

Spot Market Power and Futures Market Trading

Alexander Muermann and Stephen H. Shore*

The Wharton School, University of Pennsylvania

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Abstract

When a spot market monopolist participates in the futures market, he has an incentive to adjust spot prices to make his futures market position more profitable. Rational futures market makers take this into account when they set prices. Spot market power thus creates a moral hazard problem which parallels the adverse selection problem in models with inside information. This moral hazard not only reduces the optimal amount of hedging for those with and without market power, but also makes complete hedging impossible. When market makers cannot distinguish orders placed by those with and without market power, market power provides a venue for strategic trading and market manipulation. The monopolist will strategically randomize his futures market position and then use his market power to make this position profitable. Furthermore, traders without market power can manipulate futures prices by hiding their orders behind the monopolist's strategic trades.

1 Introduction

For many goods, spot markets with market power coexist with competitive futures markets. When a spot market monopolist participates in a futures market, this participation leads to a moral hazard problem in the spot market. In particular, he has an incentive to deviate from the monopoly optimum in order to make his futures market position more profitable. For example, if a monopolist producer of oil holds a short position in an oil futures contract, he will profit if the price of oil goes down. This gives him an incentive to produce more oil than he otherwise might in order to reduce spot market prices and make his futures position more profitable. When rational futures market participants observe the monopolist's position, they will take it

*Muermann and Shore: The Wharton School of the University of Pennsylvania, 3620 Locust Walk, Philadelphia, PA 19104-6218, USA, email: muermann@wharton.upenn.edu - sshore@wharton.upenn.edu. We wish to thank Glen Taksler whose collaboration on previous work lead to deeper insights. We received valuable comments from Christian Laux, Volker Nocke, Matt White, and seminar participants at the Financial Markets Group of the LSE, the University of Frankfurt, Wharton, and the NBER. Muermann and Shore gratefully acknowledge financial support from the Risk Management and Decision Processes Center at The Wharton School.

into account when setting futures prices. When they cannot observe the monopolist's position perfectly, they take his possible presence into account when setting prices.

In this paper, we explore the impact of goods market power on financial market participation, both for those with and without market power. We examine three rationales of trading in a futures market: hedging, strategic trading, and manipulation. We show that spot market power reduces the incentive of all agents to participate in futures markets to hedge risks. However, spot market power provides the monopolist with an incentive to trade strategically – randomly taking a position in the futures market and then moving spot prices to make that position profitable. This allows the possibility that those without market power may engage in futures market manipulation – taking a position in a derivatives market and then mimicking the monopolist's futures trading to move futures market prices to make the derivatives position profitable.

The literature on market microstructure deals extensively with the effects of adverse selection when some agents have inside information. This paper will argue that the moral hazard created by spot market power will have parallel effects. As with inside information, market power deters financial market participation by those with hedging motives but provides a venue for strategic trading and market manipulation.

Section 2 relates to the hedging motive of the monopolist. We show that hedging is expensive for those with monopoly power. Monopolists may have an incentive to participate in futures market to avoid the cost of financial distress or agency problems, or because an entire economy may depend on the profitability of the monopolist. If the monopolist reduces risk by selling future production forward or buying a futures contract, he now has an incentive to increase production in the future. When he sells future production forward, he has an incentive to increase production since doing so does not reduce the price of previously sold units. When he buys a futures contract which pays off when prices are low, he has an incentive to increase production to make the futures contract more profitable. Taking this into account, futures or forward market participants will rationally set prices that are unfavorable to the monopolist. In effect, futures markets provide a venue for competition of the monopolist with his future self. Section 2 recasts the durable goods monopoly problem of Coase (1972) in terms of futures markets instead of durability.

Anderson and Sundaresan (1984) studied a similar question to the one considered in Section 2 by focusing on whether futures trading can exist in a rational expectations equilibrium under spot market power. We obtain a result equivalent to theirs: a risk-neutral monopolist does not participate in the futures market whereas a risk-averse monopolist participates in the futures market and his participation is increasing in the degree of risk aversion. While some of the results of Anderson and Sundaresan are based on second-order approximations and on futures contracts, we are able to prove these results more generally and for general derivatives contracts. A novel insight in this section is that it is impossible to completely eliminate all risk. This holds for any degree of risk aversion and for any price-contingent derivatives contract. In addition, we

show that large amounts of hedging are state-wise dominated. Just as “no trade” theorems (Milgrom and Stokey (1982)) show that there is no price at which informed agents can trade in financial markets, we prove a “no complete hedging” theorem that shows that there is no price at which agents with spot market power can eliminate all spot market price risk.

Anderson (1990) surveys the literature on futures trading when the underlying market is imperfectly competitive and suggests in his conclusion:

“The theoretical development that would be most interesting would be to reconsider some of these models described above under conditions of asymmetric information. In particular, the models reviewed have made the assumption (at least implicitly) that the futures positions of powerful agents are observed so that forecasts of future cash prices can take this into account. In practice, futures positions of agents are likely to be imperfectly observable.” (p. 246-247)

In Sections 3 and 4 of this paper, we follow exactly that route and explore strategic trading and manipulation in futures markets when market positions cannot be perfectly inferred. In Section 3, we show that the monopolist is able to strategically exploit his spot market power. If the spot market monopolist is able to hide within the futures market aggregate order flow, he will randomly participate in the futures market and then set spot prices to make his futures market position more profitable. This makes hedging more expensive for those who may be the monopolist’s counterparty. Spot market power thus discourages futures market participation for agents without market power and provides a venue for a spot market monopolist to increase profits by trading strategically.

This section shows that results similar to Kyle’s (1985) “noise trader” model are obtained when there is spot market power instead of inside information. In our model, there are no informed traders who have private information about future prices at the time of trading. However, after taking a position in the futures market, the monopolist has the market power to set spot market prices, thereby making his futures market position more profitable. Market power thus creates a moral hazard problem in our model, whereas private information leads to an adverse selection problem in the Kyle model. Note that the monopolist is only able to exploit his position strategically if it cannot be inferred perfectly from the aggregate order flow. In our model, agents without market power respond optimally to the monopolist’s futures market presence by reducing their futures market participation (see Spiegel and Subrahmanyam (1992) for the analogous extension of the Kyle model).

Sections 2 and 3 integrate the durable goods monopoly problem of Coase from the industrial organization literature and the “noise trader” model of Kyle from the market microstructure literature to show that spot market power discourages futures market participation for participants with and without market power.

In Section 4, we show that traders are able to move (i.e. “manipulate”) futures prices even when they do not have market power. If a futures market manipulator without market power takes a position in the derivatives market, he has an incentive to trade and thereby move future underlying prices to make his initial position profitable. He will be successful in moving prices if the market believes that his subsequent trades could have been submitted by the monopolist. While these subsequent trades are unprofitable, this cost is outweighed by benefit of moving prices to make the initial position more profitable.

Section 4 relates to the literature on manipulation in capital markets (e.g. Hart (1977), Jarrow (1992), Allen and Gale (1992), Kumar and Seppi (1992)). Compared with this literature, the novelty of our model is that it does not require that agents have private information about prices; instead, we show that market power serves a similar function. For example, Kumar and Seppi develop a model in which uninformed manipulators are able to profit in the futures market by manipulating spot market prices. While they have no inside information, they are able to move spot prices because spot market makers are unable to differentiate the uninformed manipulator’s order flow from the informed trader’s order flow. This model requires the potential presence of informed traders, while our model relies on the presence of traders with market power to serve the same function.

In summary, spot market power reduces agents’ incentive to participate in futures markets. For those with market power, hedging is expensive because its price takes into account the impact of the futures market position on spot market prices. For those without market power, the presence of market power makes futures market participation more expensive by introducing the possibility of strategic trading or manipulation. As a result, we would predict that futures markets with underlying monopoly spot markets would be relatively small and illiquid when compared to the spot market.¹

2 Hedging by Spot Market Monopolists

In this section, we consider the impact of spot market power on a monopolist’s ability to hedge spot market price risk. If a spot market monopolist is risk-averse, he has an incentive to hedge against fluctuations in profits. In a world with demand shocks - when high profits coincide with high prices - the payoff of a hedging contract will be negatively related to the underlying spot market price. Entering into such a contract ex-ante creates moral hazard. The monopolist has an incentive to increase production, thereby reducing spot market prices to make the hedging contract more profitable. This moral hazard problem increases the cost of the hedging contract and reduces monopoly power. The monopolist faces a trade-off

¹Our model excludes the possibility that the good may be stored. Over time horizons short enough that storage is cost-effective, substantial futures markets may exist as storage will limit the monopolist’s power to move prices. Of course, storage will also erode monopoly power.

between hedging risk and maintaining monopoly profits. This section will show that this effect reduces the optimal quantity of hedging; a risk-neutral monopolist will not want to hedge risk in the financial markets, while a risk-averse one will hedge less than he would if he had no market power. We will show that as the degree of risk-aversion increases, the optimal amount of hedging will increase. Relative to other research (e.g. Anderson and Sundaresan (1984)), this section is innovative because it shows that complete hedging is impossible and large amounts of hedging can be state-wise dominated.

The impact of futures market in reducing the amount of monopoly power can be interpreted in light of the durable goods monopoly of Coase (1972). In a Coasian setting, the durability of the good provides a venue for the monopolist effectively to compete with his future self. The monopolist has an incentive to produce sooner rather than later if he has a positive discount rate or if consumers prefer to own the good sooner rather than later. If the monopolist cannot commit not to produce additional units in the next period, his production of the durable good today competes with his production of the good tomorrow. This reduces his monopoly power. In our setting, futures markets serve the same role as durability. The monopolist has an incentive to sell production forward or short a futures contract if he is risk-averse. If he cannot commit to condition production on realized demand, his participation in the forward market - selling tomorrow's production today - creates competition with his sales in tomorrow's spot market.

