Rational Trader Risk*

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May 2004

Abstract

Allowing for a richer information structure than usual, we show that rational traders' calculation with short-term price fluctuations may heavily influence their behaviour even if the interim price is not influenced by non-rational agents i.e. there is no noise trader risk. Instead, traders expect that new rational entrants with different information in the interim period will drive the price against them. Consequently, rational traders in the first period will hesitate to trade on their private information or - in the extreme - will trade against their private information i.e. buy more of the risky asset when they consider it worse. In the first part we develop a microstructure model with learning where the above effect will result in severe inefficiency and mispricing. In the second part, we discuss the critical properties of the information structure which are expected to result in similar findings in general models.

JEL classification: D4, D8, G11, G12.

1 Introduction

Thanks to the dotcom bubble in the second half of the 90s, we have a vast amount of anecdotal evidence that even if a large group of professional investors are aware of the mispricing of certain assets, they do not necessarily trade on this information. Often cited examples where new-technology stocks were obviously not in line with their fundamental value include Priceline.com, whose market capitalization reached \$30 million, or America Online which was worth roughly \$636 billion in April 1999, but there are similar figures on eBay, Yahoo, GeoCities or Lycos. All of them had a market value, which dwarfed those of established traditional industry stocks, which – unlike the majority of new-technology firms – actually made profits. However, this phenomenon is not exclusive for the dotcom bubble. For example, it is worth to have a look on Shleifer's (2000) discussion on Frankel and Froot (1988) survey evidence on similar finding of the US dollar market in the eighties.

*We are grateful for the help received from Hyun Song Shin, Antoine Faure-Grimaud, Regis Breton, Gabrielle Demange, Ashraf Jaffer, Zsuzsa Elek and seminar participants at the LSE and at the DELTA-Paris-Jourdan. All remaining errors are our own. We gratefully acknowledge the EU grant "Archimedes Prize" (HPAW-CT-2002-80054).

> "Frankel and Froot evaluate the forecasts and recommendations of a number of exchange rate forecasting services during the period in the mid 1980s when the dollar had been rising for some time without widening in the U.S - rest of world interest rate differentials and with a rising U.S. trade deficit. Frankel and Froot find that during this period the typical forecaster expected the dollar to continue to appreciate over the next month but also to depreciate within a year in accordance with underlying fundamentals. Consistent with these expectations, forecasting services were issuing buy recommendations while maintaining that the dollar was overpriced relative to its fundamental value. Such trend-chasing short run expectations, combined with a belief in a long run return to fundamentals are hard to reconcile with a fully rational model" (Shleifer 2000, pp. 155)

We have chosen Shleifer's interpretation instead of the original paper, because it also reflects the general approach of behavioural finance literature on similar empirical facts (see Shleifer, 2000 or Barberis and Thaler, 2003 for surveys). They argue that it is either the irrationality of the informed trader – overconfidence (e.g. Sheinkman and Xiong, 2002), loss aversion (e.g. Barberis and Santos, 2003) etc. – or the systematic irrational bias of traders' sentiment they are trading with – like noise traders in De Long et al (1990), Shleifer and Vishny (1996) or Brunnermeier and Abreu (2003) – which can be behind prices not being in line with the fundamental information present in the market.

In this paper, we show that both the seemingly irrational expectations of Froot and Frankel's forecasters and the fact that professional traders may trade against their private information can be explained by a model where all informed traders process information rationally and where there are no noise traders with systematic bias in their sentiment. Our critical assumptions will be that new – informed – entrants are expected to participate in the market and we allow for a richer information structure than usual. In our model traders may trade against their information or ignore it completely because of short-term fluctuations caused by the new rational entrants i.e. there is a *rational trader risk*. The main properties of the information structure which drives our findings are that there is a public signal present on the value of the asset and the private information of first period traders and new entrants is weakly correlated. In the following section, we will show in a classroom example that this type of structure results in the surprising fact that the guess of a trader at the value of the asset and her guess at another trader's guess in the following period move in the opposite direction as her private information changes. In particular, as her private information shows a lower fundamental value, her guess at the guess of the other trader will increase. Hence, the worse the asset looks for the trader, the more she will pay for it, if she expects to resell it in the following period. This property will drive our results.

The problem of our traders is similar to the one in "limits to arbitrage" models of the behavioural finance literature. Just like there, the problem of our traders is that they have to deal with the fact that in the interim period the price may be driven away from the fundamental value. For example, in the seminal paper of DeLong et al (1990) and Schleifer and Vishny (1996) fad following noise traders are the cause of the divergence. In Scheinkman and Xiong (2002) overconfident traders, who process their information wrongly, cause the mispricing. In Brunnermeier and Abreu (2002,2003) the price is different from the fundamental value by assumption and the problem of the traders is that it can be driven back only if enough of them act the same time. As we mentioned, the crucial difference in our story is that early traders are afraid of rational latecomers driving the price away from this early traders' guess at the fundamental value. Furthermore, latecomers would do this for the perfectly rational reason that they have different information so their guess at the fundamental value is different.

