Signalling with Debt and Equity: A Unifying Approach and its Implications for the Pecking Order Hypothesis and Competitive Credit Rationing

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Signalling with debt and equity: a unifying approach and its implications for the Pecking-Order hypothesis and competitive credit rationing[1]

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Abstract

The paper sets out to tackle the following puzzle when insiders of a firm have more information than outside investors. The insiders' desire to sell overpriced securities creates an Adverse Selection problem leading to two contradictory results. On the one hand, it leads to Myers & Majluf (1984)'s Pecking-Order hypothesis that says that debt finance dominates equity finance. On the other hand it leads to Stiglitz & Weiss (1981)' credit rationing whose consequence is that equity finance dominates debt finance.

The paper resolves the puzzle by allowing firms to issue both debt and equity together and by having a general notion of what it is that insiders know more about. Then the Pecking-Order hypothesis and credit rationing only emerge as two, mutually exclusive, special cases.

The paper shows that combinations of debt and equity can be used to credibly signal information for a wide range of parameters. Thus, it provides a generalisation of the existing financial signalling and rationing literatures and helps to explain some contradictory theoretical and empirical results.

JEL-Classification codes: G32, D82

1 Introduction

Firms seeking outside funds to finance their investment opportunities naturally face an Adverse Selection problem as insiders of a firm know more about its operations than outside investors do.[1] Outside investors anticipate insiders' desire to sell overpriced securities and therefore react negatively if firms announce to issue new securities.[2]

Myers & Majluf (1984) argue that the Adverse Selection problem is particularly severe if firms issue equity to finance their investments. Firms should therefore issue debt when they can and only issue equity if their debt capacity is exhausted. In other words there is a Pecking-Order of financial instruments in which debt dominates equity.[3]

Stiglitz & Weiss (1981) however show that when there is asymmetric information between insiders of a firm and outside investors (banks) then the Adverse Selection problem created by debt finance may lead to rationing, i.e. a situation in which firms' demand for funds is not fully satisfied. Since firms may not obtain the funds they need via debt finance they should seek equity finance instead.[4] Equity then dominates debt since it avoids rationing.

The starting point of our analysis therefore is a puzzle: when insiders of a firm have more information than outside investors then the desire to sell overpriced securities leads to two contradictory results. On the one hand, it leads to the Pecking-Order in which debt dominates equity and on the other hand it leads to Credit Rationing with the consequence that equity dominates debt.

In this paper we resolve the puzzle by allowing firms to issue both debt and equity together and by having a general notion of what it is that insiders know more about. The previous Pecking-Order and rationing literatures have either considered debt or equity separately (Bester (1987), Hellmann & Stiglitz (2000)) or they have made very strong assumptions about the nature of asymmetric information (Brennan & Kraus (1987), Constantinides & Grundy (1989), Nachman & Noe (1994)) or they have done both (Stiglitz & Weiss (1981), Myers & Majluf (1984), Bester (1985), Besanko & Thakor (1987a), de Meza & Webb (1987)).

Our main result is that combinations of debt and equity can be used to credibly signal information to the market. Contrary perhaps to one's intuition, firms with safe investments issue more equity and less debt than firms with risky investments. The reason is that a financing decision can only be a useful signal if it is credible since insiders have an incentive to sell overvalued claims. Equity credibly signals safe investment projects since firms with risky projects find it too costly to use it. Equity is a convex claim so that its value increases with the risk of the underlying assets. Since the value of a claim is a cost to the firm, equity is particularly costly for risky firms. Similarly, debt credibly signals that investment projects are risky. Debt is a concave claim whose value decreases with risk. Since safe firms' debt is very valuable they find it very costly to issue. As a result, safe firms are unwilling to mimic risky firms' decision to issue debt.

As inside information is credibly transmitted to outside investors, there will be no general Adverse Selection effect and the financing decision will be efficient, i.e. all investment projects with a positive Net Present Value are fully funded. In our model, the Pecking-Order and rationing only emerge as two, mutually exclusive special cases. In other words, they emerge

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[1] The Adverse Selection problem has first been discussed by Akerlof (1970).

[2] There is plenty of evidence that the stock price reacts negatively to the announcement of a new security issue, see for example Asquith & Mullins (1986), Masulis & Korwar (1986) or Mikkelson & Partch (1986).

[3] Ever since Myers (1984) argued that a Pecking-Order theory best explains firms' capital structures, it has been elevated to a folklore proposition of corporate finance and features prominently in standard corporate finance textbooks, for example (Grinblatt & Titman 1998, p.592).

[4] Stiglitz and Weiss do not explicitly consider equity finance since they focus on credit markets. But they argue against equity finance on the grounds that "in those principal-agent problems [...] [i]n general, revenue sharing arrangements such equity finance [...] are inefficient" (p. 409). Principal-agent problems, or Moral Hazard problems, however are distinct from Adverse Selection problems.

only when we make very specific assumptions about what it is that insiders know more about. This explains the initial puzzle. Debt dominates equity for one set of parameter values and equity dominates debt for another set of parameter values. But neither dominance result is general in the sense that is occurs for all permitted parameter values.

What is general however is that safe investments are financed with more equity and less debt than risky investments. And this results leads to an interesting by-product of our analysis of the Pecking-Order and rationing. So far, the financial signalling literature has not achieved a consensus on how firms should finance investments whose quality is unknown to outside investors. Some argue that firms should use straight debt (Nachman & Noe (1994)) or collateralised debt (Bester (1985)). Some argue argue that firms should use equity in conjunction with debt repurchases (Brennan & Kraus (1987) section II). Others propose more involved financial instruments such as convertibles (Brennan & Kraus (1987) section III), maybe together with stock repurchases (Constantinides & Grundy (1989)).[5]

In our model we can reproduce each of those possibilities by restricting ourselves to specific parameter values that describe the quality of the investment projects. Our paper shows that despite the diversity of assumptions and results there is a common underlying logic to the existing literature: safe investments should be financed with more equity and less debt than risky investments.

Finally, the paper comments on some of the empirical literature on firms' capital structures. First, Jung et al. (1996) have criticised explanations of firm's capital structures based on asymmetric information since Myers and Majluf's Pecking-Order does not fit their data. But we argue that the Pecking-Order is only a very special case so that rejecting it should not automatically lead to a rejection of explanations based on asymmetric information in general. Secondly, the main studies of stock price reactions to security issues such as Asquith & Mullins (1986), Masulis & Korwar (1986) or Mikkelson & Partch (1986) contain roughly one third of data that is not in line with the predictions of the Pecking-Order hypothesis. Again, noting that the Pecking-Order is only a special case helps to interpret that portion of data without having to abandon asymmetric information models. Third, our paper can help to explain Kim & Sorensen (1986)'s finding that firms with high business risk are more levered, a result for which there was previously no theoretical explanation.

Section 2 presents an example to illustrate the issues that we develop in the rest of the paper. Section 3 introduces the formal signalling model. Before analysing the model, we discuss our general notion of asymmetric information in section 4 and show how it relates to well-known risk-concepts and how it comprises the assumptions made by many existing models as special cases. Section 5 solves the model for equilibrium outcomes. We discuss the results and their relevance for the Pecking-Order, rationing and, more generally, for the financial signalling literature in section 6. We look at some extension in section 7 and conclude in section 8.

2 An example

A simple example should give a better understanding of the more formal model that is developed in the rest of the paper. Moreover, it illustrates that private information about the quality of investment projects can be credibly transmitted to uninformed outside investors by an appropriate choice of debt and equity finance.

Suppose a firm needs to raise one unit of outside capital right now to undertake an investment project. The firm must turn to outside investors since it has no financial slack

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[5]In the light of such diversity, it is no surprise that the comprehensive survey of the corporate finance literature in Harris & Raviv (1992) cannot give a synthesis of the predictions of asymmetric information theories. Moreover, they identify the asymmetric information theories around Myers and Majluf as one of the few areas in corporate finance where there is genuine disagreement.

and its assets-in-place, which are worth 10 units, are not marketable right now. There are two types of investment projects: safe ones and risky ones. A risky investment project fails more often than a safe project but if it succeeds, it returns more. More specifically, suppose that a safe project fails and returns nothing 15% of the time while a risky project fails and returns nothing 25% of the time. A successful safe project returns 1.3 units and a successful risky project returns 1.8 units. Hence, the net rate of return is 10.5% for a safe project and 35% for a risky project.

Let us first consider a benchmark. Assume for a moment that outside investors know whether they are financing a safe or a risky investment project. Then the fair price for a zero coupon bond that finances a risky project is a repayment of $1.\bar{3}$ units (since then the expected repayment is equal to the amount provided, $1.\bar{3} \cdot 0.75 = 1$). If a safe project is financed the fair price is 1.18 ($1.18 \cdot 0.85 = 1$). A fair financing arrangement with equity requires that investors end up owning 8.8% of the firm ($0.088 \cdot (0.75 \cdot 1.8 + 10) = 1$) if the project is risky and 9% if the project is safe.

Whether a risky project is financed with debt requiring a repayment of $1.\bar{3}$ or with equity requiring a 8.8% ownership does not affect the value of the firm. In both cases, the firm with a risky project is expected to be worth 10.35 units (assets-in-place worth 10 plus the investment return of 35% on 1 unit of capital). Similarly, the expected value of a firm with a safe investment project is independent of how the project is financed - it is always 10.105 units. Hence, if there is no asymmetric information then the financing decision is indeterminate.

One often hears the argument that risky investments should be financed with equity and that safe investments should be financed with debt. The basis for that argument is that equity protects investors from the risk of potential losses and that one should avoid bankruptcy costs associated with debt.[6]

Suppose then debt finances safe projects and equity finances risky projects, and see what happens under the natural assumption that firms know more about their investment projects than outside investors do. Although investors cannot directly observe the quality of investment projects, they can infer it correctly from the firm's choice between debt and equity. So when they observe a debt issue they think it comes from a safe firm and when they observe an equity issue they think it comes from a risky firm. Based on that inference the fair price for debt is a repayment of 1.18 and for equity it is a stake of 8.8% in the firm. But at these prices for debt and equity, firms are given an opportunity to sell overvalued securities. Risky firms, which normally would have to offer a repayment of $1.\bar{3}$ on debt, can now *mimic* the issue decision of safe firms and offer a repayment of only 1.18. Similarly, safe firms have an incentive to sell overvalued equity by mimicking risky firms.

But investors are not irrational. They know that firms have an incentive to sell overvalued claims so that they do not accept a debt repayment of 1.18 and an equity stake of 8.8% to begin with. Hence, financing risky projects with equity and safe ones with debt cannot be an equilibrium situation.

