

Long-Term Care Insurance, Annuities and Asymmetric
Information: The Case for Bundling Contracts.¹

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Abstract

Within an asymmetric information set-up in which individuals differ in terms of their risk aversion and can choose whether or not to take preventative action, we illustrate in a unified framework the equilibrium possibilities with stand-alone long-term care insurance and annuity contracts. With costs of administering insurance, so that insurance is unfair, we show the existence of an equilibrium in which the risk averse type, who take more preventative action, obtain more of both types of insurance, even though their probability of using long-term care coverage is lower than the less risk averse. Hence, we show that the empirical observations of Finkelstein and Poterba (2004) and Finkelstein and McGarry (2003) are consistent with simultaneous separating equilibria in the two markets. A key finding of the paper is that as individuals who take care will be relatively low risk in the long-term care insurance market but high risk in the annuities market, with the opposite being the case for those who take less preventative action, an equilibrium exists in bundled contracts that Pareto dominates the outcome with stand-alone contracts.

1 Introduction

This paper is concerned with the markets for annuities and long-term care insurance. It investigates the effects of asymmetric information about ex-ante mortality and ill-health in old age, together with preventative actions to improve life-expectancy and health in old age, for the supply and demand for these insurance products. The paper provides a simple unified treatment of market equilibrium for stand-alone annuities and long-term care insurance, which generates outcomes consistent with recent empirical findings. The main objective of the paper is to investigate the potential advantages of insurance contracts that bundle the two types of insurance and to compare equilibrium in such contracts to alternative outcomes in stand-alone insurance contracts.

As background motivation for the analysis in the paper we note some recent empirical findings. Finkelstein and Poterba (2002 and 2004) have investigated the implications of asymmetric information for the supply and demand for annuities. They state “.....findings of differential mortality experience for annuitants who purchase different types of policies are consistent with standard models of adverse selection in which individuals have private information about their risk type and this private information influences their choice of insurance contract.” (2004). Finkelstein and Poterba (2004) go on to note that risk averse individuals may both take more care of their health and live longer and demand more mortality related insurance. These findings are complementary to survey-based studies, such as Hamermesh (1985) and Hurd and McGarry (forthcoming), which suggest that individuals have informed, and plausible, views about their potential life expectancy, beyond the information that can be gleaned from their observable characteristics”.¹

Finkelstein and McGarry (2003) provide an innovative analysis of the market for long-term care insurance that highlights the role of asymmetric information. The results show that individuals who undertake a greater fraction of potential preventive health activity (more cautious individuals) are less likely to enter a nursing home. Their results also indicate that those who undertake more preventive health activity are also more likely to have more long-term care insurance.²

Turning to bundled contracts, Murtaugh, Spillman and Warshawsky (2001) examined the implications of the positive correlation between mortality and disability for the benefits of combining an immediate annuity, purchased with a lump-sum of accumulated saving, with long-term care insurance. Looking at the histories of a sample of 16,587 people dying in 1986, they simulated pools of persons likely or eligible to purchase annuity and long-

¹We note that Cawley and Philipson (1999) find that in life-insurance markets, where the insured are insuring against early death, rather than long life in annuity markets, those individuals who buy larger policies do not display above-average mortality risk. The finding that the correlation between risk and insurance demand is negative in the insurance market is investigated by deMeza and Webb (2001) and Chiappori et al (forthcoming).

²They find that conditional on the insurance company’s risk classification, individuals have private information that predicts their subsequent nursing home use. They also find a substantial variation in the fraction of gender-appropriate potential preventive activity actually undertaken.

term care insurance at age 65 and 75. They found that by offering bundled products to pools of individuals, combining say low-life expectancy with higher than average likelihood of needing long-term care, significantly reduced premiums by 3 to 5 percent relative to the stand-alone products. They showed that the bundled product could have a significant impact on the proportion of people participating in the market. In particular, they found that minimal underwriting, excluding only those who would be eligible for disability benefits at purchase, would increase the potential market to 98 percent of 65-year-olds, compared with only 77 percent under “current” long-term care insurance underwriting practice.

The existing theoretical literature lacks an integrated model of the two types of insurance described above and does not provide a formal framework for evaluating the role and efficiency properties of bundled contracts. To do this, we propose a framework that extends the insurance market model of Rothschild and Stiglitz (1976) along similar lines to de Meza and Webb (2001). It considers a population of individuals made up of high and low risk aversion types. Individuals can take preventative action that affects both longevity and health status in old age. Risk averse individuals place a higher premium on smoothing consumption between present and future consumption; and smoothing between a poor and sick future versus a healthy and wealthy future. This type also takes relatively more preventative action.³ Insurance contracts are supplied competitively on the basis of information available to the insurance company, but because of administrative costs, contracts are priced unfairly.

The paper is structured as follows. It begins by presenting two separate asymmetric information insurance games, played simultaneously by the same population of two types of individual, differing in their risk aversion. The two markets considered trade stand-alone contracts in long-term care insurance and annuities. We illustrate the general set of equilibrium possibilities with stand-alone contracts. However, with unfairly priced insurance, we show the existence of an equilibrium in which the more risk averse type, who take more preventative action, obtain more of both types of insurance, even though their probability of using long-term care insurance is lower than that of the less risk averse type. Hence, we show that the empirical observations of Finkelstein and Poterba (2004) and Finkelstein and McGarry (2003) are consistent with simultaneous separating equilibria in the two markets. We also investigate the possibility of pooling equilibrium in each market.

The paper then considers a market for bundled insurance products. Unlike Murtaugh, Spillman and Warshawsky (2001), we consider deferred and not immediate annuities. The key insight is that individuals who take care are relatively low risk in the long-term care insurance market but high risk in the annuities market, with the opposite being the case for those who take less preventative action. This raises the possibility that combining an annuity and long-term care insurance, purchased when young (not on becoming old), makes

³Hurd, McFadden and Merrill (2001) find that health status and longevity are strongly associated, particularly among the elderly.

the individual incentive-compatibility conditions easier to satisfy and that bundled contracts are profitable. The final part of the paper investigates the existence of equilibria in bundled insurance products. It is shown that this equilibrium Pareto dominates the equilibrium allocation in stand-alone contracts. Moreover, the equilibrium in bundled contracts is stable with respect to the introduction of stand-alone contract offers. Finally, we briefly discuss the potential public policy implications of this finding.

2 The Basic Problem

We begin by outlining the basic problem of individual choice over long-term care insurance and annuities supplied by competitive insurance companies.

2.1 Individual Preferences

The individual knows both his type, which is related to family history and other given elements of health status. However, he also has private information on the amount of preventative action he takes, which impacts upon his longevity and his chance of ill health in old age.⁴ Individuals, indexed by i are born at date $t = 0$ and live to either $t = 1$ or $t = 2$. They have preferences over consumption, C_{it} , at dates $t = 1$ and $t = 2$, with increasing strictly concave utility index $U_i(C_{ti})$, with $U'_i > 0$, $U''_i < 0$; and the utility cost of preventative action, H_i .⁵ Modelling preventative action as a utility rather than a monetary cost is done purely for tractability. We assume that individuals come in two types: high (R) and low (r) absolute risk aversion, where type $i \in \{R, r\}$ is private information. Both types' utility functions exhibit decreasing absolute risk aversion.⁶

2.2 Longevity and Long-Term Care Risk

The probability of survival to date $t = 2$, $0 < q(H_i) < 1$ and the probability of a healthy old age, $p(H_i)$, are both functions of the level of preventative action. In the case of ill-health in

⁴Brown (2000) notes that “if the insurance company could accurately determine the expected mortality of each individual applicant, it could price each policy in a manner that was appropriate and profitable. In practice, life insurance companies estimate mortality for life insurance contracts, using medical exams and health histories to separate individuals into risk classes. This estimation is almost never done for annuity products, however.”

⁵The utility specification assumes that the utility function is the same for the states of health and ill-health. Of course the consumption bundle of an ill person may differ significantly from that of a healthy one. This would suggest that the utility function should be state dependent. Although this is realistic and will have an effect on equilibrium, the principal results are the same, so we abstract from this consideration. For a model that allows for different utility functions in the states of health and ill-health; see Philippon and Becker (1998). In particular the marginal utility of consumption in the healthy state is higher in the healthy than the unhealthy state.

⁶This is a property of the constant relative risk aversion utility function. The coefficient of relative risk aversion for the R group being higher than for the r group. With this specification, there is a link between wealth and absolute risk aversion.

old age, the individual suffers a cost of long-term care equal to X . The effect of preventative action on survival and health probabilities, although positive can differ significantly. We assume that the preventative action is a binary variable. If the individual invests $H_i = \widehat{H}$ in preventative actions, then his probability of survival is q_H ; if he does not, $H_i = 0$, and his probability of survival is $q_L < q_H$. Similarly, if the individual invests $H_i = \widehat{H}$ in preventative action, then his probability of needing long-term care in the event of survival is $(1 - p_G)$, if he does not, $H_i = 0$, and his probability of needing long-term care is $(1 - p_B)$, with $p_G > p_B$. Note that this does not mean that an individual who takes preventative actions will have a lower expected utilisation of long-term care as it is possible that $q_H(1 - p_G) > q_L(1 - p_B)$.

Assumption 1. We assume that $q_H(1 - p_G) < q_L(1 - p_B)$, which means that the expected use of long-term care is determined by health status given survival.⁷

2.3 Individuals Opportunities

All individuals have the same initial wealth of $A > 0$. Individuals can purchase an annuity to cover old age consumption and insurance to cover a fraction $0 \leq \lambda \leq 1$ of the cost of long-term care if they need it. At the planning date, $t = 0$, the individual allocates his assets of A between consumption at dates $t = 1$ and $t = 2$. Resources are used to purchase an annuity at a premium P and payment Y_2 (the individual cannot save in any other way) and long-term care insurance at a premium Q that covers a fraction λ of the cost of care X . We assume that long-term care insurance is bought before the realisation of life expectancy. Then

$$A = C_{1i} + P + Q, \tag{1}$$

$$C_{2i} = Y_2, \text{ if no long-term care needed,} \tag{2}$$

and

$$C_{2i} = Y_2 - X + \lambda X, \text{ if long-term care needed.}$$

An individual i , who takes preventative action $H_i \in \{0, \widehat{H}\}$, gains utility at given annuity contract (P, Y_2) and long-term care insurance contract (Q, λ) of

$$V_i = U_i(C_{1i}) - H_i + q(H_i)[p(H_i)U_i(Y_2) + (1 - p(H_i))U_i(Y_2 - X + \lambda X)]. \tag{3}$$

This specification of preferences is consistent with that in Yaari (1965), in which the probability of survival to date $t = 2$ takes the same form as discounting.