2.1 Model Setup

We envision a model with one good and two periods, $t = 0, 1$. A monopolist with utility function u is the sole producer of the good, and the good is produced only in the last period. The monopolist chooses quantity, Q , to maximize his profits. Demand is uncertain and realizes in between the two periods. The demand curve is given by $P = f(Q, D)$ where D is the realization of demand. The cost of production is normalized to zero. In the initial period, the monopolist can enter into a derivatives contract. The payoff of this contract could be any function of the spot price, $g(P) - k$, where k is the strike price.² Given a competitive derivatives market with risk-neutral market makers, the price of the derivatives contract must be equal to the expected payoff of the contract, i.e. $k = E[g(P)]$. The monopolist's profits are then

$$\pi = C(g(P) - E[g(P)]) + QP, \tag{1}$$

²If the derivatives contract is a linear future, so that $g(P) = P$, then this problem is equivalent to one in which the monopolist could sell $t = 1$ production forward at $t = 0$.

where C is the number of derivatives contract bought by the monopolist in the first period.³ In the last period, after demand is realized, the monopolist chooses a quantity, Q , to maximize profits, i.e.

$$Q^* = \arg \max_Q (C(g(P) - k) + QP).$$

The spot market first-order condition (FOC) is given by

$$\begin{aligned} \frac{\partial \pi}{\partial Q} &= \frac{\partial}{\partial Q} (C(g(f(Q, D)) - k) + Qf(Q, D)) \\ &= (Cg' + Q)f_1 + f = 0, \end{aligned} \quad (2)$$

where we assume that the second order condition is satisfied. In the initial period, the monopolist will set C to satisfy the derivatives market FOC,

$$\frac{\partial E[u(\pi)]}{\partial C} = E \left[\frac{\partial \pi}{\partial C} u'(\pi) \right] = 0, \quad (3)$$

where

$$\frac{d\pi}{dC} = g - E[g] + C \left(\frac{dQ}{dC} f_1 g' - E \left[\frac{dQ}{dC} f_1 g' \right] \right) + \frac{dQ}{dC} f + Q \frac{dQ}{dC} f_1. \quad (4)$$

Substituting the spot market FOC (2) into (4) yields:

$$\frac{d\pi}{dC} = g - E[g] - CE \left[\frac{dQ}{dC} f_1 g' \right]. \quad (5)$$

Given that the spot market quantity, Q^* , is chosen optimally, the derivatives market FOC is found by substituting (5) into (3):

$$\frac{\partial E[u(\pi)]}{\partial C} = E \left[\left(g - E[g] - CE \left[\frac{dQ}{dC} f_1 g' \right] \right) \cdot u'(\pi) \right] = 0 \quad (6)$$

where we can set $E[g(P)] = 0$ without loss of generality. Market power introduces the $E \left[\frac{dQ}{dC} f_1 g' \right]$ term into this FOC. This has an important impact on the optimal amount of hedging, C^* . When setting C , the monopolist takes into account that hedging will induce him to deviate from the spot market optimum, $\frac{dQ}{dC}$, that this deviation will change spot prices, f_1 , and that this change in spot prices will change the payoff of the derivatives contract, g' . Importantly, he takes into account that market makers know this when setting prices.

³In this section, we assume that C is perfectly observable by all market participants. In sections 3 and 4, we relax this assumption which gives room for strategic trading and manipulation in the derivatives market.

In the following subsections, we will examine optimal monopolist behavior for varying degrees of risk aversion. Risk-neutral monopolists will not want to hedge price risk (Proposition 1). Risk-averse monopolists will hedge completely only if derivatives contracts can be conditioned on the realization of demand (Proposition 2). Otherwise, risk-averse monopolists will hedge some but not all of their risk (Proposition 3). Furthermore, the amount of hedging will be increasing in the degree of risk aversion (Proposition 4). However, even an infinitely risk-averse monopolist will not find it optimal to eliminate risk completely as doing so is state-wise dominated (Proposition 5).

2.2 Risk-Neutral Monopolists Don't Hedge

Proposition 1 *A risk-neutral monopolist does not participate in the derivatives market, i.e. $C^* = 0$, and sells the monopoly quantity at the monopoly price in the spot market.*

Proof. See Appendix A.1. ■

The monopolist faces a trade-off between reducing risk and maintaining monopoly profits. A risk-neutral monopolist does not value risk reduction. It is therefore optimal for him to maintain full monopoly profits by not participating in the derivatives market. Now we examine the behavior of a risk-averse monopolist. Risk aversion provides an incentive for the monopolist to reduce risk. If the monopolist can buy a derivatives contract with a payoff contingent on realized demand, all risk can be eliminated without giving up monopoly profits. However, when demand-contingent derivatives contracts are not feasible, the monopolist will reduce but not completely eliminate his exposure to risk and will give up some but not all of his monopoly profits.

2.3 Risk-Averse Monopolists with Commitment

Proposition 2 *If a derivatives contract can be made contingent on realized demand and if any such contract can be written, then a risk-averse monopolist will eliminate all risk and maintain full monopoly profits in the spot market.*

Proof. See Appendix A.2. ■

A demand-contingent contract is ideal for the monopolist as it allows him to completely eliminate risk while maintaining monopoly profits. The optimal demand-contingent contract will pay off most in states where demand is lowest. These are the states with the lowest equilibrium prices. Since contract payoffs are contingent on realized demand and not prices, the monopolist has no incentive to change prices from the monopoly optimum since doing so has no impact on the derivative contract's payoff. Note that the monopolist's ability to eliminate all risk while maintaining market power depends critically on the verifiability

of demand – on existence of a contractible proxy for risk that the monopolist cannot change. However, in the real world, demand may not be observable. Even when demand is observable, it is almost certainly not verifiable. One could get around that problem by writing a contract contingent on both price and quantity. However, if quantity is not verifiable, a price-contingent contract may be the only mechanism available for a monopolist to reduce risk. The next subsection will show that using this mechanism comes at the cost of reduced market power.⁴

2.4 Risk Averse Monopolists without Commitment

Buying a derivatives contract allows the monopolist to transfer wealth from high-price states to low-price states. For such a contract to be useful in reducing risk, spot prices must be correlated with good or bad states. However, doing so comes at the cost of reduced monopoly profits. In the rest of this section, we assume for compactness that the monopolist faces demand shocks. Put another way, good states have both high prices and high profits while bad states have both low prices and low profits.⁵

Proposition 3 *A risk-averse monopolist will participate in the derivatives market and will give up monopoly profits to do so. If the payoff of the derivatives contract is increasing (decreasing) in the spot price, the monopolist will go short (long), i.e. $C^* < 0$ (i.e. $C^* > 0$). He will sell a larger total quantity in the spot market, receive a lower price, and make lower expected profits than if he did not participate in the derivatives market.*

Proof. See Appendix A.3. ■

When the monopolist takes a position in the derivatives market, this gives him an incentive to deviate from the spot market optimum to make that position more profitable. This reduces his expected spot market profits. If the derivatives market is competitive and if the monopolist's derivatives market position is perfectly observable, then market makers will set prices so that the monopolist earns zero expected profit in the derivatives market. Using the derivatives market to reduce risk is costly since it reduces expected spot market profits while expected derivatives market profits are always zero. However, these costs are small when derivatives market participation is limited. As a result, a risk-averse monopolist will find it optimal to use the derivatives market to reduce some but not all risk, and will face reduced expected profits.

⁴The durable goods monopoly literature considers similar issues. Bulow (1982) shows that a monopolistic producer of a durable good could avoid the durable goods monopoly problem by leasing the good. Stokey (1981), Gul, Sonnenschein, and Wilson (1986), and Ausubel and Deneckere, (1989) examine the durable goods monopoly problem in an infinite-period setting. If the monopolist is not able to build up reputation, then the price of the durable good drops to marginal cost. However, by building up reputation the monopolist may be able to recover part of his monopoly power. Allaz and Vila (1993) show that the price in a Cournot duopoly without uncertainty but with access to a forward market converges to the competitive price as the number trading periods increases. In our setting, we consider a two-period model and therefore do not allow for multiple trading in the futures market.

⁵Note that this assumption rules out supply shocks where increased prices correspond to lower quantity and lower monopoly profits. Parallel results obtain when the monopolist faces supply shocks.

2.4.1 Hedging Increases with Risk Aversion

Not only do risk-averse monopolists participate in the derivatives market, but the degree of futures market participation increases with the degree of risk aversion.

Proposition 4 *The more risk-averse a monopolist is, the more derivatives contracts he will go long (short) if the payoff of the derivatives contract is increasing (decreasing) in the spot price.*

Proof. See Appendix A.4. ■

The monopolist faces a trade-off between reducing risk and maintaining monopoly profits. As the monopolist becomes more risk averse, risk reduction becomes relatively more important. Since the cost of reducing risk – the deviation from the spot market monopoly optimum – is increasing in the amount of risk reduction, a more risk-averse monopolist will be willing to reduce risk to a degree that a less risk-averse monopolist would find too costly.

2.4.2 Complete Hedging is Impossible

We have shown that the amount of hedging increases with the degree of risk aversion. While one might think that an infinitely risk-averse monopolist would eliminate all risk at any cost, this subsection will document that doing so is impossible. Complete hedging is impossible even when the monopolist can choose between any price-contingent derivatives contract. Furthermore, large amounts of hedging are state-wise dominated and therefore not optimal for any monopolist, regardless of their degree of risk aversion.

Proposition 5 *There exists no price-contingent derivatives contract which can eliminate all profit risk.*

Proof. See Appendix A.5. ■

Any price-contingent contract that eliminates all risk must pay out relatively more when prices indicate a “bad” state, exactly offsetting any reduced spot market profits in these states. As a result, the monopolist will always have an incentive to set prices as if a “bad” state had occurred, even in “good” states. Since spot market profits are higher in better states – holding prices fixed – and derivatives contract payoffs must be the same in all states with the same price, the monopolist will earn higher profits in “good” states. There is no incentive compatible contract that eliminates all risk. This “no complete hedging” theorem draws an analog to the “no trade” theorems of Milgrom and Stokey (1982) and others. Just as our theorem shows that complete hedging is impossible at any price for spot market monopolists, these theorems show that agents with inside information will be unable to trade at any price.

As a simple illustration of this idea, we consider a linear demand function with a binomial demand shock. Since there are only two states of the world, a linear futures contract is sufficient to span the set of

price-contingent contracts. Demand is given by

$$P = a - bQ, \quad (7)$$

where a takes values $a^H > a^L$ with probabilities p^H and $p^L = 1 - p^H$ respectively. The payoff of each futures contract is $g(P) = P - E[P|C]$. Plugging (7) into (1), the monopolist's profits are then

$$\pi = C(a - bQ - E[a - bQ|C]) + Q(a - bQ). \quad (8)$$

Evaluating (2) with (7), the spot market FOC is

$$\frac{\partial \pi}{\partial Q} = -Cb + a - 2bQ = 0,$$

which implies

$$Q^* = \frac{a - bC}{2b}; P^* = \frac{a + bC}{2}. \quad (9)$$

Therefore, lifetime profits given optimal spot market production can be found by substituting (9) into (8):

$$\pi = C\left(\frac{a}{2} - E\left[\frac{a}{2}\right]\right) + \frac{a^2 - b^2C^2}{4b}. \quad (10)$$

Note, that $-\frac{a^L}{b} < C < \frac{a^L}{b}$ to obtain positive prices and quantities in both states. A quantity C can be chosen to maximize profits in a given state of the world. An infinitely risk-averse monopolist will maximize profits in the worst state and thus set

$$C = -p^H \frac{a^H - a^L}{b}.^6$$

An infinitely risk-seeking monopolist will maximize profits in the best state and therefore choose

$$C = (1 - p^H) \frac{a^H - a^L}{b},^7$$

while, as shown above, a risk-neutral monopolist sets $C = 0$. While any $C \in \left(-\frac{a^L}{b}, \frac{a^L}{b}\right)$ is feasible, any $C \notin \left[-p^H \frac{a^H - a^L}{b}, (1 - p^H) \frac{a^H - a^L}{b}\right]$ is state-wise dominated. Outside of this range the monopolist can increase profits in both states of the world by bringing C closer to zero.

⁶This value is only feasible if $(1 + p^H) a^L > p^H a^H$. Otherwise, the monopolist will choose the lowest feasible level, $-\frac{a^L}{b}$.

⁷This value is only feasible if $(1 - p^H) a^H < (2 - p^H) a^L$. Otherwise, the monopolist will choose the highest feasible level, $\frac{a^L}{b}$.