That debt finance of safe projects gives firms with risky projects an incentive to mimic them is the driving force behind Stiglitz & Weiss (1981)'s credit rationing result. Similarly, the driving force behind Myers & Majluf (1984)'s Pecking-Order and underinvestment results is that equity finance of risky (and high expected value) projects gives firms with safe (and low expected value) projects the incentive to mimic them.

Debt finance of safe projects and equity finance of risky projects is not an equilibrium, but the reverse is. With debt finance of risky projects and equity finance of safe projects,

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[6] See, for example, a recent article in *The Economist* (31.12.1999) on how the introduction of limited liability freed risk capital or note the following quote from a standard textbook in corporate finance: "...financial distress is costly. Other things equal, distress is more likely for firms with high business risk. That is why such firms generally issue less debt." (Brealey & Myers 1991, p. 448) To be fair, we should add that these arguments remain valid when there is no significant information advantage of firms since then an outside investor does not have to draw inferences about the quality of securities issued by firms.

the debt repayment is $1.\bar{3}$ and the equity stake given to outside investors is 9%. Now no firm has an incentive to mimic another firm. Furthermore, firms still obtain fair prices for their securities, i.e. there is no loss of efficiency compared to the full information benchmark.

The situation is only sub-optimal in the following sense. Equity is a convex security so that it would be more valuable to outside investors if it finances risky projects. Debt on the other hand is a concave security whose value is higher if it finances safe projects. But it is precisely this suboptimality (from the point of view of outside investors) that prevents firms from mimicking each other and deviating from equilibrium. Any deviation would then be profitable to outside investors and so, by definition, costly to firms.

3   The Model

In this section we present the formal model of a situation where a firm needs to issue new securities to an outside investor in order to finance its investments since the firm has no financial slack. The key feature of the model is that a firm knows the quality of its investment project while the outside investor does not. It is a signalling model in which the informed party, the firm, proposes a financing contract to the uninformed party, the outside investor, who can only accept or reject the proposal.

3.1   Financing investment opportunities with debt and equity

Suppose that a firm has access to an investment project of quality t. To take advantage of the investment opportunity and to undertake the project, the firm has to invest an amount I. The investment is risky: it succeeds with probability p_t returning an amount x_t and it fails with probability $1 - p_t$ returning nothing. An investment project always has positive Net Present Value, $p_t x_t > I$.

For simplicity we assume that there are just two sorts of investment projects: safe ones, $t = s$, and risky ones, $t = r$.[7] The safe project succeeds more often than the risky project but in the case of success the safe project returns less than the risky project: $p_s \geq p_r$ and $x_s \leq x_r$ with not both as an equality. The ex-ante probability that the firm has a safe investment project is q.

Although the firm has publicly known assets-in-place worth $W > I$, they cannot be used to pay for the investment project since they are not marketable right now and the investment project cannot be postponed.[8] Furthermore, the project is indivisible so that it cannot be partially funded. The firm therefore needs to raise the entire amount I from an outside investor. In exchange for the funds he provides today, the firm gives him a mix of debt and equity claims to the firm's future value. Debt claims are zero-coupon bonds with face value F and equity claims give the outside investor an α%-stake in the firm. Note that we do not require F and α to be positive so that we allow for repurchases of debt and equity respectively. The expected value of a combination of debt and equity claims (F, α) which finances a project of quality t is:

$$v_t(F, \alpha) = p_t[F + \alpha(W + x_t - F)] + (1 - p_t)[\alpha W] \qquad (1)$$

If the project succeeds the debt will be repaid, if it fails the debt is worthless.[9] Note that equity gives a right to a fraction α of what the firm is worth after debt has been repaid.

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[7]In section 7.2 we show that a more complicated model with many quality types does not add much our arguments.

[8]Assuming $W > I$ allows us to extend the signalling role of equity to the case when safe projects are more valuable than risky projects. In that case, safe firms have to issue a lot of equity since risky firms have a very strong incentive to mimic them. But safe firms can only issue a lot of equity when they have a lot of assets-in-place.

[9]In fact, the debt contract specifies a payment $\min\{F, \tilde{x}\}$ where $\tilde{x}$ is the random return of the project. In

3.2 A pure signalling model

In order to focus only on the impact of asymmetric information between the firm and the outside investor we make a number of assumptions. First, there are no taxes. Second, we normalise the interest rate to zero. Third, there is universal risk-neutrality. Fourth, there are no direct costs of bankruptcy such as legal or liquidation costs. And fifth, there is no conflict of interests between debt-holders and equity- holders since a single outside investor can hold both debt and equity. Finally, we assume that there is no conflict of interests between the management of the firm and its initial owners. In fact, we make no distinction between them and talk instead of the insider of the firm.[10]

Under these assumptions, the insider, acting in his own interest, maximises the expected value of the firm net of the expected value of what is given to the outside investor via debt and equity:

$$
\begin{aligned}
U_t(F, \alpha) &= p_t(W + x_t) + (1 - p_t)W - v_t(F, \alpha) \\
&= (1 - \alpha)(W + p_t(x_t - F))
\end{aligned}
\tag{2}
$$

The insider who knows the quality of his investment project approaches the uninformed outside investor and asks him whether he is willing to provide an amount I in return for a combination of debt and equity (F, α). If the investor could directly observe the quality of the project he has just been asked to finance, he would provide the funds if $v_t(F, \alpha) \geq I$.

But the outside investor cannot directly observe the quality of the investment project. He only sees the combination of debt and equity (F, α) that he is being offered. But from that observation, he can make an informed guess about the quality of the investment project. Hence, he accepts to provide the investment capital when the *expectation across project qualities* of the value of the offered debt-equity portfolio, $V(F, \alpha)$, exceeds his opportunity cost:

$$
V(F, \alpha) = \mu(F, \alpha)v_s(F, \alpha) + (1 - \mu(F, \alpha))v_r(F, \alpha) \geq I
\tag{3}
$$

The probability $\mu(F, \alpha)$ describes his belief that the debt-equity portfolio (F, α) that he is being offered comes from an insider with a safe project.

If the investor accepts the financing contract (F, α) then he provides the funds and the insider undertakes the investment project. Some time later, the project delivers its returns, which are publicly known, and the investor is paid off according to the amount of debt and equity he holds. If the investor rejects the financing contract then the game ends. Figure 1 summarises the sequence of events.

The outside investor can only accept or reject the financing contract that the insider proposes. This is equivalent to assume that capital markets are perfectly competitive, i.e. outside investors have no bargaining power at all.[11]

To achieve an equilibrium, the behaviour of the insider and the outside investor must satisfy two conditions. First, they must maximise their expected payoffs. Second, the guess by the outside investor about the quality of the project must be a rational inference.[12]

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this simple two-point distribution however, debt returns F if there is success (since $F \leq x_t$; otherwise firms would make losses using debt finance) and 0 if there is failure. In section 7.1 we modify this formulation to allow for collateral.

[10] As a result of these assumptions we set aside many important issues in corporate finance. For an extensive treatment of these issues and a comprehensive list of references see Harris & Raviv (1992). We should add that although we do not allow for direct bankruptcy costs, which a priori makes risky firms avoid debt, section 7.1 deals with an example of indirect bankruptcy costs: loosing the collateral.

[11] For a discussion of the equivalence of the equilibrium outcomes of signalling/contract-proposal games to the equilibrium outcomes of competitive screening models (e.g. Rothschild & Stiglitz (1976)) see Maskin & Tirole (1992).

[12] Formally, we are solving for a Perfect-Bayesian equilibrium in pure strategies.
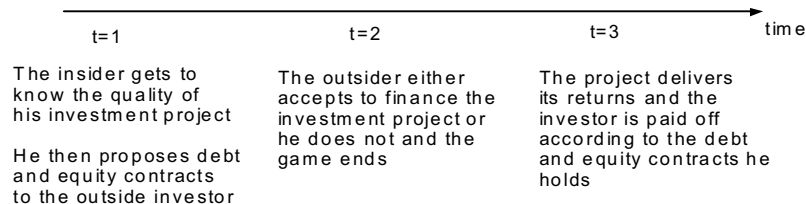
Figure 1: The sequence of events in the financing game

Condition 1 *a) Given the quality of his investment opportunity and given the investor's accept-or-reject decision, the insider issues debt and equity in order to maximise the expected value of the firm net of any expected future payments to outsiders (shown in equation (2)). b) Given the insider's issue choice of debt and equity and given the outside investor's rational belief about the quality of projects μ, the investor's accept-or-reject decision maximises the expected value of his debt-equity portfolio (shown in equation (1)).*

Condition 2 *For all debt-equity choices observed in equilibrium, Bayes' Law describes the outside investor's rational belief $\mu(F, \alpha)$.*

4 On the quality difference of investment opportunities

So far p_t and x_t describe the quality of investment projects in absolute terms. It will however be useful to conduct the analysis in terms of relative quality differences instead:

Definition 1 *The relative difference in success probabilities γ is given by*

$$\gamma = \frac{p_s - p_r}{p_s}$$

Definition 2 *The relative difference in returns ε is given by*

$$\varepsilon = \frac{x_r - x_s}{x_r}$$

Figure 2 illustrates how changes in x_t and p_t translate into changes in ε and γ. In the figure we fix the characteristics of the safe project, x_s and p_s, and vary the characteristics of the risky project, x_r and p_r. For example, if $p_r = p_s$ and $x_r > x_s$(risky project A) then $\gamma = 0$ and $\varepsilon > 0$. Alternatively, if $p_r < p_s$ and $x_r = x_s$ (risky project B) then $\varepsilon = 0$ and $\gamma > 0$.

Figure 3 demonstrates the usefulness of these measures. When there is no difference in return, $\varepsilon = 0$, then the safe project dominates the risky project by First-Order Stochastic Dominance.[13] The reverse is true when there is no difference in success probabilities, $\gamma = 0$. The safe project dominates the risky project by Second-Order Stochastic Dominance when $\varepsilon \leq \gamma$. When $\varepsilon = \gamma$ we have a special case of Second-Order Stochastic Dominance: projects then are Mean-Preserving Spreads. Finally, neither First- nor Second-Order Stochastic Dominance apply when $\varepsilon > \gamma$.