A type i individual's incentive constraint for choosing preventative action, \widehat{H} , at the

⁷This assumption limits the set of possibilities in our model in a particular way, which helps it fit the facts and enables a straightforward treatment of bundled contracts in the latter part of the paper. However, it should not be taken to mean that the opposite is uninteresting or untractable.

annuity contract (P, Y_2) and long-term care insurance contract (Q, λ) is

$$\begin{aligned} U_i(A - P - Q) - \widehat{H} + q_H(p_G U_i(Y_2) + (1 - p_G)U_i(Y_2 - X + \lambda X)) \geq \\ U_i(A - P - Q) + q_L(p_B U_i(Y_2) + (1 - p_B)U_i(Y_2 - X + \lambda X)) \text{ for } i = R, r. \end{aligned} \quad (4)$$

Then, the care-incentive constraint is written as

$$\widehat{H} \leq (q_H p_G - q_L p_B)[U_i(Y_2) - U_i(Y_2 - X + \lambda X)] + (q_H - q_L)U_i(Y_2 - X + \lambda X) \text{ for } i = R, r. \quad (5)$$

Note that preventative actions are of more value to the risk averse and that this is more so the greater the gap between q_H and q_L , and p_G and p_B . Moreover, because of decreasing absolute risk aversion, the threshold value of \widehat{H} is increasing in risk aversion. In what follows we are interested in understanding the implications of different types having different incentives to take preventative action.

There is a significant and important difference between mortality insurance and long-term care insurance. The moral hazard in the annuity market is that an individual who buys an annuity on the basis of not taking preventative action then has an incentive to take action to increase longevity. In the long-term care insurance market, the more insurance coverage the less incentive the individual has to take care. Hence, the incentive-care constraint plays a quite different role in the two markets. Geometrically, in the annuity market, the incentive to take preventative action is greater, the closer the contract is to the 45-degree line, whilst the opposite is the case in the long-term care market.

Assumption 2. To keep matters as simple as possible, rather than give conditions under which type r do not take preventative action, we only examine parts of the contract space in which they do not. Hence we need only consider (5) as a relevant constraint for type R .

Now consider the different types indifference curves. Evaluate (3) at a pair of contracts (P, Y_2) , (Q, λ) . The marginal rate of substitution between P and Y_2 (the absolute value of the slope of indifference curve in (C_1, C_2) space) is given by dY_2/dP ,

$$MRS_{P, Y_2} = \frac{dY_2}{dP} = \frac{U'_i(A - P - Q)}{q_i[p_i U'_i(Y_2) + (1 - p_G)U'_i(Y_2 - X + \lambda X)]}. \quad (6)$$

The indifference curves of individuals are kinked at the point of indifference to preventative action, and steeper for $H_i = 0$ than for $H_i = \widehat{H}_i$. Moreover, we have:

Condition 1. When the R type take preventative action and the r do not, given λ , we

have single crossing in the annuity dimension:

$$\frac{U'_R(A - P - Q)}{q_H[p_G U'_R(Y_2) + (1 - p_G)U'_R(Y_2 - X + \lambda X)]} > \frac{U'_r(A - K)}{q_L[p_B U'_r(Y_2) + (1 - p_B)U'_r(Y_2 - X + \lambda X)]}. \quad (7)$$

Thus the ordering of slopes of indifference curves on this dimension is determined by the ordering of the survival probabilities. This means that at some allocation with incomplete annuity cover, type r 's indifference curve is steeper than that of type R , and that as annuity coverage is increased this is maintained.

In the long-term care dimension, the marginal rate of substitution between Q and λ the absolute value of the slope of indifference curve in $(C_2 \text{ health}, C_2 \text{ ill-health space})$ is given by $d\lambda/dQ$,

$$MRS_{Q,\lambda} = \frac{d\lambda}{dQ} = \frac{U'_i(A - P - Q)}{q_i[(1 - p_i)U'_i(Y_2 - X + \lambda X)X]}. \quad (8)$$

Again indifference curves are kinked at the point of indifference to preventative action and less steep for $H_i = 0$ than for $H_i = \hat{H}_i$. Then, given Assumption 1, we have:

Condition 2. When type R take preventative action and type r do not, given Y_2 , we have double-crossing of indifference curves:

$$\frac{U'_R(A - P - Q)}{q_H[(1 - p_G)U'_R(Y_2 - X + \lambda X)X]} \gtrless \frac{U'_r(A - P - Q)}{q_L[(1 - p_B)U'_r(Y_2 - X + \lambda X)X]}. \quad (9)$$

This implies that at low levels of long-term care coverage, λ is low, the risk aversion effect dominates and type R indifference curve is flatter than that of type r (they will at the margin pay a larger amount to avoid a small amount of health care costs). However, at higher levels of coverage, the higher probability of suffering ill-health means that the indifference curve of the r type is flatter. Thus we have two regions:

- (i) When λ is low the left hand-side of (9) is less than the right hand side.
- (ii) When λ is high the left hand-side of (9) is greater than the right hand side.

This property is referred to as double-crossing and is seen in Figure 1.

In the analysis above, we exclude the possibility of (unobserved) saving. This can be added to the analysis and will have implications for the nature of the equilibria analysed but not in ways that affect the main conclusions.

2.4 Insurance Companies

There is a continuum of individuals, a proportion $0 < \alpha < 1$ are of type R and a proportion $(1 - \alpha)$ are of type r . These individuals apply for insurance contracts supplied by insurance

companies engaged in competition to supply contracts. Contracts comprise a premium and a level of insurance cover, either long-term care cover or an annuity payment. In offering insurance contracts, insurance companies know market characteristics but cannot identify individuals' types directly. Throughout the analysis we make assumptions that individuals cannot fake a long life and that their second period health status is verifiable. Of the two the latter is the more contentious.

Assumption 3. There is a fixed cost of administering each copy of each insurance contract.

These costs are used to introduce some element of (empirically observed) unfairness in insurance contracts that is used to explain non-participation at the margin for certain contracts. Of course administration costs may have a proportional and a fixed element, so we also discuss the implications of proportional costs towards the end of Section 3 (3.4).

The fixed cost of administering long-term care insurance contracts is denoted by $k_T \geq 0$, so that if $k_T > 0$, insurance is unfair. An Insurance company's expected profit from a long-term care insurance contract (Q, λ) is given by,

$$\Pi = Q - q^*(1 - p^*)\lambda X - k_T \geq 0, \quad (10)$$

where q^* and p^* are the insurance company's forecast of the applicants' life expectancy and health in old age. Annuity contracts incur a fixed cost of $k_Z \geq 0$. An insurance company's expected profit from a contract (P, Y_2) is given by

$$\pi = P - q^*Y_2 - k_Z \geq 0, \quad (11)$$

3 Stand-Alone Contracts Equilibrium

We now address the nature of equilibrium when individuals can participate in both the long-term care insurance market and the annuity market. The study of the simultaneous play in the stand-alone long-term care insurance and annuity market games serves two purposes. First it allows us to offer an explanation of the various empirical findings in the literature that we noted earlier. Secondly, it allows for a richer analysis of the market for contracts that bundle the two types of insurance together. In presenting the simultaneous market games we assume that preventative actions affect both longevity and the probability of a healthy old age. At this stage of the paper we assume that market for bundled contracts is closed.⁸

⁸We note that Brown, Davidoff and Diamond (2005) show how in an incomplete markets setting uninsured medical expenses affect the demand for annuities.

The model consists of two market games: an annuity market game and a long-term care insurance game. In playing each game we assume that players take the outcome of the other game as given. We assume that the insurance companies in the two games do not exchange information. They know that individuals participate in both markets but do not know the specific outcome obtained in the other market. That is we assume that individuals cannot provide verifiable information on the product purchased in one market to improve the product terms they are offered in the other market, should it be in their interests to do so.

3.1 The Game

Each market is characterised by a three-stage game.

At Stage 1, each of two insurance companies engaged in price-quantity competition offer insurance contracts. Each individual applies for one contract from one insurance company in this market, given the insurance contract in the other market, and decides upon a level of preventative action. If both insurance companies offer the same contract, each applicant tosses a fair coin to determine the insurance company to which he applies (if he applies at all). At stage 2, each insurance company forms an estimate of the care taken by the applicants for each contract and decides whether to accept or reject applications. Each insurance company withdraws contracts that will be unprofitable given beliefs and the contract offers of the other insurance company. At stage 3, if individuals find the contract they applied for is not funded, they select one of the contracts still on offer, or else do not demand a contract.

The equilibrium is established as follows. We look for a Nash equilibrium with belief consistency (the insurance companies correctly estimate the probabilities of insurance claims of the set of applicants for each contract) at the third stage of the game. We check that equilibrium contract offers are stable with respect to all possible perturbations in offers at the first stage. The equilibrium of the game we analyse has a Wilson (1977) equilibrium. Hellwig (1987) has shown (applying the stability argument of Kohlberg and Mertens (1987)) that (unlike the two-stage game in Rothschild and Stiglitz (1976)) the Wilson equilibrium is a unique perfect-Bayesian equilibrium of this game.^{9,10}

3.2 Separating Equilibrium

Competitive separating equilibria arise in the markets for the stand-alone annuity and long-term care insurance when profitable contracts can be offered that only attract one type In

⁹Kreps (1990) pp 644-645 contains a very useful discussion of issues surrounding the use of Wilson and Riley (1979) equilibrium concepts.