Figure 1 plots profits in the “bad” L -state on the y-axis against profits in the “good” H -state on the x-axis for the case when $b = 1$, $a^H = 2$, $a^L = 1$, and $p^H = p^L = 0.5$. Any point on the 45° line represents the total elimination of risk. The dashed line represents the wealth levels in the two states if the monopolist can write state-contingent contracts. The slope of this line is the rate at which he can transfer wealth from the bad state to the good state; it is constant and equal to $-\frac{p^H}{1-p^H}$. When only price-contingent contracts are possible the rate at which wealth can be transferred depends upon the amount of wealth transferred between the two states. Put another way, the marginal cost of additional hedging increases with the amount being hedged; it becomes infinite well before all risk is eliminated. This feature is represented by the curved line. Note that any hedging quantity between -1 and $+1$ is feasible. However, only hedging quantities between $-\frac{1}{2}$ and $\frac{1}{2}$ are not state-wise dominated. An infinitely risk-averse monopolist would thus choose $C = -\frac{1}{2}$, an infinitely risk-seeking monopolist would choose $C = \frac{1}{2}$, and a risk-neutral monopolist would choose $C = 0$.

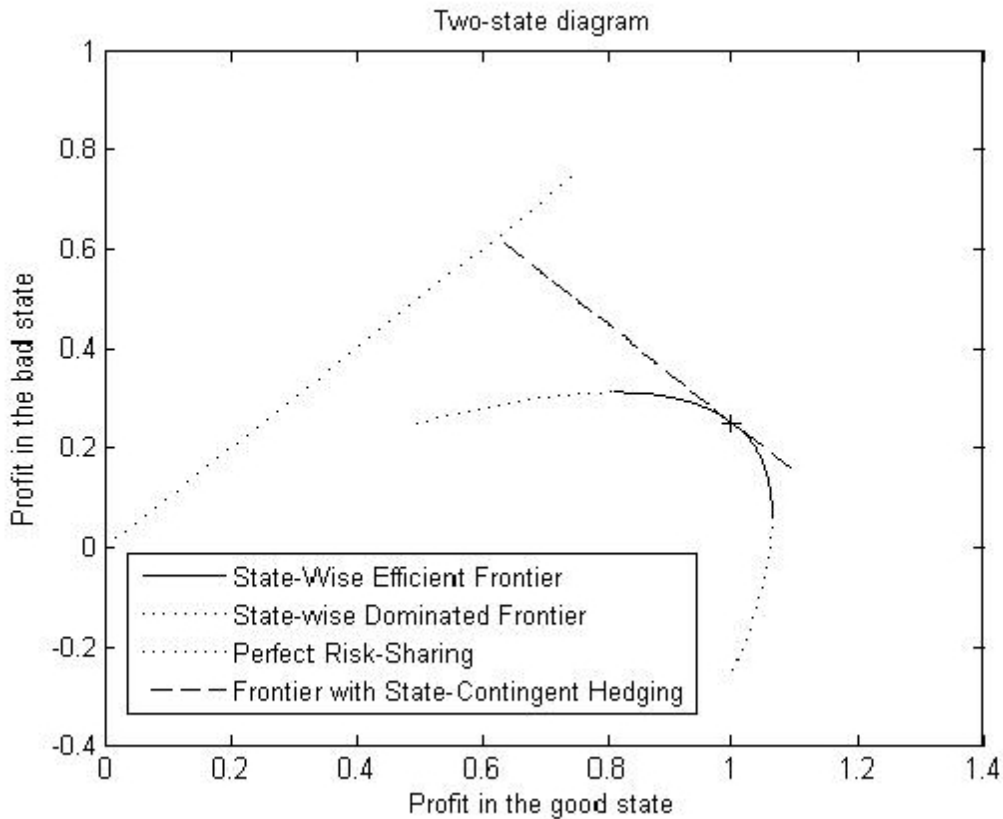


Figure 1

In this section, we documented that a spot market monopolist faces a trade-off between reducing risk through a futures market and maintaining monopoly profits. Since reducing risk through the futures market creates a moral hazard problem in the spot market, the monopolist will find it optimal to hedge only some of his risk. Furthermore, eliminating all risk is impossible.

3 Strategic Trading by Spot Market Monopolists

The last section documented that spot market power reduces the ability of a monopolist to hedge price risk. In this section, we show how spot market monopolists may trade strategically in the futures market to exploit their market power even when they are risk-neutral. This trade will discourage futures market participation by others. Agents who want to participate in the futures market fear that the monopolist may be their counterparty or the counterparty of someone with a similar position in the futures market. The monopolist will exert spot market power to make his futures position more profitable, thereby reducing the profitability of his counterparties.⁸

This section builds on the work of Kyle (1985) who shows that agents with inside information can profitably exploit their informational advantage by hiding behind the order flow of uninformed “noise traders”. In our model, the aggregate hedging demand of agents without market power is stochastic just as the number of “noise traders” is stochastic in the Kyle model. Observing only aggregate order flow, market makers cannot perfectly determine the monopolist’s futures market position – just as market makers cannot observe the orders placed by informed traders in the Kyle model.

The monopolist can increase his profits because market makers cannot take the impact of the monopolist’s futures market position on expected spot prices fully into account when setting futures market prices. While the monopolist’s expected spot market profit is reduced by deviating from the monopoly optimum, his expected profit in the futures market more than makes up for it. Since market makers earn zero expected profits, the monopolist’s expected futures market profits imply expected futures market losses for other market participants. When other agents participate in the futures market, they receive unfavorable prices since market makers believe that the order flow they generate could have come from the monopolist. This increased cost deters these agents from hedging price risk as much as they otherwise might.

Unlike the “noise traders” in the Kyle model who act mechanically, the agents in our model respond optimally to the presence of the monopolist in the futures market (see Spiegel and Subrahmanyam (1992) for the analogous extension of the Kyle model). It is also important to note that in our model, unlike in the Kyle model, there is no private information at the time of trading. In our model, spot market power serves the role performed by inside information in the Kyle model. In the Kyle model, informed agents hide behind aggregate order flow to take financial market positions consistent with their inside information; in our model, monopolists hide behind aggregate order flow by submitting random financial market positions

⁸Storage may reduce the ability of the monopolist to trade strategically. When storage is inexpensive, agents without market power may purchase and store the good in anticipation of higher prices in the future. This limits the ability of the monopolist to raise prices, as excess capacity will prevent prices from increasing. In this sense, storage is like durability in Coase (1972) in that it provides competition for the monopolist. Here, we assume that storage costs are high enough that no storage takes place in equilibrium and that monopoly power is not eroded.

and then exert market power to make these positions profitable.

3.1 Model Setup

There are three types of agents in this market. Two types of agents are identical to the ones described in Section 2. First, there is a spot market monopolist. The monopolist controls the spot price by setting the spot market quantity to maximize profits. Here, we assume that the monopolist is risk-neutral. Because the monopolist is risk-neutral, he has no incentive to participate in the futures market unless he can increase expected profits by doing so. Second, the price in the futures market is set by competitive risk-neutral market makers. These agents observe the aggregate demand for futures contracts and set prices accordingly. In addition, we introduce risk-averse agents whose payoff depends on the price realized in the spot market. They have an incentive to participate in the futures market because doing so allows them to reduce their exposure to spot price risk. We assume that the number of these agents is stochastic and unobservable.

The timing of events is as follows. First, nature chooses a number of risk-averse agents. Then the monopolist and these risk-averse agents simultaneously submit orders to the futures market. Observing the aggregate order flow, the sum of the order flows submitted by the monopolist and the risk-averse agents, market makers set the futures price equal to the spot market price they expect. Next, demand is realized and the monopolist chooses spot market quantity to maximize profits.

We assume a linear demand curve, so that spot prices are given by (7), $P = a - bQ$, where a is stochastic and $b > 0$.⁹ Again, the cost of production is assumed to be zero. The futures market is characterized by linear cash-settled contracts with payoff $P - k$ per contract. The monopolist chooses a number of contracts C^m . Given C^m and demand realization, a , the monopolist sets spot market price and quantity to maximize profits

$$\pi = C^m (a - bQ - k) + (a - bQ) Q. \quad (11)$$

The spot market FOC is

$$\frac{\partial \pi}{\partial Q} = -bC^m + a - 2bQ = 0.$$

Note that the SOC is satisfied, yielding an optimal quantity and price

$$\begin{aligned} Q^* &= \frac{1}{2b} (a - bC^m) \\ P^* &= \frac{1}{2} (a + bC^m). \end{aligned} \quad (12)$$

⁹The choice of a linear demand function is for analytic tractability. While a much broader class of functions will obtain similar results, not all demand functions will obtain the same results. In particular, convex demand curves will provide an even stronger incentive for the monopolist to strategically trade in the futures market as large changes in the spot price lead to relatively small changes in monopoly profits. Concave demand curves provide a weaker incentive for strategic trading.

We assume that all risk-averse agents are identical and that the number of such agents, N , is stochastic and uniformly distributed on $[0, 1]$. Each agent chooses a number of contracts C^n . This number will be determined optimally based on their preferences. The total number of contracts submitted by these agents, NC^n , is therefore stochastic. Market makers only observe the aggregate order flow, $NC^n + C^m$. They have beliefs about the order flow submitted by the monopolist and the risk-averse agents and set the futures price, k , accordingly.

In this setup, we look for equilibria in the futures market given optimal subsequent behavior in the spot market. We assume a set of actions and beliefs for all agents and explore whether any agent has an incentive to deviate. This section explores equilibria in which the monopolist hides his futures market participation by randomizing the order flow he submits. When the monopolist submits a positive (negative) order flow – with plans to drive up (down) spot prices to make this position profitable – market makers are unsure if it is the monopolist or other traders (without market power) who are submitting the order. This imperfect inference allows the monopolist to receive favorable futures market prices, at the expense of other agents in the market.

In this setting, a subgame perfect equilibrium consists of

1. beliefs held by market makers about C^n and the distribution of \tilde{C}^m , and a price schedule, $k(\cdot)$ for which market makers earn zero expected profits,
2. beliefs held by the monopolist about $k(\cdot)$ and C^n , and a set of possible values for C^m where each yields the same expected profit given those beliefs, and no other values for C^m yield higher expected profits,
3. beliefs held by the risk-averse agents about $k(\cdot)$ and the distribution of \tilde{C}^m , and a value of C^n that maximizes expected utility given those beliefs, and
4. off-equilibrium-path beliefs held by market makers about the monopolist's order flow when the observed aggregate order flow is inconsistent with their beliefs – given prices set competitively based on these beliefs, the monopolist will not choose to submit an off-equilibrium-path order flow quantity.

Here, the beliefs of all agents must be consistent with one another, and with the actions of other agents.

3.2 Beliefs and Prices of Market Makers

There are many sets of beliefs that market maker could hold about the monopolist's futures market participation that imply that the monopolist's order flow cannot be perfectly inferred from the aggregate order flow. Here, we look for an equilibrium involving the simplest set of such beliefs. Suppose market makers

believe that each risk-averse agent submits an order C^n and that the monopolist randomizes between $+x$ and $-x$ with equal probability where $0 \leq x < \frac{1}{2}C^n$. Based on their beliefs, they set actuarially fair prices. Off-equilibrium-path, we assume that market makers set prices based on the most punitive beliefs.