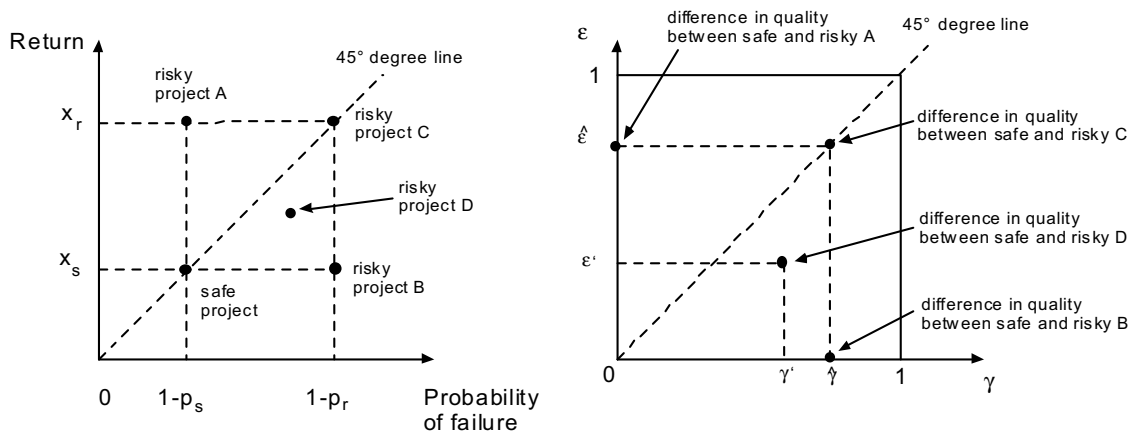[13]For a definition and discussion of these concepts see Laffont (1989).

Figure 2: How absolute quality differences translate into relative differences measured by ε and γ
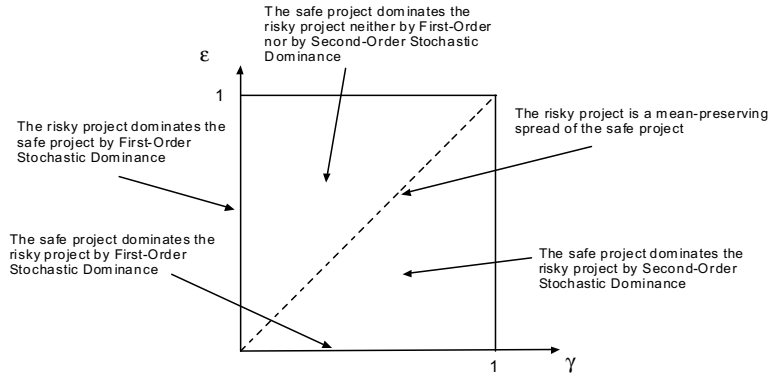

Figure 3: Relative differences in project quality and Stochastic Orderings

Finally, figure 4 shows how important contributions to the financial signalling and the credit rationing literature make different assumptions about the difference in quality between investment projects. Moreover, the figure shows that these assumptions are special cases of our more general set-up.[14] For example Brennan & Kraus (1987) assume First-Order Stochastic Dominance, i.e. $\gamma = 0$ or $\varepsilon = 0$, in the first half of their paper and Mean-Preserving Spreads, i.e. $\varepsilon = \gamma$, in the second half.[15] Stiglitz & Weiss (1981) assume Mean-Preserving Spreads too. We have put a question mark behind the classification of Myers & Majluf (1984) since in their original analysis investment projects are not risky. In the case when investments are risky, we believe that the most natural analogue to Myers and Majluf is when there is no difference in success probabilities, $\gamma = 0$, i.e. when projects differ only in their mean return.

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[14] To be fair, some papers need strong assumptions since they are cast in a continuous-type framework. There is trade-off between assuming many types and narrow assumptions about quality differences versus assuming few types but allowing for general quality differences. We favour the latter approach since we believe that it is important to show that the same logic drives a lot of apparently different results. Moreover, full separation or full pooling when there are many types are results that are usually not very robust. Semi-pooling or semi-separation are much more realistic outcomes whose logic can be exposed using relatively few types.

[15] We have both possibilities since there is an ambiguity under First-Order Stochastic Dominance: should a dominating type be called "safe" or should a dominated type be called "safe"? This ambiguity also applies to Constantinides & Grundy (1989) and Nachman & Noe (1994).

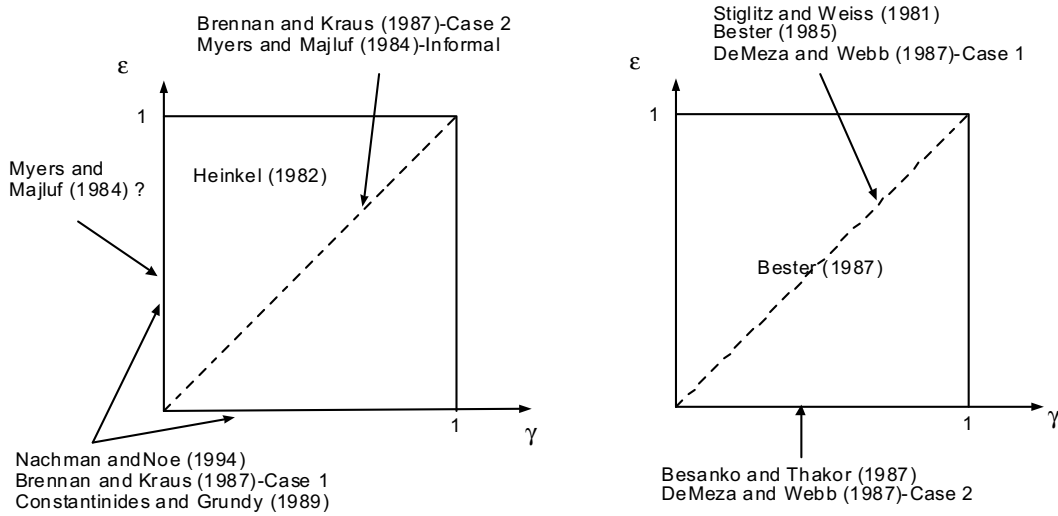Note that only Bester (1987) has assumptions that are as general as ours.

Figure 4: Assumptions in the financial signalling and the credit rationing literature

5 Analysis

In this section we analyse the signalling game. First, we present the full information benchmark. Second, we derive the incentive compatible financing contracts, i.e. those contracts which do not allow mimicking. Third, we present what we call believe-independent contracts. They are combinations of debt and equity for which the investor does not care what type of firm issues them. They play a very important role in the rest of the paper. We then put these elements together to characterise the separating and pooling equilibria of the signalling game. The section closes with a graphical illustration of the analysis.

5.1 The full information benchmark

A useful benchmark is obtained when we suppose for a moment that the outside investor knows the quality of the investment project too. He then accepts to finance a project of quality t when the proposed financing contract (F, α) satisfies $v_t(F, \alpha) \geq I$, or solving for the debt repayment, when

$$F \geq \frac{I - \alpha(W + p_t x_t)}{p_t (1 - \alpha)} \tag{4}$$

Insiders choose the combination of debt and equity that maximises their objective function subject to the condition that outside investors accept to finance the project. Since it is costly to issue debt or equity (the objective function is decreasing in F and α) an insider will issue just enough debt and equity so that the investor is indifferent between accepting or rejecting to finance the project, i.e. (4) holds as an equality. Substituting for F in the insider's objective function yields

$$U_t(F, \alpha) = W + p_t x_t - I$$

The insider's choice between debt and equity finance is indeterminate. The indeterminacy reflects the Modigliani-Miller Theorem in the context of our model: under full information (and all the other assumption that we have made) the decision to undertake a project is

independent of the decision of how to finance the project. Furthermore, all the surplus from investing is captured by the insider, i.e. the outside capital market is competitive.

Proposition 1 *If the outsider is as well informed about the quality of investment projects as an insider then the choice between debt and equity is indeterminate. Furthermore all the surplus from investing is captured by the insider.*

Proposition 1 describes the First-Best situation. All possible debt-equity choices are First-Best choices. This is different from standard signalling models such as Spence (1973) where there is a unique First-Best choice: sending no signal. The difference comes from the fact that standard signalling models assume an exogenous cost of sending the signal. Only sending no signal incurs no dead-weight costs and hence, it is only First-Best choice. Here, there is no exogenous cost of signalling via debt and equity and hence, all debt-equity choices are first-best choices.

The distinction between standard, costly, signalling models and our, costless, signalling model will be encountered a few more times in the remainder of the paper.

5.2 Incentive compatible contracts

We saw in the example that when debt finances safe projects and equity finances risky projects then a firm with a safe project has an incentive to mimic a firm with a risky project and vice versa. In that case, the debt-equity choice was not incentive compatible.

Incentive compatibility is a necessary condition for a signalling equilibrium. Condition 1 says that the debt-equity choice of an insider with a project of quality t, (F_t, α_t), must maximise his objective function $U_t(F, \alpha)$, i.e. it must be that

$$(1 - \alpha_t)(W + p_t(x_t - F_t)) \geq (1 - \alpha_{t'})(W + p_t(x_t - F_{t'})) \quad \text{for any } (F_t, \alpha_t), (F_{t'}, \alpha_{t'})$$

In our simple two-types case, the condition becomes

$$(1 - \alpha_s)(W + p_s(x_s - F_s)) \geq (1 - \alpha_r)(W + p_s(x_s - F_r)) \tag{5}$$
$$(1 - \alpha_r)(W + p_r(x_r - F_r)) \geq (1 - \alpha_s)(W + p_r(x_r - F_s)) \tag{6}$$

These are the incentive compatibility constraints which describe those debt-equity choices which will not be imitated by others. From an analysis of the incentive compatibility constraints we arrive at the following important result.

Proposition 2 *An equilibrium requires that a safe investment project will be financed with weakly more equity and weakly less debt than a risky investment project: $\alpha_s \leq \alpha_r, F_r \geq F_s$.*

Unfortunately, the proof gives little intuition for the result and it is therefore relegated to the appendix. But we gain a lot of insight from the insider's preferences. His Marginal Rate of Substitution of equity for debt, which tells us how much more equity an insider must issue to compensate him for a one-unit reduction in debt, is

$$\text{MRS}_t(F, \alpha) = \frac{\frac{\partial U_t(F, \alpha)}{\partial \alpha}}{\frac{\partial U_t(F, \alpha)}{\partial F}} = \frac{W + p_t(x_t - F)}{p_t(1 - \alpha)} \tag{7}$$

The key observation is that an insider with a safe project has a higher Marginal Rate of Substitution of equity for debt than an insider with a risky project:

$$\text{MRS}_s(F, \alpha) - \text{MRS}_r(F, \alpha) = \frac{p_s x_s \varepsilon(1 - \gamma) + W\gamma(1 - \varepsilon)}{p_s(1 - \alpha)(1 - \gamma)(1 - \varepsilon)} > 0 \tag{8}$$

In other words, an insiders with a risky project needs to issue less equity than an insider with a safe project in order to compensate for a reduction of debt. Thus, equity must be more valuable when issued by an insider with a risky project. Equity is a convex claim that is indeed more valuable if the underlying asset is riskier. The idea of incentive compatibility is to make signals credible. If a signal is mimicked then it is no longer credible. Intuitively, for a signal to be credible, it must be too costly to mimic. Now it becomes clear why a necessary condition for a signalling equilibrium is that the insider with a safe project uses more equity. The insider with a risky project does not mimic him since his equity is riskier, i.e. more valuable and thus costlier to issue.