Probably, Allen et al (2003) is the most related work to our own. They show that the presence of a public signal and short horizons together break down the law of iterated expectations in an asymmetric information set up i.e. traders expectations on the fundamental value and their expectations of others' expectation will be different. In particular, they show that early traders will overweigh the public signal and underweight the private one. With our richer information structure this break down is more spectacular, as in our set up not only the weights but also the sign of weights will differ.

There is also a string of literature (Harrison and Kreps, 1978, Allen et al ,1992, Morris ,1995, Biais and Bossaerts, 2000) on rational speculation where the fundamental valuation and the speculative value related to the expectation of others' expectations is different. But unlike our work, those papers rely on heterogeneous priors and their assumptions result in abstract models, which are hard to link to standard asset-pricing frameworks.

Our paper is structured as follows. In the next section, we present a classroom example which makes the intuition behind our model very clear in a very simple set up. In Section 3, we present a modified version of Kyle's (1989) share-auction model, which will show that our intuition works in a standard microstructure framework with learning. We also discuss the effect of our story on the efficiency of the market and the interim mispricing of the risky asset. In Section 4, we analyse the critical assumptions in our model with special attention on the properties of the information structure. Finally, we state our conclusions.

2 Wallet game with a public signal

Inspired by Klemperer's (1998) wallet game, in this section we present a classroom example which illustrates the intuition behind our model. The game is as follows. Two students – a boy and a girl – are asked to participate. They do not know each other – probably they are from different classes –, but the teacher knows both of them. Each student has a wallet in his or her pocket with some money in it. The teacher announces first her own guess at the sum of the money in the two wallets. Then she asks the students to write two numbers on a sheet of paper. They both have to guess the sum and what the other will guess as the sum. So the girl has to guess what the boy will guess as the sum and the other way around. The better their guesses, the greater the payoff they receive from the teacher. Let us consider the reasoning of one of them with the supposition that the teacher announced 50 pounds.

When the female student has to guess the sum, she will think in the following way: "As the teacher knows both of us, her guess cannot be very far from the truth. Apart from that, if I have little money today with me, I should guess less, and if I have a lot, I should guess more. So my guess will be positively related to both the teacher's announcement and the money in my pocket."

When she has to guess his guess at the sum, she will think in this way: "He will use the same reasoning about the sum, as I have used. So his guess will be positively related to both the teacher's announcement and the money in his pocket. So what is my guess about the amount of his money? As the teacher knows both of us, her guess cannot be far from the true sum. But if I have 20, and the sum is 50 then the other student has some money around 30. But if I have 30 he has to have 20 to make the sum right. Hence, the more money I have, the less my guess at his money will be. But as a clear consequence, the more money I have, the less my guess at his guess will be, as his guess and his money is positively related."

We see that the presence of the public signal causes opposite movements in her two guesses as the amount of money she has changes. If she has a lot, she will think that the sum is big, but he has less of it, consequently, his guess will be less. Of course, the optimal guesses of the students depend on the precise nature of these signals, the correlations between them and their distributions. We will be very explicit on this issue in section 4. At this point, it is only important to see that if the relationship between the information of the two students is weak enough – as in our example, where we assumed that they do not know each other – then the announcement of some noisy aggregate of the two induces a conditional negative relationship between her private information and her guess at his guess i.e. the second order expectation of the sum. However, her guess at the aggregate – her first order expectation of the sum – still remains positively related to her information. Our next question is what this observation leads to in a financial model.

2.1 Wallet game in financial markets

We will use the intuition behind our simple example in a financial setting. We will use a two-period model. We will think of the two wallets as two factors determining the fundamental value of a firm. Hence, the sum of money in the two pockets will be the fundamental value of a risky asset connected to the value of this firm. We will think of the students as traders of this asset. However, instead of two students we will have two groups of traders and none of them will observe the value of the factors perfectly. Instead, each trader in the first group will observe a noisy private signal on the first factor, while each trader in the second group will observe a noisy private signal on the second factor.[1] Apart from their information, investors in the two groups also differ in the time of their entry. The first group will enter earlier, we may think of them as professional investors getting information – or allowed to trade – before others. The second group will enter in the second period only; they are the new entrants. There will be a public signal observed by every participant in the market at the moment of the entry: the announcement of the teacher.

As the fundamental value of the asset will be realized at the end of the second period, new entrants will care about the fundamental value only. Hence, their demand will positively depend on their private information; just like in the classroom example. Consequently, – if new entrants' demand dominates aggregate demand in the second period – the second period price will be positively related to the fundamental expectation of the second group: their guess on the sum. Our focus will be on the

[1] With independently and normally distributed factors and error terms, this is the simplest structure which makes the intuition of our classroom example work. However, in section 4 we will show that much less restriction on the information structure is sufficient.

demand function of the first group in the first period. In the spirit of our example, their fundamental expectation – their guess on the sum – will be positively related to their private signal, but their expectation of the interim price – their guess on the guess of the second group on the sum – will be negatively related to their private signal.

