function is a map $p : \mathcal{S} \to \mathcal{P}$, where $\mathcal{P} := I\!R^{J-1} \times \{1\}$. Note that we normalize the price of asset J to one: $p_J(s) = 1$, for every s. To describe the information revealed by prices, it is convenient to associate with any price function p, the partition $\mathcal{S}^p$ of $\mathcal{S}$ induced by p. A generic element of $\mathcal{S}^p$ is denoted by $\mathcal{S}^p_s$, which is the cell of $\mathcal{S}^p$ that contains s. Let S^p be the number of cells in $\mathcal{S}^p$, and S^p_s the number of states in $\mathcal{S}^p_s$. We say that p is fully revealing if $S^p = S$; otherwise it is partially revealing.

DEFINITION 1. *A rational expectations equilibrium (REE) consists of an allocation* $\{y^h\}$, *and a price function* $p : \mathcal{S} \to \mathcal{P}$, *satisfying the following two conditions:*

(AO) Agent optimization: $\forall h \in \mathcal{H}$, $s^h \in \mathcal{S}^h$, *and* $s \in \mathcal{S}$, $y^h(s^h, s)$ *solves*

$$\max_{y \in I\!R^J} \; Eu^h[\omega^h(s^h, \tilde{s}, \tilde{t}) + r(\tilde{s}, \tilde{t}) \cdot y \mid s^h, \mathcal{S}^p_s]$$

subject to $p(s) \cdot y = 0$.

(RF) Resource feasibility: $\forall s \in \mathcal{S}$,

$$\sum_{h, s^h} \pi(s^h \mid s)\, y^h(s^h, s) = 0.$$

This is the standard definition of an REE with asymmetric information (for example, as in Radner (1979)). Agents know the equilibrium price function and this allows them to make inferences from prices.

An inspection of the agents' optimization problem (AO) and a simple revealed preference argument show that equilibrium portfolios satisfy the following constraints:

(BC_p) Budget constraints: $\forall h \in \mathcal{H}$, $s^h \in \mathcal{S}^h$, and $s \in \mathcal{S}$,

$$p(s) \cdot y^h(s^h, s) = 0.$$

($\mathrm{M}_{\mathcal{S}^p}$) Measurability:[11] $\forall h \in \mathcal{H}$, and $s^h \in \mathcal{S}^h$,

$$y^h(s^h, \tilde{s}) \text{ is } p\text{-measurable.}$$

($\mathrm{IR}_{\mathcal{S}^p}$) Individual rationality: $\forall h \in \mathcal{H}$, $s^h \in \mathcal{S}^h$, and $\mathcal{S}^p_s \in \mathcal{S}^p$,

$$Eu^h(\omega^h + r \cdot y^h \mid s^h, \mathcal{S}^p_s) \geq Eu^h(\omega^h \mid s^h, \mathcal{S}^p_s).$$

($\mathrm{IC}_{\mathcal{S}^p}$) Incentive compatibility: $\forall h \in \mathcal{H}$, $s^h, \hat{s}^h \in \mathcal{S}^h$, and $\mathcal{S}^p_s \in \mathcal{S}^p$,

$$Eu^h[\omega^h(s^h, \tilde{s}, \tilde{t}) + r(\tilde{s}, \tilde{t}) \cdot y^h(s^h, \tilde{s}) \mid s^h, \mathcal{S}^p_s]$$

$$\geq Eu^h[\omega^h(s^h, \tilde{s}, \tilde{t}) + r(\tilde{s}, \tilde{t}) \cdot y^h(\hat{s}^h, \tilde{s}) \mid s^h, \mathcal{S}^p_s].$$

In particular, an equilibrium allocation is incentive compatible (see Laffont (1985), Proposition 2.2). The ($\mathrm{M}_{\mathcal{S}^p}$), ($\mathrm{IR}_{\mathcal{S}^p}$) and ($\mathrm{IC}_{\mathcal{S}^p}$) constraints depend on the price function p only through the partition $\mathcal{S}^p$ induced by it; hence they are indexed by $\mathcal{S}^p$ instead of p. As $\mathcal{S}^p$ becomes finer, ($\mathrm{M}_{\mathcal{S}^p}$) becomes weaker while ($\mathrm{IR}_{\mathcal{S}^p}$) and ($\mathrm{IC}_{\mathcal{S}^p}$) become tighter.

The purpose of this paper is to provide a characterization of the efficiency properties of rational expectations equilibria, whether fully or partially revealing, and in particular to show that various sources of inefficiency are typically present. However, as is well known, the existence of partially revealing equilibria, for an open set of economies, is problematic. In our setup, since the state space $\mathcal{S}$ is finite, a partially revealing REE generically does not exist (see Pietra and Siconolfi (1998)). Hence no generic welfare statements can properly be made for these equilibria. This leads us to consider the following weaker equilibrium notion:

Definition 2. *A pseudo-rational expectations equilibrium (P-REE) consists of an allocation* $\{y^h\}$, *and a price function* $p : \mathcal{S} \to \mathcal{P}$, *satisfying (AO) and*

$$\sum_{s' \in \mathcal{S}^p_s} \pi(s') \sum_{h, s^h} \pi(s^h \mid s')\, y^h(s^h, s) = 0 \qquad \forall \mathcal{S}^p_s \in \mathcal{S}^p.$$

[11] Even though $y^h(s^h, \cdot)$ is written as a function of s, the agent's objective is the same for all s in $\mathcal{S}^p_s$, so that the same vector y in $\mathbb{R}^J$ is optimal for all such s.

Henceforth, we often refer to a P-REE simply as an equilibrium. A P-REE differs from an REE in that resource feasibility is required to hold only on average within cells of the partition $\mathcal{S}^p$, rather than for every $s \in \mathcal{S}$. Note that $y^h(s^h, s)$ is invariant with respect to s within any cell of the partition $\mathcal{S}^p$. We can equivalently restate the above condition as follows:

($\mathrm{RF}_{\mathcal{S}^p}$) Resource feasibility given $\mathcal{S}^p$: $\forall \mathcal{S}^p_s \in \mathcal{S}^p$,

$$\sum_{h,s^h} \pi(s^h, \mathcal{S}^p_s)\, y^h(s^h, s) = 0.$$

Generic existence and determinacy of a P-REE can be established using standard arguments:

Lemma 3.1. *For any given partition of $\mathcal{S}$, there generically exists a a finite set of P-REE such that the equilibrium price function induces this partition.*

The definition of a fully or partially revealing P-REE is analogous to that of a fully or partially revealing REE. In the fully revealing case, REE and P-REE are identical. On the other hand, while a partially revealing REE is a partially revealing P-REE, the converse is in general not true.

Lemma 3.2. *Any REE is a P-REE. Also, a fully revealing P-REE is a fully revealing REE.*

At a P-REE agents solve the same optimization problem (AO) as at an REE. Hence the properties of an REE that follow from (AO) also hold for a P-REE:

Lemma 3.3. *Consider a P-REE with price function p. Then the equilibrium allocation satisfies ($RF_{\mathcal{S}^p}$), ($M_{\mathcal{S}^p}$), ($IR_{\mathcal{S}^p}$), ($IC_{\mathcal{S}^p}$), and (BC_p).*

In what follows, we provide a characterization of the welfare properties of both fully and partially revealing P-REE. We identify the precise conditions under which a Pareto improvement is, or is not, possible. In so doing we exercise care to ensure that inefficiency is not due to the fact that the resource feasibility condition for P-REE is weaker than that for REE. Basically, a P-REE serves as a reference point, which generically exists, from which we can examine the possibility of improving reallocations, when exact feasibility (RF) is imposed on these reallocations (see Section 5 for details). In the light of Lemma 3.2, our results for P-REE allow us to determine the sources of inefficiency of REE (and also, as a special case, of equilibria with symmetric information).

4. Welfare Analysis: An Overview

As is well-known for economies in which agents have some information prior to trading, a number of efficiency criteria can be defined depending on the stage at which utilities are evaluated (see Holmström and Myerson (1983)). The ex-ante stage refers to the time before the arrival of any information. At the interim stage agents have observed their private signals. The $\mathcal{S}^p$-posterior stage is when agents have their private information and have in addition observed a public signal, such as an REE price function, which induces the partition $\mathcal{S}^p$ of $\mathcal{S}$. Finally, at the ex-post stage, agents know the realization of their own private signal, as well as of $\tilde{s}$, *i.e.* all the payoff relevant information contained in other agents' signals, but not the realization of $\tilde{t}$.[12]

We compare equilibrium allocations to those that can be implemented by an allocation rule satisfying certain constraints. An allocation rule is a direct mechanism, *i.e.* a map that assigns to each agent a portfolio that depends on his type, on the aggregate state s, and on the signal reported by the agent. An allocation rule is feasible if it satisfies the resource feasibility constraint ($\text{RF}_{\mathcal{S}^p}$), for some partition $\mathcal{S}^p$. As is standard, we will in general restrict attention to feasible allocation rules that respect the following interim incentive constraint:

(IC) Incentive compatibility: $\forall h \in \mathcal{H}$, and $s^h, \hat{s}^h \in \mathcal{S}^h$,

$$Eu^h[\omega^h(s^h, \tilde{s}, \tilde{t}) + r(\tilde{s}, \tilde{t}) \cdot y^h(s^h, \tilde{s}) \,|\, s^h] \geq Eu^h[\omega^h(s^h, \tilde{s}, \tilde{t}) + r(\tilde{s}, \tilde{t}) \cdot y^h(\hat{s}^h, \tilde{s}) \,|\, s^h].$$

As we shall see, an equilibrium allocation is generically inefficient (whatever the welfare criterion) relative to the set of such allocation rules. The reason is apparent from Lemma 3.3, which lists a number of constraints imposed by the market mechanism. An equilibrium allocation satisfies the $\mathcal{S}^p$-posterior incentive constraint ($\text{IC}_{\mathcal{S}^p}$), which is tighter than (IC), as well as the additional constraints ($\text{M}_{\mathcal{S}^p}$), ($\text{IR}_{\mathcal{S}^p}$), and ($\text{BC}_p$). Any inefficiency can be traced to one or more of these constraints.

To determine the precise sources of inefficiency, we impose alternative sets of additional constraints, analogous to those that are satisfied by competitive equilibria, on feasible allocation rules. This leads to various notions of constrained efficiency. For instance, an allocation

[12] Since there is no trading at the ex-ante stage, using ex-ante efficiency as the welfare criterion may be too strong (see Holmström and Myerson (1983)). All of our inefficiency results pertain to interim or ex-post inefficiency.

$\{c^h\}$ is ($\text{IC}_{\mathcal{S}^p}$, $\text{M}_{\mathcal{S}^p}$)-constrained $\mathcal{S}^p$-posterior efficient if there does not exist a feasible allocation $\{\hat{c}^h\}$ satisfying ($\text{IC}_{\mathcal{S}^p}$) and ($\text{M}_{\mathcal{S}^p}$) such that $Eu^h(\hat{c}^h \mid s^h, \mathcal{S}^p_s) \geq Eu^h(c^h \mid s^h, \mathcal{S}^p_s)$ for every $h, s^h, \mathcal{S}^p_s$, with strict inequality for some $h, s^h, \mathcal{S}^p_s$.

We show that there are two kinds of inefficiency that are generically present at an equilibrium: *price inefficiency* and *spanning inefficiency.* Price inefficiency arises at the interim stage whenever prices reveal some information. This inefficiency notion formalizes and generalizes the so-called Hirshleifer effect, *i.e.* the effect of the resolution of uncertainty on risk-sharing possibilities evaluated from a prior perspective. Spanning inefficiency, on the other hand, is present even if welfare is evaluated ex-post, as long as equilibrium prices are not fully revealing and asset markets are incomplete relative to the uncertainty faced by agents at the time of trade. It captures the Blackwell effect, which in our context is the effect of more information in increasing the set of achievable state-contingent payoffs, and the adverse selection effect, which arises when an increase in information reduces informational asymmetries in the economy. We provide a precise formalization of these effects by identifying the constraints that are satisfied by equilibrium allocations, and showing how the effects operate by relaxing some these constraints.