Note that the only part of the insider's utility that depends on F and α is v_t, the expected value of the firm that is transferred to the outside investor. Equation (7) therefore also describes the investor's Marginal Rate of Substitution of equity for debt when he finance a project of quality t. The equality of the insider's and outsider's Marginal Rate of Substitution is explained by the absence of an exogenous cost of signalling via debt and equity.

In fact, equation (8) is a discrete version of the Single-Crossing Condition which plays a central role in all signalling games. But note that in standard, costly, signalling models such as Spence (1973) the Condition holds by assumption. In our, costless, signalling model such an assumption is not needed. The Condition is an endogenous feature derived from the characteristics of debt and equity.

5.3 Belief-independent combinations of debt and equity

Besides incentive compatibility, another important element in our signalling game is the investor's belief about the quality of the investment project he has been asked to finance. In the example we saw that investors are not irrational. When being offered a certain combination of debt and equity, they try to figure out which insider has an incentive to propose that combination. A natural question then is: are there combinations of debt and equity for which the outside investor does not care who proposes them?

The investor does not care about the quality of the project if the financing contract $(\bar{F}, \bar{\alpha})$ makes his payoff independent of his beliefs, i.e. if $(\bar{F}, \bar{\alpha})$ satisfies

$$\frac{\partial V(\bar{F}, \bar{\alpha})}{\partial \mu} = 0$$

Solving for the debt repayment $\bar{F}$ and using the definitions of γ and ε we obtain the following result.

Proposition 3 *Some combinations of debt and equity give the same pay-off to the outside investor no matter what he believes about the quality of the investment project. These combinations of debt and equity satisfy*

$$\bar{F} = x_s \frac{\bar{\alpha}(\varepsilon - \gamma)}{(1 - \bar{\alpha})(1 - \varepsilon)\gamma} \tag{9}$$

The proposition has some appealing implications. When projects are Mean-Preserving Spreads then they have the same mean and differ in risk only. In that case we would expect pure equity, i.e. a security that is linear, to be the belief-independent financial arrangement. When $\varepsilon = \gamma$, i.e. projects are indeed Mean-Preserving Spreads, that is exactly what proposition 3 says: $\bar{F} = 0$ which means that pure equity is the belief-independent contract.

When $\varepsilon < \gamma$ the projects are ordered by Second-Order Stochastic Dominance and the safe project has a higher mean. Proposition 3 tells us that now the believe-independent security is convex since it involves a debt repurchase. A linear security is no longer appropriate. Now that the safe project has a higher mean the investor would prefer it if the linear security

were issued by the safe firm. In order to "tilt the balance back" in favour of the risky firm, the belief-independent security must be convex (a convex security becomes more valueable to hold if the underlying asset is more risky).

The revers holds when

5.4.1 Separating equilibria

According to proposition 2, a necessary condition for an equilibrium is that safe projects are financed with weakly more equity and weakly less debt than risky projects. If we focus on separating equilibria we can strengthen this necessary condition.

Lemma 1 *In a separating equilibrium safe projects are financed with strictly more equity and strictly less debt than risky projects.*

   Proof. Suppose on the contrary that there is a separating equilibrium with $\alpha_s = \alpha_r$ (proposition 2 rules out $\alpha_s < \alpha_r$). Then the incentive compatibility constraints (5) and (6) simplify to

$$\begin{aligned} F_s &\geq F_r \\ F_r &\geq F_s \end{aligned}$$

   which is equivalent to $F_s = F_r$. This contradicts the assumption that $\alpha_s = \alpha_r$ occurs in a separating equilibrium since $\alpha_s = \alpha_r$ and $F_s = F_r$ is pooling . Similarly, $F_s = F_r$ in a separating equilibrium implies $\alpha_s = \alpha_r$. ∎

Next, we show that the investor makes zero profits in a separating equilibrium. In a separating equilibrium, firms choose different debt and equity contracts so that the investor can perfectly infer the unobserved project quality from the observed financing decision. The investor therefore considers two different sorts of profits: profits that he makes on financing a safe project and profits he makes on financing a risky project. It is impossible that the investor makes positive profits on either project. If he did, then the insider is not acting optimally. He could propose an alternative contract and deviate from equilibrium by reducing either the debt repayment or the equity stake. The alternative contract is still accepted by the outside investor and it is clearly more profitable for the investor. The formal version of that argument is found in the appendix.[17]

Lemma 2 *In any separating equilibrium investors make no profits.*

   A separating contract a) allows the investor to infer the project quality perfectly and b) requires him to break even. At equilibrium, a separating contract therefore produces an outcome that is identical to the full information/First-Best outcome described in section 5.1. As a corollary of lemma 2 we have:

Corollary 3 *Separating equilibria are efficient.*

   Corollary 3 points to an important difference to standard signalling models such as Spence (1973). In these models, the signal is costly so that separation is wasteful. Here, in contrast, the signal is not costly so that separation does not incur a dead-weight loss.

Lemma 2 requires that combinations of debt and equity that allow insiders to communicate their private information, to satisfy the investor's zero-profit constraints

$$\begin{aligned} p_s[F_s + \alpha_s(W + x_s - F_s)] + (1 - p_s)[\alpha_s W] &= I \qquad (12) \\ p_r[F_r + \alpha_r(W + x_r - F_r)] + (1 - p_r)[\alpha_r W] &= I \qquad (13) \end{aligned}$$

Solving for α_s and α_r, substituting into the incentive compatibility constraints and using the definitions of ε and γ yields a simple pair of inequalities for F_s and F_r :

$$F_s \leq \bar{F}(I) \leq F_r \qquad (14)$$

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[17]Note that the argument cannot be made if our starting point is a pooling contract since the deviation to an alternative and more profitable contract breaks the pooling.

where $\bar{F}(I)$ is the belief-independent debt contract that gives zero profits to the investor. Similarly, one derives a pair of inequalities for α_s and α_r :

$$\alpha_s \geq \bar{\alpha}(I) \geq \alpha_r \qquad (15)$$

where $\bar{\alpha}(I)$ is the belief-independent equity contract that gives zero profits to the investor. Moreover, lemma 1 requires that one inequality in both (14) and (15) to be strict.

Equations (14) and (15) say that safe and risky firms optimally separate when a safe firm issues a security that is more convex than the belief-independent security while a risky firm issues a more concave security.

The belief-independent debt and equity contracts play an important role in the characterisation of separating equilibria. Since these contracts are accepted no matter what type of project they finance, they are possible separating equilibrium contracts for either safe or risky firms. For example, an insider with a safe project could issue the belief-independent combination of debt and equity $(\bar{F}(I), \bar{\alpha}(I))$. Then the risky project must be financed with $F_r > \bar{F}(I)$ and $\alpha_r < \bar{\alpha}(I)$, and (F_r, α_r) must also satisfy the investor's zero-profit constraint for risky projects. Alternatively, an insider with a risky project could issue $(\bar{F}(I), \bar{\alpha}(I))$. Now a safe project must be financed with $F_s < \bar{F}(I)$ and $\alpha_s > \bar{\alpha}(I)$, and (F_s, α_s) must satisfy the zero-profit constraint for safe projects.

There is however a possible problem. Proposition 3 says that the belief-independent financing arrangement involves debt repurchases when $\varepsilon < \gamma$. If debt repurchases are not allowed, for example because the firm has no outstanding debt, then belief-independent financing arrangements do not exist. Then separating equilibria do not exit either. We will see in section 5.5 what sort of equilibria do exist when debt repurchases are not allowed.

The last, albeit technical, element that completes the characterisation of separating equilibria is about how the investor interprets deviations from equilibrium contracts. There is only one possible interpretation:

Lemma 3 *Deviations with more equity than the amount of belief-independent equity $\bar{\alpha}(I)$ must be interpreted as coming from an insider with a safe project. Deviations with less or equal equity than the amount of belief-independent equity $\bar{\alpha}(I)$ must be interpreted as coming from an insider with a risky project.*

Proof. In the appendix. ∎

The proof makes use of corollary 2. The corollary says that the investor makes more profits on safe projects than on risky projects if the financing contract involves less equity (and more debt) than belief-independent contracts. This means that a safe firm can obtain cheaper finance than risky firms (less equity and less debt issued for a given amount of profits to the investor) if it uses such a contract. Thus, a risky firm wants to mimic a safe firm that uses little equity. In order to counter a risky firm's incentive to deviate the investor must correctly interpret deviations that involve little equity as coming from a risky firm.

The following proposition summarises the results on separating equilibria.

Proposition 4 *If a repurchase of debt is allowed then there exists a continuum of efficient separating equilibria in which i) risky projects are financed with strictly more debt and strictly less equity than safe projects: $F_s \leq \bar{F}(I) \leq F_r$ and $\alpha_s \geq \bar{\alpha}(I) \geq \alpha_r$ (with at least one strict inequality each), ii) the investor makes zero profits and iii) a deviation with more equity than $\bar{\alpha}(I)$ must be interpreted as coming from insiders with safe projects and vice versa. If a repurchase of debt is not allowed then these efficient separating equilibria only exist when $\varepsilon \geq \gamma$.*

5.4.2 Pooling equilibria

One pooling equilibrium is easy to spot. It consists of both risky and safe projects being financed with the belief-independent combination of debt and equity that leaves no profits to investors, $(\bar{F}(I), \bar{\alpha}(I))$. Note that there is no dead-weight loss in this pooling equilibrium. Just like the separating equilibria it is efficient too. Again, lemma 3 describes the only possible interpretation of deviations.

If the belief-independent combination of debt and equity is admissible (remember that it may not if debt repurchases are not allowed) then the pooling equilibrium is unique. Clearly, the insider would not pool on belief-independent combinations of debt and equity that leave the investor with positive profits. More importantly, an insider would not pool on non-belief-independent combinations of debt and equity. To see this, suppose the contrary and let $(F_p, \alpha_p) \neq (\bar{F}(V), \bar{\alpha}(V))$ be the pooling contract. Condition 2 means that in a pooling equilibrium the investor's belief is given by his priors, i.e. $\mu(F_p, \alpha_p) = q$. Condition 1 requires that the investor must at least break even. We therefore write

$$V(F_p, \alpha_p) = q v_s(F_p, \alpha_p) + (1-q) v_r(F_p, \alpha_p) \geq I$$

The inequality implies that the investor makes profits on the financing of at least one type of project. Consequently, the cost of the pooling contract (F_p, α_p) is more than I to some insider. This insider would deviate to $(\bar{F}(I), \bar{\alpha}(I))$ which costs only I and is always accepted by the investor. Hence, (F_p, α_p) cannot be an equilibrium contract.