The critical question is how these two opposite effects enter first-period demand functions. A rational trader's first-period demand in a two-period model consists of two main parts. Firstly, she will try to use her information to earn profits on the price change between the two periods. Secondly, she will hedge her second period demand. Let us take the example of the competitive equilibrium of a CARA-normal model with long-term traders, where first period traders' wealth is given as the sum of the profits in the two periods. By backward induction, after substituting in the optimal second-period demand, these two parts enter linearly into the first-period demand function in the following way (see Brunnermeier 2001, pp. 110):

$$A\left[E\left(p_2|I_1^i\right)-p_1\right]+B\left[E\left(\theta|I_1^i\right)-E\left(p_2|I_1^i\right)\right], \tag{1}$$

where p_1, p_2 are the prices, θ is the fundamental value, I_t^i is the information set of trader i in period t, A and B are positive constants determined by terms in the variance-covariance matrix of the random variables θ, $p_2, E\left(p_2|I_2^i\right)$ and the risk-aversion parameter. The first term reflects the short-term profit motive, while the second term reflects the hedging motive. If the intuition in our class-room example works, the hedging part will depend positively and the short-term profit part will depend negatively on the private signal. Hence, depending on the relative size of A and B , first period-traders may buy more or less of the asset as their private information improves or – if the two effects cancel out – they may ignore their private information all together.[2]

Alternatively, we may reinterpret the nature of the relative forces of our two opposing effects, if we transform (1) as follows:

$$(A-B)\left[E\left(p_2|I_1^i\right)-p_1\right]+B\left[E\left(\theta|I_1^i\right)-p_1\right]. \tag{1'}$$

Now, the short-term profit part in the first pair of brackets is contrasted to the long-term profit on the asset in the second pair of brackets. Apart from the risk-aversion of agents embodied in A and B when there are no inefficiencies in the market, in reality, there can be many factors which affect the weights of short-term and long-term motives. Probably, the most obvious one is that consumers smooth their consumption, so price fluctuation will matter. Another simple argument is that they may be impatient having a high subjective discount rate. They can also be long-term investors who are subject to liquidity shocks in the short-term. It is also possible that they are maximizing short-term gains but they are uncertain whether the fundamental value will be realized in the short run, or not. Uncertain execution time can also cause similar effects. Alternatively – as in Shleifer and Vishny

[2] Bacchetta and van Wincoop (2002) present numerical simulations arguing that generally, the hedging part is small for relevant parameter values, hence the bulk of the results in models with short-lived traders would generalize to models with long-lived traders. To the extent that this is true, it means that readers interested in results reflecting the pure long-term case without any other inefficiencies, should focus their attention – in our model presented in the next section – to the results when μ is close to 1.

(1997) – they can be agents getting continuation funds from investors in the second period, only if their first period performance is good enough. Or they are paid by their short-term performance, but their future employment possibilities depend on their reputation of being able to pick the stocks with high fundamental value. Even trustees who profess long-term investment goals to their fund managers will, in turn, suffer from similar agency problems vis-à-vis their principals, the beneficiaries of the pension fund.

From an intuitive point of view, the logic of our model will go through with any of these stories including the standard long-term investor case. In all cases the positive relationship between the private information and the fundamental valuation is offset partly or wholly by the negative relationship between the private information and the expected interim price resulting in mispricing and informational inefficiency. However, different stories would result in models with different tractability. Hence, we choose to work with two simplifying assumptions. Firstly, we do not build up the relative size of the two opposite effects – the constants A and B – from the primitives of the model, but we take it as exogenously given and analyse the effect of its change. Secondly, we assume that our first-period traders trade strategically only once, in the first period. The resulting model will allow us to analyse the equilibrium consequences of these forces taking into account the effect of imperfect competition, signalling and learning, for the expense of not being able to address how primitives affect the relative sizes of the short-term effect and the hedging effect in the demand function. However, in Proposition 7, we will show that – in any linear model – even if we have long-term traders trading in each period, if there are enough new entrants with private information not too correlated with the private information of first period traders, $E\left(p_2|I_1^i\right)$ and $E\left(\theta|I_1^i\right)$ will move to the opposite direction as early traders' information changes, hence our results are expected to hold in a much more general context.

Our assumptions correspond to the delegation story. For example, our first period traders may be portfolio managers whose problem is to make a trading decision when new entrants are expected to arrive in the interim period causing price fluctuations. Even if managers have some information on the fundamental value of the asset, their payoff may be linked to the short-term profit as well, as investors may judge their abilities by their interim performance. As timing the market perfectly is impossible, they make a once-and-for-all portfolio decision taking into account the trade-off between long term and short-term gains.[3]

In the next sections we formalize our intuition.