¹⁰Smart (2000) and Wambach (2000) both provide general models that demonstrate the nature of equilibrium in models with asymmetric information about heterogeneous preferences.

each market a pair of contracts are offered that satisfy an incentive-compatibility (or self-selection) constraint for that type and must be profitable for insurance companies on a stand alone basis.

Let $Z_i = (P_i, Y_{2i})$ denote the annuity contract for type i ; and $T_i = (Q_i, \lambda_i)$ is the long-term care contract for each type. The utility at each contract offer is written conditional upon the contract taken by this type in the other market, denoted by an asterisk. The incentive-compatibility (or self-selection) constraints in the long-term care insurance game are

$$V_R(T_R|Z_R^*) \geq V_R(T_r|Z_R^*), \quad (12)$$

and

$$V_r(T_r|Z_r^*) \geq V_r(T_R|Z_r^*). \quad (13)$$

In the annuity market game these are

$$V_R(Z_R|T_R^*) \geq V_R(Z_r|T_R^*), \quad (14)$$

and

$$V_r(Z_r|T_r^*) \geq V_r(Z_R|T_r^*). \quad (15)$$

The insurance companies' profitability conditions for long-term care insurance contracts are given in (10) those for annuity contracts in (11).

3.2.1 Long-Term Care Insurance

We first examine a separating equilibrium in the long-term care insurance market. In such an equilibrium, each insurance company offers two long-term care insurance contracts: $T_R = (Q_R, \lambda_R)$ and $T_r = (Q_r, \lambda_r)$. These contracts must satisfy (12) and (13) and must be profitable,

$$\Pi_i = Q_i + q_i^*(1 - p_i^*)\lambda_i X - k_T \geq 0, \quad i = R, r. \quad (16)$$

Competition implies that equilibrium long-term care insurance contracts earn zero-expected profit. In forming their estimate of p_i^* and q_i^* the insurance companies have to forecast whether at the contract offer the type chooses to take care. If it does, then belief consistency requires $p_i^* = p_G$, $q_i^* = q_H$ and if not $p_i^* = p_B$, $q_i^* = q_L$ for $i = R, r$.

The space of state contingent consumption is divided into two regions by a curve labeled $H_i H_i$, $i = R, r$. To the right of this curve the individual takes preventative action and to the left he does not. Moreover, with decreasing-absolute-risk aversion, this curve is convex and for the less risk averse type lies to the right of that for the more risk-averse type. The indifference curves of individuals are kinked at the point of indifference to preventative action and to the left of the $H_i H_i$ curve are less steep than to the right and satisfy the property in Condition 2. Given Assumption 2, the care-incentive constraint for the r type

lies to the right of the endowment point, so that this type never takes preventative action.

Given Assumption 2, from (16) as equalities, under complete information (we drop the incentive-compatibility conditions), the long-term care insurance premia for maximum coverage (subject to (5) being satisfied for type R) against the loss X for the two types have the following ordering, $\bar{Q}_R < \bar{Q}_r$. Denote the complete-information contracts by (\bar{T}_R, \bar{T}_r) . Then we have the following condition:

Condition 3. $V_r(\bar{T}_R|Z_r^*) > V_r(\bar{T}_r|Z_r^*)$.

This condition precludes the complete information allocation from existing as an equilibrium under asymmetric information. The reason is that type r prefer the complete information contract for type R to that for its own type.

Proposition 1. Given Condition 3, if the proportion of type R , $0 < \alpha < 1$, is not too high, there exist two different separating equilibria: (i) If the administration cost, k_T is below a threshold, \hat{k}_T , the separating equilibrium has complete long-term care insurance for the r type and partial cover for type R . (ii) If the administration cost, k_T is above \hat{k}_T , given Condition 2, the r type switch to no insurance, and choose to stay at the endowment point.

Proof. Consider first part (i), illustrated in Figure 1: Condition 3 means that potential separating equilibria maximise (3) for type R subject to (12) and (13) and the two zero-profit conditions in (16). The pair of contracts satisfy (13) strictly and (12) weakly, that is $V_r(T_r|Z_r^*) = V_r(T_R|Z_r^*)$ and $V_R(T_R|Z_R^*) > V_R(T_r|Z_R^*)$ and the two zero profit conditions in (16). Type R take preventative action and obtain the contract at T_R , with incomplete cover, $\lambda_R < 1$, on the offer curve $F_R F_R$, with $p_i^* = p_G$ and $q_i^* = q_H$. The r type individuals obtain the complete-information contract for their type, $T_r = \bar{T}_r$, with complete insurance, $\lambda_r = 1$, on the offer curve $F_r F_r$, but do not take preventative action, $p_i^* = p_B$ and $q_i^* = q_L$.

We show the $\lambda_R < 1$ property formally. The above optimisation problem is to choose T_R to solve (noting that (5) does not bind and can be dropped):

$$\max\{U_R(A - Q_R - P_R^*) + q_H(p_G U_R(Y_{2R}^*) + (1 - p_G)U_R(Y_{2R}^* - X + \lambda_R X))\} \quad (17)$$

subject to

$$U_r(A - Q_R - P_R^*) + q_H[p_G U_r(Y_{2R}^*) + (1 - p_G)U_r(Y_{2R}^* - X + \lambda_R X)] \leq V_r(T_r|Z_r^*) \quad (18)$$

and

$$Q_R = q_H(1 - p_G)\lambda_R X + k_T \quad (19)$$

Let $\mu \geq 0$ and $\phi \geq 0$ be the multipliers on the respective constraints. Then choose λ_R and Q_R and the multipliers to solve the problem. The first-order conditions are

$$q_H(1 - p_G)U'_R(Y_{2R}^* - X + \lambda_R X)X - \mu q_L(1 - p_B)U'_r(Y_{2R}^* - X + \lambda_R X)X - \phi q_H(1 - p_G)X = 0 \quad (20)$$

$$-U'_R(A - Q_R - P_R^*) + \phi = 0 \quad (21)$$

and the two constraints. Inspection shows that (18) binds and $\mu > 0$ so that $\lambda_R < 1$.

The property $k_T < \widehat{k}_T$ ensures that the terms of contracts are superior to non-participation. In Figure 1, A is the endowment point and the distance AB represents the administration cost per contract and as can be seen the contract \bar{T}_r is preferred to A by type r . The assumption that α is not too high means that the cross-subsidy from type R to type r is so high that there are no profitable pooling offers that dominate the separating offers.

The second case, (ii), illustrated in Figure 2, occurs when $k_T > \widehat{k}_T$ (in the Figure, again the distance AB measures administration costs), so that insurance is sufficiently unfair that the r type switch to no insurance, at the null contract T'_r (at the endowment point). But for some range of costs, the more risk averse, more careful type, still demand insurance at the contract T_R , which is the best contract on $F_R F_R$ that is not demanded by type r . As can be seen in the Figure, this occurs because of the double-crossing of the indifference curves (Condition 2), which means that although type r value complete insurance more than type R (Condition 3) as they are not very risk averse they are fairly indifferent to varying the amount of cover near the endowment point. It is at this margin that the participation decision is made, so they may drop out of the market. Finally, if the administration cost exceeds some higher threshold, both types drop out of the market. QED.

Figures 1 and 2 here.

At the contract T_R , type R obtain limited cover, which is consistent with the findings of Brown and Finkelstein. They observe that long-term-care insurance typically covers only one-third of the present value of long-term care costs for sixty five year olds.

3.2.2 Annuities

Eckstein et al., (1985a,b) provide a detailed treatment of annuity market equilibrium under conditions of asymmetric information. The analysis below is similar to theirs, but is included here in a brief form because it provides both a contrast with the equilibrium in the market for stand-alone long-term care insurance contracts described above and also, and more importantly, because it is needed in order to understand some of the properties of the market for bundled contracts examined later in the paper.

By Assumption 2, when the R type take preventative action the r type do not. To keep matters simple, and in this case without loss, we restrict attention to parts of the contract space where type R take preventative action, so that (5) is satisfied for this type.

A separating equilibrium pair of contracts (Z_R, Z_r) satisfy (14) and (15) and must be profitable,

$$\pi_i = P_i - q_i^* Y_{2i} - k_Z \geq 0, \quad i = R, r, \quad (22)$$

Denote the complete-information equilibrium annuity contracts by (\bar{Z}_R, \bar{Z}_r) , again where (5) is satisfied for type R . Then we have the following condition:

Condition 4. $V_R(\bar{Z}_r|T_R^*) > V_R(\bar{Z}_R|T_R^*)$.

This condition precludes the existence of a complete information allocation under asymmetric information and means that the problem is that the R type prefer the complete information contract for type r to that for its own type.

Proposition 2. Given Assumption 2 and Condition 4, if the proportion of type R , $0 < \alpha < 1$, is not too low, there exists a separating equilibrium in the annuity market. There are two possibilities: (i) If the administration cost, k_Z is below some threshold, \hat{k}_Z , in the separating equilibrium type R have full cover and type r partial cover. (ii) Given Condition 1, if $k_Z > \hat{k}_Z$ type r drop out of the market.

Proof. Consider part (i). Condition 4 means that potential separating equilibria maximise (3) subject to (14) and (15) and the two zero profit conditions in (16). The pair of contracts satisfy (14) strictly and (15) weakly, the pair of annuity contracts satisfy $V_R(Z_R|T_R^*) = V_R(Z_r|T_R^*)$ and $V_r(Z_r|T_r^*) > V_r(Z_R|T_r^*)$, and the two zero profit conditions. Hence, the contract demanded by the r type, who do not take preventative action and have shorter life expectancy, has under-annuitisation. The contract for type R who take preventative action, fully annuitise on fair terms.