The aggregate order flow, $\theta \equiv C^m + C^n N$, can indicate that the monopolist has successfully hidden, that he has been caught for sure with $+x$ or $-x$ given market maker beliefs, or that aggregate order flow is inconsistent with market makers' beliefs. We categorize the aggregate order flow into the following five groups and specify the price schedules for all possible values of θ .¹⁰

$$\begin{aligned}
A1. k(\theta) &= \frac{1}{2}E[a] + \frac{1}{2}b\theta \text{ if } \theta > x + C^n \\
A2. k(\theta) &= \frac{1}{2}E[a] + \frac{1}{2}bx \text{ if } -x + C^n < \theta \leq x + C^n \\
A3. k(\theta) &= \frac{1}{2}E[a] \text{ if } x \leq \theta \leq -x + C^n \\
A4. k(\theta) &= \frac{1}{2}E[a] - \frac{1}{2}bx \text{ if } -x \leq \theta < x \\
A5. k(\theta) &= \frac{1}{2}E[a] + \frac{1}{2}b(\theta - C^n) \text{ if } \theta < -x
\end{aligned} \tag{13}$$

In ranges A1 and A5, market makers know that the monopolist submitted an order flow inconsistent with market makers' expectations. In range A1, it must have been the case that $C^m > x$, and they assume that $N = 0$. In range A5, it must have been the case that $C^m < -x$, and they assume $N = 1$. Prices are set accordingly. In ranges A2 and A4, market makers believe that the monopolist submitted $+x$ and $-x$, respectively. Prices are set accordingly. In range A3, the monopolist hides successfully within the aggregate order flow. In this region, market makers believe that $-x$ and $+x$ are equally likely.

Prices are set competitively. In other words, if the monopolist and risk-averse agents take actions that conform to the beliefs of market makers, then no market maker will have an incentive to deviate. Note that this equilibrium behavior on the part of market makers takes as given the order flow of each risk-averse agent, C^n . Next, we examine optimal behavior on the part of the monopolist given the beliefs and price schedule of market makers.

¹⁰If $x = 0$, prices are set as:

$$\begin{aligned}
A1. k(\theta) &= \frac{1}{2}E[a] + \frac{1}{2}b(\theta) \text{ if } \theta > C^n \\
A3. k(\theta) &= \frac{1}{2}E[a] \text{ if } 0 \leq \theta \leq C^n \\
A5. k(\theta) &= \frac{1}{2}E[a] + \frac{1}{2}b(\theta - C^n) \text{ if } \theta < 0
\end{aligned}$$

3.3 Beliefs and Actions of Monopolist

The monopolist takes as given the order flow of risk-averse agents, C^n , as well as the futures price schedule, $k(\cdot)$, set by market makers given the aggregate order flow. As shown in Proposition 1, in any equilibrium in which the risk-neutral monopolist does not try to disguise his order flow he will not participate in the futures market. In this case, his expected profits are

$$E[\pi|C^m = 0] = \frac{1}{4b}E[a^2].$$

On the other hand, if the monopolist finds it optimal to randomize in a way consistent with market makers' beliefs, he must earn the same expected profits whether he submits an order flow $+x$ or $-x$. Otherwise, he would only play one of the strategies and his actions would be incompatible with market makers' beliefs. Given the futures price schedule, $k(\cdot)$, we now find the optimal behavior on the part of the monopolist.

Proposition 6 *Given that market makers set $k(\cdot)$ as in (13), the monopolist will maximize expected profits by submitting either $C^m = +x$ or $-x$, where $0 \leq x < \frac{1}{4}C^n$. The monopolist's expected profits will be*

$$\begin{aligned} E[\pi|x] &= E[\pi|-x] = \frac{1}{4b}E[a^2] + \frac{1}{4}bx^2 - \frac{b}{C^n}x^3 \\ &> E[\pi|C^m = 0] \text{ for } x > 0. \end{aligned}$$

Proof. See Appendix A.6. ■

When market makers set $k(\cdot)$ consistent with the belief that the monopolist randomizes between $+x$, and $-x$, the monopolist will find it optimal to act consistently with those beliefs. Note that there are many possible equilibria, one for each x . In an equilibrium in which $x = 0$, the monopolist does not participate in the futures market. For larger x , the monopolist profits in the futures market at the expense of risk-averse agents.

3.3.1 Beliefs and Actions of Risk-Averse Agents

The risk-averse agents know that it is optimal for the monopolist to hide within the aggregate order flow by randomizing C^m . The monopolist will then set spot market prices optimally given his futures position, thereby increasing expected profits. Risk-averse agents are risk-averse in the domain of their profits, $\pi^n = \pi^n(P)$. Their preferences are represented by a concave utility function, u . To reduce their exposure to spot market price risk, a given risk-averse agent will participate in the futures market by purchasing C units of the futures contract. C^n is the number of contracts purchased by the average risk-averse agent in the

market. C is then set optimally by each agent according to the following optimization problem:

$$C^{n*} = \arg \max_C E [u(C(P - k) + \pi^n(P))].$$

Note that any given risk-averse agent is too small to affect aggregate order-flow and thus takes prices as given. We assume that profits are linear in the spot market price, i.e. $\pi^n(P) = c_0 + c_1P$, with $c_1 < 0$, so that higher spot prices imply lower profits. This implies that

$$\begin{aligned} C^{n*} &= \arg \max_C E [u(C(P^* - k) + c_0 + c_1P^*)] \\ &= \arg \max_C E \left[u \left(C \left(\frac{1}{2}(a + bC^m) - k \right) + c_0 + c_1 \left(\frac{1}{2}(a + bC^m) \right) \right) \right] \end{aligned} \quad (14)$$

where $k(\cdot)$ is set consistent with (13). As before, risk-averse agents believe that C^m can take on two values, $+x$ and $-x$, with equal probability. We have shown above that for a given C^m there exist equilibria with $0 \leq x < \frac{1}{4}C^m$.

When a risk-averse agent wants to hedge, this provides him with information that the expected aggregate hedging demand is high. He then acts rationally taking this information into account. Since risk-averse agents are identical, none is more or less likely to hedge than any other. Therefore, the distribution of the number of hedgers, conditional on a given agent wanting to hedge, is $f(N) = 2N$.¹¹

First, we examine optimal hedging in the absence of the monopolist, i.e. if $x = 0$. In this case,

$$C^{n*} = \arg \max_C E \left[u \left(C \left(\frac{1}{2}a - \frac{1}{2}E[a] \right) + c_0 + c_1 \frac{1}{2}a \right) \right].$$

The FOC is then

$$E \left[\left(\frac{1}{2}a - \frac{1}{2}E[a] \right) u' \left(C \left(\frac{1}{2}a - \frac{1}{2}E[a] \right) + c_0 + c_1 \frac{1}{2}a \right) \right] = 0.$$

Note that the SOC is satisfied. For $C^{n*} = -c_1$, the FOC is satisfied and it is a global maximum. Without the monopolist's participation in the futures market, it is optimal for risk-averse agents to eliminate all risk. We now examine optimal hedging when the monopolist participates in the futures market, i.e. when $x > 0$.

Proposition 7 *In an equilibrium in which the monopolist participates in the futures market with order flow $+x$ and $-x$ with equal probability where $0 < x < \frac{1}{4}C^m$, risk-averse agents maximizing (14) will participate in the futures market, though will participate less than they would if the monopolist did not participate, i.e.*

¹¹Similarly, agents without hedging needs would update their beliefs about the expected number of hedgers accordingly. These agents will participate in the futures market to exploit their information. This will mitigate but not eliminate the effect we discuss. If all agents do not take into account the information contained in their own hedging demand, these agents will not believe that they face unfavorable prices on average, and will not reduce their hedging demand. However, their expected profits will be lower if they hold these naïve beliefs.

$$0 < C^{n*} < -c_1.$$

Proof. See Appendix A.7. ■

If risk-averse agents believe that the monopolist trades strategically in the futures market, they are concerned that the monopolist will hold an opposite position and move spot prices against them. A given risk-averse agent knows that he is more likely to want to hedge precisely at the wrong times as he is more likely to hedge when aggregate order flow from risk-averse agents is large. In this case, either the monopolist also submits a large order flow and is spotted – in which case futures prices are set fairly – or the monopolist submits a small order flow and hides successfully – in which case the monopolist gains at the risk-averse agents’ expense. This makes hedging more expensive for agents without market power and thus discourages their participation in the futures market.

The following proposition shows that a subgame perfect equilibrium exists with the beliefs and actions as specified above.

Proposition 8 *Given the market structure described in Subsection 3.1, there exists a subgame perfect equilibrium in which futures market prices are set as in (13), risk-averse agents each submit an order of C^n , where $0 < C^n < -c_1$, and the monopolist submits an order flow of either $+x$ or $-x$ with equal probability, where $0 < x < \frac{1}{4}C^n$.*

Proof. *Proposition 6 shows that the monopolist has no incentive to deviate from this equilibrium. Proposition 7 shows that risk-averse agents have no incentive to deviate from this equilibrium. Market makers earn zero profits and none has an incentive to offer another price schedule.* ■

Here, a spot market monopolist is able to increase profits by trading strategically in the futures market. The monopolist takes a futures market position randomly, then deviates from the spot market monopoly optimum to move spot market prices and make this position more profitable. If the monopolist’s futures market position were perfectly observable, market makers would set futures market prices anticipating these actions. In this case, the monopolist would not want to participate in the futures market since doing so would decrease expected spot market profits without increasing expected futures market profits. However, when there are other traders in the market, the futures market position submitted by the monopolist cannot be perfectly inferred by observing the aggregate order flow. In this case, market makers set prices based on the rational belief that the orders they receive could have come from either the monopolist or from other agents without market power. As a result, trades submitted by the monopolist move prices less than they would had they been observable. Just as an informed trader in the Kyle model profits at the expense of “noise traders”, the monopolist earns positive expected profits in the futures market at the expense of the

other market participants. This makes futures market participation expensive, and reduces the optimal hedging of risk-averse agents.

4 Futures Market Manipulation under Spot Market Power

The last section showed that a spot market monopolist can profitably exploit spot market power in the futures market. This section documents that even those without market power can profit in the futures market when another agent has spot market power. This section relies on the insight of Kumar and Seppi (1992), that agents without inside information can manipulate a market if they are mistaken for agents with inside information. Here, we show that the same can be said of market power: agents without market power who hold futures market positions can use later trading to manipulate prices to make the original position profitable when market makers believe they might have market power.

4.1 Model Setup with Manipulators

4.1.1 Timing and Markets

While the models developed earlier in this paper have markets in only two periods, this model requires trade in three periods. The last two periods mirror our earlier setup. In this section, we add an initial period in which agents trade contracts whose payoffs are contingent on futures prices in the next period. Presenting the markets in reverse chronological order:

$t = 2$: There is a spot market at time $t = 2$. Production in this period is controlled by a monopolist, who faces a linear demand curve, i.e. spot prices are given by (7), $P = a - bQ$, where $a, b > 0$.¹² The cost of production is zero.

$t = 1$: There is a futures market at time $t = 1$, characterized by linear cash-settled contracts based on the spot price in the next period, with payoff $P - k_1$ per contract.

$t = 0$: There is a futures market at time $t = 0$, characterized by linear cash-settled contracts based on the futures strike price in the next period, with payoff $k_1 - k_0$ per contract.¹³

¹²While the assumption of linear demand is critical to obtain simple analytic results, the same intuition obtains with a convex demand curve. While losing analytic tractability, these demand functions have the advantage that the monopolist has a strict benefit from participating in the futures market. When demand is linear, the increased profits in the futures market that come with futures market participation are exactly offset by lower spot market monopoly profits.