3 The model

We modify Kyle's (1989) share-auction model in order to show that the intuition of our wallet game with public signal example goes through in a fully-fledged microstructure model, which incorporates rational learning. Kyle (1989) uses a static set-up where a finite number of informed agents and a finite

[3]A spectacular real-life example of the severity of fund managers' problem to go for short-term gains even if the asset is overpriced is – borrowed from Brunnermeier and Abreu (2003) – that two legendary hedge fund managers Julian Robertson of Tiger Hedge Fund and Stanley Druckenmiller of Quantum Fund lost their jobs due to this trade-off. The earlier because he decided not to invest in new-technology stock, so his Fund could not keep up with the profit of others, and the latter because he decided to ride the bubble, but he did not exit before it burst.

number of uninformed agents submit demand curves to buy up a random supply of a risky asset. He shows that there is a unique symmetric linear Bayesian Nash equilibrium of the game, where traders submit linear demand curves, which are increasing in their signals and decreasing in the price. Within the periods, our set up will be virtually the same as that model, with the only exception that we drop the uninformed agents, as they would only complicate our analysis without any added value. However the structure of the supergame, the payoffs and the information structure will be different.

The focus of our attention will be the portfolio choice of n_1 informed fund managers (early traders), who enter the market in the first period and bid for the u_1 random supply of the risky asset. They know that the true value, θ, will be realized at the end of the second period only. Based on their information, they suspect that the interim price, p_2, probably will not be in line with their fundamental valuation. They trade only in the first period, and they are faced with the problem of whether to ride on this mispricing for short-term gains, or to take a position against it, risking interim loss. We assume that their payoff reflects this trade-off in the following way:

$$\begin{aligned} EU_i &= E(e^{-\rho W_i}) \\ W_i &= \mu d_1^i (p_2 - p_1) + (1-\mu)\, d_1^i (\theta - p_1) = d_1^i \left(\mu p_2 + \theta\,(1-\mu) - p_1\right) = d_1^i \,(s - p_1) \end{aligned}$$

where s is the weighted average of the true value and the interim price and μ is the measure of the trade off. We look at W_i as purely monetary, where the first part is received by the manager when p_2 is realized as the remuneration for the short-term gains/losses and the second part is received when θ is realized as the bonus – or penalty – for the ability to guess the true value. The constant μ reflects the relative weight of these two aspects in the remuneration package of the manager. Note, that $\mu = 0$ is the long-term investor case – who can trade only once – and when the $\mu = 1$ the investor is myopic. Just as in Kyle, they submit demand curves and the market clears at p_1.

Now, until this point the insight of the model looks similar to many limits to arbitrage models like Shleifer and Visny (1997) or DeLong et al (1990). There are traders with an interest in the interim price. The main novelty of this paper is that the price in the second period is not driven by irrational traders i.e. there is no noise trader risk. The second period price, p_2 is formed in a similarly rational way as the first one. There are n_2 informed traders (new entrants) that enter and bid for the u_2 aggregate random supply of the asset. The true value is realized at the end of the second period. The second period traders maximize a CARA utility function over their profit $W_2^j = d_2^j\,(\theta - p_2)$ with the same risk-aversion coefficient ρ as the first period traders. They are not faced with trade-off. Just as in the first period, traders in the second period submit demand curves and p_2 is the market-clearing price.

The driving force of the model is the information structure. We follow the strategy that in this section we use the simplest signal structure which is consistent with our story and we delay the discussion of a general class of signal structures which is expected to result in similar effects until the next section. Hence, here we assume that the true value, θ, is determined as the sum of two independent factors, θ_1 and θ_2. We think of them as projects of the firm – which produce the cash flows that the risky asset is a claim of – which are generating signals in different time periods, probably because they are not started at the same time or they generate profit in different horizons. Consequently, each trader in the first period observes a private signal on the first factor, and each trader in the second period observes a private signal on the second factor. Additionally, there is a public signal, y, on θ observed by all players when they enter. We think of the public signal as a firm or industry specific earning forecast, a credit rating or other indicator which gives information on the aggregate value of the firm's projects announced by the firm itself or other agencies. We assume that all noisy terms, the random supply and factors are iid normal. Formally:

$$\theta = \theta_1 + \theta_2, \quad y = \theta + \eta,$$
$$x_i = \theta_1 + \varepsilon_i, \quad x'_j = \theta_2 + \varepsilon_j$$
$$\theta_1, \theta_2 \tilde{} N(0, \frac{1}{\gamma}), \quad u_1 \tilde{} N(0, \frac{1}{\delta_1}), \quad u_2 \tilde{} N(0, \frac{1}{\delta_2})$$
$$\eta \tilde{} N(0, \frac{1}{\beta}), \quad \varepsilon_i \tilde{} N(0, \frac{1}{\alpha})$$

Although we will argue in the next section that our story is robust to a large class of signal structures, it is important to stress that even this specific example may catch an important insight of the nature of information in financial markets. Our argument is that public signals, like quarterly reports, credit rating updates and forecasts are tend to come in regular intervals tend to contain information on the whole company. In contrast, private signals are just generated from time to time when there is a leakage of information and they are frequently about parts of the value of the company: some new projects, business plans etc. Hence, the situation that some public information is announced at the beginning of the period and then insiders with information on some factors have to calculate with the possibility of new entrants with information on other factors can naturally arise in many markets. Furthermore, economists think of the value of a firm as the sum of value of the activities of the firm; a view which is consistent with our formulation of θ.