The contract Z_r solves the following problem:

$$\max\{U_r(A - Q_r^* - P_r) + q_L(p_B U_r(Y_{2r}) + (1 - p_B)U_r(Y_{2r} - X + \lambda_r^* X))\} \quad (23)$$

subject to

$$U_R(A - Q_r^* - P_r) + q_H[p_G U_R(Y_{2r}) + (1 - p_G)U_R(Y_{2r} - X + \lambda_r^* X)] \leq V_R(Z_R|T_R^*) \quad (24)$$

and

$$P_r = q_L Y_{2r} + k_Z \quad (25)$$

Let $\hat{\mu} \geq 0$ and $\hat{\phi} \geq 0$ be the multipliers on the respective constraints. Then choose Y_{2r}

and P_r and the multipliers to solve the problem. The first-order conditions are

$$q_L(p_B U'_r(Y_{2r}) + (1 - p_B)U'_r(Y_{2r} - X + \lambda_r^* X)) \quad (26)$$

$$+ \widehat{\mu}[q_H[p_G U'_R(Y_{2r}) + (1 - p_G)U'_R(Y_{2r} - X + \lambda_{2r}^* X)] - \widehat{\phi} q_L] = 0$$

$$-U'_r(A - Q_r^* - P_r) + \widehat{\phi} = 0 \quad (27)$$

and the two constraints. Inspection shows that (24) binds and $\widehat{\mu} > 0$ so that $Y_{2r} < \bar{Y}_{2r}$.

The property $k_Z < \widehat{k}_Z$ ensures that the terms of contracts are superior to non-participation. The assumption that α is not too low (the proportion of type r is not too high) means that the cross-subsidy from type r to type R in pooling is too high for them, so that there are no profitable pooling offers that dominate the separating offers. Given the single crossing property in Condition 1, the case in the second part of the proposition follows immediately. QED.

Finally note that as both of our market games are played simultaneously we have the consistency requirement that in equilibrium, the constraint value of utility for a type in one market must equal the optimal value in the other. For type R , $V_R(Z_R|T_R^*) = V_R(T_R|Z_R^*)$, where the left-hand-side is the value of utility achieved in the annuity market that features on the right hand side of the incentive constraint (24); the right-hand-side is in the value of utility achieved in the long-term care insurance market. In this case $Z_R = Z_R^*$ and $T_R = T_R^*$. For type r , $V_r(T_r|Z_r^*) = V_r(Z_r|T_r^*)$, where the left-hand-side is the value of utility achieved in the long-term care insurance market that features on the right hand side of the incentive constraint (18); the right-hand-side is the value of utility achieved in the annuity market. In this case $Z_r = Z_r^*$ and $T_r = T_r^*$.

Thus if $k_T > \widehat{k}_T$ but $k_Z < \widehat{k}_Z$, we have an equilibrium in which the high-risk aversion, careful type have a higher than average incentive to take long-term care insurance, but also have a greater than average incentive to take care of their health; so they will have a lower than average utilisation of long-term care. This prediction is consistent with the findings of Finkelstein and McGarry (2003). The R type also has a high demand for annuities as in Finkelstein and Poterba (2004).

The prediction rests upon the relative magnitudes of administrative costs in the two markets. In the annuity market empirical evidence for the United States such as that in Mitchell et al (2002) shows, using the life-expectancy of the average sixty-five year old in the population, loadings are about fifteen per-cent of premium. However, once selection effects have been taken into account, this figure drops to below ten per cent. For long-term care insurance, Brown and Finkelstein (2004) report loadings of approximately eighteen per cent of premium, which would appear to be even higher once selection effects, which run in the opposite direction to those in the annuity market, are removed.¹¹ If cost differences

¹¹It should be noted that there are big gender biases in these loadings. Loadings of forty four per cent are

are not sufficient to support the empirical finding, then if the probability difference between types is relatively bigger in the annuity market, the same possibility emerges.

3.3 Pooling Equilibrium

Now for completeness, consider the possibility of pooling equilibria in the dual games, in which in each market both types obtain the same contract. This type of equilibrium is obtained if the pooling contract Pareto dominates the separating pair of contracts. There is the possibility of pooling equilibria in both games and mixtures of pooling and separating equilibria.¹²

First consider simultaneous pooling equilibria, (Z_p, T_p) . As is well known, in a pooling equilibrium better types subsidise poorer types. Therefore a necessary condition for the proposed equilibrium at the contract pair (Z_p, T_p) is that the care-incentive constraint (5) is satisfied for one type and not the other, that is for type R but not for type r . In the long-term care game, the incentive-compatibility conditions are,

$$V_R(T_p|Z_R^*) \geq V_R(T_R|Z_R^*), \quad (28)$$

$$V_r(T_p|Z_r^*) \geq V_r(T_r|Z_r^*). \quad (29)$$

In the annuity market game the incentive-compatibility conditions are,

$$V_R(Z_p|T_R^*) \geq V_R(Z_r|T_R^*), \quad (30)$$

$$V_r(Z_p|T_r^*) \geq V_r(Z_r|T_r^*), \quad (31)$$

The pooling contracts also satisfy the relevant zero profit conditions. In long-term care insurance the pooling contract, denoted by $T_p = (Q_p, \lambda_p)$, satisfies

$$Q_p + q^*(1 - p^*)\lambda_p X - k_T = 0, \quad (32)$$

In the annuity market the equilibrium pooling contract, denoted by $Z_p = (P_p, Y_p)$, satisfies

$$P_p + q_i^* Y_{2p} - k_Z = 0, \quad (33)$$

reported for men and better than actuarially fair prices (negative loadings) for women. The finding that the take up rate on this type of insurance by women is so low leads Brown and Finkelstein to suggest that the explanation of low levels of coverage may be to do with demand factors rather than supply.

¹²The possibility of a pooling equilibrium in each market for some range of values of the population parameter α , arises because of the assumption in our game structure that insurance companies can withdraw contracts that will be unprofitable given beliefs and the contract offers of the other insurance companies. Of course, if they cannot do this, then for this range of α , we either have non-existence as in the Rothschild and Stiglitz (1976) two-stage game or separating equilibria as in the tree-stage game of Engers and Fernandez (1987).

where $p_i^* = \alpha p_R^* + (1 - \alpha)p_r^*$, with $(p_R^*, p_r^*) \in \{p_B, p_G\}$ and $q_i^* = \alpha q_R^* + (1 - \alpha)q_r^*$ with $(q_R^*, q_r^*) \in \{q_H, q_L\}$.

3.3.1 Long-Term Care Insurance

Pooling emerges as an equilibrium possibility in the long-term care insurance market for standard reasons. That is when the cost of separation due to under-insurance exceeds the cross-subsidy in the pooling equilibrium contract.

Proposition 3. Given the values of the probabilities of survival and needing long-term care, Assumption 2 and Condition 3, if α is sufficiently high there exists a pooling equilibrium in the long-term care insurance market.

Proof. Given Assumption 2 and Condition 3, the pooling contract T_p maximises the utility of type R , subject to (32). To check that the contract T_p constitutes a pooling equilibrium of the postulated game we have to show that it is robust to possible perturbations. At T_p , as α is high (the zero-profit curve in (32) is close to the full-information curve for type R), $V_R(T_p|Z_R^*) > V_R(T_R|Z_R^*)$ and that $V_r(T_p|Z_r^*) > V_r(T_r|Z_r^*)$, so the separating contracts are dominated. Hence, if at the first stage, T_p were offered alongside (T_R, T_r) , both types will demand the contract and it will just break even. Now, suppose that at the first-stage, contracts such as T' are introduced alongside T_p , such that $V_R(T'|Z_R^*) > V_R(T_p|Z_R^*)$, but $V_r(T'|Z_r^*) < V_r(T_p|Z_r^*)$. T' is profitable if only type R take it. At the second stage, however, T' would be demanded by the R type, leaving only type r demanding T_p . But then T_p would be unprofitable and would be withdrawn. As a result, type r must demand T' , which is itself unprofitable if taken by both types. Hence neither insurance company has an incentive to introduce T' .

The precise nature of the pooling equilibrium contract depends upon the location of the tangent of the R types indifference curve and the pooling offer curve satisfying (32). If the tangent of the type's R indifference curve with the pooling offer curve, $F_p F_p$, lies to the right of $H_R H_R$, and to the left of $H_r H_r$, the R will take care, $p_R^* = p_G$ and $q_R^* = q_H$ with the r type not doing so with $p_r^* = p_B$ and $q_r^* = q_L$. Then this contract, which maximises the utility of the R type, is the pooling equilibrium contract. However, if the tangency lies to the left of $H_R H_R$, the care-incentive constraint for type R must be strictly binding and the pooling equilibrium lies on this constraint at the intersection with the pooling offer curve, $F_p F_p$.¹³ QED.

¹³The pooling equilibrium survives because the separating offers that upset the profitability of the pooling contract are unprofitable if the pooling offer is withdrawn. Engers and Fernandez (1987) have examined a similar game structure to that above but with successive rounds of contract offers at the second stage that cannot be withdrawn. This allows the separating pair of contracts to survive in equilibrium. The reason is that pooling contracts that require the withdrawal of other offers for them to be profitable will not be offered.

Figure 3 here.

Figure 3 shows the pooling contract that obtains in the first case. Here the contract T_p on the pooling offer curve, $F_p F_p$, is preferred by both types to the separating pair of contracts (T_R, T_r) and can be shown to be the unique equilibrium of the game. If this contract is taken by both types then all individuals receive the same amount of long-term care insurance, a prediction that runs counter to the findings of Finkelstein and McGarry (2003).¹⁴ Finally, notice in Figure 3 that the double-crossing of the indifference curves in Condition 2 does not in this case preclude the possibility that if administration costs increase it is the R type who drop out of the market first.

3.3.2 Annuities

Pooling emerges an equilibrium possibility in the annuity market for the same reasons as given at the beginning of the last Subsection. Hence we can state the following proposition immediately:

Proposition 4. Given the values of the probabilities of survival and needing long-term care, Assumption 2 and Condition 4, if α is sufficiently low there exists a pooling equilibrium in the annuity market.