We begin with an example, essentially due to Hirshleifer (1971), of a fully revealing equilibrium which is interim inefficient.

Example 1: The Hirshleifer effect I

The aggregate uncertainty is as follows: $\mathcal{S} = \{s_1, s_2\}$ and $\mathcal{T} = \{\bar{t}\}$, with $\pi(s_1) = \pi(s_2) = \frac{1}{2}$. There are two types: $\mathcal{H} = \{1, 2\}$. Private signals are symmetric and (almost) completely uninformative[13]: $\mathcal{S}^1 = \mathcal{S}^2 = \{\hat{s}_1, \hat{s}_2\}$ with $\pi(s_1 \mid \hat{s}_1) = \pi(s_2 \mid \hat{s}_2) = \frac{1}{2} + \epsilon$, where ϵ is a small positive number. Thus agents are (almost) symmetrically informed. Their endowments depend only on the aggregate state s and are given by $\omega^1(s_1) = \omega^2(s_2) = \omega_H$ and $\omega^1(s_2) = \omega^2(s_1) = \omega_L$, with $\omega_H > \omega_L$. Asset markets are complete, so that the measurability constraint is irrelevant. In a fully revealing rational expectations equilibrium, there is no trade. For sufficiently small ϵ, the equilibrium allocation is interim Pareto dominated by the allocation in which all agents consume the ex-ante expected value of their endowment, $\frac{1}{2}(\omega_H + \omega_L)$. Thus information revelation destroys risk-sharing, as in Hirshleifer's (1971) original example with a public information signal. The REE is ($\text{IR}_{\mathcal{S}^q}$, $\text{IC}_{\mathcal{S}^q}$)-constrained interim inefficient, where $\mathcal{S}^q = \{\mathcal{S}\}$ is the partition associated with any nonrevealing price

[13] See footnote 10.

function q.[14] It is, however, ($\mathrm{IR}_{\mathcal{S}^p}$)-constrained ex-ante efficient, where $\mathcal{S}^p = \{\{s_1\}, \{s_2\}\}$ is the partition induced by the equilibrium price function p. $\|$

In this example, there is too much information revealed in equilibrium in the sense that a nonrevealing allocation rule can do better by exploiting trades that are individually rational at the interim stage but not at the ex-post stage. More precisely, the example shows that the source of the inefficiency of the market outcome is the ex-post individual rationality constraint ($\mathrm{IR}_{\mathcal{S}^p}$), which must be satisfied in equilibrium, as opposed to the weaker interim individual rationality constraint ($\mathrm{IR}_{\mathcal{S}^q}$) that applies to a nonrevealing allocation rule.

Thus, in order to capture the possible welfare gains associated with a decrease in agents' information, it is crucial that the individual rationality and incentive constraints imposed on an allocation rule are conditional on the information used by that allocation rule. An allocation rule that uses less information than is revealed in equilibrium is accordingly subject to weaker individual rationality and incentive constraints. We will of course also adhere to this principle when considering the converse case: an allocation rule that uses more information than revealed by prices should satisfy tighter individual rationality and incentive constraints than those that apply in equilibrium. A mechanism must satisfy such an information disclosure constraint if it is to be implemented in a decentralized way. In particular, any equilibrium allocation has this property, with market prices providing agents with the information on which their portfolios depend.[15]

Example 1 is rather special in one respect—the only ex-post individually rational allocation is the one in which there is no trade. We now consider a more general example in which the set of ex-post individually rational allocations is not a singleton.

Example 2: The Hirshleifer effect II

We modify Example 1 by introducing residual uncertainty $\mathcal{T} = \{t_1, t_2\}$, with $\pi(t_1) = \frac{3}{4}$ and $\pi(t_2) = \frac{1}{4}$. Asset markets are complete. Agents' private information is as in Example 1. Their endowments are as follows:

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[14] Since there is no idiosyncratic uncertainty affecting endowments, and private information about $\tilde{s}$ is nonexclusive, incentive constraints do not impose any restriction.

[15] More generally, an important feature of a common value environment is that an agent's consumption may depend on common information that the agent himself is not endowed with. In such an environment, information disclosure is a desirable feature of a mechanism: if the information used by the mechanism is not made available to agents when they evaluate their allocation, renegotiation opportunities may arise. See Forges (1994a, 1994b).

	ω^1	ω^2
(s_1, t_1)	ω_H	ω_L
(s_1, t_2)	ω_L	ω_H
(s_2, t_1)	ω_L	ω_H
(s_2, t_2)	ω_H	ω_L

There is a fully revealing rational expectations equilibrium (with price function p) in which agents of type 1 consume $\frac{3}{4}\omega_H + \frac{1}{4}\omega_L$ in state s_1, and $\frac{1}{4}\omega_H + \frac{3}{4}\omega_L$ in state s_2, while the consumption of agents of type 2 is the reverse across states. Thus agents are able to smooth consumption across the residual uncertainty parameterized by $\mathcal{T}$, but not across the uncertainty described by $\mathcal{S}$. For sufficiently small ϵ, the equilibrium allocation can be interim Pareto dominated by transferring a small quantity η from type 1 to type 2 agents in state s_1, and doing the opposite transfer in state s_2. As in Example 1, the equilibrium allocation is ($\mathrm{IR}_{\mathcal{S}^q}$, $\mathrm{IC}_{\mathcal{S}^q}$)-constrained interim inefficient, where q is a nonrevealing price function. But in this case, if η is sufficiently small, the dominating allocation satisfies the ex-post individual rationality constraints of the agents. The equilibrium allocation is, therefore, inefficient in a stronger sense: it is ($\mathrm{IR}_{\mathcal{S}^p}$, $\mathrm{IC}_{\mathcal{S}^p}$)-constrained interim inefficient. It is possible to bring about a Pareto improvement by using a fully revealing allocation rule. In other words, the fully revealing REE is inefficient even conditional on the information it transmits. At the same time, it is clear that an interim efficient allocation which, for sufficiently small ϵ, entails almost perfect consumption smoothing across states, will in general violate $\mathrm{IR}_{\mathcal{S}^p}$, and therefore can only be implemented by a nonrevealing allocation rule. ‖

In Examples 1 and 2 we see that one source of interim inefficiency of an REE is that revelation of information restricts the transfers of wealth that agents can achieve across the states $\mathcal{S}$. In Example 1 this inefficiency is captured by the constraint ($\mathrm{IR}_{\mathcal{S}^p}$): it is possible to improve upon the equilibrium allocation only by weakening this constraint. In Example 2, on the other hand, ($\mathrm{IR}_{\mathcal{S}^p}$) is not binding, and it is possible to find a Pareto improvement while still respecting ($\mathrm{IR}_{\mathcal{S}^p}$).

More generally, as long as there is some trade in equilibrium, the ($\mathrm{IR}_{\mathcal{S}^p}$) and ($\mathrm{IC}_{\mathcal{S}^p}$) constraints are slack (see Fact 2 in the Appendix). Thus if an equilibrium allocation is inefficient, the inefficiency cannot be due only to these constraints. A more careful analysis is needed to isolate the precise source of inefficiency.

A particular feature of Examples 1 and 2 is that asset markets are complete. Hence revelation of information is not helpful in terms of expanding the agents' trading possibilities; there is no Blackwell effect. On the other hand, if markets are incomplete and prices are not fully revealing, an allocation rule that uses more information may be able to effect a Pareto improvement. There is a cost associated with using more information, however, since agents' individual rationality and incentive constraints become tighter once the additional information is disclosed to them. The following example illustrates this point.

Example 3

The economy is the same as in Example 2, except that markets are incomplete. There are three assets with the following payoffs:

	r_1	r_2	r_3
(s_1, t_1)	1	0	1
(s_1, t_2)	1	0	0
(s_2, t_1)	0	1	1
(s_2, t_2)	0	1	0

Thus markets are complete with respect to s, and with respect to t, conditional on s. Both types of agent have the same utility function: $u(c) = ac - \frac{1}{}$

employing an allocation rule that uses more information than agents have in equilibrium, in this case information about the actual realization of s. If such information is disclosed to agents, however, the improving allocation must respect their ex-post individual rationality constraints, which are tighter, and this may not be possible.

Consider, for instance, the possibility of a Pareto improvement conditional on s_1. An ex-post efficient allocation smoothes consumption completely across t_1 and t_2, so in looking for a possible ex-post improvement, we may restrict attention to allocations of consumption that are invariant with respect to t. Consider agents of type 1 who receive the signal $\hat{s}_2$. At the P-REE these agents transfer wealth from state s_1 to state s_2, both because they are better endowed in s_1 and because they consider this state less likely. Their expected consumption, conditional on s_1, is

$$E(c^1(\hat{s}_2)\,|\,s_1) = (\frac{3}{4}\omega_H + \frac{1}{4}\omega_L) - \frac{1}{4}(\omega_H - \omega_L) - \frac{\epsilon}{2}(\omega_H + \omega_L) - 2a\epsilon.$$

The (nonrandom) level of consumption that a Pareto improving allocation can assign to these agents in state s_1 is at most $E(c^1(\hat{s}_2)\,|\,s_1) + \kappa$, where κ is the maximum amount of resources that other agents (*i.e.* agents of type 2, and agents of type 1 who received the signal $\hat{s}_1$) are willing to give up in the aggregate in order to be insured against the variability of their equilibrium consumption across t_1 and t_2, given s_1. Note that κ can be made arbitrarily small by choosing a sufficiently large, so that agents are close to risk neutral. But for small κ, agents of type 1 with signal $\hat{s}_2$ are better off consuming their endowment (which gives them a significantly higher level of expected consumption, $\frac{3}{4}\omega_H + \frac{1}{4}\omega_L$). Thus the proposed improvement violates their ex-post individual rationality constraint. The P-REE allocation is consequently ($\text{IR}_{\mathcal{S}^q}$)-constrained ex-post efficient, where $\mathcal{S}^q$ is the partition $\{\{s_1\},\{s_2\}\}$. $\|$

Example 3 illustrates the two countervailing effects of more information. On the one hand, there is the Blackwell effect, whereby using more information weakens the measurability constraint, thus expanding the set of attainable allocations. On the other hand, if the additional information is disclosed to agents, there is also the Hirshleifer effect, captured by a tighter individual rationality constraint which restricts the set of attainable allocations. In this example, the latter effect dominates; no improvement is possible if the additional information is disclosed.

We now proceed with a more systematic investigation of the causes of inefficiency of competitive equilibria. We begin the analysis by identifying sets of constraints on attainable allocations under which an equilibrium allocation cannot be improved upon.

Consider a P-REE with price function p. The price function induces a partition of the state space. Each cell of this partition can be identified with a "subeconomy," which is essentially a self-contained economy, in which agents maximize expected utility conditional on being in that cell, subject to a single budget constraint. If we restrict ourselves to allocation rules that satisfy the same measurability constraint that applies to agents in equilibrium, then we should expect the equilibrium allocation to be efficient within each subeconomy.