We will search for Perfect Bayesian Nash equilibria of the game i.e. we want to find , $d_1(x_i, y, p_1)$ and $d_2\left(x'_j, y, p_2, p_1\right)$demand schedules of first period traders and second period traders respectively, satisfying

$$\begin{aligned}
E(e^{-\rho(d_2^j(x'_i,y,p_1)(\theta-p_2))}) &\geq E(e^{-\rho(d_2^{j\prime}(x'_i,y,p_1)(\theta-p_2))}) \qquad \nabla j, \nabla d_2^{j\prime} && (2)\\
E(e^{-\rho(d_1^i(x_i,y,p_1)(s-p_1))}) &\geq E(e^{-\rho(d_1^{i\prime}(x_i,y,p_1)(s-p_1))}) \qquad \nabla i, \nabla d_2^{i\prime} && (3)\\
\sum_{i=1}^{n_1} d_1^i(x_i, y, p_1) &= u_1 && (4)\\
\sum_{j=1}^{n_2} d_2^j\left(x'_j, y, p_1\right) &= u_2 && (5)
\end{aligned}$$

and second period beliefs are updated by Bayes Rule[4]. Just as Kyle (1989), we will restrict our

[4]Kyle (1989) discusses in length the necessary technical conditions on the demand functions and the market-clearing rules to have a well defined equilibrium concept. The demand function restrictions will not be relevant here as we focus our attention to linear demand functions. Regarding the market clearing conditions, Kyle specifies arbitrary rules on

attention to symmetric equilibria where the demand functions are linear in the signals and in the price. To find the equilibria we use the method of undetermined coefficients. The details of the derivation are in appendix A, here we just give an intuition of the main steps. Firstly, we assume that demand functions have the following form:

$$d_i^1 = b_1 x_i + c_1 y - e_1 p_1 \tag{6}$$
$$d_j^1 = b_2 x'_j + c_2 y + f_2 p_1 - e_2 p_2 \tag{7}$$

where $b_1, c_1, e_1, b_2, c_2, f_2,$ and e_2 are – not necessarily positive – coefficients. Secondly, using the market clearing conditions (4) and (5), we determine the equilibrium price for the given coefficients. It is shown in Kyle (1989) that given the CARA-Normal framework and the oligopolistic structure of the market the equilibrium demand functions will have the form of

$$d_i^1 = \frac{E(s|p_1, x_i, y) - p_1}{\rho var(s|p_1, x_i, y) + \lambda_1} \tag{8}$$
$$d_j^2 = \frac{E(\theta|p_2, x'_j, y, p_1) - p_2}{\rho var(\theta|p_2, x'_j, y, p_1) + \lambda_2} \tag{9}$$

where $\lambda_1 = \frac{1}{(n_1-1)e_1}$ and $\lambda_2 = \frac{1}{(n_2-1)e_2}$ are the slope of the residual supply curve in the first and in the second period respectively[5]. Furthermore, the second order conditions for the first and second periods are

$$\rho var(s|p_1, x_i, y) + 2\lambda_1 > 0 \tag{10}$$
$$\rho var(\theta|p_2, x'_j, y, p_1) + 2\lambda_2 > 0. \tag{11}$$

Thirdly, we derive the conditional expectation and conditional variance of the random variables θ and s needed for (8) and (9) given the coefficients. Note, that the properties of normally distributed random variables secures that expectation terms are linear and variance terms are constants in signals and the price. Hence, in the final step, we can equate the coefficients of p_1, x_i and y in (8) and (6) and those of p_2, x'_j, y and p_1 in (9) and (7). As the outcome of these steps, equilibrium is given by two second-order conditions and seven equations – one for each coefficient – all of them are given in the appendix.

Although we do not have a general proof of the existence and uniqueness of the equilibrium, we can show it in certain important subcases. We present them in the following two propositions.

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the choice of equilibrium price of the auctioneer when more than one price clears the market and when the aggregate demand would be infinite for any price. We refer the interested reader to the original paper noting that none of these cases will happen in equilibrium in our model.

[5] Note, that the first period demand – apart from the λ term of the imperfect competition structure – corresponds to the first-period demand function (1) of a long-term, twice-trading speculator in the standard model with the substitution of μ satisfying $\frac{\mu}{(1-\mu)} = \frac{(A-B)}{B}$. Hence, we arrived to the same form of first period demand functions by exogenously assuming the trade-off between short-term and long term gains, instead of deriving them from the primitives. In this sense, our analysis incorporates the standard, long-term CARA-Normal case (see also Proposition 7).

Proposition 1 *The game has a unique linear symmetric Perfect Bayesian Nash equilibrium in the following subcases.*

1. *When* $\mu = 0$ *or*
2. *when* $\delta_1 \to 0$.