¹³Note that a futures contract whose payoff is based on the price of another futures contract is unusual. However, there are many options whose payoff is based on a futures contract. While we use a linear futures contract and not an options contract at $t = 0$ for analytical tractability, our result that manipulative trading exists in equilibrium is robust to changes in the contractual structure. Furthermore, many futures markets based on the spot price of a storable commodity are effectively a future on a future, as storability links current spot and forward prices.

The manipulator is willing to accept expected losses in the futures market at $t = 1$ for the same reason that the monopolist is willing to accept lower expected profits in the spot market at $t = 2$. Just as the monopolist sets spot prices at $t = 2$ to make his futures market position at $t = 1$ profitable, the manipulator trades in the futures market at $t = 1$ in order to move futures prices, thereby making his futures market position at $t = 0$ profitable. Just as the monopolist earns expected profits at the expense of the manipulator at $t = 1$, the manipulator earns expected profits at the expense of noise traders at $t = 0$.

4.1.3 Spot Market Prices

For a given futures market position, the monopolist's profit is

$$\begin{aligned}\pi &= C_1^m (P - k_1) + PQ \\ &= C_1^m (a - bQ - k_1) + (a - bQ) Q.\end{aligned}$$

Note that prices and quantities are optimally set at

$$\begin{aligned}Q^* &= \frac{1}{2b} (a - bC_1^m) \text{ and} \\ P^* &= \frac{1}{2} (a + bC_1^m),\end{aligned}$$

so that optimal profit will be

$$\pi = -C_1^m k_1 + \frac{1}{4b} (a + bC_1^m)^2.$$

As in Section 3, futures market participation causes the monopolist to deviate from the spot market monopoly optimum. He moves prices to make the futures market position profitable.

4.2 Equilibrium with Manipulation

Here, we propose an equilibrium with manipulation:

At $t = 0$, the manipulator randomizes between $+x$ and $-x$ with equal probability where $x = \min(W, \frac{2}{3}(C^{n+} - C^{n-} - W))$. Note that W is the liquidity constraint faced by the manipulator, the maximum order flow in absolute value that the manipulator can submit. C^{n+} and C^{n-} are the maximum and minimum order flows that could be submitted by the noise traders. Market makers set

$$k_0 = \frac{1}{2}a$$

regardless of the aggregate order flow submitted.

At $t = 1$, the equilibrium will take three different forms depending on aggregate order flow $\theta_0 = C_0^h + C_0^n$ at $t = 0$. If $\theta_0 > C^{n+} - x$ then market makers know that the manipulator must have submitted $C_0^h = +x$. In this case, the monopolist will not participate in the futures market, i.e. $C_1^m = 0$. The manipulator submits the same order as in the previous period, i.e. $C_1^h = C_0^h$, and market makers set the futures price as

$$k_1 = E [P^* | \theta_1] = \frac{1}{2}a + \frac{1}{2}b(\theta_1 - x).$$

If $C^{n-} + x \leq \theta_0 \leq C^{n+} - x$ then the manipulator has successfully hidden his order flow in the previous period. The monopolist randomizes over $C_1^m \in \{-\frac{1}{2}x, \frac{1}{2}x\}$ with equal probability, and the manipulator sets $C_1^h = \frac{1}{2}C_0^h$. Market makers set the futures price as

$$k_1 = E [P^* | \theta_1] = \frac{1}{2}a + \frac{1}{4}b\theta_1.$$

If $\theta_0 < C^{n-} + x$ then market makers know that the manipulator must have submitted $C_0^h = -x$. The monopolist will not participate, i.e. $C_1^m = 0$, and the manipulator submits the same order as in the previous period, i.e. $C_1^h = C_0^h$. The futures price is then set as

$$k_1 = E [P^* | \theta_1] = \frac{1}{2}a + \frac{1}{2}b(\theta_1 + x).$$

At $t = 2$ monopolists sets $P^* = \frac{1}{2}(a + bC_1^m)$ and $Q^* = \frac{1}{2b}(a - bC_1^m)$.

Proposition 9 *The actions and beliefs described above constitute a subgame perfect equilibrium.*

Proof. See Appendix A.8. ■

Financial market manipulation is possible when agents without market power can be mistaken for those with market power. An agent without market power can profit by taking a random position in the initial futures market at $t = 0$. When this random position is not spotted, he has an incentive to move subsequent futures prices at $t = 1$ to make this initial position more profitable. For example, if he takes a long position in the initial futures market, this position becomes profitable if subsequent futures market prices are high. As a result, he has an incentive to take a long position in the subsequent futures market to drive up prices. When market makers observe this long position, they believe it could have been submitted by the monopolist, who would then use his monopoly power to raise spot prices. Therefore, market makers rationally set higher futures prices in response to the long aggregate order flow they observe. Since the manipulator's trade at $t = 1$ moves prices without altering the underlying contract payoff, this trade is unprofitable. By taking a larger position in the initial futures market than in the subsequent one, the profits he earns in the initial

futures market by moving subsequent prices exceed his losses from subsequent trading.

In Section 3, a monopolist's trades could not be differentiated from those submitted by risk-averse agents. He was able to profit because the futures market trades he submitted moved prices by *less* than they would have had they been observable. In this section, a manipulator's trades cannot be differentiated from those of the monopolist. The manipulator is able to profit because the futures market trades he submits move prices by *more* than they would have had they been observable. As in Section 3, the monopolist profits from the manipulator's presence at $t = 1$ since this causes the trades he submits to move prices by *less* than they would otherwise.

5 Conclusions

In this paper, we have shown how spot market power impacts three rationales for trading in futures contracts. First, agents with and without spot market power will participate in futures market to hedge their risk. Second, agents with spot market power trade in the futures market and then strategically set spot prices to make their futures position more profitable. Last, agents without market power may manipulate futures prices to make their earlier futures market positions profitable. In a rational expectations equilibrium, these three motives provide a reason for futures markets to exist even if the underlying spot market is monopolistic.

When futures market positions can be perfectly inferred by market participants, trades will only take place to satisfy hedging needs. If, however, the futures market positions of individual agents are not perfectly observable, strategic and manipulative motives for trade are also possible. In the case of strategic trading, the monopolist profits by hiding behind the trades of agents without market power. In the case of manipulation, agents without market power profit from hiding behind the trades of the monopolist.

Rational market makers set futures prices taking into account the moral hazard problem created by the monopolist's adjustment of spot market prices given his futures market position. This makes hedging expensive, and therefore reduces futures market participation for agents with and without market power. We have shown this makes it impossible for the monopolist to eliminate all risk.

Many existing futures market whose underlying spot markets are imperfectly competitive exhibit very low participation relative to the importance of those markets. In particular, markets for longer term contracts are very illiquid. Given the moral hazard problems discussed in this paper, several markets – including weather and insurance derivatives – have emerged to avoid the inefficiencies caused by market power. The trading activity in futures markets on oil, for example, is very low. Our paper suggests that this can be explained by the imperfectly competitive structure of the oil spot market. Weather derivatives provide an index-hedge against extreme temperatures, and therefore against oil demand risk. However, these contracts

are not susceptible to the moral hazard issue discussed in this paper and thus improve market efficiency. The market for insurance derivatives (e.g. catastrophe bonds) offers investors the opportunity to trade on the impact of natural catastrophes. One common feature that almost all insurance derivatives share is the inclusion of an index trigger that, for example, relates to the strength of a hurricane or an industry index of insured property losses. These triggers contrast to indemnity triggers – which are based on the size of an insurer’s loss – which are prone to moral hazard. For example, the issuing insurance company could change the profile of risks it underwrites or its claim-settlement process. While these index-related contracts solve the moral hazard problem, they introduce basis risk, the potential mismatch between the underlying risk and the payoff of these index contracts.¹⁶

¹⁶Doherty and Richter (2002) show that it is optimal to supplement an index hedge by “gap insurance” which provides insurance against basis risk.

A Appendix: Proofs

A.1 Proof of Proposition 1

Risk neutrality implies that $u'(\pi)$ is constant. The first derivative of expected utility with respect to the number of derivatives contracts can be written as

$$\begin{aligned}\frac{\partial E[u(\pi)]}{\partial C} &= E \left[\left(g - E[g] - CE \left[\frac{dQ}{dC} f_1 g' \right] \right) \cdot u'(\pi) \right] \\ &= u'(\pi) E \left[\left(g - E[g] - CE \left[\frac{dQ}{dC} f_1 g' \right] \right) \right]\end{aligned}$$

Differentiation the FOC for the last period

$$(Cg' + Q) f_1 + f = 0$$

with respect to C yields

$$\begin{aligned}0 &= \left(g' + C \frac{dQ}{dC} f_1 g'' + 2 \frac{dQ}{dC} \right) f_1 + (Cg' + Q) \frac{dQ}{dC} f_{11} = 0; \\ \frac{dQ}{dC} &= - \frac{g' f_1}{(2 + C f_1 g'') f_1 + (Cg' + Q) f_{11}}.\end{aligned}$$

$\frac{\partial E[u(\pi)]}{\partial C}$ can then be rewritten as

$$\frac{\partial E[u(\pi)]}{\partial C} = u'(\pi) CE \left[\frac{(g' f_1)^2}{(2 + C f_1 g'') f_1 + (Cg' + Q) f_{11}} \right].$$

For $C = 0$, the FOC is satisfied. The SOC in the last period,

$$(C f_1 g'' + 2) f_1 + (Cg' + Q) f_{11} < 0,$$

implies that

$$\frac{(g' f_1)^2}{(2 + C f_1 g'') f_1 + (Cg' + Q) f_{11}} < 0.$$

Therefore $\frac{\partial E[u(\pi)]}{\partial C} > 0$ for all $C < 0$, and $\frac{\partial E[u(\pi)]}{\partial C} < 0$ for all $C > 0$. We conclude that $C^* = 0$ is a global maximum.

A.2 Proof of Proposition 2

In this proof, we assume without loss of generality that $E[g(D)] = 0$. Profits are then given by $\pi = g(D) + QP$. The FOC in the final period is

$$\frac{\partial \pi}{\partial Q} = Q f_1 + f = 0.$$

Note, that this FOC is identical to the one without futures market participation. The monopolist therefore maintains full market power in the spot market. The FOC determines an optimal quantity, $Q^*(D)$, and a price, $f(Q^*(D), D)$. Assume the monopolist designs a futures contract with the following payoff

$$g(D) = E[Q^*(D) f(Q^*(D), D)] - Q^*(D) f(Q^*(D), D).$$

In this case, profits are constant and risk is eliminated completely. Note, that any perturbation to g reduces expected utility for the risk-averse monopolist.