Proof. Given Assumption 2 and Condition 4, the pooling contract Z_p maximises the utility of type r , subject to (33). The argument showing that this is the unique equilibrium follows along the same lines as in the case of the long-term care insurance game. In this equilibrium, the R type under-annuitise to gain the cross-subsidy in the pooled annuity contract. On the other hand, as α is low, the r type find the cost of under-annuitisation at the best separating contract too high relative to the cross-subsidy they pay in the pooling equilibrium and also participate. QED.

We have focussed on simultaneous pooling equilibria. However, there are other possibilities. Suppose $(p_G - p_B)$ is large and $(q_H - q_L)$ is small, but with (5) still satisfied for type R but not for type r . Then for α not too large the configuration can be a separating equilibrium in the long-term care insurance market and pooling in the annuity market. If $(q_H - q_L)$ is large and $(p_G - p_B)$ is small, the reverse configuration is a possibility.¹⁵

¹⁴However, in drawing this conclusion we cannot rule out the possibility that the theoretical correlation between risk aversion and taking preventative measures is proxying for unmeasured aspects of socio-economic status which themselves have a causal effect on long-term care utilization. Individuals with more children are both less likely to have insurance and less likely to use nursing home care.

¹⁵Finally note that if the annuities market involves separation, then it is the relatively short-lived r type that are constrained to under-annuitise. To achieve further consumption smoothing, if it is possible they may undertake unobserved saving. If the annuities market involves pooling, then the incentive to take care and increase mortality is higher for type R , because they benefit from the cross-subsidy in financing future consumption. However, the annuity then under-provides for future consumption for this type. If the terms of the pooling contract are relatively poor, because there are simply not enough low-survivors participating,

3.4 Proportional Administration Costs

Finally, consider the alternative of proportional administration costs (with fixed costs set at zero). In the long-term care insurance market, in equation (10) set $k_T = 0$ and let a proportional cost be applied to either the payment X or to the collection of premium, Q . Suppose the latter and denote the proportional cost by $\kappa_T > 0$. In the case of the annuity market, in equation (11) set $k_Z = 0$ and let the proportional cost, denoted by κ_Z , be applied the premium P . The introduction of proportional transaction costs means that the indifference curves of both types will now be steeper than the slopes of the offer curves at the 45-degree line. In particular, in the diagrams drawn for long-term care insurance, the distance AB is now zero, but the offer curves are less steep.

The main implications of the switch to proportional administration costs in the current analysis are in the long-term care insurance market. Therefore, consider this market and focus on the separating equilibrium. The indifference curve of type r , that is tangent to the offer curve $F_r F_r$ at the contract T_r can now cut the offer curve for type R , $F_R F_R$, in two places, one to the right of T_r (as before) but also to the left of T_r . The latter is possible with proportional administration costs as the odds ratio is now unfair.

In the first case matters are much the same as with low fixed administration costs, as illustrated in Figure 1. The only difference is that at the contract T_r , the r type obtain incomplete coverage.

However, in the second case, when there is an intersection to the left of T_r (a contract at this point will for type R dominate one at an intersection to the right of T_r) we have that both types demand incomplete insurance, but with R type obtaining a higher level of coverage than type r . If this cutting point of the type r indifference on $F_R F_R$ is to the right of the care-incentive constraint $H_R H_R$, this point determines the location of the contract for type R , T_R . On the other hand, if the cutting point on $F_R F_R$ is to the left of $H_R H_R$, the contract T_R is the best contract on $F_R F_R$ at which type R take preventative action. This lies at the intersection of $F_R F_R$ and $H_R H_R$. In either of these two sub-cases, type R take preventative action and also obtain more long-term care insurance cover than type r , which again yields a prediction that is consistent with the findings of Finkelstein and McGarry (2003).

Finally we note what happens in the above we introduce fixed administration costs alongside proportional costs. In the first case as these costs rise, analogous to the model with only fixed costs, they reach a level at which the r type drop out of the market. In the second case, as these costs rise, the first thing to happen is that the cutting point of the type r indifference curve (tangent to $F_r F_r$) with $F_R F_R$ to the left of T_r disappears and the equilibrium is the same as in the first case, with type r exiting the market when costs reach a certain threshold.

then preventative actions would be wasted as the level of coverage would be too low and they would have to do too much additional saving.

4 Bundling

In the above we assumed that individuals only have access to stand-alone long-term care and annuity contracts traded on separate markets. However, in the market place there exist financial instruments that are packages of long-term care insurance linked to a single premium deferred annuity. The idea is that some or all of the earnings on the annuity pay for long-term care. Thus an annuity that would normally yield $x\%$ might only yield $y\% < x\%$ when combined with long-term care insurance.¹⁶ The key contribution of this paper is to show that contracts that combine or bundle the two types of insurance can be supported as a separating equilibrium that Pareto dominates the equilibrium in stand-alone contracts. Moreover, separating equilibria will exist for a wider range of values than the population proportion parameter, α . This result arises because the selection effects in the stand-alone long-term care insurance and annuity markets work in opposite directions. An individual who takes care will be relatively low risk in the long-term care insurance market but high risk in the annuity market, with the opposite being the case for those who take less preventative action. The terms of bundled contracts are set so as to exploit this asymmetry and satisfy a single pair of incentive (self-selection) constraints.

Instead of separate markets for stand-alone contracts we now consider an alternative formulation in which there is only one competitive market for packages of insurance contracts that bundle annuities and long-term care insurance. Contract offers take the general form $W = (K, \tilde{Y}_2 + \tilde{\lambda}X)$, where K is a single premium. The game structure with these contracts is the same as in Section 3.1. In presenting the analysis we assume that the administration cost of bundled contracts, $k_B \geq 0$, is no more than the sum of the administration costs of the two stand-alone contracts ($k_B \leq k_T + k_Z$).

4.1 Separating Equilibrium

In presenting the basic analysis we assume that markets for stand-alone contracts are closed and show the existence of a separating equilibrium in bundled contracts. We then compare the efficiency properties of this equilibrium to the equilibrium with only stand-alone insurance contracts. Having done this we stand-alone insurance markets being open along-side the market for bundled contracts.

The bundled contracts for the two types are given by $W_i = (K_i, \tilde{Y}_{2i} + \tilde{\lambda}_i X)$, $i = R, r$. The care-incentive constraint is still given by (5), written as

$$\hat{H} \leq [q_H p_G - q_L p_B][U_i(\tilde{Y}_{2i}) - U_i(\tilde{Y}_{2i} - X + \tilde{\lambda}_i X)] + (q_H - q_L)U_i(\tilde{Y}_{2i} - X + \tilde{\lambda}_i X), \quad (34)$$

¹⁶One advantage of this arrangement is that long-term-care insurance premiums are paid with tax deferred earnings but since they are expensed inside the policy, premiums become tax free. Another advantage could be the perception that no money is lost to a long-term-care insurance policy that may never be used. A possible perceived disadvantage is that the money is tied up. Removing money will kill the long-term-care insurance.

which can only be satisfied for $\lambda_i < 1$. As before we focus on situations where Assumption 2 holds. Then if (34) is satisfied for type R , $q_R = q_H$; $q_r = q_L$; $p_R = p_G$ and $p_r = p_B$. The proposed pair of bundled contracts satisfy the respective profitability conditions, one for each contract

$$\Pi_{Bi} = K_i - q_i^*(\tilde{Y}_{2i} + (1 - p_i^*)\tilde{\lambda}_i X) - k_B \geq 0, i = R, r. \quad (35)$$

The two incentive-compatibility constraints for the two types are given by

$$V_R(W_R) \geq V_R(W_r), \quad (36)$$

and

$$V_r(W_r) \geq V_r(W_R). \quad (37)$$

Under complete information (we drop the incentive-compatibility conditions) the annuity payment on the bundled contract for each type, \tilde{Y}_{2i} , are given by $\tilde{Y}_{2i} = \bar{Y}_{2i}$, $i = R, r$, and the amounts of long-term care cover for each type, $\tilde{\lambda}_i$, are given by $\tilde{\lambda}_i = \bar{\lambda}_i = 1$. We solve the two equations in (35), subject to (34) being satisfied for type R , for the complete-information premia \bar{K}_R and \bar{K}_r . Then $\bar{K}_R - \bar{K}_r = (q_H \bar{Y}_{2R} - q_L \bar{Y}_{2r}) + (q_H - q_L)X - (q_H p_G - q_L p_B)X \leq 0$. There are two cases: (a) If $(q_H - q_L)$ is large relative to $(p_G - p_B)$ and $\bar{Y}_{2R} - \bar{Y}_{2r}$ is large relative to X , then $\bar{K}_R - \bar{K}_r > 0$; (b) If $(q_H - q_L)$ is small relative to $(p_G - p_B)$ and $\bar{Y}_{2R} - \bar{Y}_{2r}$ is small relative to X , then $\bar{K}_R - \bar{K}_r < 0$. Complete contracts denoted by (\bar{W}_R, \bar{W}_r) satisfy the following condition:

Condition 5. (i) If $\bar{K}_R - \bar{K}_r > 0$, then for the complete information contracts $V_R(\bar{W}_r) > V_R(\bar{W}_R)$. (ii) $\bar{K}_R - \bar{K}_r < 0$ then for the complete information contracts $V_r(\bar{W}_R) > V_r(\bar{W}_r)$.

Moreover, evaluating (3) for each type at the respective bundled contracts $(K_i, \tilde{Y}_{2i} + \tilde{\lambda}_i X)$, $i = R, r$, the trade-offs in both dimensions of the indifference surfaces are given by (6) and (8) and satisfy Condition 1 and 2 for the two types. We now show the following:

Proposition 5. There exists a separating equilibrium in bundled contracts. (i) If Condition 5.(i) applies and α is relatively high, then type R obtain maximum insurance subject to having the incentive to take preventative action, whilst the r type obtain incomplete long-term care cover and incomplete annuity cover. In this case, the burden of separation is in incomplete annuity cover. (ii) If Condition 5.(ii) applies and α is relatively low, then type r achieve complete insurance, whilst type R obtain incomplete annuity cover and incomplete long-term care insurance. But in this case the burden of separation is in incomplete long-term care insurance.