PROPOSITION 4.1. *A P-REE with price function p is ($\mathrm{M}_{\mathcal{S}^p}$)-constrained $\mathcal{S}^p$-posterior efficient.*

Proof. Consider a P-REE with price function p and portfolio allocation $\{y^h\}$. In the subeconomy corresponding to the cell $\mathcal{S}^p_s$, the indirect utility over portfolios (in $I\!R^J$) of agent (h, τ) is

$$V^h_{\mathcal{S}^p_s}(y; \tilde{s}^{h\tau}) := Eu^h[\omega^h + r \cdot y \,|\, \tilde{s}^{h\tau}, \mathcal{S}^p_s].$$

For this subeconomy consider a competitive equilibrium $(\overline{p} \in \mathcal{P}, \{\overline{y}^h : \mathcal{S}^h \to I\!R^J\})$ wherein, for every $s^h \in \mathcal{S}^h$,

$$\overline{y}^h(s^h) \in \arg\max_{y \in I\!R^J} V^h_{\mathcal{S}^p_s}(y; s^h) \quad \text{s.t.} \quad \overline{p} \cdot y = 0,$$

and asset markets clear:

$$\sum_{h, s^h} \pi(s^h, \mathcal{S}^p_s)\, \overline{y}^h(s^h) = 0.$$

Since $(p, \{y^h\})$ is a P-REE for the overall economy, $(p(s), \{y^h(s^h, s)\})$ is an equilibrium in the subeconomy associated with the cell $\mathcal{S}^p_s$. Furthermore, along the lines of the first welfare theorem, the equilibrium in this subeconomy is Pareto efficient relative to the preferences $\{V^h_{\mathcal{S}^p_s}\}$. Hence the P-REE is $\mathcal{S}^p$-posterior efficient relative to the set of feasible allocations satisfying ($\mathrm{M}_{\mathcal{S}^p}$). ∎

If markets are complete (with respect to the state space $\mathcal{S} \times \mathcal{T}$) the measurability constraint is vacuous. With incomplete markets, this constraint is weaker the more revealing is p, and is no longer binding when p is fully revealing. Therefore, we have:

COROLLARY 4.1.1. *A fully revealing REE is ex-post efficient.*

COROLLARY 4.1.2. *If markets are complete, a nonrevealing P-REE is interim efficient.*

Proposition 4.1 leaves open the possibility of finding an improving allocation if agents' utilities are evaluated conditional on coarser information than is revealed by prices (even if we restrict ourselves to allocation rules satisfying ($M_{\mathcal{S}^p}$)). In particular, when prices reveal some information, agents effectively face a collection of distinct subeconomies at the interim stage, with no assets to reallocate consumption across them. Accordingly, we should expect that an interim Pareto improvement can be found by redistributing resources across these subeconomies. At a competitive equilibrium, transfers of income across subeconomies are precluded by the separate budget constraints that agents must satisfy in each subeconomy. If we impose the same budget constraints on the set of attainable allocations, in addition to ($M_{\mathcal{S}^p}$), a Pareto improvement is impossible to achieve, even ex-ante:

PROPOSITION 4.2. *A P-REE with price function p is ($M_{\mathcal{S}^p}$, BC_p)-constrained ex-ante efficient.*

Proof. Consider a P-REE $(p, \{c^h, y^h\})$. If $\{c^h\}$ is not ($M_{\mathcal{S}^p}$, BC_p)-constrained ex-ante efficient, there exists a feasible allocation $\{\hat{c}^h\}$ which satisfies ($M_{\mathcal{S}^p}$) and (BC_p), with $Eu^h(\hat{c}^h) > Eu^h(c^h)$ for some h. But this means that the P-REE allocation $\{c^h\}$ violates the agent optimization condition (AO), a contradiction. ■

In fact, it is easy to check that, for any given equilibrium price function p, the set of ($M_{\mathcal{S}^p}$, BC_p)-constrained ex-ante efficient allocations is a singleton—it is the unique P-REE allocation associated with p. This is true whether or not p reveals any information.

Propositions 4.1 and 4.2 identify restricted notions of efficiency that are satisfied by P-REE. In a sense, these propositions are rather obvious, the first being essentially an application of the first welfare theorem, and the second being a direct consequence of agent optimization. Their usefulness lies in providing a benchmark for the inefficiency results to follow. We will show that the restrictions in these propositions are tight—if we relax any of the constraints imposed on feasible allocation rules, equilibrium allocations can typically be Pareto improved. In particular, under appropriate conditions, if either ($M_{\mathcal{S}^p}$) or (BC_p) is relaxed, it is generically possible to achieve a local Pareto improvement in the neighborhood of an equilibrium allocation. This is true even if we require the improving allocation to satisfy the agents' individual rationality and incentive constraints conditional on the information revealed by the allocation rule.

We first consider the case where (BC_p) is not imposed, *i.e.* we allow wealth transfers across states that are not budget-feasible at equilibrium prices (while maintaining ($M_{\mathcal{S}^p}$)).

We know from Proposition 4.1 that if an improvement is possible in this case it can only be at the interim or ex-ante stage. In Proposition 5.1 we show that, if attainable allocations are not restricted to satisfy equilibrium budget constraints, then generically an equilibrium in which some information is revealed can be interim Pareto dominated. This is the case even if the improving allocation must satisfy the same individual rationality and incentive constraints that agents face in equilibrium.

Thus the inefficiency due to the Hirshleifer effect that we saw in Example 2 holds for a generic set of economies. Moreover, this inefficiency can be ascribed to the (BC_p) constraint. Revelation of information results in the imposition of multiple budget constraints on agents, one constraint for each cell of the partition induced by the price function. Agents choose portfolios that are optimal for them in each subeconomy, leaving some interim risk-sharing gains across subeconomies unexploited. To put it differently, asset markets allow agents to reallocate risks, subject to the incompleteness of markets, that are resolved after the trading stage, but no asset is available to transfer income across events that are already known at the time of trade. The economy can thus be viewed as an incomplete markets economy with three periods, but no assets traded at the initial date. A Pareto improvement can then be achieved essentially by "creating" new assets at this date.

We know, however, from Hart's well-known example (Example 1 in Hart (1975)) that such an economy may have Pareto-ranked equilibria. This suggests that it may be possible to improve upon an equilibrium allocation even without being able to "create" new assets. We show in Proposition 5.2 that this is indeed the case, generically, when prices reveal some information: we can achieve an interim Pareto improvement by altering prices across subeconomies, and choosing portfolios of the existing assets that respect budget constraints (at the modified prices) for each agent in each subeconomy. Thus we can further refine our understanding of the inefficiency of competitive equilibria that arises from information revelation. Since agents are subject to a separate budget constraint in each subeconomy, equilibrium prices affect insurance possibilities across these subeconomies. This gives rise to a pecuniary externality. In this sense, the inefficiency due to the Hirshleifer effect is a price inefficiency.

We now consider the case where the measurability constraint ($M_{\mathcal{S}^p}$) on equilibrium allocations is relaxed. In Proposition 6.1 we show that, if markets are sufficiently incomplete, a partially revealing equilibrium allocation can be generically Pareto dominated ex-post by an allocation that violates ($M_{\mathcal{S}^p}$) while satisfying ($IR_{\mathcal{S}^p}$), ($IC_{\mathcal{S}^p}$), and (BC_p). However, the improving allocation may not respect the information disclosure condition: the proposed allocation may not be individually rational or incentive compatible, once agents take into account the additional information that they can infer from the allocation rule (as was shown in Example 3).

The question then arises whether a Pareto improvement can be achieved by using more information than is revealed by prices, but without violating the information disclosure condition. Proposition 6.2 answers this question in the affirmative: if markets are sufficiently incomplete and the equilibrium is not fully revealing, it is generically possible to achieve an ex-post Pareto improvement by relaxing the measurability constraint, even if the additional information used by the mechanism is disclosed to agents and their (tighter) individual rationality and incentive constraints are respected.

We call this spanning inefficiency since relaxing the measurability constraint is equivalent to the introduction of new securities (this time at the $\mathcal{S}^p$-posterior stage, and at the cost of tighter individual rationality and incentive constraints), increasing the asset span. Such a spanning effect will typically be present if markets are incomplete, even in a symmetric information economy, in which case it can be viewed as the Blackwell effect. However, in an economy with asymmetric information, the measurability constraint also captures an adverse selection effect. In Section 6 we provide an example (Example 4) in which the measurability constraint imposes no restriction (in equilibrium) if all agents are symmetrically informed. Thus there is no Blackwell effect in the absence of private information. With asymmetric information, on the other hand, at a partially revealing equilibrium, the measurability constraint does bind. The equilibrium allocation is ex-post inefficient, and this can be viewed as arising purely from adverse selection.

To sum up, the source of spanning inefficiency at an equilibrium is the measurability constraint. Relaxing this constraint provides scope for an ex-post Pareto improvement, which is due to the Blackwell effect (all agents have more information to construct portfolios) and/or an adverse selection effect (the asymmetry of information is reduced).

If an equilibrium is price inefficient, we may interpret this as being due to prices revealing "too much" information, while in the case of spanning inefficiency, prices reveal "too little" information. Such an interpretation is often convenient, but it should be kept in mind that a given equilibrium allocation may be both price inefficient and spanning inefficient (so we simultaneously have "too much" and "too little" information revelation). Also, in the case of price inefficiency, an equilibrium allocation can be Pareto dominated even conditional on the information it transmits, by perturbing prices while keeping fixed the partition generated by the equilibrium price function.

5. Price Inefficiency

We proceed now to a formal analysis of price inefficiency. We assume in what follows that $J_{\mathcal{S}_s^p} \geq 2$, for every $\mathcal{S}_s^p \in \mathcal{S}^p$, where $J_{\mathcal{S}_s^p}$ denotes the number of linearly independent assets in the subeconomy corresponding to the cell $\mathcal{S}_s^p$.[16] This is simply to ensure that nontrivial trades are possible in each subeconomy; in particular Example 1 is ruled out. As a preliminary step we state the following result:

PROPOSITION 5.1. *For a generic subset of economies, any P-REE with price function p is ($M_{\mathcal{S}^p}$, $IR_{\mathcal{S}^p}$, $IC_{\mathcal{S}^p}$)-constrained interim inefficient, provided $S^p > 1$.*

This proposition generalizes Laffont's (1985) Proposition 3.2, where it is shown by means of an example that fully revealing equilibria may be interim inefficient. The result is generic and the source of inefficiency is that asset markets are effectively incomplete whenever information is revealed before trading takes place. Note, however, that the result applies even if markets are complete in the usual sense, *i.e.* even if the asset payoff matrix has full rank ST. The proof of the proposition is fairly obvious, and is omitted.[17]

As we pointed out in the previous section, Proposition 5.1 can be considerably sharpened. A Pareto improvement can be achieved even if the improving allocation is subject to budget constraints in each subeconomy comparable to those that apply in equilibrium. More precisely, given a P-REE $\{p, \{y^h\}\}$, we restrict the set of attainable allocations $\{y^h + \Delta y^h\}$ to satisfy budget constraints evaluated at prices that have the same informational content as p:

($\mathrm{BC}_{\mathcal{S}^p}$) Information-preserving budget constraints: $\forall h \in \mathcal{H}$, $s^h \in \mathcal{S}^h$, and $s \in \mathcal{S}$,

$$q(s) \cdot [y^h(s^h, s) + \Delta y^h(s^h, s)] = 0$$

for some function $q : \mathcal{S} \to \mathcal{P}$, such that $\mathcal{S}^q = \mathcal{S}^p$.

[16] We continue to use p to denote the price function and $\{y^h\}$ the portfolios of the subset of nonredundant assets in each subeconomy.

[17] It is in fact analogous to establishing that equilibria with incomplete markets are generically Pareto inefficient. We also exploit the fact that the ($\mathrm{IR}_{\mathcal{S}^p}$) and ($\mathrm{IC}_{\mathcal{S}^p}$) constraints are generically non-binding in a neighborhood of an equilibrium allocation.