The following proposition summarises the discussion on pooling equilibria.

Proposition 5 *If a repurchase of debt is allowed then there exists a unique and efficient pooling equilibrium in which all projects are financed with the belief-independent combination of debt and equity that leaves no profits to the investor $(\bar{F}(I), \bar{\alpha}(I))$. Any deviation with more equity must be interpreted as coming from insiders with safe projects and vice versa. If a repurchase of debt is not allowed then the efficient pooling equilibrium only exists when $\varepsilon \geq \gamma$.*

5.5 Equilibria without repurchases of debt

Separating equilibria and the efficient pooling equilibrium do not exist when debt repurchases are not allowed and $\varepsilon < \gamma$. Instead, there exists a continuum of inefficient pooling equilibria since $(F_p, \alpha_p) \neq (\bar{F}(V), \bar{\alpha}(V))$ can no longer be ruled out by deviating to the belief-independent contract $(\bar{F}(I), \bar{\alpha}(I))$.

A full characterisation of these inefficient pooling equilibria is tedious and does not add anything substantial to what follows in the rest of the paper. The only element we need is that these inefficient pooling equilibria must guarantee a minimum level of utility to a safe firm. If they did not then a safe firm has an incentive to deviate. But what would it deviate to? To answer this, note that the investor's worst possible interpretation of a safe firm's deviation $(\hat{F}, \hat{\alpha})$ is that it comes from a risky firm, i.e. $\mu(\hat{F}, \hat{\alpha}) = 0$. The safe firm knows that the deviation will give the investor an expected return of $V(\hat{F}, \hat{\alpha}) = v_r(\hat{F}, \hat{\alpha})$. The safe firm has no interest in leaving profits to the investors so that the deviation $(\hat{F}, \hat{\alpha})$ satisfies $v_r(\hat{F}, \hat{\alpha}) = I$. Now which of these combinations of debt and equity $(\hat{F}, \hat{\alpha})$ will give the highest payoff to the safe firm? It is the pure equity contract $(0, \hat{\alpha})$. Equity is a convex security that, if issued by a safe firm, is least valuable to the investor who buys it and thus least costly for the firm who sells it. In that sense $(0, \hat{\alpha})$ is the best deviation for a safe firm given that the investor makes the worst interpretation (from the safe firm's point of view) of a deviation.

Proposition 6 *When $\varepsilon < \gamma$ and debt repurchases are not allowed then only inefficient pooling equilibria exist. These inefficient pooling equilibria must give safe firms a utility of at least $U_s(0, \hat{\alpha})$ where $\hat{\alpha}$ is defined by $v_r(0, \hat{\alpha}) = I \iff \hat{\alpha} = \frac{I}{W + p_r x_r}$.*

5.6 A graphical exposition

Figure 5 and 6 illustrate the arguments on separating and pooling equilibria. The figures depict the investor's indifference curves when he finances a risky project and when he finances a safe project. Figure 5 shows the case when $\varepsilon > \gamma$ and figure 6 shows the case when $\varepsilon < \gamma$. The figures assume that the investor makes zero profits, e.g. he just accepts a pure debt proposal from risky firms if $F = \frac{I}{p_r}$ (point A in both figures). The investor's indifference curve is steeper when he finances a risky project than if he finances a safe project. Note that the curves also depict the safe and the risky firm's indifference curves when issuing a claim worth I.

Consider first figure 5. A risky firm for example is indifferent between offering any contract on the curve ABC'. The investor is indifferent along the same curve if he knows that it is the risky firm that proposes the contract. This illustrates why the financing decision is indeterminate under full information.

Under asymmetric information the situation is different. In the introductory example we argued that financing risky projects with equity and safe ones with debt cannot be an equilibrium. In that situation the safe firm is at point A' and the risky firm is point C'. But at point A' the safe firm has an incentive to mimic the risky firm and deviate to point C' which is on a lower (and thus more profitable) indifference curve. Similarly, the risky firm wants to deviate from C' to A'. In a separating equilibrium there must be no incentives to deviate, so that a risky firm must be on curve AB and a safe firm must be on curve BC. This is exactly what equations (14) and (15) say.

Point B is special since at that point the indifference curves cross each other. But to say that the investor's indifference curves, which both give him zero profits, cross at point B is to say that he is indifferent between financing a safe or a risky project. Hence, the contract at point B is a belief-independent contract. It is the contract $(\bar{F}(I), \bar{\alpha}(I))$ which is important in characterising the separating equilibria and the efficient pooling equilibrium.

The figure also illustrates why the efficient pooling equilibrium is unique when $\varepsilon > \gamma$. An alternative pooling equilibrium is represented by some point between the curves AB and A'B or BC and BC'. But for any such point, some firm would be on a lower indifference curve than at B and would therefore deviate to B.
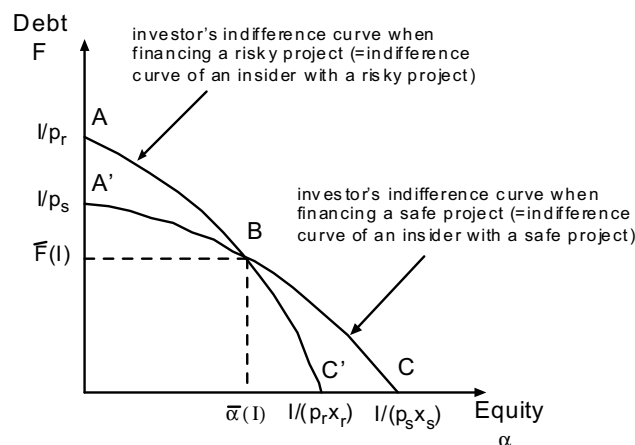

Figure 5: Firms and investor's indifference curves when $\varepsilon > \gamma$

Figure 6 illustrates the situation when $\varepsilon < \gamma$. Point B now lies below the x-axis, i.e. it represents a belief-independent financial arrangement with a debt repurchase. If such a repurchase is not permitted, i.e.

space of feasible contracts and no separating equilibrium and no efficient pooling equilibrium exist. There are however many inefficient pooling equilibria that are represented by a point between the two indifference curves. Its precise location depends on the parameters of the model such as q, the ex ante probability of the firm having access to a safe project.
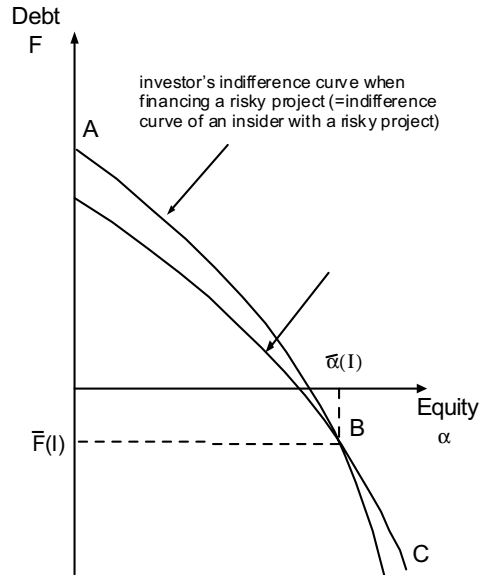

Figure 6: Firms' and investor's indifference curves when $\varepsilon < \gamma$

5.7 Summary

The general picture that emerges is that for all values of ε and γ, a safe firm can credibly communicate its private information (and hence obtain a efficient financing contract) by issuing more equity and less debt than a risky firm.

If a debt repurchase is permitted then separating and pooling equilibria are efficient. Combinations of debt and equity that give the investor an expected return which is independent of what he thinks about the project quality, play an important role in the characterisation of both types of efficient equilibria.

If a debt repurchase is not permitted and if $\varepsilon < \gamma$ then no separating equilibrium exists. Pooling equilibria will be inefficient but must guarantee a safe firm a minimum payoff $U_s(0, \hat{\alpha})$ where $(0, \hat{\alpha})$ is the best deviation for a safe firm given that the investor makes the worst interpretation of a deviation.

6   Discussion

In this section we use the general picture that emerges from the analysis to comment on three related issues. The first issue concerns the multitude of results found in the financial signalling literature. We explain that the results are in fact special cases that correspond to particular values of ε and γ. For example Myers & Majluf (1984)'s original Pecking-Order hypothesis corresponds to the special case $\gamma = 0$.

The other two issues are about well known inefficiencies that arise due to a pooling of investment projects: Myers & Majluf (1984)'s underinvestment and Stiglitz & Weiss (1981) credit rationing result.

6.1 Financial signalling

6.1.1 Theoretical issues

Figure 7 shows how various results of the financial signalling literature can be interpreted as special cases. Let us start with the case when $\varepsilon = \gamma$, i.e. projects are Mean-Preserving spreads. In the second half of their paper, Brennan & Kraus (1987) consider this case and conclude that convertible bonds allow full separation. In a example they argue more precisely that convertibles financing riskier investment projects require a higher face value of the bond and less equity into which the bond can be converted. This is analogous to the separating result in our, simpler, two-type model. Risky firms generally issue more debt and when $\varepsilon = \gamma$, then safe firms issue a lot of equity and buy back debt.

Yet, Brennan & Kraus (1987) do not consider pooling equilibria. Had they done so, they should have concluded, as we do, that there exists an efficient pooling equilibrium in which all projects are financed with pure equity. That particular outcome has been alluded to, but not explicitly modelled, by Myers & Majluf (1984).[18] Efficient pooling on equity has been obtained outside the financial signalling literature by de Meza & Webb (1987). Since they are more concerned about credit rationing, we comment on their analysis in section 6.3.

The opposite to pooling on pure equity is of course pooling on pure debt. It occurs when $\gamma = 0$, i.e. when the risky project dominates the safe project by First-Order Stochastic Dominance. Again, the pooling equilibrium is efficient. Indeed, one of Myers and Majluf's central results was that debt finance dominates equity finance, i.e. there exists a pooling equilibrium in which all firms issue debt. The problem with their argument was that it was not cast rigorously in a signalling model and that investment projects are not risky. Nachman & Noe (1994) remedies the problems by showing that if the cash flows from investment projects can be ordered by a strong version of First-Order Stochastic Dominance then debt is indeed the optimal security to issue. Optimality there means minimising mispricing when all projects are financed by the same security, i.e. finding the most efficient pooling equilibrium.[19]

We saw in figure 4 that Nachman & Noe (1994), part I of Brennan & Kraus (1987) and Constantinides & Grundy (1989) all assume that investment projects are ordered by First-Order Stochastic Dominance. But each paper comes up with a different optimal capital structure under asymmetric information. Constantinides & Grundy (1989) argue that stock repurchases allow insiders to communicate their private information to outside investors when investment projects are ordered by First-Order Stochastic Dominance. At first glance, it is difficult to reconcile their result with Nachman & Noe (1994)'s optimality of debt finance. Figure 7 however shows that when $\gamma = 0$, Nachman & Noe (1994)'s result corresponds to the pooling equilibrium while Constantinides & Grundy (1989)'s result corresponds to the separating equilibria which require a repurchase of equity by risky firms.[20]

The first part of Brennan & Kraus (1987) argues that debt repurchases signal the quality of investment projects to investors. Again, we can reconcile their result with the other literature. Cash flows are not only ordered by First-Order Stochastic Dominance when $\gamma = 0$ but also when $\varepsilon = 0$. In the latter case, it is the safe project that dominates the risky project and figure 7 confirms that separating equilibria then indeed require debt repurchases.