A.3 Proof of Proposition 3

The FOC in the initial period is

$$\frac{\partial E[u(\pi)]}{\partial C} = E \left[\left(g - E[g] - CE \left[\frac{dQ}{dC} f_1 g' \right] \right) \cdot u'(\pi) \right] = 0.$$

The first derivative of expected utility evaluated at $C = 0$ is given by

$$\begin{aligned} \frac{\partial E[u(\pi)]}{\partial C} \Big|_{C=0} &= E[(g - E[g]) \cdot u'(QP)] \\ &= Cov(g, u'(QP)). \end{aligned}$$

Here, we make an assumption about the nature of demand shocks. In particular, we assume that a shock to demand that increases equilibrium prices will at least weakly increase equilibrium quantities. Put another way, any demand shock that increases prices will increase monopoly profits. Formally, for any demand shock such that

$$\frac{\partial f(Q, D)}{\partial D} = f_1 \frac{\partial Q}{\partial D} + f_2 > 0$$

we have

$$\frac{\partial Q}{\partial D} \geq 0.$$

This implies that

$$\frac{\partial \pi}{\partial D} = \frac{\partial Q}{\partial D} f + Q \left(f_1 \frac{\partial Q}{\partial D} + f_2 \right) > 0.$$

In this case, a shock to demand has the following impact on marginal utility

$$\frac{\partial u'(QP)}{\partial D} = \left(\frac{\partial Q}{\partial D} f + Q \left(f_1 \frac{\partial Q}{\partial D} + f_2 \right) \right) \cdot u''(QP) < 0.$$

The impact on the payoff of the derivatives contract is

$$\frac{\partial g(P)}{\partial D} = \left(\frac{\partial Q}{\partial D} f_1 + f_2 \right) g'.$$

Therefore, if the payoff of the derivatives contract is increasing in the spot price then $Cov(g, u'(QP)) < 0$. This yields $\frac{\partial E[u(\pi)]}{\partial C} \Big|_{C=0} < 0$, which implies that $C^* < 0$. It is thus optimal for the monopolist to sell derivatives contracts. The effect on quantity is

$$\frac{\partial Q}{\partial C} = - \frac{g' f_1}{(2 + C f_1 g'') f_1 + (C g' + Q) f_{11}} < 0$$

and the effect on spot prices is

$$\frac{\partial f}{\partial C} = - \frac{g' (f_1)^2}{(2 + C f_1 g'') f_1 + (C g' + Q) f_{11}} > 0.$$

Note that the spot market SOC implies that the denominators in both fractions are negative. He thus increases quantity and sells at a lower price as $C^* < 0$. The derivative of expected profits with respect to C is

$$E \left[\frac{\partial \pi}{\partial C} \right] = -CE \left[\frac{dQ}{dC} f_1 g' \right].$$

In the situation above, we get $E \left[\frac{\partial \pi}{\partial C} \right] > 0$ for all $C < 0$. This implies that the monopolist's expected profits when optimally participating in the derivatives market with $C^* < 0$ are lower than if he did not participate.

If the payoff of the derivatives contract is decreasing in the spot price then $Cov(g, u'(QP)) > 0$. This yields $\frac{\partial E[u(\pi)]}{\partial C} \Big|_{C=0} > 0$, which implies that $C^* > 0$. It is thus optimal for the monopolist to buy strictly

positive number of derivatives contracts. The effect on quantity is

$$\frac{\partial Q}{\partial C} = -\frac{g' f_1}{(2 + C f_1 g'') f_1 + (C g' + Q) f_{11}} > 0$$

and the effect on spot prices is

$$\frac{\partial f}{\partial C} = -\frac{g' (f_1)^2}{(2 + C f_1 g'') f_1 + (C g' + Q) f_{11}} < 0.$$

Again, the monopolist increases quantity and sells at a lower price. In this case, $E \left[\frac{\partial \pi}{\partial C} \right] < 0$ for all $C > 0$. This implies that the monopolist's expected profits when optimally participating in the derivatives market with $C^* > 0$ are lower than if he did not participate.

A.4 Proof of Proposition 4

In this proof, we assume that the payoff of the derivatives contract is increasing in the spot price.¹⁷ Here, we compare two potential monopolists with utility functions u and v . If the monopolist with utility function v is more risk averse than the one with u , then there exist an increasing, concave function h such that $v = h \circ u$. Let C^{u*} denote the optimal number of derivatives contracts bought by monopolist u . In this case, C^{u*} satisfies the FOC

$$\frac{\partial E [u(\pi(C))] }{\partial C} \Big|_{C=C^{u*}} = 0. \quad (15)$$

The first derivative of expected utility of monopolist v with respect to C evaluated at C^{u*} is

$$\begin{aligned} \frac{\partial E [v(\pi(C))] }{\partial C} \Big|_{C=C^{u*}} &= E \left[v'(\pi(C^{u*})) \frac{\partial \pi(C^{u*})}{\partial C} \right] \\ &= E \left[h'(u(\pi(C^{u*}))) u'(\pi(C^{u*})) \frac{\partial \pi(C^{u*})}{\partial C} \right]. \end{aligned}$$

Recall that

$$\frac{\partial \pi}{\partial C} = \left(g - E[g] - CE \left[\frac{dQ}{dC} f_1 g' \right] \right), \quad (16)$$

and let

$$\bar{P} \equiv E[g] + CE \left[\frac{dQ}{dC} f_1 g' \right]. \quad (17)$$

Note that $\frac{\partial \pi}{\partial C} > 0$ iff $g < \bar{P}$. Recall that by assumption

$$\begin{aligned} \frac{\partial f}{\partial D} &> 0 \text{ and} \\ \frac{\partial Q}{\partial D} &\geq 0, \end{aligned} \quad (18)$$

which implies

$$\frac{\partial \pi}{\partial D} > 0. \quad (19)$$

We have $\frac{\partial g(P)}{\partial D} = \frac{\partial f}{\partial D} g' > 0$. Let \bar{D} be a level of demand for which $g(f(Q(\bar{D}), \bar{D})) = \bar{P}$. As g is increasing in the level of demand, there can be at most one such point. If $\frac{\partial \pi}{\partial C}$ could be either positive or negative, such a point exists. If $\frac{\partial \pi}{\partial C}$ never switches sign, set $\bar{D} = 0$ or ∞ . Together,

$$g(f(Q(D), D)) < \bar{P} \text{ iff } D < \bar{D}.$$

¹⁷The proof for a contract whose payoff is decreasing in the spot price is equivalent.

Therefore,

$$D < \bar{D} \text{ iff } \frac{\partial \pi}{\partial C} > 0. \quad (20)$$

In this case, we can split $\frac{\partial E[v(\pi(C, D))]}{\partial C}$ into two parts,

$$\begin{aligned} \frac{\partial E[v(\pi(C))]}{\partial C} &= \int_{D=0}^{\bar{D}} \left[h'(u(\pi(C, D))) u'(\pi(C, D)) \frac{\partial \pi(C, D)}{\partial C} \right] dD \\ &\quad + \int_{D=\bar{D}}^{\infty} \left[h'(u(\pi(C, D))) u'(\pi(C, D)) \frac{\partial \pi(C, D)}{\partial C} \right] dD. \end{aligned}$$

(19) implies that

$$\frac{\partial h'(u(\pi(C, D)))}{\partial D} = h''(u(\pi(C, D))) u'(\pi(C, D)) \frac{\partial \pi}{\partial D} < 0,$$

and therefore

$$h'(u(\pi(C, \bar{D}))) < h'(u(\pi(Q_0, D))) \text{ iff } D < \bar{D}. \quad (21)$$

This means that

$$\begin{aligned} \frac{\partial E[v(\pi(C))]}{\partial C} &= \int_{D=0}^{\bar{D}} \left[h'(u(\pi(C, D))) u'(\pi(C, D)) \frac{\partial \pi(C, D)}{\partial C} \right] dD \\ &\quad + \int_{D=\bar{D}}^{\infty} \left[h'(u(\pi(C, D))) u'(\pi(C, D)) \frac{\partial \pi(C, D)}{\partial C} \right] dD \\ &> g'(u(\pi(C, D))) \int_{D=0}^{\bar{D}} \left[u'(\pi(C, D)) \frac{\partial \pi(C, D)}{\partial C} \right] dD \\ &\quad + g'(u(\pi(C, D))) \int_{D=\bar{D}}^{\infty} \left[u'(\pi(C, D)) \frac{\partial \pi(C, D)}{\partial C} \right] dD. \end{aligned}$$

The last inequality follows from (21) and (20). Rejoining the two parts of the integral,

$$\frac{\partial E[v(\pi(C))]}{\partial C} > g'(u(\pi(C, D))) E \left[u'(\pi(C, D)) \frac{\partial \pi(C, D)}{\partial C} \right].$$

(15) implies that $\frac{\partial E[v(\pi(C))]}{\partial C} \Big|_{C=C^{u^*}} > 0$. Therefore, at the optimal number of derivatives contracts for monopolist u , expected utility of monopolist v could be increased by increasing C . If expected utility is concave, namely if $\frac{\partial^2 E[v(\pi(C))]}{\partial C^2} < 0$, then $C^{v^*} > C^{u^*}$.

A.5 Proof of Proposition 5

Let $g(P)$ be the payoff of a price-contingent derivatives contract to the monopolist and let $Q(P, D)$ be the inverse demand function. Suppose demand D is indexed such that $Q(P, D_i) < Q(P, D_j)$ if and only if $D_i < D_j$ and $P > 0$. For a demand realization D_i profits are given by

$$\pi(D_i) = g(P) + Q(P, D_i) P.$$

Suppose there exists a price-contingent contract $g(P)$ that eliminates all risk and whose price is set competitively. Then the monopolist sets prices optimally contingent on realized demand. Therefore, for any two demand realization D_i and D_j

$$g(P(D_i)) + Q(P(D_i), D_i) P(D_i) = g(P(D_j)) + Q(P(D_j), D_j) P(D_j). \quad (22)$$

The contract must be incentive compatible in the sense that

$$g(P(D_j)) + Q(P(D_j), D_j) P(D_j) \geq g(P(D_i)) + Q(P(D_i), D_j) P(D_i) \quad (23)$$

for all D_i and D_j . Now, assume that D_j is the realized level of demand but that the monopolist set prices equal to $P(D_i)$ for some $D_i < D_j$. Profits are then

$$\pi = g(P(D_i)) + Q(P(D_i), D_j) P(D_i).$$

Since $Q(P(D_i), D_i) < Q(P(D_i), D_j)$

$$g(P(D_i)) + Q(P(D_i), D_j) P(D_i) > g(P(D_i)) + Q(P(D_i), D_i) P(D_i).$$

(22) then implies

$$g(P(D_i)) + Q(P(D_i), D_j) P(D_i) > g(P(D_j)) + Q(P(D_j), D_j) P(D_j).$$

This violates the incentive compatibility constraint (23) as the monopolist gets a higher profit by pretending to be in demand state D_i when realized demand is D_j . Therefore, no derivatives contracts exists that eliminates all risk.