Furthermore, since we do not wish to exploit the fact that ($\mathrm{RF}_{\mathcal{S}^p}$) is weaker than exact feasibility state by state, we require that any deviation from the equilibrium portfolio allocation does satisfy exact feasibility, *i.e.*

$$(\overline{\mathrm{RF}}) \qquad \sum_{h,s^h} \pi(s^h \mid s)\, \Delta y^h(s^h, s) = 0, \qquad \forall s \in \mathcal{S}.$$

Clearly ($\overline{\mathrm{RF}}$) is a stronger restriction than ($\mathrm{RF}_{\mathcal{S}^p}$).[18]

Definition 3. *A P-REE $\{p, \{y^h\}\}$ is price efficient if $\{y^h\}$ is interim efficient relative to the set of allocations $\{y^h + \Delta y^h\}$ that satisfy ($\overline{RF}$), ($M_{\mathcal{S}^p}$), ($IR_{\mathcal{S}^p}$), ($IC_{\mathcal{S}^p}$), and ($BC_{\mathcal{S}^p}$).*

Note that the constraints ($\mathrm{M}_{\mathcal{S}^p}$), ($\mathrm{IR}_{\mathcal{S}^p}$), ($\mathrm{IC}_{\mathcal{S}^p}$), and ($\mathrm{BC}_{\mathcal{S}^p}$) are imposed on $\{y^h + \Delta y^h\}$, while ($\overline{\mathrm{RF}}$) applies to $\{\Delta y^h\}$. Let $J_{\mathcal{S}^p} := \sum_{\mathcal{S}^p_s \in \mathcal{S}^p} J_{\mathcal{S}^p_s}$. We can now state the main result of this section:

Proposition 5.2. *For a generic subset of economies, any P-REE with price function p is price inefficient, provided $S^p > 1$, and $S^p_s < \overline{S} \leq J_{\mathcal{S}^p} - S^p$, for every $\mathcal{S}^p_s \in \mathcal{S}^p$.*

The proof is in the Appendix. The main difference relative to Proposition 5.1 is that reallocations must now also satisfy budget constraints. The condition $\overline{S} > \mathcal{S}^p_s$ simply ensures that the set of reallocations satisfying ($\overline{\mathrm{RF}}$) has a nonempty interior. For a Pareto improvement to be (generically) possible, the number of assets ($J_{\mathcal{S}^p}$) must be large relative to the number of agents ($\overline{S}$) whose welfare has to be improved.[19]

Proposition 5.2 shows that competitive equilibria are inefficient due to the presence of a pecuniary externality. This inefficiency is similar to the one pointed out by Stiglitz (1982) and later formalized by Geanakoplos and Polemarchakis (1986). However, our notion of constrained optimality is different.

Given a competitive equilibrium, Geanakoplos and Polemarchakis consider a reallocation of agents' portfolios at the initial date, after which spot commodity markets open and agents trade at (the new) market-clearing prices. This way of inducing changes in spot

[18] We do not impose exact feasibility on $\{y^h + \Delta y^h\}$, for then the P-REE allocation $\{y^h\}$ itself would be unattainable in general.

[19] Note, however, that this result strictly improves upon Proposition 5.1 only if $\overline{S} > J_{\mathcal{S}^p_s}$, for some $\mathcal{S}^p_s$. Otherwise, the budget constraints we have imposed on attainable allocations have no bite—for any feasible allocation, we can always find a price vector in each subeconomy at which the associated portfolios are budget-feasible for every agent.

prices, and corresponding allocations, does not apply in our setting since there are no assets to reallocate at the initial date. Instead we perturb equilibrium prices directly, and choose a portfolio allocation that is budget-feasible for agents at the perturbed prices (which precludes any direct transfer of income across subeconomies). For instance, we could think of perturbing prices away from their equilibrium values and then clearing markets via a rationing scheme.[20] Thus, unlike Geanakoplos and Polemarchakis, attainable allocations are not in general ex-post optimal for the agents, nor are they ex-post efficient. Nevertheless, we show that an improvement can generically be achieved by trading off ex-post efficiency against some interim gains from trade.

6. Spanning Inefficiency

In this section we present our formal analysis of spanning inefficiency. First, we need a further piece of notation. Recall that R_s is the asset payoff matrix conditional on state s. For $\hat{s} \in \mathcal{S}_s^p$, let

$$R_{\mathcal{S}_s^p,\hat{s}} := \left(\begin{pmatrix} \vdots \\ R_{s'} \\ \vdots \end{pmatrix}_{s' \in \mathcal{S}_s^p} \begin{array}{|c} 0 \\ \vdots \\ R_{\hat{s}} \\ \vdots \\ 0 \end{array} \right),$$

where $R_{\hat{s}}$ is the only nonzero row block in the submatrix on the right, and it appears in the same position in the left submatrix. The matrix $R_{\mathcal{S}_s^p,\hat{s}}$ is the asset payoff matrix faced by an agent in the subeconomy $\mathcal{S}_s^p$ when he is allowed to violate the measurability constraint in state $\hat{s}$ only (*i.e.* to choose portfolios contingent on $\hat{s}$ and the complement of $\hat{s}$ in $\mathcal{S}_s^p$). As is evident, relaxing measurability in this way is equivalent to introducing a particular set of new securities.

As a preliminary step to our main result on spanning inefficiency, we consider the possibility of an ex-post Pareto improvement, without imposing the information disclosure condition.

[20] We do not explicitly consider any such mechanism. We conjecture however that a result similar to Proposition 5.2 would hold (with a tighter bound on the number of agents, possibly) even if a prespecified rationing scheme determines the reallocation associated with any price change. An interesting analogy then emerges with the results of Herings and Polemarchakis (2000), who show that, in a standard two-period incomplete market economy, competitive equilibria may be Pareto dominated by allocations supported by disequilibrium prices and rationing.

PROPOSITION 6.1. *Suppose $\overline{S} \geq 3$. Then, for a generic subset of economies, any P-REE with price function p is ($\overline{RF}$, $IR_{\mathcal{S}^p}$, $IC_{\mathcal{S}^p}$, BC_p)-constrained ex-post inefficient, provided there is a cell $\mathcal{S}^p_s$, and a state $\hat{s} \in \mathcal{S}^p_s$, such that rank $(R_{\mathcal{S}^p_s, \hat{s}}) \geq J_{\mathcal{S}^p_s} + 3$.*

The proof is in the Appendix. A P-REE satisfies ($\overline{\mathrm{RF}}$), ($\mathrm{M}_{\mathcal{S}^p}$), ($\mathrm{IR}_{\mathcal{S}^p}$), ($\mathrm{IC}_{\mathcal{S}^p}$), and ($\mathrm{BC}_p$). The proposition states that it is (generically) possible to achieve a Pareto improvement ex-post by relaxing ($\mathrm{M}_{\mathcal{S}^p}$), in particular by allowing portfolios to be contingent on one state within one subeconomy. Thus the equilibrium allocation can be Pareto dominated by an allocation rule that uses more information to construct portfolios than the information that can be inferred from the price function p. This amounts to adding new securities that increase the rank of the asset payoff matrix in the subeconomy $\mathcal{S}^p_s$ by at least 3, as the condition *rank* $(R_{\mathcal{S}^p_s, \hat{s}}) \geq J_{\mathcal{S}^p_s} + 3$ says. For this to be possible, asset markets must be sufficiently incomplete, and the P-REE must be partially revealing.

Proposition 6.1 generalizes Laffont's (1985) Proposition 4.2 which shows, again by means of an example, that partially revealing equilibria may be ex-post inefficient. Note that the additional information used by a Pareto dominating allocation rule is not made available to the agents themselves. If this information had to be made public, a Pareto improvement is not possible in general without violating the agents' tighter individual rationality and incentive constraints, as shown in Example 3. Nevertheless, we can exploit the fact that generically these constraints are not binding in the neighborhood of an equilibrium. To do so we will restrict attention to marginal changes in information, where only a small amount of extra information is used (and made public), and local perturbations from an equilibrium allocation. The problem is that with the partitional representation of information over the finite set $\mathcal{S}$, no change in information is "small." This leads us to consider information signals that induce a more general conditional probability over $\mathcal{S}$.

For ease of exposition, we restrict ourselves to nonrevealing P-REE. The extension to the general partially revealing case is immediate. Let the equilibrium price and allocation, both of which are constant over $\mathcal{S}$, be $\{\overline{p}, \{\overline{y}^h(s^h)\}\}$. We consider allocation rules of the form $\{\overline{y}^h(s^h) + \Delta y^h(s^h, \tilde{\sigma})\}$, where $\tilde{\sigma}$ is the information used by the mechanism, and may be chosen optimally. The signal $\tilde{\sigma}$ may be correlated with $\tilde{s}$, without necessarily inducing a partition of $\mathcal{S}$. We assume that it is informative only about $\tilde{s}$: it is independent of $\tilde{t}$ and, conditionally on $\tilde{s}$, also of $\tilde{s}^{h\tau}$, for all (h, τ), (*i.e.* $\pi(\tilde{s}^{h\tau}, \tilde{\sigma} \mid \tilde{s}) = \pi(\tilde{s}^{h\tau} \mid \tilde{s})\, \pi(\tilde{\sigma} \mid \tilde{s})$). We denote the support of $\tilde{\sigma}$ by Σ, which (without loss of generality) has the same cardinality as $\mathcal{S}$. We parameterize $\tilde{\sigma}$ by $\Pi := \{\pi(s,\sigma)\}_{s\in\mathcal{S},\,\sigma\in\Sigma}$. Perturbing the probabilities Π allows us to perturb the information of agents in a smooth way. In doing so, we do not change the support of $\tilde{\sigma}$.

For a given $\tilde{\sigma}$, we consider the set of allocations satisfying the following constraints:

($\overline{\text{RF}}_{\tilde{\sigma}}$) Resource feasibility: $\forall h \in \mathcal{H}$,

$$\overline{c}^h = \omega^h + r \cdot (\overline{y}^h + \Delta y^h)$$

and, $\forall s \in \mathcal{S}, \sigma \in \Sigma$,

$$\sum_{h,s^h} \pi(s^h \mid s)\, \Delta y^h(s^h, \sigma) = 0.$$

($\text{IR}_{\tilde{\sigma}}$) Individual rationality: $\forall h \in \mathcal{H}$, $s^h \in \mathcal{S}^h$, and $\sigma \in \Sigma$,

$$Eu^h(\overline{c}^h \mid s^h, \sigma) \geq Eu^h(\omega^h \mid s^h, \sigma).$$

($\text{IC}_{\tilde{\sigma}}$) Incentive compatibility: $\forall h \in \mathcal{H}$, $s^h, \hat{s}^h \in \mathcal{S}^h$, and $\sigma \in \Sigma$,

$$Eu^h\left[\omega^h(s^h, \tilde{s}, \tilde{t}) + r(\tilde{s}, \tilde{t}) \cdot [\overline{y}^h(s^h) + \Delta y^h(s^h, \sigma)] \mid s^h, \sigma\right]$$

$$\geq Eu^h\left[\omega^h(s^h, \tilde{s}, \tilde{t}) + r(\tilde{s}, \tilde{t}) \cdot [\overline{y}^h(\hat{s}^h) + \Delta y^h(\hat{s}^h, \sigma)] \mid \hat{s}^h, \sigma\right].$$

($\text{BC}_{\overline{p},\tilde{\sigma}}$) Budget constraints: $\forall h \in \mathcal{H}$, $s^h \in \mathcal{S}^h$, and $\sigma \in \Sigma$,

$$\overline{p} \cdot \Delta y^h(s^h, \sigma) = 0.$$

Conditions ($\text{IR}_{\tilde{\sigma}}$) and ($\text{IC}_{\tilde{\sigma}}$) are essentially the same as ($\text{IR}_{\mathcal{S}^p}$) and ($\text{IC}_{\mathcal{S}^p}$) respectively, except that $\tilde{\sigma}$ replaces p. Conditions ($\overline{\text{RF}}_{\tilde{\sigma}}$) and ($\text{BC}_{\overline{p},\tilde{\sigma}}$) are the analogues of ($\overline{\text{RF}}$) and ($\text{BC}_p$) respectively. Note that now the aggregate demand for assets in the economy depends on both the aggregate state s and the signal σ; hence ($\overline{\text{RF}}_{\tilde{\sigma}}$) requires resource feasibility to be satisfied for all s and σ. The constraint ($\text{BC}_{\overline{p},\tilde{\sigma}}$) implies that, at every s, and for every agent-type (h, s^h), the portfolio $\overline{y}^h(s^h) + \Delta y(s^h, \sigma)$ is budget-feasible at equilibrium prices.