Finally, Heinkel (1982) has a signalling model in which firms can only issue debt and

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[18] At some moment in their analysis they were troubled by having left no room at all for equity issues. They speculated that if there were only asymmetric information about risk then equity would dominate debt (Myers & Majluf 1984, p. 209).

[19] Innes (1990) and DeMarzo & Duffie (1999) obtain the same result as Nachman & Noe (1994) in slightly different contexts. Innes (1990) considers the optimal design of a security when there is Moral Hazard between the owner and the manager of the firm. DeMarzo & Duffie (1999) consider an Adverse Selection problem in which the designer of a security is not identical to its seller.

[20] When $\gamma = 0$ then the belief-independent equity contract is $\bar{\alpha}(\cdot) = 0$. Together with equation (15) this means that the risky firm must repurchase equity if the safe firm issues only debt.
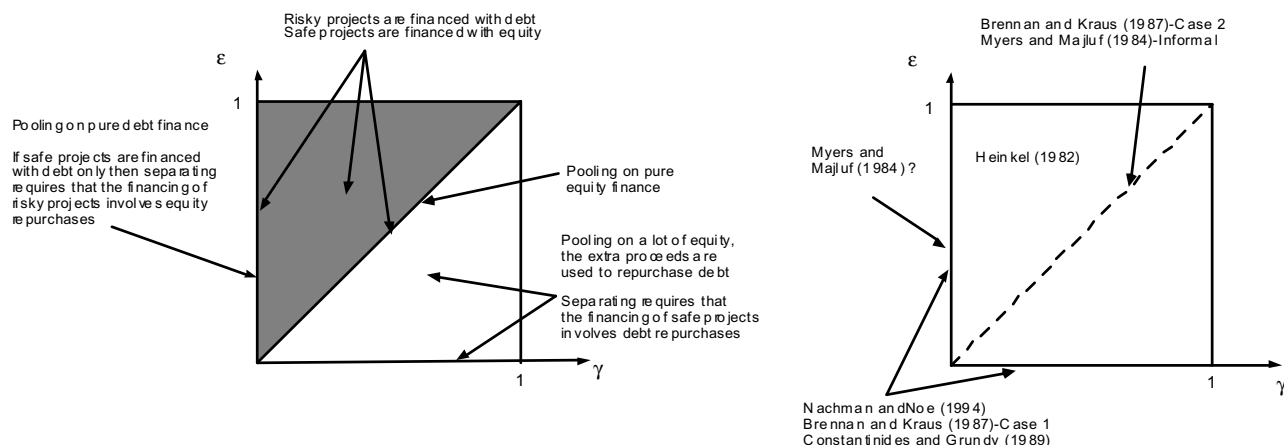
Figure 7: Overview of equilibrium outcomes of the financial signalling model and comparison to the financial signalling literature

equity. His result is that the amount of debt used by a firm is increasing in its unobservable true value. How does his result relate to the other signalling results and to our model? Heinkel makes no explicit assumption about the ordering of investment cash flows. Instead he imposes that high-value firms have low-value debt. At first sight this seems to be a strong assumption. But in fact it corresponds to the case when $\varepsilon > \gamma$. Then a risky, and now high-value firm issues more debt than a safe, low value, firm. Thus, when $\varepsilon > \gamma$ then the high-value firm is a risky firm whose debt, due to its concavity, has little value. Heinkel's assumption was not so strong after all. It only captures those instances where cash flows cannot be ordered by First- or Second-Order Stochastic Dominance.

So we see that although this part of the corporate finance literature is initially confusing since different risk concepts are used and different equilibrium outcomes are compared, there is a common underlying logic.

6.1.2 Empirical issues

One standard argument in the financial signalling literature is that the Pecking-Order hypothesis fits very well the empirical evidence on stock price reactions to issue decisions.[21] A well documented pattern in these stock price reactions is that the stock price drops following an announcement to issue new securities, i.e. there is an Adverse Selection discount. More importantly, the drop is stronger the more junior and equity-like the issued security is (see for example Asquith & Mullins (1986) or Masulis & Korwar (1986)). The Pecking-Order hypothesis fits that pattern since it says that debt is a more favourable signal than equity.

Recently, people have argued that the pattern in stock price reactions is not as conclusive as previously thought. For example, roughly a third of the firms in Asquith & Mullins (1986) and Masulis & Korwar (1986) exhibit a positive stock price reaction on the announcement of an equity issue. Bradford (1987) argues that many of the classic empirical studies contain a statistically significant sample of inconclusive stock price reactions. Moreover, if the Pecking-Order hypothesis is correct then we should expect a negative relation between the rating of debt or convertibles and the stock price reaction. But Mikkelson & Partch (1986) find that there is only a weak negative relation for debt issues and a positive relation for convertibles. Finally, Jung et al. (1996) use a combination of cross-sectional and stock-price reaction data

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[21] See any textbook on corporate finance, e.g. Ch. 18 in Grinblatt & Titman (1998).

to show the following puzzle. In their sample there are firms that issue equity although they should issue debt since they share the characteristics of a typical debt-issuing firm, i.e. low market-to-book ratio and high tax payments. A Pecking-Order explanation of that puzzle would be that these firms face little information asymmetries. But this is not the case since Jung et al. (1996) show that when these firms issue equity then the stock price reacts very negatively.[22]

We argued that the original Pecking-Order hypothesis, as formulated by Myers and Majluf, should be viewed as a special case. In our model, both positive and negative stock price reactions are possible. When $\varepsilon > \gamma$ then low value (safe) firms issue more equity than high value (risky) firms. The reverse holds when $\varepsilon < \gamma$. Our model can also offer an explanation for the puzzle in Jung et al. In our model, all firms are identical but for the risk of future cash flows from investment projects. It is that risk which determines whether a firm issues debt or equity. This risk is not tested for in Jung et al.

A direct test of our model would have to examine how the risk of investment projects relates to debt and equity finance. It is not clear to us how one could measure the *risk of future cash flows.* No study does it to our knowledge. Kim & Sorensen (1986) come close by relating a firm's *operating risk* to its capital structure. They find that firms with a high business risk, measured by variations of EBIT (earnings before interest and taxes), have a high leverage. In their comprehensive overview of the financial contracting literature, Harris & Raviv (1992) argue that there is no theoretical explanation for such a positive relationship.[23] Our model offers a possible explanation: firms with risky but profitable investment projects credibly signal this information to capital markets by financing their projects with a lot of debt.

Finally, we would like to mention that since our model links the theoretical results of various papers it also links the pieces of empirical evidence that they cite.[24]

6.2 Underinvestment

Myers and Majluf describe how the underpricing of equity in an inefficient pooling equilibrium can lead to underinvestment. Insiders whose equity is underpriced may have to give such a large equity stake to outside investors, i.e. dilute their own stake, that more firm value is transferred to outsiders than is created for insiders by investing. In that case insiders prefer not to dilute their equity stake and decide not to invest.

If, in contrast, the equilibrium outcome is efficient then by definition the dilution cost is zero and all projects are undertaken. Inefficient (pooling) equilbria only occur when $\varepsilon < \gamma$ and when debt repurchases are not allowed. But it turns out that even then all investment projects are undertaken.

Proposition 7 *There is never underinvestment in equilibrium.*

To prove this result we first note that only a safe firm may forego its investment project when $\varepsilon < \gamma$ and when debt repurchases are not allowed. It cannot issue enough equity to prevent being mimicked by a risky firm and hence, there is pooling. If pooled, a safe firm issues underpriced securities which makes investing costly while risky firms issue overpriced

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[22]To solve the puzzle they use Moral-Hazard arguments along the lines of Jensen (1986): managers of these abnormal firms issue equity to avoid the disciplinary effects of debt.

[23]In their paper Kim and Sorensen argue that Myers (1977) debt-overhang argument offers an explanation. An investment is more valuable if it is more risky since it should be viewed as a call option. But more valuable firms suffer less from the debt-overhang and can therefore issue more debt.

[24]For example, Brennan & Kraus (1987) cite evidence that a debt repurchase leads to a positive stock price reaction while Constantinides & Grundy (1989) cite evidence that a stock repurchase is good news.

securities which makes investing cheap.[25]

Proposition 6 tells us that although a safe firm issues underpriced securities in an inefficient pooling equilibrium, it will still obtain a utility of $U_s(0, \hat{\alpha})$. This level of utility is sufficient to guarantee that safe firms still undertake their investment projects since it exceeds their reservation utility:

$$U_s(0, \hat{\alpha}) > W \tag{16}$$

This equivalent to

$$\hat{\alpha} < \alpha_s^{IR}$$

where α_s^{IR} is the pure equity contract that gives an insider with a safe project his reservation utility W. In other words, the dilution cost of equity is not big enough to deter a safe firm from investing.[26]

To see that the inequality indeed holds, rewrite it explicitly as

$$\frac{p_s x_s + W}{p_r x_r + W} < \frac{p_s x_s}{I}$$

At first glance not obvious that the statement is true since $\varepsilon < \gamma \iff p_s x_s > p_r x_r$. Yet the left-hand side is decreasing in W and its highest value, achieved at $W = 0$, is not high enough to contradict the inequality.

6.3 Credit rationing in competitive markets

Inefficient pooling equilibria can also be responsible for credit rationing.[27] Stiglitz & Weiss (1981) show how inefficient pooling can lead to sticky prices in a competitive credit market, i.e. sticky interest rates, so that an excess demand is not cleared away by raising interest rates. Banks are unwilling to change interest rates since doing so changes the composition of the pool of borrowers. More precisely, higher interest rates attract borrowers with risky projects to the pool since they know that they can rarely repay their debt. Similarly, borrowers with safe projects leave the pool since they know that they often have to repay their debt. This Adverse Selection effect may outweigh the increased revenue from higher interest repayments so that a competitive bank may prefer to ration borrowers even though they would accept higher interest rates.