Proof. Proof is in the appendix. ■

Proposition 2 *For each combination of the parameters other than* μ*, there is a unique* $\mu = \mu^*$ *which corresponds to an equilibrium with* $b_1 = 0$*. This* μ^* *is in the interior of the unit interval.*

Proof. Proof is in the appendix. ■

When $\mu = 0$, first period traders care about the fundamental value only, so from their perspective they are in a Kyle(1989)-type one period model with the only difference being of the presence of the public signal. So it is not surprising that we have the same result of existence and uniqueness as in the original model. The case of $\delta_1 \to 0$, is much more interesting. It shows that when the supply shock in the first period is very large – when we effectively assume away learning from first period prices –, then we can prove existence and uniqueness. From the point of our story, the existence result in proposition 2 is of crucial importance. It shows that even if there is learning, there will always be an equilibrium where the trade-off between short-term fluctuations and fundamental value is such that first period traders ignore their private information completely ($b_1 = 0$). It means that even if their private information tells them that the price is too high or too low compared to the fundamental value, they will not trade on this information fearing that price fluctuations in the short run will cause too much short-term losses. The main point is that this fear comes from the understanding of the interaction between the information structure and new entrants' rational trading decisions in the second period. Early traders know that first period prices will not transmit enough information to second period traders to make them drive the price to the fundamental value. Just the opposite: rational traders in the second period drive away the price from first period traders' estimate of the true value. This is the rational trader risk.

In the next proposition we describe the behaviour of the coefficients in any equilibrium.

Proposition 3 *In any linear equilibrium of the game* b_2 *and* e_2 *are positive. Regarding the signs of the rest of the coefficients, there are the following subcases.*

1. *When* $\frac{\gamma e_2}{(\gamma e_2 + \beta b_2)} > \mu$ *– the small* μ *case –,* f_2, b_1, c_1, e_1 *are also positive.*
2. *When* $\frac{\gamma e_2}{(\gamma e_2 + \beta b_2)} = \mu$ *– the* $\mu = \mu^*$ *case –,* $f_2 = 0$, $b_1 = 0$ *and* c_1, e_1, c_2 *are positive.*
3. *When* $\frac{\gamma e_2}{(\gamma e_2 + \beta b_2)} < \mu$ *– the large* μ *case –,* $b_1 < 0$*, and depending on the parameters either*
   (a) $f_2 < 0$ *and* $e_1, c_1, c_2 > 0$ *or*
   (b) $f_2 > 0$ *and* $e_1 < 0$.
   Furthermore, as $\delta_1 \to 0$ *– first period price becomes uninformative – the system converges to 3.a in the large* μ *case.*

Proof. Proof is in the appendix. ■

These results are very intuitive. Second period traders are interested in the true value only, so they behave as it is usual in traditional models, the higher their private signal and the lower the price, the more they buy of the asset ($b_2, e_2 > 0$). However, in the first period the situation is different. Let us consider first the two polar cases: when $\mu = 0$ first period traders are interested in the true value only. Just as in the second period and as in our wallet game, when they have to guess the true value, the higher their private information, the more they buy of the asset ($b_1 > 0$). In the other polar case, when $\mu = 1$, they interested in the interim price only. They behave as our students in the wallet game when they had to guess the guess of the other player. They do so, because second period prices are very much related to the guess of second period traders, so if first period traders want to receive short-term gains, they have to guess what latecomers will guess. Furthermore, this second order expectation will be negatively related to the private signal of the trader by the same logic as in the wallet game ($b_1 < 0$). They observe the public signal on the aggregate value, so the higher the factor which they have private information on, the lower the other factor has to be to keep the aggregate value in line with the public signal. And the higher the other factor, the higher the guess of traders in the second period will be. Between the two polar cases, the two opposite effects are weighted differently, and the sign of b_1 changes accordingly. Hence, there must be a particular μ^* where the two forces exactly cancel out ($b_1 = 0$). The signs of c_1 and c_2 are not of much interest, when we know something about them, then their signs are positive. What is more surprising is the behaviour of f_2 and e_1 when b_1 is negative. Case 3.a. is simpler to understand. Here, f_2 is negative, so second period traders demand less if they learn that the equilibrium price in the previous period was high. In standard models it cannot happen, as there higher prices mean higher private signals, which in turn mean higher fundamental value. But here, first period prices work as inverse signals. As b_1 is negative, the information content of first period prices will be just the opposite of the standard case. If first period price is high, it means high demand, which must be the result of low private signals. Therefore, a rational trader in the second period will infer that the asset is bad and demand less of it. In this case, positive e_1 shows that demand functions are negatively sloped as usual. However, in case 3.b, e_1 will be negative i.e. the higher the current price, the more first period traders want to buy. In this model, the only possible reason, if the positive information content in the current price outweighs the negative effect that actually they have to pay more for the asset. It can happen when first period price is a strong positive signal for second period traders i.e. when f_2 is high and positive. Hence, in these high-μ equilibria first period traders expect that higher current price will increase second period prices enough to make it rewarding to pay more for them. But for this f_2 has to be positive even if b_1 is negative which seems to be in contradiction with our argument on negative f_2 in the 3.a case. It is not, because these are equilibria with positively sloped demand curves in the first period ($e_1 < 0$): the auctioneer works in the opposite way as usual. If at the actual price demand was too high, she would *increase* the price. Hence, high first period price means low first period demand which means high first period signals. So first period price is a positive signal of the true value. This is equilibrium logic: f_2 can be positive, because e_2 is negative, and e_2 can be negative, because f_2 is very high and positive. Given that it is a signalling argument, it is intuitive that when the supply shock is too noisy – so learning is impossible – we cannot have an equilibrium of the 3.b type. We end up in 3.a. To sum up, when the weight on short-term fluctuations, μ, is high enough, traders trade against their fundamental information because they expect the second period price to move in the opposite direction. In this case, first period prices might become inverse signals of the true value, or first period demand functions may be positively sloped because the expected signalling effect makes it worth to buy more of an asset with a higher price. We illustrate these analytical results with numerical computer simulations shown in Figures 1-3 (3.a case for large μ) and in Figures 4-6 (3.b case for large μ).