The equilibrium allocation $\{\overline{y}^h\}$ clearly satisfies the above constraints, with an appropriate choice of $\tilde{\sigma}$: it can be achieved by setting $\Delta y^h = 0$, and $\pi(s,\sigma) = \pi(s)\,\pi(\sigma)$, *i.e.* $\tilde{\sigma}$ independent of $\tilde{s}$.

Definition 4. *A P-REE $\{\overline{p}, \{\overline{y}^h\}\}$ is spanning efficient if $\{\overline{y}^h\}$ is ex-post efficient relative to the set of allocations $\{\overline{y}^h + \Delta y^h\}$ satisfying ($\overline{RF}_{\tilde{\sigma}}$), ($IR_{\tilde{\sigma}}$), ($IC_{\tilde{\sigma}}$), and ($BC_{\overline{p},\tilde{\sigma}}$), for some $\tilde{\sigma}$.*

Proposition 6.2. *Suppose* $\overline{S} \geq S+2$, *and there are states* $\check{s}$ *and* $\hat{s}$ *in* $\mathcal{S}$, *such that* *min* $[rank\ (R_{\mathcal{S},\check{s}}), rank(R_{\mathcal{S},\hat{s}})] \geq J + 2(S+2)$. *Then, for a generic subset of economies, any nonrevealing P-REE is spanning inefficient.*

The proof is in the Appendix. The proposition says that a nonrevealing equilibrium is generically ex-post inefficient, even if attainable allocations are restricted by the information disclosure condition. As in Proposition 6.1, asset markets must be sufficiently incomplete. Since admissible reallocations are further restricted here, the market incompleteness condition is more stringent.

7. Adverse Selection

The class of economies we have analyzed in this paper is characterized by the presence of adverse selection (except in the limiting case of symmetric information). Yet none of the sources of inefficiency we have described depends on this feature. In particular, our results on price and spanning inefficiency hold even in the absence of informational asymmetries. In this section we provide an example in which an adverse selection effect can be clearly identified. It arises in equilibrium from the measurability constraint ($M_{\mathcal{S}^p}$). The resulting inefficiency is thus an example of spanning inefficiency.

Example 4: Adverse selection[21]

The aggregate uncertainty is given by $\mathcal{S} = \{s_1, s_2\}$ and $\mathcal{T} = \{t_1, t_2\}$, with $\pi(s_1) = \frac{1}{2}$, and $\pi(t_1 \,|\, s_1) = \pi(t_2 \,|\, s_2) = \frac{3}{4}$. There are two types: $\mathcal{H} = \{I, U\}$. The private information of agents is as follows: $\mathcal{S}^I = \{s_1, s_2\}$ and $\mathcal{S}^U = \{\overline{s}^U\}$. In other words, type I is perfectly informed about the aggregate state s, while type U is uninformed. Both types have log utility. Agents' endowments depend only on the aggregate uncertainty. There are two assets. Endowments and asset payoffs are as follows:

[21] In this example, for expositional reasons, we depart slightly from the general information structure described in Section 2. First, we do not take $\tilde{s}$ and $\tilde{t}$ to be independent, even though we could easily satisfy this assumption by redefining the state space. Second, we do not impose the full support restriction on private signals.

	r_1	r_2	ω^I	ω^U
(s_1, t_1)	1	0	ω_L	ω_H
(s_1, t_2)	0	1	ω_H	ω_L
(s_2, t_1)	1	0	ω_L	ω_H
(s_2, t_2)	0	1	ω_H	ω_L

where $\omega_H > \omega_L$.

In this economy, if agents can trade before the arrival of private information, the equilibrium allocation is an ex-ante Pareto optimum (with perfect consumption smoothing for all agents). Thus, in the absence of private information, there is no spanning role for information even though markets are incomplete. Now consider what happens when trading occurs at the interim stage but prices do not reveal any information. There exists a nonrevealing P-REE of this economy in which $p_1 = p_2 = 1$. The equilibrium consumption of the uninformed agents is $\frac{1}{2}(\omega_H + \omega_L)$ in every state, while the consumption of the informed agents is $\frac{3}{4}(\omega_H + \omega_L)$ in states (s_1, t_1) and (s_2, t_2) and $\frac{1}{4}(\omega_H + \omega_L)$ in the other two states (thus it is different from the equilibrium with no information). At this stage, if the true state (s_1 or s_2) is announced and markets are reopened, there will be further trade. Provided the uninformed agents' consumption is ex-post individually rational (which is the case if ω_H is large enough relative to ω_L), this implies that the P-REE allocation is ($\overline{\text{RF}}$, $\text{IR}_{\mathcal{S}^q}$, $\text{IC}_{\mathcal{S}^q}$)-constrained ex-post inefficient, where q is a fully revealing price function.

Note that the fully revealing REE for this economy, in which agents consume their expected endowment conditional on s, is ex-post efficient, but does not Pareto dominate the nonrevealing equilibrium allocation. In the fully revealing equilibrium, s_1-informed agents are better off while s_2-informed agents are worse off. ||

In this example the nonrevealing P-REE is ex-post Pareto inefficient because of the measurability constraint that applies to the uninformed. As we have pointed out, there is no Blackwell effect of information revelation (since efficiency can be achieved with no information). Thus we can attribute the inefficiency to adverse selection. The situation is analogous to a pooling allocation in a prototypical insurance economy with adverse selection *á la* Rothschild and Stiglitz (1977). In the candidate pooling equilibrium of Rothschild and Stiglitz, the "bad" type of informed agent imposes a negative externality on the "good" type, just as in the nonrevealing equilibrium of Example 4 above, where s_1-informed agents "subsidize" s_2-informed agents. The separating equilibrium in Rothschild and Stiglitz, on the other hand, may be ex-post inefficient (due to the exclusivity of agents' private information), unlike a fully revealing equilibrium in our economy.

Other examples have been proposed in the literature to study the adverse selection effect in economies such as ours (see, for instance, Marín and Rahi (2000)). In these examples, there are only two agents, an informed and an uninformed agent. At a partially revealing equilibrium, the same measurability constraint applies to both agents (due to the market-clearing condition). Hence the equilibrium is identical to the one that would obtain in a symmetric information economy. This is not the case in our example, where at a nonrevealing equilibrium the trades of the informed agent depend nontrivially on his information.

APPENDIX

The proofs of the various inefficiency propositions are based on a common idea. We identify necessary conditions satisfied by an equilibrium allocation on the one hand and by a constrained efficient (in the appropriate sense) allocation on the other. We then show that, generically, these conditions cannot hold simultaneously. In other words, we demonstrate that it is generically possible to achieve a local Pareto improvement in the neighborhood of an equilibrium allocation. In all of the results except Proposition 6.2 the only necessary conditions we need to consider are the first order conditions. In the Proposition 6.2, however, these do not suffice, so we also use the second order conditions for constrained efficiency.

In order to proceed with the proofs, we need to introduce some more notation. Let

$$c^h_{s^h,s} := [\omega^h(s^h, s, t) + r(s,t) \cdot y^h(s^h, s)]_{t \in \mathcal{T}}$$

be the vector of state-contingent consumption of agent (h, τ), conditional on the event $(\tilde{s}^{h\tau} = s^h,\, \tilde{s} = s)$. Define the function $U^h_{s^h,s} : I\!R^T$

and subject to $y^h(s^h, \cdot)$ being p-measurable. Under Assumption 2, the solutions of (AO) are characterized by the following system of first order conditions:

$$\sum_{s' \in \mathcal{S}_s^p} R_{s'}^\top DU^h_{s^h,s'} - \lambda^h(s^h, s)\, p(s) = 0, \qquad \forall h \in \mathcal{H}, s^h \in \mathcal{S}^h, s \in \mathcal{S}, \tag{A.1}$$

$$p(s) \cdot y^h(s^h, s) = 0, \qquad \forall h \in \mathcal{H}, s^h \in \mathcal{S}^h, s \in \mathcal{S}, \tag{A.2}$$

where $\lambda^h(s^h, \cdot) : \mathcal{S} \to I\!R$ is a p-measurable function. By Walras' law, for each $\mathcal{S}_s^p \in \mathcal{S}^p$, the market-clearing equation for one asset is redundant. Hence, the resource feasibility condition can be written as

$$\sum_{h,s^h} \pi(s^h, \mathcal{S}_s^p)\, \hat{y}^h(s^h, s) = 0, \qquad \forall s \in \mathcal{S}, \tag{A.3}$$

where $\hat{y}^h(s^h, s)$ is the vector obtained from $y^h(s^h, s)$ by deleting the last element.

A P-REE can be described as a solution to the equations (A.1)–(A.3) with respect to $\{y^h\}, \{\lambda^h\}$, and p. In order to write this equation system more compactly, let

$$\xi_{\mathcal{S}_s^p} := [y^h(s^h, s),\, \lambda^h(s^h, s),\, \hat{p}(s)]_{h \in \mathcal{H}, s^h \in \mathcal{S}^h} \quad \in I\!R^{\overline{S} J_{\mathcal{S}_s^p}} \times I\!R^{\overline{S}} \times I\!R^{J_{\mathcal{S}_s^p} - 1},$$

where $\hat{p}(s)$ is the vector obtained by deleting the last element of $p(s)$, and[23]

$$\xi := [\xi_{\mathcal{S}_s^p}]_{\mathcal{S}_s^p \in \mathcal{S}^p} \quad \in I\!R^{\overline{S} J_{\mathcal{S}^p}} \times I\!R^{\overline{S} S^p} \times I\!R^{J_{\mathcal{S}^p} - S^p}.$$

Thus ξ is a complete specification of the endogenous variables of (A.1)–(A.3). Let $f(\xi; \omega) = 0$ denote the set of equations given by (A.1), and $g(\xi; \omega) = 0$ the equations (A.2)–(A.3). Then, ξ is a P-REE if and only if

$$F(\xi; \omega) := \begin{pmatrix} f(\xi; \omega) \\ g(\xi; \omega) \end{pmatrix} = 0.$$

This system has $\overline{S} J_{\mathcal{S}^p} + \overline{S} S^p + J_{\mathcal{S}^p} - S^p$ equations, which is equal to the dimension of ξ. We denote the components of F corresponding to the cell $\mathcal{S}_s^p$ by $F_{\mathcal{S}_s^p}(\xi_{\mathcal{S}_s^p}; \omega_{\mathcal{S}_s^p})$, where

$$\omega_{\mathcal{S}_s^p} := [\omega^h(s^h, s', t)]_{h \in \mathcal{H}, s^h \in \mathcal{S}^h, s' \in \mathcal{S}_s^p, t \in \mathcal{T}} \quad \in I\!R_{++}^{\overline{S} S_s^p T}.$$

The functions $f_{\mathcal{S}_s^p}$ and $g_{\mathcal{S}_s^p}$ are defined analogously.

Given a collection $\{z(a)\}_{a \in A}$ of vectors or matrices, we denote by $\operatorname{diag}_{a \in A}[z(a)]$ the (block) diagonal matrix with typical entry $z(a)$, where a varies across the diagonal entries.