A.6 Proof of Proposition 6

In this case, when submitting C^m , there are 7 ranges the monopolists order flow, C^m , could be in. These are categorized according to which possible prices, $A1 - A5$, the monopolist could face, depending upon the realization of N :

M1 $C^m > x + C^n$ always $A1$ “caught up off-equilibrium”

$$\begin{aligned} E[\pi|C^m] &= -C^m E[k|C^m] + \frac{1}{4b} E[(a + bC^m)^2] \\ E[\pi|C^m] &= -C^m \int_0^1 \left(\frac{1}{2} E[a] + \frac{1}{2} b(C^m + C^n N) \right) dN + \frac{1}{4b} E[(a + bC^m)^2] \\ &= \frac{1}{4b} E[a^2] - \frac{1}{2} bC^m \left(\frac{1}{2} C^m + \frac{1}{2} C^n \right) \\ &< E[\pi|0] = \frac{1}{4b} E[a^2] \text{ if } C^n > -C^m \end{aligned}$$

M2 $-x + C^n < C^m \leq x + C^n$ either $A1$ “caught up off-equilibrium” or $A2$ “caught up on-equilibrium”

- (a) $A1$ if $C^n + x - C^m < C^n N \leq C^n$
- (b) $A2$ if $0 \leq C^n N \leq C^n + x - C^m$

$$\begin{aligned} E[\pi|C^m] &= -C^m E[k|C^m] + \frac{1}{4b} E[(a + bC^m)^2] \\ E[\pi|C^m] &= -C^m \frac{1}{2} E[a] - C^m \int_{1+\frac{x-C^m}{C^n}}^1 \frac{1}{2} b(C^m + C^n N) dN - C^m \int_0^{1+\frac{x-C^m}{C^n}} \frac{1}{2} bxdN + \frac{1}{4b} E[(a + bC^m)^2] \\ &= \frac{1}{4b} E[a^2] - \frac{1}{4} bC^m \left(C^m + \frac{(x - C^m)^2}{C^n} \right) \\ &< E[\pi|0] = \frac{1}{4b} E[a^2] \text{ if } C^m > 0 \end{aligned}$$

M3 $x < C^m \leq -x + C^n$ either $A1$ “caught up off-equilibrium”, $A2$ “caught up on-equilibrium”, or $A3$ “hidden”

- (a) $A1$ if $C^n + x - C^m < C^n N \leq C^n$

- (b) A2 if $C^n - x - C^m \leq C^n N \leq C^n + x - C^m$
(c) A3 if $0 \leq C^n N \leq C^n - x - C^m$

$$\begin{aligned} E[\pi|C^m] &= -C^m E[k|C^m] + \frac{1}{4b} E[(a + bC^m)^2] \\ E[\pi|C^m] &= -C^m \frac{1}{2} E[a] - C^m \int_{1+\frac{x-C^m}{C^n}}^1 \frac{1}{2} b (C^m + C^n N) dN - C^m \int_{1-\frac{x+C^m}{C^n}}^{1+\frac{x-C^m}{C^n}} \frac{1}{2} b x dN + \frac{1}{4b} E[(a + bC^m)^2] \\ &= \frac{1}{4b} E[a^2] - \frac{1}{2} b C^m \left(\frac{1}{2} C^m - \frac{1}{2} \frac{x^2 - C^m^2}{C^n} - x + 2 \frac{x^2}{C^n} \right) \end{aligned}$$

$$\begin{aligned} \frac{dE[\pi|C^m]}{dC^m} &= -\frac{1}{2} b \left(C^m - x + \frac{3}{2} \frac{x^2}{C^n} + \frac{3}{2} \frac{C^m^2}{C^n} \right) \\ \frac{d}{dC^m} E[\pi|x < C^m \leq -x + C^n] &< 0 \quad \forall C^m > x \end{aligned}$$

M4 $-x \leq C^m \leq x$ either A2 “caught up on-equilibrium”, A3 “hidden”, or A4 “caught down on-equilibrium”

- (a) A2 if $C^n - x - C^m \leq C^n N \leq C^n$
(b) A3 if $x - C^m \leq C^n N \leq C^n - x - C^m$
(c) A4 if $0 \leq C^n N < x - C^m$

$$\begin{aligned} E[\pi|C^m] &= -C^m \frac{1}{2} E[a] - C^m \int_{1-\frac{x+C^m}{C^n}}^1 \frac{1}{2} b x dN + C^m \int_0^{\frac{x-C^m}{C^n}} \frac{1}{2} b x dN + \frac{1}{4b} E[(a + bC^m)^2] \\ &= \frac{1}{4b} E[a^2] + b C^m^2 \left(\frac{1}{4} - \frac{1}{C^n} x \right) \end{aligned}$$

For $0 < x < \frac{1}{4} C^n$ – as in the proposed equilibrium – $E[\pi|C^m]$ is maximized at $C^m = x$ and $C^m = -x$.

M5 $x - C^n \leq C^m < -x$ either A3 “hidden”, A4 “caught down on-equilibrium”, or A5 “caught down off-equilibrium”

- (a) A3 if $x - C^m \leq C^n N \leq C^n$
(b) A4 if $-x - C^m \leq C^n N < x - C^m$
(c) A5 if $0 \leq C^n N < -x - C^m$

By analogy to M3, $\frac{d}{dC^m} E[\pi|x - C^n \leq C^m < -x] > 0$

M6 $-x - C^n \leq C^m < x - C^n$ either A4 “caught down on-equilibrium” or A5 “caught down off-equilibrium”

- (a) A4 if $-x - C^m \leq C^n N < C^n$
(b) A5 if $0 \leq C^n N < 0 - x - C^m$

By analogy to M2, $E[\pi|-x - C^n \leq C^m < x - C^n] < E[\pi|0]$

M7 $C^m < -x - C^n$ always A5 “caught up off-equilibrium”

By analogy to M1, $E[\pi|C^m < -x - C^n] < E[\pi|0]$.

Given the values of $E[\pi|C^m]$ given above, $E[\pi]$ is maximized for $C^m = x$ and $C^m = -x$ for $0 < x < \frac{1}{4}C^n$, so that

$$E[\pi|x] = E[\pi|-x] = \frac{1}{4b}E[a^2] + \frac{1}{4}bx^2 - \frac{b}{C^n}x^3 > E[\pi|0].$$

Therefore, the monopolist is indifferent between submitting $C^m = x$ and $C^m = -x$, and the market makers can rationally believe that the monopolist randomizes between these two values with equal probability. Given these beliefs, prices are set competitively and no market maker has an incentive to change k .

A.7 Proof of Proposition 7

Risk-averse agents maximize the following objective function:

$$\begin{aligned} C^{n*} &= \arg \max_C E[u(C(P^* - k) + c_0 + c_1P^*)] \\ &= \arg \max_C E\left[u\left(C\left(\frac{1}{2}(a + bC^m) - k\right) + c_0 + c_1\frac{1}{2}(a + bC^m)\right)\right] \end{aligned}$$

where the risk averse agent takes as given the order flow submitted by the average risk-averse agent, C^n , and by the monopolist, C^{m*} . The first and second derivative of expected utility are given by

$$\begin{aligned} \frac{\partial Eu}{\partial C} &= E\left[\left(\frac{1}{2}(a + bC^m) - k\right) u'\left(C\left(\frac{1}{2}(a + bC^m) - k\right) + c_0 + c_1\frac{1}{2}(a + bC^m)\right)\right] \\ \frac{\partial^2 Eu}{\partial C^2} &= E\left[\left(\frac{1}{2}(a + bC^m) - k\right)^2 u''\left(C\left(\frac{1}{2}(a + bC^m) - k\right) + c_0 + c_1\frac{1}{2}(a + bC^m)\right)\right] < 0. \end{aligned}$$

Expected utility of risk averse agents is therefore a concave function in C .

$$\begin{aligned} &\frac{\partial Eu}{\partial C} \\ &= \left[\begin{aligned} &\frac{1}{2} \int E\left[\left(\frac{1}{2}(a + bx) - k(x + NC^n)\right) u'\left(C\left(\frac{1}{2}(a + bx) - k(x + NC^n)\right) + c_0 + c_1\frac{1}{2}(a + bx)\right)\right] 2NdN \\ &+ \frac{1}{2} \int E\left[\left(\frac{1}{2}(a - bx) - k(-x + NC^n)\right) u'\left(C\left(\frac{1}{2}(a - bx) - k(-x + NC^n)\right) + c_0 + c_1\frac{1}{2}(a - bx)\right)\right] 2NdN \end{aligned} \right] \\ &= \left[\begin{aligned} &\frac{1}{2} \left(1 - \left(1 - \frac{2x}{C^n}\right)^2\right) E\left[\left(\frac{1}{2}(a + bx) - \frac{1}{2}E[a] - \frac{1}{2}bx\right) u'\left(C\left(\frac{1}{2}(a + bx) - \frac{1}{2}E[a] - \frac{1}{2}bx\right) + c_0 + c_1\frac{1}{2}(a + bx)\right)\right] \\ &\quad + \frac{1}{2} \left(1 - \frac{2x}{C^n}\right)^2 E\left[\left(\frac{1}{2}(a + bx) - \frac{1}{2}E[a]\right) u'\left(C\left(\frac{1}{2}(a + bx) - \frac{1}{2}E[a]\right) + c_0 + c_1\frac{1}{2}(a + bx)\right)\right] \\ &\quad + \frac{1}{2} \left(1 - \left(\frac{2x}{C^n}\right)^2\right) E\left[\left(\frac{1}{2}(a - bx) - \frac{1}{2}E[a]\right) u'\left(C\left(\frac{1}{2}(a - bx) - \frac{1}{2}E[a]\right) + c_0 + c_1\frac{1}{2}(a - bx)\right)\right] \\ &\quad + \frac{1}{2} \left(\frac{2x}{C^n}\right)^2 E\left[\left(\frac{1}{2}(a - bx) - \frac{1}{2}E[a] + \frac{1}{2}bx\right) u'\left(C\left(\frac{1}{2}(a - bx) - \frac{1}{2}E[a] + \frac{1}{2}bx\right) + c_0 + c_1\frac{1}{2}(a - bx)\right)\right] \end{aligned} \right] \end{aligned}$$

If we set $x = \lambda C^n$ for $0 < \lambda < \frac{1}{4}$ and $C = C^n$, we can define the function $g(\cdot)$ such that

$$\begin{aligned} g(C) &\equiv \frac{\partial Eu}{\partial C} \Big|_{C^n=C} \\ &= \left[\begin{aligned} &\frac{1}{2} \left(1 - (1 - 2\lambda)^2\right) E\left[\left(\frac{1}{2}a - \frac{1}{2}E[a]\right) u'\left(C\left(\frac{1}{2}a - \frac{1}{2}E[a]\right) + c_0 + c_1\frac{1}{2}(a + b\lambda C)\right)\right] \\ &\quad + \frac{1}{2} (1 - 2\lambda)^2 E\left[\left(\frac{1}{2}(a + b\lambda C) - \frac{1}{2}E[a]\right) u'\left(C\left(\frac{1}{2}(a + b\lambda C) - \frac{1}{2}E[a]\right) + c_0 + c_1\frac{1}{2}(a + b\lambda C)\right)\right] \\ &\quad + \frac{1}{2} \left(1 - (2\lambda)^2\right) E\left[\left(\frac{1}{2}(a - b\lambda C) - \frac{1}{2}E[a]\right) u'\left(C\left(\frac{1}{2}(a - b\lambda C) - \frac{1}{2}E[a]\right) + c_0 + c_1\frac{1}{2}(a - b\lambda C)\right)\right] \\ &\quad + \frac{1}{2} (2\lambda)^2 E\left[\left(\frac{1}{2}a - \frac{1}{2}E[a]\right) u'\left(C\left(\frac{1}{2}a - \frac{1}{2}E[a]\right) + c_0 + c_1\frac{1}{2}(a - b\lambda C)\right)\right] \end{aligned} \right] \end{aligned}$$

First, we evaluate $g(0)$

$$g(0) = E\left[\left(\frac{1}{2}a - \frac{1}{2}E[a]\right) u'\left(c_0 + c_1\frac{1}{2}a\right)\right]$$

Note that this expression is positive as $c_1 < 0$. As a result, when other risk-averse agents do not hedge, any given risk-averse agent can increase utility by hedging. Therefore, there is no equilibrium in which

no hedging occurs unless $c_1 = 0$, in which case the agents have no incentive to hedge. Next, we evaluate $g(-c_1)$. In this case, we get

$$\begin{aligned} g(-c_1) &= 2\lambda u' \left(c_1 \frac{1}{2} E[a] + c_0 \right) \frac{1}{2} b \lambda c_1 (1 - 2\lambda) \\ &< 0 \text{ for } c_1 < 0. \end{aligned}$$

As expected, $g(-c_1) < 0$.