Bester (1985) explained that an inefficient pooling equilibrium, and hence credit rationing, cannot exist when competitive banks can screen borrowers. He focuses on collateral requirements as a screening device. Banks know that risky borrowers prefer debt contracts with a high interest repayment and little collateral while safe borrowers prefer debt with a low interest repayment and much collateral. By observing a borrower's choice from a menu of

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[25]It makes investing so attractive for risky firms that they would even undertake projects with a negative Net Present Value.

[26]The equality says that the worst pure equity contract for a safe firm $\hat{\alpha}$ (which is the equity that the investor just accepts when he thinks the contract is offered by a risky firm) is cheaper than the pure equity contract that just induces a safe firm to forego the investment opportunity α_s^{IR}. This is different to Myers and Majluf's model where it could be that $\hat{\alpha} > \alpha_s^{IR}$. The reason for this difference is that in their model a firm's assets-in-place W are not observable by outside investors and that they make the important assumption that W is positively related to the expected return of an investment project. Their underinvestment result therefore relies crucially on large firms having more profitable investment opportunities.

Besides, Myers and Majluf do not consider the possibility that debt finance could lead to underinvestment too. This possibility exists in our model since $\hat{F} = \frac{I}{p_r} > F_s^{IR} = x_s$ which is equivalent to $\gamma > r$ where $r = \frac{p_s x_s - I}{p_s x_s}$, i.e. there is underinvestment by safe firms under pure debt finance when the degree of asymmetric information in success probabilities exceeds the rate of return of the safe project.

[27]There are other reasons for credit rationing too. See for example Besanko & Thakor (1987a).

debt contracts with different combinations of interest-repayment and collateral, a bank infers which type of borrower it is facing. Safe and risky borrowers are no longer pooled together so that raising interest rates, together with an appropriate change in collateral, allows to clear an excess demand separately on safe and risky borrowers. There is no longer an Adverse Selection cost and rationing never occurs.

Whether firms signal their private information by choosing contracts from a menu or by proposing these contracts themselves does not matter.[28] So we know from Bester's work that if firms can signal the quality of their investment projects, for example through an appropriate debt-equity choice, then firms can never be rationed.

From propositions 4 and 5 we therefore obtain the following result:

Proposition 8 *Firms that use debt and equity to finance their investment projects can never be rationed when $\varepsilon \geq \gamma$. If a repurchase of debt is not allowed and if $\varepsilon < \gamma$ then rationing is possible.*

Our comment on credit rationing is stronger than Bester's comment for three reasons. First, for any signal to be credible, it must be costly to imitate. Bester assumes that there is an exogenous cost of collateralisation for borrowers, e.g. legal costs. But this is not uncontroversial and indeed, other papers assume alternatively that the cost of collateralisation lies with banks, e.g. liquidation costs, or that banks and borrowers have different preferences towards risk.[29] The bottom line is that arguments that use the collateral as a screening device need to make a strong assumption about an exogenous costs of using collateral. Our model, in contrast, does not need such an assumption.

Second, using collateral as a screening device leads to the counter-intuitive result that safe borrowers use more collateral than risky borrowers. Moreover, if the Adverse Selection problem is severe then a borrower's wealth/assets-in-place may not suffice to provide the amount of collateral he needs to signal his private information.[30] Third, Bester only considers the case when investment returns are Mean-Preserving Spreads, i.e. when $\varepsilon = \gamma$.

The only other paper (apart from this one) which discusses the role of equity finance in relation to credit rationing is de Meza & Webb (1987). They compare rationing in debt markets to rationing in equity markets and conclude that rationing never occurs. Either the assumption in Stiglitz & Weiss (1981) about the ordering of projects' cash flows holds but then equity is optimal since it avoids rationing or the assumption does not hold but then debt is optimal. De Meza and Webb, however, obtain their strong result not by using the separating properties of combinations of debt and equity (since they consider debt and equity finance separately) but by considering rather special cases.[31] When they argue that there is no rationing in debt markets they effectively assume that $\gamma = 0$. In that case the efficient pooling equilibrium indeed requires pure debt finance. When they argue that there is no rationing in equity markets they use Stiglitz and Weiss' original assumption of Mean-Preserving Spreads, i.e. $\varepsilon = \gamma$. Then the efficient pooling equilibrium indeed requires pure equity finance. In both cases, the equilibrium is efficient so that rationing cannot occur even though it is a pooling equilibrium.

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[28] See Maskin & Tirole (1992).

[29] See Besanko & Thakor (1987*b*) and Bester (1987).

[30] See Besanko & Thakor (1987*b*).

[31] This has also been noted recently by Hellmann & Stiglitz (2000). Their framework however is different from ours since i) they, as de Meza and Webb, consider debt and equity finance separately and ii) their investment projects differ in success probabilities and *mean*.

7 Extensions

In this section we present some extensions of the basic model. The first extension concerns the assets-in-place W. We relax the assumption that a firm's assets-in-place are not marketable. First, we allow these assets to be marketable before an investment project is undertaken so that firms can reduce the amount of outside finance I. Then we allow these assets to be marketable after an investment project is undertaken so that firms can use them as collateral. The aim is to show that cash and collateral cannot be alternative signalling instruments to equity. Moreover, we show that the use of cash or collateral cannot rule out credit rationing when $\varepsilon < \gamma$ since we can still not obtain separating equilibria without debt repurchases for those values of ε and γ.

The second extension concerns our restriction to only two types of investment projects. We show that the restriction is a convenient one since none of our arguments about the Pecking-Order, rationing or underinvestment changes fundamentally when we add a third type of investment project to the model. What changes is that full separation can no longer be obtained when $\varepsilon < \gamma$ (even when debt repurchases are allowed).

7.1 Making better use of wealth: inside cash and collateral

Suppose then that firms can sell some assets-in-place to raise an amount of cash C before the project is undertaken, so that only $I - C$ must be raised from an outside investor. To keep the problem interesting we assume that only some assets can be sold so that a project cannot be fully financed internally, i.e. $C < I$. The expected value of a combination of debt and equity with cash C (F, α, C) which finances a project of quality t is:

$$v_t(F, \alpha, C) = p_t[F + \alpha(W - C + x_t - F)] + (1 - p_t)[\alpha(W - C)]$$

$$\bar{F}(V) = x_s \frac{V(\varepsilon - \gamma)}{p_s x_s (1 - \gamma)\varepsilon + (W - V - C)(1 - \varepsilon)\gamma} \tag{10'}$$

$$\bar{\alpha}(V) = \frac{V(1 - \varepsilon)\gamma}{p_s x_s (1 - \gamma)\varepsilon + (W - C)(1 - \varepsilon)\gamma} \tag{11'}$$

The difference to equations (10) and (11) is intuitive: contributing cash reduces the amount of debt and equity in equal proportions. But when $\varepsilon < \gamma$ then $\bar{F}(I) < 0$ (under reasonable parameter constellations[33]) so that we still need debt repurchases to obtain separating equilibria.

We repeat the exercise for the case when assets-in-place not marketable at the time the investment project is undertaken but they can still be used as collateral. So C now denotes the collateral that firms put up. Again we impose $C < I$, i.e. firms cannot borrow the entire investment outlay against their assets, in order to keep the problem interesting. To conform to standard practice we assume that $C \leq F$ so that a firm can at most sell enough of the collateralised assets to repay the debt fully.[34] The expected value of a portfolio of equity and collateralised debt that finances a project of quality t is

$$v_t(F, \alpha, C) = p_t[F + \alpha(W + x_t - F)] + (1 - p_t)[C + \alpha(W - C)] \tag{1''}$$

The crucial difference to (1) and (1') is that if the project fails then the debt contract returns C instead of zero.[35]

The firm's objective function then becomes

$$\begin{aligned} U_t(F, \alpha, C) &= p_t(W + x_t) + (1 - p_t)(W) - v_t(F, \alpha, C) \\ &= (1 - \alpha)(W + p_t(x_t - F) - (1 - p_t)C) \end{aligned} \tag{2''}$$

and the investor accepts the offered contract when

$$V(F, \alpha, C) = \mu(F, \alpha, C)v_s(F, \alpha, C) + (1 - \mu(F, \alpha, C))v_r(F, \alpha, C) \geq I \tag{3''}$$

Propositions 1 and 2 are unchanged. Again the financing decision is indeterminate under full information and again a safe firm needs to issue weakly more equity than risky firms. The relationship between belief-independent debt and equity contracts in equation (9) of proposition 3 however changes to

$$F = x_s \frac{\alpha(\varepsilon - \gamma)}{(1 - \alpha)(1 - \varepsilon)\gamma} + C \tag{9''}$$

which is intuitive since the investor always recovers at least C from debt contracts. Again we set $\alpha = 0$ in this relationship to see whether there can be separating equilibria without

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[33]A sufficient condition for $\bar{F}(I) < 0$ when $\varepsilon < \gamma$ is that the amount of non-marketed assets $W - C$ is bigger than the investment outlay I. If this is not the case, the following must hold to have debt repurchases: $\frac{\gamma - \varepsilon}{\gamma(1 - \varepsilon)} < \frac{p_s x_s - I}{p_s x_s} + \frac{W - C}{p_s x_s}$. A safe firm needs to repurchase debt since it must issue more equity than is needed for investment purposes when $\varepsilon < \gamma$. The condition now says that with inside cash the safe firm does not need to repurchase debt when, for example, γ is very big, the rate of return on the safe project is low and the amount of non-marketed assets is small. This is intuitive since under these conditions mimicking the safe firm is not very attractive for the risky firm and hence the former needs to issue less equity to avoid be mimicking by the latter.

[34]The US legal system treats $C > F$ as an unenforceable penalty clause (see Posner (1992), p.128ff).

[35]Alternatively we could have $v_t = \dots + (1 - p_t)[\min\{F, C\} + \alpha(W - \min\{F, C\})]$. In other words if the amount of collateralised assets exceeds the debt repayment then the firm does not default. This is because selling some of the collateralised assets raises enough money to repay the debt although the project has failed. One can show that this alternative formulation does not change our argument since no additional separating equilibria can be obtained.

equity. The answer is no since the safe firm would then have to use more collateral than what is needed to repay the debt fully, $C > F_s$ (see equation (14)).

Furthermore, collateral cannot help to rule out competitive credit rationing when $\varepsilon < \gamma$. Belief-independent debt and equity contracts with collateral are given by

$$\bar{F}(V) = x_s \frac{(V - C)(\varepsilon - \gamma)}{p_s x_s(1 - \gamma)\varepsilon + (W - V)(1 - \varepsilon)\gamma} + C \tag{10"}$$

$$\bar{\alpha}(V) = \frac{(V - C)(1 - \varepsilon)\gamma}{p_s x_s(1 - \gamma)\varepsilon + (W - C)(1 - \varepsilon)\gamma} \tag{11"}$$

Now $\varepsilon < \gamma$ yields $\bar{F}(I) < C$ so that the collateral again exceeds what is needed to repay the debt fully.