Now, we turn our attention towards the effect of these informational problems to market efficiency and pricing. We measure informational efficiency of the prices by the additional percentage decrease in the conditional variance of the fundamental value due to the observation of the price. Our findings are presented in the following proposition.

Proposition 4 *The information content of p_1, defined by $1-\frac{var(\theta|y,p_1)}{var(\theta|y)}$, the reduction in the conditional variance as a result of observing first period price, is positively related to absolute value of b_1, and it reaches its minimum value zero at μ^*. The information content of p_2, defined by $1-\frac{var(\theta|y,p_1,p_2)}{var(\theta|y,p_1)}$ the additional reduction in the conditional covariance as a result of observing the second period price, is positively related to the absolute value of b_1 and to the absolute value of b_2.*

Proof. Proof is in the appendix. ■

Our result shows that we can expect a u-shaped pattern in the efficiency as μ increases. We know that at $\mu=0$ b_1 is positive, there is a μ^* where $b_1=0$, and then at $\mu=1$ b_1 is negative. Hence, the information content of the first period price is positive when μ is small enough and when it is large enough. However, at μ^* it is zero, which makes sense as this is the point where demand does not depend on private information, consequently the price will not aggregate any of the private information in the market. As second period prices depend on b_1, we expect similarly u-shaped patterns, but the outcome is ambiguous as the measure depends on b_2 as well. The second price measure will be always positive. At μ^*, even though it does not contain any private information from the first period, it still incorporates private information from the second period as b_2 is always positive. Figures 7 illustrates our results[6].

We also asses how first period price over- or underweight different elements of information which are present at the market. Our benchmark is the "perfectly aggregating market", where price is just the expected fundamental value conditional on all public and private information in the joint information set of participants. From the market clearing condition (4) and the linear demand curves (6), it is apparent that the expected price in the first period will be a linear function of the public signal and the average private signal. Namely, the coefficient of y will be $\frac{c_1}{e_1}$ and the coefficient of the average private signal will be $\frac{b_1}{e_1}$. Hence, we can compare these coefficients with the coefficients of the "perfectly aggregating" linear regression of $E(\theta|y,\frac{1}{n_1}\sum x_i)$. When the difference is positive we

[6] For results considering the information content of prices and the over- and underreaction of signals, we present figures corresponding to a parameter combination which results in equilibria of the 3.a type for large μ values. Figures for the case when there are also equilibria of the 3.b type would be very similar.

will conclude that the public or the private piece of information is overweighed. A given piece of information can be over- or underweighted for four separate reasons:

1. Average expectations effect. Even if individual demand curves would reflect the best individual estimates of the fundamental value, the average of the individual guesses reflected in the price will not equal the guess using the average private signal. This is related to the fact that the average private signal is more precise than individual signals. This problem is highlighted in Allen et al (2003).

2. Imperfect competition effect. The price may be influenced by bid-shading of traders, who are aware that their trades affect prices.

3. Signalling effect. Traders with short-term motives will try to influence the price, when second period traders learn from it.

4. Rational trader risk effect. Because of our information structure, traders expect that second period prices will move against them. This affects their trades, which influences first period price formation.

The fourth effect is the most important from the perspective of this paper. In order to separate out these effects, we will first calculate the same measure for the case when the supply is so noisy that the first period price becomes uninformative i.e. $\delta_1 \to 0$, which excludes the signalling effect. We also work out the measure for the hypothetical case when – in addition to the uninformative first period price – traders do not expect that their trades will affect prices, so – instead of (8) – they form their demand by

$$d_i^{1pc} = \frac{E(s|p_1, x_i, y) - p_1}{\rho var(s|p_1, x_i, y)},$$

where the subscript *pc* refers to perfect competition. This assumes away the imperfect competition effect. The following Proposition summarizes the analytical results.