[23] Note that all the components of $\xi_{\mathcal{S}_s^p}$ are invariant with respect to s in the cell $\mathcal{S}_s^p$.

For a given vector or matrix z, $\text{diag}_{a\in A}[z]$ is the diagonal matrix with the term z repeated $\#A$ times. Finally, we define $y_{\mathcal{S}_s^p} := \{y^h(s^h, s)\}_{h\in\mathcal{H}, s^h\in\mathcal{S}^h}$ and $y := [y_{\mathcal{S}_s^p}]_{\mathcal{S}_s^p\in\mathcal{S}^p}$.

It is easily seen that $D_{\xi,\omega}F$ has a diagonal structure:

$$D_{\xi,\omega}F = \text{diag}_{\mathcal{S}_s^p\in\mathcal{S}^p}[D_{\xi_{\mathcal{S}_s^p},\omega_{\mathcal{S}_s^p}}F_{\mathcal{S}_s^p}].$$

Furthermore,

$$D_{\xi_{\mathcal{S}_s^p},\omega_{\mathcal{S}_s^p}}F_{\mathcal{S}_s^p} = \begin{pmatrix} D_{\xi_{\mathcal{S}_s^p}}f_{\mathcal{S}_s^p} & D_{\omega_{\mathcal{S}_s^p}}f_{\mathcal{S}_s^p} \\ D_{\xi_{\mathcal{S}_s^p}}g_{\mathcal{S}_s^p} & 0 \end{pmatrix},$$

with

$$D_{\omega_{\mathcal{S}_s^p}}f_{\mathcal{S}_s^p} = \text{diag}_{h,s^h}[\{\ldots R_{s'}^\top D^2U^h_{s^h,s'}\ldots\}_{s'\in\mathcal{S}_s^p}]$$

and

$$D_{\xi_{\mathcal{S}_s^p}}g_{\mathcal{S}_s^p} = \begin{pmatrix} \text{diag}_{h,s^h}[p(s)^\top] & | & 0 & | & Y_{\mathcal{S}_s^p}^\top \\ \text{———————} & & \text{—} & & \text{—} \\ \{\ldots\pi(s^h,\mathcal{S}_s^p)\,\hat{I}^\top\ldots\}_{h,s^h} & | & 0 & | & 0 \end{pmatrix},$$

where

$$\hat{Y}_{\mathcal{S}_s^p} := [\ldots\hat{y}^h(s^h,s)\ldots]_{h,s^h}$$

is the $((J_{\mathcal{S}_s^p}-1)\times\overline{S})$ matrix of agents' portfolios in the cell $\mathcal{S}_s^p$, and $\hat{I}$ is the $(J_{\mathcal{S}_s^p}\times(J_{\mathcal{S}_s^p}-1))$ matrix defined by

$$\hat{I} := \begin{pmatrix} I_{(J_{\mathcal{S}_s^p}-1)} \\ \text{——} \\ 0 \end{pmatrix}.$$

The following two results can be established using standard arguments (see, for instance, Citanna, Kajii, and Villanacci (1998)):

Fact 1. *The matrices $D_{\omega_{\mathcal{S}_s^p}}f_{\mathcal{S}_s^p}$ and $D_{\xi_{\mathcal{S}_s^p}}g_{\mathcal{S}_s^p}$ have full row rank. Hence, so do $D_{\xi_{\mathcal{S}_s^p},\omega_{\mathcal{S}_s^p}}F_{\mathcal{S}_s^p}$ and $D_{\xi,\omega}F$, as well as $D_\omega f$ and $D_y g$.*

Fact 2. *For a generic subset of economies, at any P-REE with price function p, the constraints ($IR_{\mathcal{S}^p}$) and ($IC_{\mathcal{S}^p}$) are satisfied with strict inequality, for every $h\in\mathcal{H}$, $s^h,\hat{s}^h \in \mathcal{S}^h$ ($s^h\neq\hat{s}^h$), and $\mathcal{S}_s^p\in\mathcal{S}^p$.*

In the proofs of Propositions 5.2, 6.1, and 6.2, we restrict endowments to be in the generic subset for which Fact 2 holds. The next lemma is a preliminary step to proving Proposition 5.2.

LEMMA A.1. *For a generic subset of economies, at any P-REE* $\{p, \{y^h\}\}$, *the row rank of* $\hat{Y}_{\mathcal{S}^p_s}$ *is* $min\,(\overline{S}-1, J_{\mathcal{S}^p_s}-1)$, *for every* $\mathcal{S}^p_s \in \mathcal{S}^p$.

Proof. Fix a partition $\mathcal{S}^p$ and a cell $\mathcal{S}^p_s$ of this partition. We first consider the case where $\overline{S} \geq J_{\mathcal{S}^p_s}$. We will show that generically, at a P-REE that induces the partition $\mathcal{S}^p$, there is no solution δ to the equations $\delta^\top \widehat{Y}_{\mathcal{S}^p_s} = 0$ and $\delta \cdot \delta = 1$, where $\widehat{Y}_{\mathcal{S}^p_s}$ is obtained from $\hat{Y}_{\mathcal{S}^p_s}$ by deleting its first column. Consider the equation system

$$\Gamma_{\mathcal{S}^p_s}(\xi_{\mathcal{S}^p_s}, \delta; \omega_{\mathcal{S}^p_s}) := \begin{pmatrix} F_{\mathcal{S}^p_s}(\xi_{\mathcal{S}^p_s}; \omega_{\mathcal{S}^p_s}) \\ \delta^\top \widehat{Y}_{\mathcal{S}^p_s} \\ \delta \cdot \delta - 1 \end{pmatrix} = 0.$$

Its Jacobian is

$$D_{\xi_{\mathcal{S}^p_s}, \delta, \omega_{\mathcal{S}^p_s}} \Gamma_{\mathcal{S}^p_s} = \left(\begin{array}{cc|c} D_{\xi_{\mathcal{S}^p_s}} f_{\mathcal{S}^p_s} & 0 & D_{\omega_{\mathcal{S}^p_s}} f_{\mathcal{S}^p_s} \\ \hline D_{\xi_{\mathcal{S}^p_s}} g_{\mathcal{S}^p_s} & 0 & 0 \\ D_{\xi_{\mathcal{S}^p_s}}(\delta^\top \widehat{Y}_{\mathcal{S}^p_s}) & \widehat{Y}^\top_{\mathcal{S}^p_s} & 0 \\ 0 & 2\delta^\top & 0 \end{array}\right). \qquad (A.4)$$

Note that the matrix

$$D_{y_{\mathcal{S}^p_s}} \begin{pmatrix} g_{\mathcal{S}^p_s} \\ \delta^\top \widehat{Y}_{\mathcal{S}^p_s} \end{pmatrix} = \begin{pmatrix} \text{diag}_{\,h, s^h}\,[p(s)^\top] \\ \{\ldots \pi(s^h, \mathcal{S}^p_s)\,\hat{I}^\top \ldots\}_{h, s^h} \\ \big(0 \quad \text{diag}_{\,\{s^1_2, \ldots, s^H_{S^H}\}}[(\delta^\top \; 0)]\big) \end{pmatrix}$$

is row-equivalent to

$$\begin{pmatrix} p(s)^\top & 0 & \ldots & 0 \\ \pi(s^1_1, \mathcal{S}^p_s)\,\hat{I}^\top & \pi(s^1_2, \mathcal{S}^p_s)\,\hat{I}^\top & \ldots & \pi(s^H_{S^H}, \mathcal{S}^p_s)\,\hat{I}^\top \\ 0 & p(s)^\top & \ldots & 0 \\ 0 & (\delta^\top \; 0) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & p(s)^\top \\ 0 & 0 & \ldots & (\delta^\top \; 0) \end{pmatrix}, \qquad (A.5)$$

where s^h_j is the j-th element of $\mathcal{S}^h$. Since $p_J(s) = 1$ and, at any zero of $\Gamma_{\mathcal{S}^p_s}$, $\delta \neq 0$, the matrix (A.5) has full row rank, and hence so does the lower left block of $D_{\xi_{\mathcal{S}^p_s}, \delta, \omega_{\mathcal{S}^p_s}} \Gamma_{\mathcal{S}^p_s}$ (as partitioned in (A.4)). Furthermore, by Fact 1, $D_{\omega_{\mathcal{S}^p_s}} f_{\mathcal{S}^p_s}$ has full row rank. Hence, the whole matrix $D_{\xi_{\mathcal{S}^p_s}, \delta, \omega_{\mathcal{S}^p_s}} \Gamma_{\mathcal{S}^p_s}$ has full row rank. By the transversality theorem, for $\omega_{\mathcal{S}^p_s}$ in a generic subset $E_{\mathcal{S}_s^p}$ of $I\!R_{++}^{\overline{S}S_s^p T}$, the same is true for $D_{\xi_{\mathcal{S}_s^p},\delta}\Gamma_{\mathcal{S}_s^p}$ at all zeros of $\Gamma_{\mathcal{S}_s^p}(\xi_{\mathcal{S}_s^p},\delta;\omega_{\mathcal{S}_s^p})$.[24] But this system has more independent equations, $(\overline{S}J_{\mathcal{S}_s^p}+\overline{S}+J_{\mathcal{S}_s^p}-1)+\overline{S}$, than unknowns, $(\overline{S}J_{\mathcal{S}_s^p}+\overline{S}+J_{\mathcal{S}_s^p}-1)+J_{\mathcal{S}_s^p}-1$, since, by hypothesis, $\overline{S}>J_{\mathcal{S}_s^p}-1$. So $\Gamma_{\mathcal{S}_s^p}(\xi_{\mathcal{S}_s^p},\delta;\omega_{\mathcal{S}_s^p})=0$ has no solution, for any $\omega_{\mathcal{S}_s^p}\in E_{\mathcal{S}_s^p}$.

Since the Cartesian product of generic sets is generic in the product space, it follows that for a generic subset $E:=\times_{\mathcal{S}_s^p\in\mathcal{S}^p}E_{\mathcal{S}_s^p}$ of $I\!R_{++}^{\overline{S}ST}$ there is no solution to $\Gamma_{\mathcal{S}_s^p}(\xi_{\mathcal{S}_s^p},\delta;\omega_{\mathcal{S}_s^p})=0$ for any $\mathcal{S}_s^p\in\mathcal{S}^p$. This establishes the result for the case of $\overline{S}\geq J_{\mathcal{S}_s^p}$. If $\overline{S}<J_{\mathcal{S}_s^p}$, we mimic the above argument by replacing $\widehat{Y}_{\mathcal{S}_s^p}$ by the submatrix of $\widehat{Y}_{\mathcal{S}_s^p}$ consisting of its first $\overline{S}-1$ rows. ∎

Proof of Proposition 5.2. We restrict endowments to be in the generic subset for which the rank condition of Lemma A.1 holds. Consider a P-REE with price function p and portfolio allocation $\{y^h\}$. If it is price efficient, then $\Delta y:=\{\Delta y^h(s^h,s)\}_{h,s^h,s}=0$ is a solution to the following program, for some strictly positive weights $\mu:=\{\mu^h(s^h)\}_{h,s^h}$:

$$\max_{\Delta y,q}\sum_{h,s^h}\mu^h(s^h)U^h_{s^h,s}(\overline{c}^h_{s^h,s})$$

subject to

$$\overline{c}^h_{s^h,s}=[\omega^h(s^h,s,t)+r(s,t)\cdot(y^h(s^h,s)+\Delta y^h(s^h,s))]_{t\in\mathcal{T}},\qquad\forall h\in\mathcal{H},s^h\in\mathcal{S}^h,s\in\mathcal{S};$$

and the constraints $(\overline{\mathrm{RF}})$, $(\mathrm{M}_{\mathcal{S}^p})$, $(\mathrm{IR}_{\mathcal{S}^p})$, and $(\mathrm{IC}_{\mathcal{S}^p})$ on $\{y^h+\Delta y^h\}$. Given that $\overline{S}>S_s^p$, for all $\mathcal{S}_s^p\in\mathcal{S}^p$, it is easy to check that constraint qualification holds after deleting any redundant constraints in $\overline{\mathrm{RF}}$. By Fact 2, in a neighborhood of a P-REE, the constraints $(\mathrm{IR}_{\mathcal{S}^p})$ and $(\mathrm{IC}_{\mathcal{S}^p})$ are not binding. Therefore, the first order conditions, evaluated at $\Delta y=0$, give us

$$\mu^h(s^h)\sum_{s'\in\mathcal{S}_s^p}R_{s'}^{\top}DU^h_{s^h,s'}=\sum_{s'\in\mathcal{S}_s^p}\nu(s')\,\pi(s^h\mid s')+\gamma^h(s^h,s)\,p(s),\qquad\forall h\in\mathcal{H},s^h\in\mathcal{S}^h,s\in\mathcal{S},\tag{A.6}$$

$$\sum_{h,s^h}\gamma^h(s^h,s)\,\hat{y}^h(s^h,s)=0,\qquad\forall s\in\mathcal{S},\tag{A.7}$$

for some $\nu(s)\in I\!R^{J_{\mathcal{S}_s^p}}$, for every s, and a p-measurable function $\gamma^h(s^h,\cdot):\mathcal{S}\to I\!R$. The equations (A.6) and (A.7) must be satisfied in addition to the equations (A.1)–(A.3) defining a P-REE.