This means that g switches signs between $C = 0$ and $C = -c_1$. Furthermore, g is a smooth function. Therefore, there must exist a C^* between zero and $-c_1$ such that $g(C^*) = 0$. As shown above, expected utility is a concave function in the amount of hedging, C , which implies that the first derivative of expected utility, g , is decreasing in C . The solution C^* to $g(C^*) = 0$ is therefore unique. Note that this implies that in the proposed equilibria above there is some hedging by the risk averse agents but hedging is reduced relative to the case of no monopolist participation in the market.

A.8 Proof of Proposition 9

We first examine the optimal behavior given the beliefs about the manipulator's behavior.

A.8.1 Monopolist

We showed that at $t = 2$ the monopolist maximizes his profits by setting price and quantity as $P^* = \frac{1}{2}(a + bC_1^m)$ and $Q^* = \frac{1}{2b}(a - bC_1^m)$. At $t = 1$, the monopolist maximizes expected profits given the price schedule he faces and the beliefs he holds about the manipulator's trading behavior. His objective function at $t = 1$ depends on the aggregate order flow $\theta_0 = C_0^h + C_0^m$ at $t = 0$.

If $\theta_0 > C^{n+} - x$ then his expected profits are

$$E[\pi] = E \left[-C_1^m \left(\frac{1}{2}a + \frac{1}{2}b(C_1^m + C_1^h - x) \right) + \frac{1}{4b}(a + bC_1^m)^2 \right].$$

The FOC is

$$\frac{\partial E[\pi]}{\partial C_1^m} = E \left[-\frac{1}{2}b(C_1^h - x) - \frac{1}{2}bC_1^m \right].$$

Note that given $\theta_0 > C^{n+} - x$ and the beliefs about the manipulator's trade at $t = 0$ we have $C_1^h = x$, which implies $C_1^m = 0$. The SOC is

$$\frac{\partial^2 E[\pi]}{\partial C_1^m^2} = -\frac{1}{2}b < 0.$$

If $C^{n-} + x \leq \theta_0 \leq C^{n+} - x$ then his expected profits are

$$E[\pi] = E \left[-C_1^m \left(\frac{1}{2}a + \frac{1}{4}b(C_1^m + C_1^h) \right) + \frac{1}{4b}(a + bC_1^m)^2 \right]$$

The FOC is

$$\frac{\partial E[\pi]}{\partial C_1^m} = E \left[-\frac{1}{4}bC_1^h \right].$$

Note that the monopolist believes that the manipulator will randomize between $+\frac{1}{2}x$ and $-\frac{1}{2}x$ with equal probability, so that $E[C_1^h] = 0$, and $\frac{\partial E[\pi]}{\partial C_1^m} = 0$. Therefore, the monopolist is indifferent between submitting an order flow and therefore willing to submit $C_1^m \in \{-\frac{1}{2}x, \frac{1}{2}x\}$ with equal probability.¹⁸

¹⁸Here the monopolist is indifferent between participating and not participating in the futures market. This result is obtained because we make the assumptions that there is no noise trading at $t = 1$ and demand is linear. If we either allow for noise trading at $t = 1$ or a convex demand function the monopolist would have a strict incentive to strategically randomize at $t = 1$.

If $\theta_0 < C^{n-} + x$ then his expected profits are

$$E[\pi] = E \left[-C_1^m \left(\frac{1}{2}a + \frac{1}{2}b(C_1^m + C_1^h + x) \right) + \frac{1}{4b}(a + bC_1^m)^2 \right]$$

In this case, the FOC is

$$\frac{\partial E[\pi]}{\partial C_1^m} = E \left[-\frac{1}{2}b(C_1^h + x) - \frac{1}{2}bC_1^m \right].$$

Note that given $\theta_0 < C^{n-} + x$ and the beliefs about the manipulator's trade at $t = 0$ we have $C_1^h = -x$. This implies

$$\frac{\partial E[\pi]}{\partial C_1^m} = -\frac{1}{2}bC_1^m = 0$$

which implies $C_1^m = 0$. The SOC is

$$\frac{\partial^2 E[\pi]}{\partial C_1^m{}^2} = -\frac{1}{2}b < 0.$$

We have thus shown that the monopolist has no incentive to deviate from the proposed equilibrium given the price schedule and his beliefs about the manipulator's actions. Next, we examine the optimal behavior of the manipulator.

A.8.2 Manipulator

At $t = 1$ the manipulator submits an order flow to maximize his expected profits which depend on the aggregate order flow at $t = 0$. His expected profits are

$$E[\pi^h] = E[C_0^h(k_1 - k_0) + C_1^h(P - k_1)].$$

If $\theta_0 > C^{n+} - x$ his expected profits are

$$\begin{aligned} E[\pi^h] &= E[C_0^h(k_1 - k_0) + C_1^h(P - k_1)] \\ &= E \left[\frac{1}{2}b(C_1^h - x)(C_0^h - C_1^h) \right] \end{aligned}$$

The FOC is

$$\frac{\partial E[\pi^h]}{\partial C_1^h} = E \left[\frac{1}{2}b(C_0^h - 2C_1^h) + \frac{1}{2}bW \right] = 0$$

and the SOC is satisfied. This implies $C_1^h = \frac{1}{2}(x + C_0^h)$. Note that if the manipulator does randomize between $+x$ and $-x$ at $t = 0$, then $\theta_0 > C^{n+} - x$ is only true if $C_0^h = x$. Thus $C_1^h = x$. His profits are then $E[\pi^h] = 0$.

If $C^{n-} + x \leq \theta_0 \leq C^{n+} - x$ his expected profits are

$$\begin{aligned} E[\pi^h] &= E[C_0^h(k_1 - k_0) + C_1^h(P - k_1)] \\ &= E \left[\frac{1}{4}bC_0^h(C_1^m + C_1^h) + \frac{1}{4}bC_1^h(C_1^m - C_1^h) \right]. \end{aligned}$$

The FOC is

$$\frac{\partial E[\pi^h]}{\partial C_1^h} = \frac{1}{4}b(C_0^h + E[C_1^m] - 2C_1^h) = 0$$

and the SOC is satisfied. This implies $C_1^h = \frac{1}{2}(C_0^h + E[C_1^m])$. Note that if the manipulator believes that the monopolist randomizes between $+\frac{1}{2}x$ and $-\frac{1}{2}x$ with equal probability at $t = 0$, then $C_1^h = \frac{1}{2}C_0^h$. His profits are then

$$E[\pi^h] = \frac{1}{16}bC_0^h{}^2 > 0$$

If $\theta_0 < C^{n-} + x$ his expected profits are

$$\begin{aligned} E[\pi^h] &= E[C_0^h(k_1 - k_0) + C_1^h(P - k_1)] \\ &= E\left[\frac{1}{2}b(C_1^h + x)(C_0^h - C_1^h)\right]. \end{aligned}$$

The FOC is

$$\frac{\partial E[\pi^h]}{\partial C_1^h} = E\left[\frac{1}{2}b(C_0^h - 2C_1^h) - \frac{1}{2}bW\right] = 0$$

and the SOC is satisfied. This implies $C_1^h = \frac{1}{2}(-x + C_0^h)$. Again, if the manipulator randomizes between $+x$ and $-x$ at $t = 0$, then $\theta_0 < C^{n-} + x$ is only true if $C_0^h = -x$. Therefore, $C_1^h = -x$ and his profits are $E[\pi^h] = 0$.

At $t = 0$ the manipulator submits an order flow C_0^h to maximize his expected profits given optimal behavior in subsequent periods. His expected profits are given as

$$E[\pi^h] = E[C_0^h(k_1 - k_0) + C_1^h(P - k_1)].$$

If $C_0^h \geq 0$ then

$$\begin{aligned} E[\pi^h] &= \left[\frac{1}{C^{n+} - C^{n-}} \int_{C^{n+} - x - C_0^h}^{C^{n+}} (C_0^h(k_1 - k_0) + C_1^h(P - k_1)) dC_0^n \right. \\ &\quad \left. + \frac{1}{C^{n+} - C^{n-}} \int_{C^{n-} - x - C_0^h}^{C^{n+} - x - C_0^h} (C_0^h(k_1 - k_0) + C_1^h(P - k_1)) dC_0^n \right] \\ &= \frac{1}{C^{n+} - C^{n-}} \frac{1}{16} b C_0^{h2} (C^{n+} - x - C_0^h - C^{n-}). \end{aligned}$$

The FOC is

$$\frac{\partial E[\pi^h]}{\partial C_0^h} = \frac{1}{C^{n+} - C^{n-}} \frac{1}{16} b C_0^h (2(C^{n+} - C^{n-}) - 2x - 3C_0^h).$$

If the FOC is satisfied, then $C_0^h = \frac{2}{3}(C^{n+} - C^{n-} - x)$. Recall the liquidity constraint, $|C_0^h| \leq W < \frac{1}{2}(C^{n+} - C^{n-})$. In an interior equilibrium, $x = C_0^h$, so that $C_0^h = \frac{2}{5}(C^{n+} - C^{n-})$. The SOC is satisfied in this case. Taking the liquidity constraint into account, it is optimal to set $C_0^h = \min(W, \frac{2}{5}(C^{n+} - C^{n-}))$.

If $C_0^h < 0$ then

$$\begin{aligned} E[\pi^h] &= \left[\frac{1}{C^{n+} - C^{n-}} \int_{C^{n-} + x - C_0^h}^{C^{n+}} (C_0^h(k_1 - k_0) + C_1^h(P - k_1)) dC_0^n \right. \\ &\quad \left. + \frac{1}{C^{n+} - C^{n-}} \int_{C^{n-} - x - C_0^h}^{C^{n-} + x - C_0^h} (C_0^h(k_1 - k_0) + C_1^h(P - k_1)) dC_0^n \right] \\ &= \frac{1}{C^{n+} - C^{n-}} \frac{1}{16} b C_0^{h2} (C^{n+} - C^{n-} - x + C_0^h). \end{aligned}$$

The FOC is

$$\frac{\partial E[\pi^h]}{\partial C_0^h} = \frac{1}{C^{n+} - C^{n-}} \frac{1}{16} b C_0^h (2(C^{n+} - C^{n-}) - 2x + 3C_0^h).$$

By logic parallel to the case where $C_0^h \geq 0$, it is optimal to set $C_0^h = -\min(W, \frac{2}{5}(C^{n+} - C^{n-}))$.

For an interior solution, expected profits are

$$E\left[\pi^h | C_0^h = \pm \frac{2}{5}(C^{n+} - C^{n-})\right] = \frac{1}{100} b (C^{n+} - C^{n-})^2 > 0;$$

for a corner solution they are

$$E[\pi^h | C_0^h = \pm W] = \frac{1}{16} b W^2 > 0.$$

Note that the expected profits are the same if the manipulator submits $+x$ or $-x$, so he will be willing to randomize with equal probability.

A.8.3 market makers

Note that prices are set competitively when market makers beliefs are consistent with the actions of the noise traders, the monopolist, and the manipulator. As a result, no market maker has an incentive to change k .

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