7.2 More than two types

In this section we introduce a third type of investment project which is safer than risky projects but riskier than safe projects. More precisely, this new medium project, $t = m$, has $p_s \geq p_m \geq p_r$ and $x_s \leq x_m \leq x_r$. There are then four measures of the relative differences in success probabilities and returns: $\gamma_{s,m}, \gamma_{m,r}, \varepsilon_{s,m}$ and $\varepsilon_{m,r}$ (for example $\varepsilon_{m,r} = \frac{x_r - x_m}{x_r}$ and $\gamma_{s,m} = \frac{p_s - p_m}{p_s}$). For simplicity we assume that the relative quality differences do not depend on the type of a project:

$$\gamma_s = \gamma_m = \gamma \text{ and } \varepsilon_s = \varepsilon_m = \varepsilon$$

The argument which characterises the equilibria in the three-type case is essentially the same as in the two-type case. What is new in the three-type case, is that a full separating equilibrium no longer exists when $\varepsilon < \gamma$. In the two-type case, separating equilibria are characterised by the pair of inequalities (14) and (15). In the three-type case this pair of inequalities becomes

$$F_s \leq \bar{F}_{s,m}(I) \leq F_m \leq \bar{F}_{m,r}(I) \leq F_r$$
$$\alpha_s \geq \bar{\alpha}_{s,m}(I) \geq \alpha_m \geq \bar{\alpha}_{m,r}(I) \geq \alpha_r \tag{17}$$

where $\bar{F}_{s,m}(I)$, $\bar{F}_{m,r}(I)$, $\bar{\alpha}_{s,m}(I)$ and $\bar{\alpha}_{m,r}(I)$ are the relevant belief-independent financing contracts. For example, $\bar{F}_{s,m}(I)$ is the debt contract which gives the investor zero profits and for which he does not care whether the debt contract is issued by a safe or a medium firm. $\bar{F}_{s,m}(I)$ and $\bar{\alpha}_{s,m}(I)$ are identical to $\bar{F}(I)$ and $\bar{\alpha}(I)$ from equations (10) and (11) whereas $\bar{F}_{m,r}(I)$ and $\bar{\alpha}_{m,r}(I)$ are given by

$$\bar{F}_{m,r}(I) = x_s \frac{I(\varepsilon - \gamma)}{p_s x_s(1 - \gamma)^2\varepsilon + (W - I)(1 - \varepsilon)^2\gamma}$$

$$\bar{\alpha}_{m,r}(I) = x_s \frac{I\gamma(1 - \varepsilon)^2}{p_s x_s(1 - \gamma)^2\varepsilon + W(1 - \varepsilon)^2\gamma}$$

Equation (17) tells us that the belief-independent debt and equity contracts and the actual separating contracts must be ordered in the same way. The amount of debt that makes the investor indifferent between financing risky and medium projects must be larger than the amount of debt that makes him indifferent between medium and safe projects.

But when $\varepsilon < \gamma$, the ordering of belief-independent contracts runs the opposite way:

Lemma 4 *When $\varepsilon < \gamma$ then $\bar{F}_{m,r}(I) < \bar{F}_{s,m}(I)$ and $\bar{\alpha}_{m,r}(I) > \bar{\alpha}_{s,m}(I)$.*

Full separation therefore cannot occur when $\varepsilon < \gamma$. This is not a serious limitation since we already saw in the two-type case that without debt repurchases no separating equilibrium existed when $\varepsilon < \gamma$. As far as pooling equilibria are concerned, it is straightforward to verify that efficient pooling equilibria only exist when $\gamma = 0$ (pooling on pure debt finance) or when $\varepsilon = \gamma$ (pooling on pure equity finance).[36]

The argument that competitive credit rationing is impossible when $\varepsilon \geq \gamma$ and possible when $\varepsilon < \gamma$ therefore extends beyond the two-type case. The argument that underinvestment never occurs with debt and equity finance also carries over. The argument relies on the possibility that a safe firm can always deviate to the worst equity contract $(0, \hat{\alpha})$ in an inefficient pooling equilibrium. We showed that this contract is not costly enough for safe firms not to invest. Nothing of that argument changes with three-types: since a safe firm, which is the most likely not to invest, always invests, all other types of firms always invest too.

8 Conclusion

The starting point of this paper was a puzzle that occurs when insiders of a firm have more information than outside investors. The insiders' desire to sell overpriced securities creates an Adverse Selection problem leading to two contradictory results. On the one hand, it leads to Myers & Majluf (1984)'s Pecking-Order hypothesis that says that debt finance dominates equity finance. On the other hand it leads to Stiglitz & Weiss (1981)' credit rationing whose consequence is that equity finance dominates debt finance.

To resolve the puzzle we proposed a signalling game in which insiders can issue combinations of debt and equity to outside investors. Moreover, we used a general notion of what it is that insiders know more about. For example, First- and Second-Order Stochastic Dominance are both considered as special cases. We explained how important contributions to the financial signalling and credit rationing literatures make different assumptions about this notion of asymmetric information and how these assumptions can be seen as special cases of our set-up.

The main result is that combinations of debt and equity can be used to credibly signal information to the market. Safe projects are financed with more equity and less debt than risky projects. Equity is a convex claim so that its value increases with the risk of the underlying assets. Since the value of a claim is a cost to the firm, equity is particularly costly for risky firms so that safe firms can use it as a credible signal. Similarly, debt, a concave claim whose value decreases with risk, credibly signals that investment projects are risky.

As inside information is credibly transmitted to outside investors, there will be no general Adverse Selection effect and the financing decision will be efficient. The Pecking-Order and rationing only emerge as two, mutually exclusive special cases which explains the initial puzzle. Debt dominates equity for one set of parameter values and equity dominates debt for another set of parameter values. But neither dominance result is general in the sense that is occurs for all permitted parameter values.

Since our model is a generalisation of the existing financial signalling and credit rationing literatures, it helps to reconcile some, apparently contradictory, results. For example both Constantinides & Grundy (1989)'s result on equity repurchases and Brennan & Kraus (1987)'s result on debt repurchases are special cases in our model. As far as empirical results are concerned, we can counter some of the recent criticism of asymmetric information models.

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[36]There are however many inefficient semi-separating equilibria. Their characterisation is tedious and unilluminating. One can still argue that safe firms issue relatively more equity than risky firms. By this we mean that either the risky and the medium firm pool and issue less equity than the safe firm or that the safe and the medium firm pool and issue more equity than the risky firm. Furthermore, one can show that safe and risky firms never pool.

The criticism is based on evidence that contradicts the Pecking-Order, for example a positive stock price reaction that follows the announcement of an equity issue. But we show that the Pecking-Order is only a special case and a more general asymmetric information model may be able to explain even such evidence.

A Appendix

A.1 Proof of proposition 2

To show that $\alpha_s \geq \alpha_r$ we divide (5) by p_s and (6) by p_r and then add them together to yield

$$[(x_r - x_s) + W(\frac{1}{p_r} - \frac{1}{p_s})](\alpha_s - \alpha_r) \geq 0$$

Since the term is square brackets is strictly positive we have $\alpha_s \geq \alpha_r$.

To show that $F_s \leq F_r$ we proceed by contradiction and assume that $F_s > F_r$. The safe firm's incentive compatibility constraint (5) can be rewritten as

$$\frac{(1 - \alpha_s)}{(1 - \alpha_r)} \geq \frac{W + p_s(x_s - F_r)}{W + p_s(x_s - F_s)}$$

The inequality does not hold with $\alpha_s \geq \alpha_r$ and $F_s > F_r$ which is the desired contradiction.

A.2 Proof of lemma 2

Condition 2 implies that in a separating equilibrium the investors believes with probality one that he is financing a project of quality t when he observes (F_t, α_t). Hence, he knows that he will make of profit of $V(F_t, \alpha_t) = v_t(F_t, \alpha_t)$. Suppose now that $v_t(F_t, \alpha_t) > I$ and consider an alternative contract with slightly reduced debt and equity payments: $(F_t - \delta_F, \alpha_t - \delta_\alpha)$. Now for δ_F and δ_α sufficiently small, $v_t(F_t - \delta_F, \alpha_t - \delta_\alpha) > I$, so that the alternative contract is still accepted by the investor. Moreover, $U_t(F_t - \delta_F, \alpha_t - \delta_\alpha) > U_t(F_t, \alpha_t)$ which means that the original financing contract (F_t, α_t) does not satisfy condition 1 which is the required contradiction. Note that the alternative contract satisfies incentive compatibility since lemma 1 says that incentive compatibility in a separating equilibrium requires $F_r > F_s$ and $\alpha_r < \alpha_s$. Hence, it is always possible to find δ_F and δ_α so that $(F_t - \delta_F, \alpha_t - \delta_\alpha)$ is incentive compatible.

A.3 Proof of lemma 3

By contradiction: suppose that (F_r^*, α_r^*) is the risky firm's choice of debt and equity at equilibrium and that a deviation (F', α') with $\alpha' < \bar{\alpha}(I)$ is interpreted with some probability as coming from a safe firm, i.e. $\mu(F', \alpha') > 0$. We show that the deviation is profitable for the risky firm.

Since there is no reason to leave the investor with positive profits, he makes zero profits on the deviation:

$$\mu(F', \alpha')v_s(F', \alpha') + (1 - \mu(F', \alpha'))v_r(F', \alpha') = I$$

The investor also makes zero profits on the risky firm's equilibrium contract (F_r^*, α_r^*) (see lemma 2 for a separating equilibrium and proposition 5 for the pooling equilibrium):

$$v_r(F_r^*, \alpha_r^*) = I$$

So that

$$\mu(F', \alpha')(v_s(F', \alpha') - v_r(F', \alpha')) \quad = v_r(F_r^*, \alpha_r^*) - v_r(F', \alpha')$$
$$> 0$$

where the inequality follows from corollary 2.

Deviations with $\alpha' > \bar{\alpha}(I)$ are ruled out similarly.

A.4 Proof of lemma 4

We just show that $\bar{F}_{m,r}(I) < \bar{F}_{s,m}(I)$:

$$\bar{F}_{m,r}(I) - \bar{F}_{s,m}(I) = \frac{(\varepsilon - \gamma)\varepsilon\gamma I x_s[p_s x_s(1-\gamma) + (W-I)(1-\varepsilon)]}{[p_s x_s \varepsilon(1-\gamma) + (W-I)\gamma(1-\varepsilon)][p_s x_s \varepsilon(1-\gamma)^2 + (W-I)\gamma(1-\varepsilon)^2]}$$

which is negative if $\varepsilon < \gamma$. Similarly, one can show that $\bar{\alpha}_{m,r}(I) - \bar{\alpha}_{s,m}(I)$ is positive when $\varepsilon < \gamma$.

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