Proof. There are two cases to consider: (i) Given Condition 5.(i), the bundled contracts are chosen to maximise (3) for type r subject to (36), (37) and the two zero profit conditions

in (35). The solution gives maximum insurance for type R subject to (34) being satisfied at the contract W_R^* and partial coverage to type r . The contract for type R lies at the intersection of (34) and the zero-profit condition in (35) for type R . The contract W_r solves the following problem:

$$\max\{U_r(A - K_r) + q_L(p_B U_r(\tilde{Y}_{2r}) + (1 - p_B)U_r(\tilde{Y}_{2r} - X + \tilde{\lambda}_r X))\} \quad (38)$$

subject to

$$U_R(A - K_r) + q_H[p_G U_R(\tilde{Y}_{2r}) + (1 - p_G)U_R(\tilde{Y}_{2r} - X + \tilde{\lambda}_r X)] \leq V_R(W_R^*) \quad (39)$$

and

$$K_r = q_L \tilde{Y}_{2r} - q_L(1 - p_B)X + k_B \quad (40)$$

Let $\theta \geq 0$ and $\gamma \geq 0$ be the multipliers on the respective constraints. Then choose $\tilde{\lambda}_r$, \tilde{Y}_{2r} and K_r and the multipliers to solve the problem. The first-order conditions are:

$$\begin{aligned} q_L(1 - p_B)U'_r(\tilde{Y}_{2r} - X + \tilde{\lambda}_r X)X \\ -\theta q_H(1 - p_G)U'_R(\tilde{Y}_{2r} - X + \tilde{\lambda}_r X)X - \gamma q_L(1 - p_B)X = 0 \end{aligned} \quad (41)$$

$$\begin{aligned} q_L(p_B U'_r(\tilde{Y}_{2r}) + (1 - p_B)U'_r(\tilde{Y}_{2r} - X + \tilde{\lambda}_r X)) \\ +\theta[q_H[p_G U'_R(\tilde{Y}_{2r}) + (1 - p_G)U'_R(\tilde{Y}_{2r} - X + \tilde{\lambda}_r X)]] - \gamma q_L = 0 \end{aligned} \quad (42)$$

$$-U'_r(A - K_r) + \gamma = 0 \quad (43)$$

and the two constraints. To achieve separation (39) must bind, $\theta > 0$, then we see that there are distortions from the first-best (which applies when $\theta = 0$) in both (41) and (42). However, as we have assumed (Assumption 1) that $q_H(1 - p_G)$ is relatively small, the distortion in (41) is small relative to that in (42) in which the term $q_H p_G U'_R(\tilde{Y}_{2r})$ is relatively large. Thus although the contract for type r sets $\tilde{Y}_{2r} < \bar{Y}_{2r}$ and $\tilde{\lambda}_r < 1$, the burden of the deviation from first-best is in setting \tilde{Y}_{2r} low. This is because the cut in \tilde{Y}_{2r} has a relatively bigger impact on type R (the opposite is true for cuts in $\tilde{\lambda}_r$).

Finally, as α is high, the cross-subsidy in any profitable pooling contract with type R is high, so that type r prefer the separating contract.

(ii) Given Condition 5.(ii), the bundled contracts are chosen to maximise (3) for type R subject to (36), (37) and the two zero profit conditions in (35). The solution gives complete insurance to type r and partial coverage to type R .

The contract W_R solves

$$\max\{U_R(A - K_R) + q_H(p_G U_R(\tilde{Y}_{2R}) + (1 - p_G)U_R(\tilde{Y}_{2R} - X + \tilde{\lambda}_R X))\} \quad (44)$$

subject to

$$U_r(A - K_R) + q_H[p_G U_r(\tilde{Y}_{2R}) + (1 - p_G)U_r(\tilde{Y}_{2R} - X + \tilde{\lambda}_R X)] \leq V_r(W_r^*) \quad (45)$$

and

$$K_R = q_H \tilde{Y}_{2R} - q_H(1 - p_B)X \quad (46)$$

Let $\hat{\theta} \geq 0$ and $\hat{\gamma} \geq 0$ be the multipliers on the respective constraints. Then choose $\tilde{\lambda}_R$, \tilde{Y}_{2R} and K_R and the multipliers to solve the problem. The first-order conditions are:

$$\begin{aligned} & q_H(1 - p_G)U'_R(\tilde{Y}_{2R} - X + \tilde{\lambda}_R X)X - \\ & \hat{\theta}q_L(1 - p_B)U'_r(\tilde{Y}_{2R} - X + \tilde{\lambda}_R X)X - \hat{\gamma}q_H(1 - p_G)X = 0 \end{aligned} \quad (47)$$

$$\begin{aligned} & q_H(p_G U'_R(\tilde{Y}_{2R}) + (1 - p_G)U'_R(\tilde{Y}_{2R} - X + \tilde{\lambda}_R X)) \\ & + \hat{\theta}[q_L[p_B U'_r(\tilde{Y}_{2R}) + (1 - p_B)U'_r(\tilde{Y}_{2R} - X + \tilde{\lambda}_R X)]] - \hat{\gamma}q_H = 0 \end{aligned} \quad (48)$$

$$-U'_R(A - K_R) + \hat{\gamma} = 0 \quad (49)$$

and the two constraints. As $\hat{\theta} > 0$, then, we can see that there are distortions from the first-best in both (47) and (48). However, by Assumption 1, $q_L(1 - p_G)$ is relatively large, the distortion in (47) is small relative to that in (48) in which the term $q_L p_B U'_r(\tilde{Y}_{2R})$ is relatively small. Thus although the contract for type R sets $\tilde{Y}_{2R} < \bar{Y}_{2R}$ and $\tilde{\lambda}_R < 1$, the burden of the deviation from first-best is in setting $\tilde{\lambda}_R$ low. This is because the cut in $\tilde{\lambda}_R$ has a relatively bigger impact on type r (the opposite is true for cuts in \tilde{Y}_{2R}).

Finally, as α is high, the cross-subsidy in any profitable pooling contract with type R is high, so that type r prefer the separating contract. QED.

As we have argued, an individual who takes care will be relatively low risk in the long-term care insurance market but high risk in the annuity market, with the opposite being the case for those who take less preventative action. The terms of bundled contracts are set to exploit this asymmetry. On a particular dimension of the contract, if one type is high risk the other is low risk, so coverage is set accordingly. For example, if type R are the high demand for total insurance group, the contract offers a high level of annuity cover, which they value highly and type r value less. At the same time the contract offers less long-term care insurance that is of relatively less value to type R but this makes the contract less attractive to type r who have a higher demand for this element of insurance. This leads quite naturally to the next proposition, which also provides some further intuition for the above result.

Under stand-alone contracts, all the costs of separation in the long-term care insurance market are borne by type R by setting λ_R low, through the term $\mu q_L(1 - p_B)U'_r(Y_{2R}^* - X + \lambda_R X)$ in equation (20). In turn, all of the costs of separation in the annuity market

are borne by type r , by setting Y_{2r} low, determined by the term $\hat{\mu}[q_H[p_G U'_R(Y_{2r}) + (1 - p_G)U'_R(Y_{2r} - X + \lambda_{2r}^* X)]]$ in equation (26). The bundled contracts dominate the stand-alone contracts because they achieve separation through greater flexibility in the allocation of income between dates and states, in ways that are attractive to heterogeneous types. The next proposition presents a comparison of the allocation with bundled contracts to that with stand-alone contracts.

Proposition 6. The bundled pair of contracts, W_R, W_r , Pareto dominates the stand-alone contracts.

Proof. Consider the first case in Proposition 5. With the bundled contracts, the cost of separation is borne by setting $\tilde{Y}_{2r} < \bar{Y}_{2r}$ and $\tilde{\lambda}_r < 1$, with type R obtaining maximum insurance (subject to taking preventative action). Hence, type R is strictly better-off with the bundled contract than the pair of stand alone contracts (T_R^*, Z_R^*) , in which they obtain only partial long-term care insurance: $V_R(W_R) > V_R(Z_R|T_R^*) = V_R(T_R|Z_R^*)$. This means that the incentive constraint (39) is easier to satisfy than that in (24), where the right-hand-side is conditional upon incomplete long-term care insurance. Type r can achieve separation with a contract that places all of the burden on cutting \tilde{Y}_{2r} to a value below \bar{Y}_{2r} , but by less than in the equilibrium with stand-alone contracts. But in addition the optimal separating bundled contract shares the cost of separation with a cut in $\tilde{\lambda}_r$, so that \tilde{Y}_{2r} is cut by less (an option that is not available with the stand-alone contracts), further raising the utility of type r with the bundled contract. Thus this type must also be better-off.

Now consider the second case in Proposition 5. With the bundled contract type r obtain complete insurance and are thus better-off than with stand-alone contracts, which deliver incomplete annuity coverage. Hence, $V_r(W_r) > V_r(T_r|Z_R^*) = V_r(Z_r|T_r^*)$, so that (47) is easier to satisfy than (18), where the right-hand-side is conditional upon incomplete annuity coverage. Type R can achieve separation with a contract that places all of the burden on cutting $\tilde{\lambda}_R$, but by less than with the stand-alone contract. But there is an added gain in that the optimal separating contract shares the cost with a cut in \tilde{Y}_{2R} also and cuts $\tilde{\lambda}_R$ by less. Thus this type must also be better-off. QED.

The above argument shows that for a given value of α , separating equilibria are more likely with bundled contracts. That is, without a formal demonstration, for some range of values below the lower cut-off value of α for a separating equilibrium in the annuity market, there will be equilibrium separation with bundled contracts. Similarly, for some range of values of α , above the upper cut-off for separating equilibrium in the long-term care insurance market, there will be equilibrium separation in bundled contracts.

Finally let us consider the implications of allowing markets for stand-alone contracts to operate side-by-side with markets for bundled contracts. The idea is to test whether the equilibrium offers in Proposition 5 are stable with respect offers of stand-alone contracts.