[24] Openness of $E_{\mathcal{S}_s^p}$ follows from a standard argument; see, for example, Citanna, Kajii, and Villanacci (1998). This argument also applies in the subsequent proofs.

It follows from (A.1) and (A.6) that

$$\mu^h(s^h)\lambda^h(s^h,s)p(s) = \sum_{s'\in\mathcal{S}^p_s} \nu(s')\,\pi(s^h\,|\,s') + \gamma^h(s^h,s)\,p(s), \qquad \forall h\in\mathcal{H}, s^h\in\mathcal{S}^h, s\in\mathcal{S}.$$

Since, for every s, $p_J(s) = 1$ and, from Walras' law, $\nu_J(s) = 0$, we obtain

$$\mu^h(s^h)\lambda^h(s^h,s) = \gamma^h(s^h,s). \tag{A.8}$$

Multiplying both sides of (A.8) by $\hat{y}^h(s^h,s)$, summing over (h,s^h), and using (A.7), we get:

$$\phi(\xi,\mu;\omega) := \sum_{h,s^h} \mu^h(s^h)\lambda^h(s^h,s)\hat{y}^h(s^h,s) = 0 \qquad \forall s\in\mathcal{S}. \tag{A.9}$$

Since $\lambda^h(s^h,\cdot)$ and $\hat{y}^h(s^h,\cdot)$ are p-measurable, (A.9) consists of $(J_{\mathcal{S}^p} - S^p)$ distinct equations.

If the P-REE is price efficient, it follows from the foregoing analysis that

$$\Phi(\xi,\mu;\omega) := \begin{pmatrix} F(\xi;\omega) \\ \phi(\xi,\mu;\omega) \end{pmatrix} = 0.$$

The Jacobian of Φ, $D_{\xi,\mu,\omega}\Phi$, is row/column-equivalent to the block triangular matrix

$$\begin{pmatrix} D_{y,\hat{p}}f & D_{\lambda,\mu}f & D_\omega f \\ D_{y,\hat{p}}\phi & D_{\lambda,\mu}\phi & 0 \\ D_{y,\hat{p}}g & 0 & 0 \end{pmatrix},$$

where the subscripts $\hat{p}$ and λ are used to denote derivatives with respect to $\{\hat{p}(s)\}_{\mathcal{S}^p_s\in\mathcal{S}^p}$ and $\{\lambda^h(s^h,s)\}_{s^h\in\mathcal{S}^h,\mathcal{S}^p_s\in\mathcal{S}^p}$ respectively. Now $D_\lambda\phi = \text{diag}_{\,\mathcal{S}^p_s\in\mathcal{S}^p}[\hat{Y}_{\mathcal{S}^p_s}\,\text{diag}_{\,h,s^h}(\mu^h(s^h))]$, which has row rank $k := \sum_{\mathcal{S}^p_s\in\mathcal{S}^p} min\,(\overline{S}-1, J_{\mathcal{S}^p_s}-1)$, by Lemma A.1. From Fact 1, $D_\omega f$ and $D_y g$ have full row rank. Therefore, $D_{\xi,\mu,\omega}\Phi$ has row rank equal to k plus the number of variables in ξ and, by the transversality theorem, for a generic subset of endowments so does $D_{\xi,\mu}\Phi$, at every solution of $\Phi(\xi,\mu;\omega) = 0$. But then this set of solutions must be empty, since $J_{\mathcal{S}^p} - S^p > \overline{S} - 1$ implies that $k > \overline{S} - 1$, so that the system $\Phi(\xi,\mu;\omega) = 0$ has more independent equations than unknowns. (Note that we can normalize one of the weights μ to be one.) ∎

Proof of Proposition 6.1. Consider a P-REE with price function p and portfolio allocation $\{y^h\}$. If it is ($\overline{\text{RF}}$, $\text{IR}_{\mathcal{S}^p}$, $\text{IC}_{\mathcal{S}^p}$, BC_p)-constrained ex-post efficient, then in particular it is ex-post efficient in the subeconomy $\mathcal{S}^p_s$ and, therefore, $\Delta y := \{\Delta y^h(s^h,s')\}_{h\in\mathcal{H},s^h\in\mathcal{S}^h,s'\in\mathcal{S}^p_s} = 0$ is a solution to the following program, for some strictly positive weights $\{\mu^h(s^h,s')\}_{h\in\mathcal{H},s^h\in\mathcal{S}^h,s'\in\mathcal{S}^p_s}$:

$$\max_{\Delta y} \sum_{h,s^h} \sum_{s' \in \mathcal{S}_s^p} \mu^h(s^h, s') U^h_{s^h,s'}(\overline{c}^h_{s^h,s'})$$

subject to

$$\overline{c}^h_{s^h,s'} = [\omega^h(s^h, s', t) + r(s', t) \cdot (y^h(s^h, s) + \Delta y^h(s^h, s'))]_{t \in \mathcal{T}}, \qquad \forall h \in \mathcal{H}, s^h \in \mathcal{S}^h, s' \in \mathcal{S}_s^p;$$

$$\sum_{h,s^h} \pi(s^h \mid s')\, \Delta y^h(s^h, s') = 0, \qquad \forall s' \in \mathcal{S}_s^p;$$

$$p(s) \cdot \Delta y^h(s^h, s') = 0, \qquad \forall h \in \mathcal{H}, s^h \in \mathcal{S}^h, s' \in \mathcal{S}_s^p;$$

as well as the constraints ($\mathrm{IR}_{\mathcal{S}^p}$) and ($\mathrm{IC}_{\mathcal{S}^p}$) on $\{y^h + \Delta y^h\}$ for the subeconomy $\mathcal{S}_s^p$. Constraint qualification can easily be established for this program. By Fact 2, in a neighborhood of a P-REE, the constraints ($\mathrm{IR}_{\mathcal{S}^p}$) and ($\mathrm{IC}_{\mathcal{S}^p}$) are not binding. Hence, the first order conditions, evaluated at $\Delta y = 0$, give us

$$\mu^h(s^h, s') R^\top_{s'} DU^h_{s^h,s'} = \nu(s')\, \pi(s^h \mid s') + \gamma^h(s^h, s')\, p(s), \qquad \forall h \in \mathcal{H}, s^h \in \mathcal{S}^h, s' \in \mathcal{S}_s^p,$$

for some functions $\nu : \mathcal{S}_s^p \to I\!R^{J_{\mathcal{S}_s^p}}$, and $\gamma^h : \mathcal{S}^h \times \mathcal{S}_s^p \to I\!R$. In particular, this implies that, for any given $s' \in \mathcal{S}_s^p$, the marginal utility vectors $R^\top_{s'} DU^h_{s^h,s'}$ lie in the two-dimensional subspace of $R^{J_{\mathcal{S}_s^p}}$ spanned by $\nu(s')$ and $p(s)$, for every (h, s^h).

Now consider three pairs (h, s^h), identifying three agent-types, indexed by $\{h_1, h_2, h_3\}$, and define

$$\psi(\xi, \eta; \omega) := \eta_1 R^\top_{\hat{s}} DU^{h_1}_{\hat{s}} + \eta_2 R^\top_{\hat{s}} DU^{h_2}_{\hat{s}} + \eta_3 R^\top_{\hat{s}} DU^{h_3}_{\hat{s}} = 0, \qquad \eta \in I\!R^3,$$

for some $\hat{s} \in \mathcal{S}_s^p$ such that *rank* $(R_{\mathcal{S}_s^p,\hat{s}}) \geq J_{\mathcal{S}_s^p} + 3$. Then, a necessary condition for the P-REE allocation to be ($\overline{\mathrm{RF}}$, $\mathrm{IR}_{\mathcal{S}^p}$, $\mathrm{IC}_{\mathcal{S}^p}$, BC_p)-constrained ex-post efficient is that

$$\Psi(\xi, \eta; \omega) := \begin{pmatrix} F(\xi; \omega) \\ \psi(\xi, \eta; \omega) \\ \eta \cdot \eta - 1 \end{pmatrix} = 0,$$

for some $\eta \in I\!R^3$. The Jacobian, $D_{\xi,\eta,\omega}\Psi$, is row-equivalent to

$$\left(\begin{array}{cc|c} D_\xi f & 0 & D_\omega f \\ D_\xi \psi & D_\eta \psi & D_\omega \psi \\ \hline 0 & 2\eta^\top & 0 \\ D_\xi g & 0 & 0 \end{array}\right),$$

and $D_\omega\binom{f}{\psi}$ is given, up to a permutation of columns, by

$$\begin{pmatrix} (\ldots R_{s'}^\top D^2 U_{s'}^{h_1}\ldots)_{s'\in\mathcal{S}_s^p} & 0 & 0 & 0 \\ 0 & (\ldots R_{s'}^\top D^2 U_{s'}^{h_2}\ldots)_{s'\in\mathcal{S}_s^p} & 0 & 0 \\ 0 & 0 & (\ldots R_{s'}^\top D^2 U_{s'}^{h_3}\ldots)_{s'\in\mathcal{S}_s^p} & 0 \\ 0 & 0 & 0 & X \\ 0\ldots\eta_1 R_{\hat{s}}^\top D^2 U_{\hat{s}}^{h_1}\ldots 0 & 0\ldots\eta_2 R_{\hat{s}}^\top D^2 U_{\hat{s}}^{h_2}\ldots 0 & 0\ldots\eta_3 R_{\hat{s}}^\top D^2 U_{\hat{s}}^{h_3}\ldots 0 & 0 \end{pmatrix},$$

where we have not specified what the block X is since it does not affect the analysis. At any zero of Ψ, η is nonzero; without loss of generality, let η_1 be nonzero. Then

$$\begin{pmatrix} (\ldots R_{s'}^\top D^2 U_{s'}^{h_1}\ldots)_{s'\in\mathcal{S}_s^p} \\ 0\ldots\eta_1 R_{\hat{s}}^\top D^2 U_{\hat{s}}^{h_1}\ldots 0 \end{pmatrix}$$

has the same row rank as $R_{\mathcal{S}_s^p,\hat{s}}^\top$, which is by assumption at least $J_{\mathcal{S}_s^p}+3$. Therefore,

$$\textit{row rank}\left[D_\omega\begin{pmatrix} f\\ \psi\end{pmatrix}\right] \geq \textit{row rank}\,(D_\omega f)+3.$$