Suppose that there is an equilibrium in the stand-alone market and consider the opening of the market for bundled contracts, then because of Proposition 6, bundled contract offers can be made that are profitable and cause each type to deviate from the respective stand-alone configuration. However, the opposite is not so straightforward.

Suppose that there is an equilibrium in bundled contracts and consider the possibility of stand-alone contracts being offered. Then we need to establish that an individual type cannot be induced to deviate to a pair of individually profitable offers of stand-alone contracts that destabilises the putative equilibrium in bundled contracts.

In presenting the following argument, we retain the three stage game structure used in the body of the paper. We assume that an individual of a particular type, in responding to the offer of a stand-alone contract in one market, anticipates the offer it will get in the other market and *visa versa*. Moreover, in making an offer of a contract in one market, to attract types to deviate from the bundled contract for its type, the insurance company anticipates the offer made simultaneously in the other market and predicts the resulting behaviour of the individual applicant. We can now demonstrate the following:

Proposition 7. The equilibrium in Proposition 5 is stable with respect to offers of stand-alone contracts.

Proof. We only consider the case in the part (ii) of Proposition 5, as essentially the same argument as below applies to part (i). In this case type r achieve complete insurance with the bundled contract and so will not deviate to a profitable offer in stand-alone contracts for their type. Thus we only need consider type R . Then in principle, insurance companies could offer this type a pair of stand-alone contracts that give it maximum insurance (subject to the care incentive constraint being satisfied) that are individually profitable if only taken by this type. However, given Condition 5.(ii), this configuration will attract type r , and so will be unprofitable and will not be offered. Hence, in this case offers of stand-alone insurance cover must offer type R less than maximum cover. The best pair of individually profitable stand-alone contracts for type R that will not attract type r , leaves type r indifferent between this pair of contracts and the bundled contract for their type, denoted by W_r^* . But if type r is indifferent, the incentive-compatibility constraint for type r in the market for bundled contracts is unchanged from the case where the stand-alone markets are closed. However, because the incentive compatibility constraint for type r with bundled contracts, (45), is easier to satisfy than the pair of constraints with stand-alone contracts, (13) and (15), type R will strictly prefer the bundled contract for their type to this pair of stand-alone contracts. Thus there does not exist a profitable pair of stand-alone contracts that attracts type R . QED.

Thus we see that if Proposition 7 holds, we do not need to propose banning stand-alone contracts to ensure that the desirable outcome in Propositions 5 and 6 obtains.

4.2 Pooling Equilibrium

A pooling equilibrium is also possible in bundled contracts. As in Section 3.3, this type of equilibrium is obtained if the pooling contract Pareto dominates the separating pair of contracts. Remember that in a pooling equilibrium, better types subsidise poorer types. But with bundled contracts there are two dimensions to type. The R type are lower risk on the long-term care dimension and higher risk on the longevity dimension, whereas for the r type the reverse is true.

Consider the putative separating equilibrium in bundled contracts in part (i) of Proposition 5, in which the burden of separation is borne by type r , through partial annuity coverage. Suppose that an insurance company was to offer a pooled contract, $W_p = \{\tilde{K}_p, \tilde{Y}_{p2} + \tilde{\lambda}_p X\}$, satisfying

$$\Pi_p = K_p - q^*[Y_{p2} + (1 - p^*)\lambda_p X] - k_B \geq 0, \quad (50)$$

where $q^* = \alpha q_H + (1 - \alpha)q_L$ and $p^* = \alpha p_G + (1 - \alpha)p_B$. This contract must satisfy

$$V_R(W_p) \geq V_R(W_R), \quad (51)$$

and

$$V_r(W_p) \geq V_r(W_r). \quad (52)$$

In this case these conditions are satisfied when α is below a lower threshold. Moreover, it can be shown that in this case the equilibrium contract, W_p , must maximise $V_r(W_p)$ subject to (50) set equal to zero and the constraint that type R take preventative action. The putative pooling equilibrium of course has to satisfy the same stability requirements as in Section 3.3. Without going into detail, it is necessary to show that the pooling offer is robust to offers of separate contracts aimed at skimming-off the low risk r type. Applying the same reasoning as in Section 3.3 we can show that it is. Moreover, although a little more intricate, we can show that the equilibrium is robust to offers of stand-alone contracts. Finally we note that the extent to which the pooling offer involves a cross-subsidy from type r to type R is restrained by the extent of the cross-subsidy from the R type to the r type on the long-term care part of the bundled contract.

If on the other hand the burden of separation is taken by type R , through lower long-term care coverage, then the equilibrium contract, W_p , must maximise $V_R(W_p)$ subject to (50) and the constraint that type R take preventative action. Then the extent to which the pooling offer involves a cross-subsidy from the R type to the r type is restrained by the extent of the cross-subsidy from the r type to the R type on the annuity part of the bundled contract.

5 Social Efficiency

If the administration cost of bundled contracts is no more than the sum of the administration costs of the two stand-alone contracts ($k_B \leq k_T + k_Z$), the incidence of any administration costs will be no higher than in the case of stand-alone contracts if both types participate. Moreover, if type r are on the margin of participation in the long-term care insurance market with stand-alone contracts but certainly participates in the stand-alone annuity market, they will definitely participate in the market with bundled contracts. Thus combining the two types of insurance has the potential to reduce the cost of insurance relative to the stand-alone contracts and make insurance affordable to more potential buyers. However, although these contracts are marketed, currently in both the United States and the United Kingdom, they make up less than 10 percent of the voluntary annuity market.

Phillopson and Becker (1998) point to the moral hazard problem in providing mortality contingent claims as limiting the market. However, in the current model there is limited scope for this effect. The failure of the market to supply the bundled contracts may though be because of complex intergenerational gaming of the type discussed by Pauly (1990) in which parents may not buy this type of insurance, so as to ensure help from their children who see their bequest being dependent upon such help. Alternatively, the market may be limited because individuals are concerned about the ‘illiquidity’ of annuities, which may be relevant if they are undecided about, amongst other things, the bequest they wish to leave.¹⁷

Given Proposition 6, the problem is to understand why bundled contracts are not more prevalent, rather than invoking government intervention. Proposition 7 means that we do not advocate the banning of stand-alone contracts as a part of a programme of encouraging the development of the market for bundled contracts. However, if the market has not developed because of costs, the argument stemming from the paper is then that retirement programmes should not be designed in isolation from long-term care insurance programmes. Chen (1994), for example, has argued that social security benefits could include a basic long-term care benefit. He argues that the State might develop markets for bundled contracts to take pressure off medicare, or State disability benefit.

The issue of the optimal, welfare-maximizing, levels of mandatory annuitisation of retirement wealth and long-term care insurance is replete with problems of implementation. On the other hand, it is worth remarking that government transfers such as means tested benefit and public provision of long-term care may provide incentives for some individuals to draw down their savings early and to take less preventative action than otherwise.

¹⁷The market might fail to supply the bundled contracts because underwriters find it difficult to hedge longevity risk and the risk of rising long-term care costs. However, this argument applies equally to the stand-alone contracts.

6 Conclusion

This paper has shown that observed behaviour in annuity and long-term care insurance markets can be explained by a model that has heterogeneous risk aversion and endogenous preventative action. The Finkelstein and McGarry (2003) finding of a positive correlation between preventative action and demand for long-term care insurance and the Finkelstein and Poterba (2003, 2004) findings on annuity markets are explained as simultaneous separating equilibria in the two markets. Administration costs in the long-term care insurance market and the interaction of differences risk aversion and the choices of preventative action explain any non-participation by the less risk-averse-reckless type. A key finding of the paper is that as the selection effects for the two types in the markets for stand-alone contracts work in opposite directions, with an individual who is high risk in one market being low risk in the other, there exists the possibility of a separating equilibrium in a single market for bundled contracts. Moreover, we showed that this equilibrium allocation Pareto dominates the stand-alone contracts allocations. Pooling equilibria are still a possibility but are less likely.

The bundled contracts expose insurance providers to two types of risk, namely longevity risk and to possible increases in the cost of long-term care. These contracts are available in the voluntary annuity market. In the light of Proposition 6 in this paper and the findings of Murtaugh, Spillman and Warshawsky (2001) noted in the introduction, the puzzle is that the market for bundled insurance contracts is so thin.

7 References

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Bad state consumption

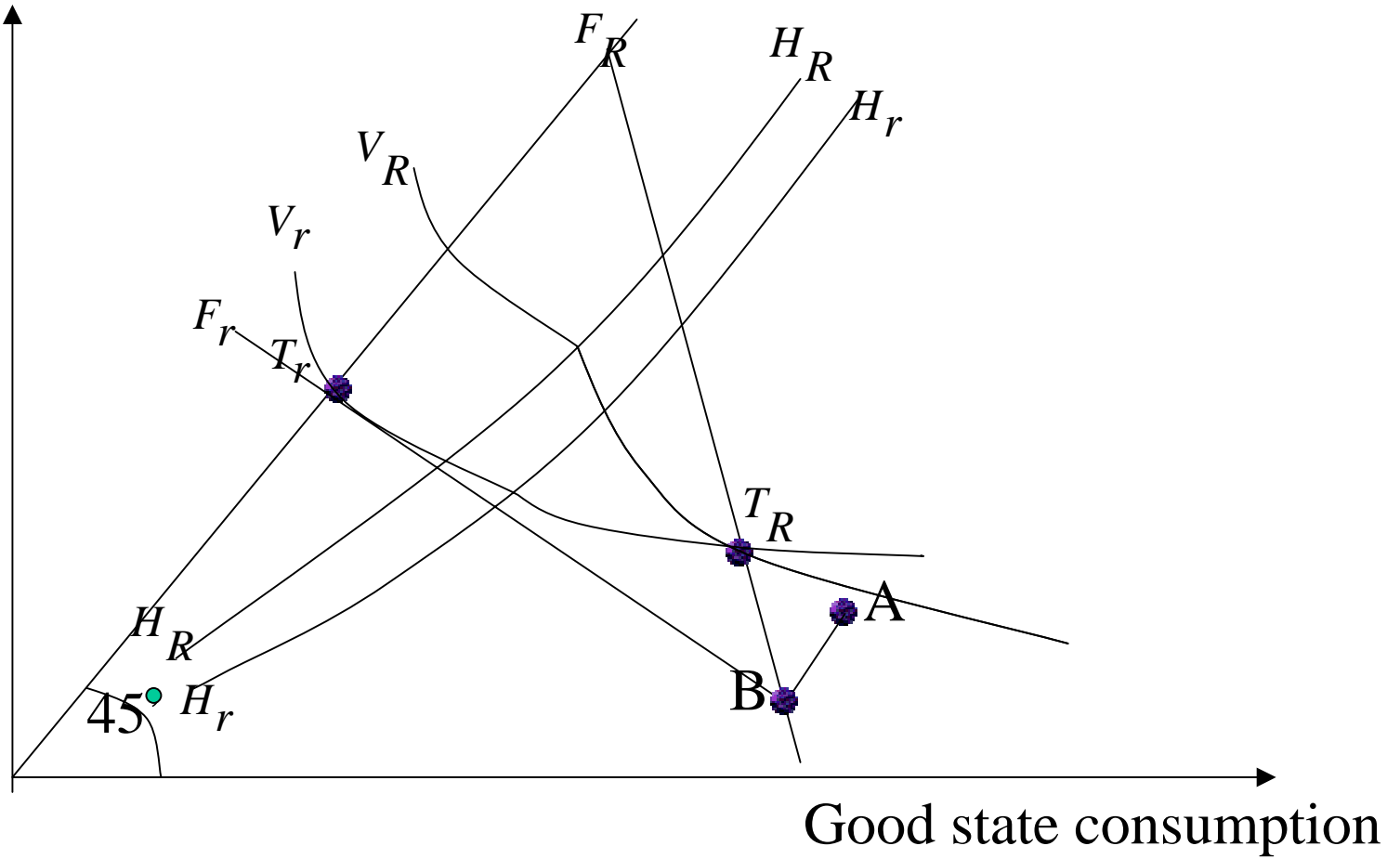


Figure 1

Bad state consumption

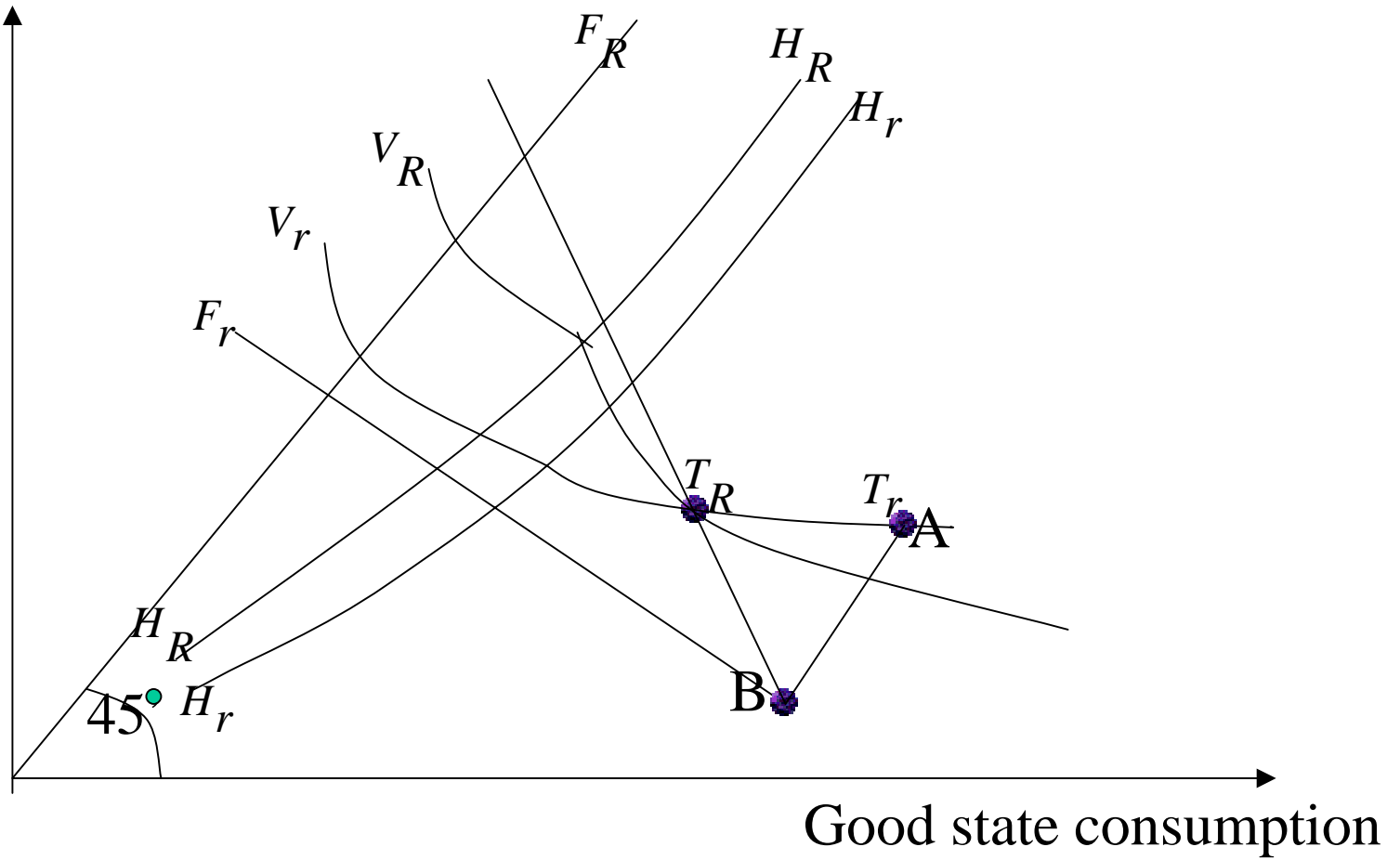


Figure 2

Bad state consumption

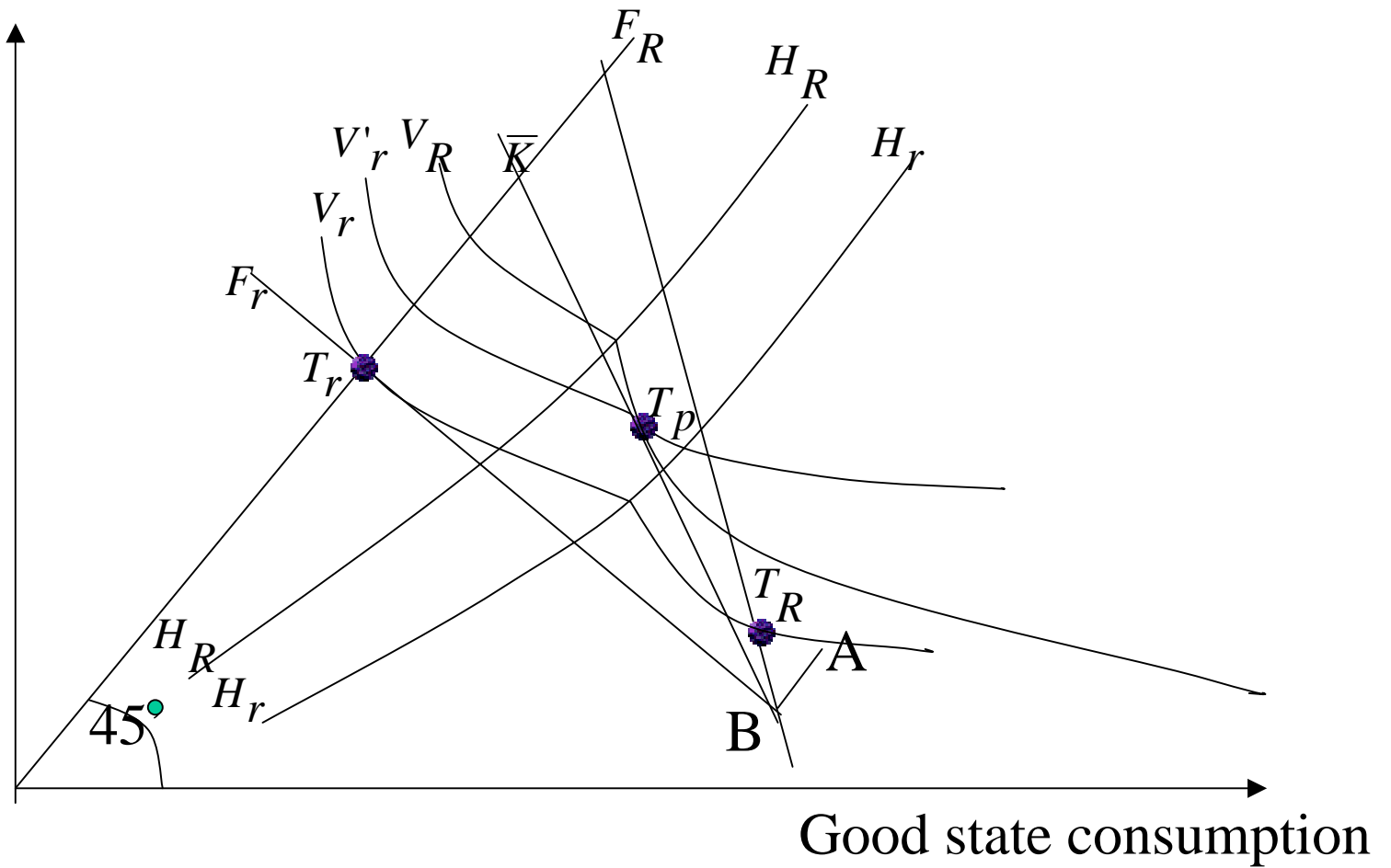


Figure 3