Since $D_\xi g$ has full row rank (by Fact 1), at any zero of Ψ,

$$\textit{row rank}\,(D_{\xi,\eta,\omega}\Psi) \geq \textit{row rank}\,(D_{\xi,\omega}F)+4.$$

In other words, relative to the equilibrium equations, the equation system $\Psi=0$ has three additional unknowns ($\eta\in I\!R^3$) and at least four additional (locally) independent equations. Generically, therefore, the system has no solution. ∎

Proof of Proposition 6.2. Consider a nonrevealing P-REE $\{\overline{p},\{\overline{y}^h\}\}$. Let $\Delta y := \{\Delta y^h(s^h,\sigma)\}_{h,s^h,\sigma}$, and

$$W := \sum_{h,s^h,t,s,\sigma} \mu^h(s^h,s)\,\pi(s^h|s)\,\pi(s,\sigma)\,\pi(t)\,u^h[\omega^h(s^h,s,t)+r(s,t)\cdot\overline{y}^h(s^h)+r(s,t)\cdot\Delta y^h(s^h,\sigma)].$$

The probabilities Π must satisfy

$$\sum_{\sigma\in\Sigma}\pi(s,\sigma)=\pi(s) \quad \forall s\neq\hat{s}, \qquad \text{and} \qquad \sum_{s\in\mathcal{S}}\pi(s,\sigma)=\pi(\sigma) \quad \forall\sigma.$$

solves

$$\max_{\Pi,\Delta y} W \qquad \text{subject to} \qquad (\overline{\text{RF}}_{\tilde{\sigma}}), (\text{BC}_{\overline{p},\tilde{\sigma}}), \text{ and } (A.10), \tag{A.12}$$

for some strictly positive weights $\{\mu^h(s^h, s)\}_{h,s^h,s}$. Fact 2 allows us to ignore the $(\text{IR}_{\mathcal{S}^p})$ and $(\text{IC}_{\mathcal{S}^p})$ constraints in a neighborhood of (A.11).

It can easily be verified that, if the weights $\{\mu^h\}$ are chosen to be invariant with respect to s, the first order conditions for (A.12) are satisfied at (A.11). The second order necessary condition is that D^2W, the Hessian of W obtained by taking derivatives with respect to $(\Pi, \Delta y)$, evaluated at (A.11), be negative semidefinite when the derivatives are restricted to the directions that satisfy the constraints of (A.12). The directions along which derivatives with respect to $\pi(s, \sigma)$ and $\Delta y^h(s^h, \sigma)$ are taken are respectively denoted by $\alpha(s, \sigma) \in I\!R$ and $\beta^h(s^h, \sigma) \in I\!R^J$. Let

$$\alpha := \begin{pmatrix} \vdots \\ \{\alpha(s,\sigma)\}_{\sigma\in\Sigma} \\ \vdots \end{pmatrix}_{s\in\mathcal{S}}, \qquad \text{and} \qquad \beta := \begin{pmatrix} \vdots \\ \{\beta^h(s^h,\sigma)\}_{h\in\mathcal{H},\, s^h\in\mathcal{S}^h} \\ \vdots \end{pmatrix}_{\sigma\in\Sigma},$$

with the set $\mathcal{S}$ ordered so that $\hat{s}$ is its last element. The directions satisfying the constraints of (A.12) are solutions to

$$\zeta_1(\alpha) := \begin{pmatrix} 1\ldots1 & 0\ldots0 & \ldots & 0\ldots0 & 0\ldots0 \\ 0\ldots0 & 1\ldots1 & \ldots & 0\ldots0 & 0\ldots0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0\ldots0 & 0\ldots0 & \ldots & 1\ldots1 & 0\ldots0 \\ I_S & I_S & \ldots & I_S & I_S \end{pmatrix} \alpha = 0, \tag{A.13}$$

and

$$\zeta_2(\beta) := \begin{pmatrix} \text{diag}_{\,\sigma} \left[\begin{pmatrix} \vdots \\ [\ldots \pi(s^h \mid s)\, \hat{I}^\top \ldots]_{h,s^h} \\ \vdots \end{pmatrix}_s \right] \\ \hline \text{diag}_{\,h,s^h,\sigma}(\overline{p}^\top) \end{pmatrix} \beta = 0. \tag{A.14}$$

Thus the second order condition for (A.12) can be stated as follows:

$$(\,\alpha^\top \quad \beta^\top\,)\, D^2W \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \leq 0 \tag{A.15}$$

for all α and β satisfying

$$\zeta(\alpha,\beta) := \begin{pmatrix} \zeta_1(\alpha) \\ \zeta_2(\beta) \end{pmatrix} = 0.$$

The number of equations in (A.13) is $2S-1$, so the set of zeros of ζ_1 is a subspace of dimension $S(S-2)+1$. Likewise ζ_2 consists of $S^2(J-1)+S\overline{S}$ equations and the set of its zeros is a subspace of dimension $S(J-1)(\overline{S}-S)$. Therefore, the set of zeros of ζ is nonempty. We assume for simplicity that the equations in (A.14) are linearly independent (if not, our argument goes through by deleting redundant equations). Then $D_{\alpha,\beta}\zeta$ has full row rank (which, in particular, implies that constraint qualification is satisfied for the program (A.12)).

Note that

$$D^2W = \begin{pmatrix} 0 & D^2_{\Pi,\Delta y}W \\ (D^2_{\Pi,\Delta y}W)^\top & D^2_{\Delta y,\Delta y}W \end{pmatrix}.$$

Therefore,

$$(\,\alpha^\top \quad \beta^\top\,)\, D^2W \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 2\alpha^\top (D^2_{\Pi,\Delta y}W)\beta + \beta^\top (D^2_{\Delta y,\Delta y}W)\beta. \qquad (A.16)$$

The second term of (A.16) is always negative (since utility functions are concave). However, if the first term is nonzero for some α,β, the whole expression in (A.16) can be made positive and arbitrarily large by appropriately rescaling α (which is always possible by (A.13)), without affecting the second term of (A.16). This will result in a violation of (A.15). Hence, for (A.11) to be a solution to (A.12) (for some μ), we must have

$$v(\alpha,\beta) := \alpha^\top (D^2_{\Pi,\Delta y}W)\beta = 0$$

for all α,β satisfying $\zeta(\alpha,\beta)=0$ (with $D^2_{\Pi,\Delta y}W$ evaluated at (A.11)). Equivalently, the set of solutions to

$$\Upsilon(\alpha,\beta) := \begin{pmatrix} v(\alpha,\beta) \\ \zeta(\alpha,\beta) \end{pmatrix} = 0$$

and to $\zeta(\alpha,\beta)=0$ must coincide. This implies that $D_{\alpha,\beta}\Upsilon$ does not have full row rank at any zero of Υ (for some μ). (Suppose not, *i.e.* suppose there is a zero of Υ, (α^*,β^*), at which $D_{\alpha,\beta}\Upsilon$ has full row rank. Then, by the local submersion theorem, $D_{\alpha,\beta}\Upsilon$ has full row rank on a neighborhood N of (α^*,β^*). Let Υ_N and ζ_N be the restriction to N of Υ and ζ respectively. Then, zero is a regular value of Υ_N and ζ_N. By the preimage theorem, the set of solutions to $\Upsilon_N(\alpha,\beta)=0$ is either empty or is a manifold of dimension one less than the manifold of the set of solutions to $\zeta_N(\alpha, \beta) = 0$. In other words, Υ and ζ do not have the same zeros.)

Straightforward computations yield

$$v(\alpha, \beta) = \sum_{h, s^h, s, \sigma} \frac{\mu^h(s^h, s)}{\pi(s)} \alpha(s, \sigma) \beta^h(s^h, \sigma)^\top (R_s^\top DU^h_{s^h, s}),$$

and

$$D_\alpha v = \left(\ldots \sum_{h, s^h} \frac{\mu^h(s^h, s)}{\pi(s)} \beta^h(s^h, \sigma)^\top (R_s^\top DU^h_{s^h, s}) \ldots \right)_{s, \sigma}.$$

Since $D_{\alpha,\beta}\Upsilon$ does not have full row rank at any zero of ζ, $D_{\alpha,\beta} v$ must lie in the row space of $D_{\alpha,\beta}\zeta$, at every zero of ζ. In particular, $D_\alpha v$ is spanned by the rows of $D_\alpha \zeta_1$, *i.e.* there exist $a \in I\!R^{S-1} \times \{0\}$, and $b \in I\!R^S$, such that

$$\sum_{h, s^h} \frac{\mu^h(s^h, s)}{\pi(s)} \beta^h(s^h, \sigma)^\top (R_s^\top DU^h_{s^h, s}) = a_s + b_\sigma, \qquad \forall s, \sigma. \tag{A.17}$$

Noting that $a_{\hat{s}}$ is zero, we evaluate (A.17) at $s = \hat{s}$ to give us

$$\sum_{h, s^h} \frac{\mu^h(s^h, \hat{s})}{\pi(\hat{s})} \beta^h(s^h, \sigma)^\top (R_{\hat{s}}^\top DU^h_{s^h, \hat{s}}) = b_\sigma, \qquad \forall \sigma.$$

Substituting this back into (A.17), we get

$$\sum_{h, s^h} \beta^h(s^h, \sigma)^\top \left[\frac{\mu^h(s^h, s)}{\pi(s)} (R_s^\top DU^h_{s^h, s}) - \frac{\mu^h(s^h, \hat{s})}{\pi(\hat{s})} (R_{\hat{s}}^\top DU^h_{s^h, \hat{s}}) \right] = a_s, \quad \forall s, \sigma,$$

which implies that, fixing some $\sigma' \in \Sigma$,

$$\sum_{h, s^h} [\beta^h(s^h, \sigma) - \beta^h(s^h, \sigma')]^\top \left[\frac{\mu^h(s^h, s)}{\pi(s)} (R_s^\top DU^h_{s^h, s}) - \frac{\mu^h(s^h, \hat{s})}{\pi(\hat{s})} (R_{\hat{s}}^\top DU^h_{s^h, \hat{s}}) \right] = 0, \quad \forall s, \sigma. \tag{A.18}$$

(A.18) must hold for all β satisfying (A.14), for some μ. Furthermore, (A.14) and (A.18) are both linear in β. Hence the coefficients of β in (A.18) must be linearly dependent on those in (A.14). In particular, this is true for some $s = \check{s}$, *i.e.* there exist $c_s \in I\!R^{J-1} \times \{0\}$, and $d^h(s^h) \in I\!R$, such that

$$\frac{\mu^h(s^h, \check{s})}{\pi(\check{s})} (R_{\check{s}}^\top DU^h_{s^h, \check{s}}) - \frac{\mu^h(s^h, \hat{s})}{\pi(\hat{s})} (R_{\hat{s}}^\top DU^h_{s^h, \hat{s}}) = \sum_{s \in \mathcal{S}} \pi(s^h \mid s)\, c_s + d^h(s^h)\, \overline{p}, \qquad \forall h, s^h. \tag{A.19}$$

This implies that the vectors on the left hand side of (A.19) lie in an $(S+1)$-dimensional subspace of R^J (the one spanned by $\{c_s\}$ and $\overline{p}$), for every (h, s^h), for some μ. By an immediate reformulation of the argument in the proof of Proposition 6.1, we can show that, generically, the vectors $\{R_{\check{s}}^{\top} DU_{s^h,\check{s}}^h,\ R_{\hat{s}}^{\top} DU_{s^h,\hat{s}}^h\}$ are linearly independent across $S+2$ agent-types. Therefore, generically, condition (A.19) cannot hold, regardless of what the weights μ are. ∎

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