Proposition 5 *Our coefficient by coefficient measure of mispricing, showing the difference between the coefficients of the optimal linear regression of the market information on the true value and the actual coefficients given by*

$$E(p_1|y, \frac{1}{n_1}\sum x_i) - E(\theta|y, \frac{1}{n_1}\sum x_i),$$

shows that in the first period, in cases 2 and 3.a of Proposition 3 and when $\mu = 0$ the auctioneer underreacts $\frac{1}{n_1}\sum x_i$. The effect of y in the first period is ambiguous.

When p_1 is uninformative – $\delta_1 \to 0$ – then in both under the perfect and imperfect competition assumptions, the auctioneer underreacts the average private signal when $\mu = 0$ and the underreaction worsens in a linear rate as μ increases. The public signal is overreacted when $\mu = 0$, it changes linearly as μ increases, but the direction is ambiguous.

Proof. Proof is in the appendix. ■

In Figure 8 we illustrate the separation of different forces affecting the under- and over reaction of signals. The first observation to make is that the imperfect competition and the perfect competition cases are practically indistinguishable, hence the imperfect competition effect is – although not zero – very small. It happens, because the coefficients b_1, c_1, e_1 are affected very similarly by bid shading, so in the fractions $\frac{b_1}{e_s}$ and $\frac{c_1}{e_1}$ most of the effect cancels out. All curves related to the same signal start from the same point. It is so, because at $\mu = 0$ neither the signalling effect nor the rational trader risk effect are in operation, as traders do not have short-term motivations at this point. Hence, at the $\mu = 0$ point, curves differ from 0 only because of the average expectation effect. Consistent with Allen et al (2003), the average expectation effect is positive in the case of the public signal and negative in the case of the private one. As μ increases, the role of the signalling effect and the rational trader risk effect increases. As it is apparent from the $\delta_1 \to 0$ case, the rational trader risk effect worsens the overreaction of the public signal and the effect looks considerably larger than the average expectation effect. This is not surprising as with larger μ, traders react more to their guess of the interim price, which can be very different from their guess on the fundamental value. Numerical simulations with different parameter values seems to indicate that the overreaction of the public signal also always worsens due to the rational trader risk effect, but it is not proved analytically. The differences between the straight lines and the corresponding non-linear curves in Figure 8 show the hectic effect of signalling. In the case of the private signal it remains negative for all μ, but it is not monotone anymore. In the case of the public signal, the aggregate effect becomes ambiguous.

4 Discussion

In the previous section we showed that even if interim prices are determined by fully rational traders who optimally update their information observing past prices, short-term traders may ignore their private information or trade against it. It must be clear now that these results are simple consequences of the fact that our first period trader's expectation on the fundamental value and their expectation on the second period price move in the opposite direction as their private information changes. In this section we will discuss the critical properties of the information structure and the role of new entrants in this phenomenon. As the normality assumption has a preponderant role in asset pricing models with asymmetric information, we restrict ourselves here to the normal case. However, in a companion paper, Kondor (2004), we show that our results can be derived from a particular modification of the affiliation concept and it can be applied to many problems outside of finance.

4.1 The information structure and the role of new entrants

In financial models with asymmetric information – just as in our model – there are factors which determine the common value of the asset and there are signals on these factors which determine the information sets of agents. Typically (see Brunnermeier, 2001 for a detailed survey), the structure used is with a normally distributed factor, private signals which are noisy versions of this factor with iid normally distributed noise terms and possibly there is a similar public signal:

$$\begin{aligned}\theta &= \theta_1,\ x_i=\theta_1+\varepsilon_i, \quad y=\theta_1+\eta \qquad (12)\\ &\theta_1\tilde{\ }N(0,\frac{1}{\gamma}), \quad \eta\tilde{\ }N(0,\frac{1}{\beta}), \quad \varepsilon_i\tilde{\ }N(0,\frac{1}{\alpha}).\end{aligned}$$

The problem with this structure is not that there is only one factor. The problem is that this results in a very rigid structure: the covariance of any two of the public signal, the private signal and the fundamental value is the same: $\frac{1}{\gamma}$, which is also the variance of the fundamental value. Let us consider instead, a general normally distributed system of the fundamental value and signals:

$$\left(\theta, x, x', y\right) \sim N^{1+n_1+n_2+1}(0,\Sigma)$$

where x and x' are vectors of signals of two groups of traders, θ is the fundamental value and y is the public signal as before. Just like in our wallet game with a public signal example, we are interested in structures where the first order guess of a trader is increasing, but the trader's guess of someone's guess in the other group is decreasing in her private signal:

$$\frac{\partial E\left(\theta|x'_j,y\right)}{\partial x'_j} > 0 \qquad (13)$$

$$\frac{\partial E(E\left(\theta|x'_j,y\right)|x_i,y)}{\partial x_i} < 0. \qquad (14)$$

Condition (13) is quite natural. If it does not hold, we cannot really call x_j a "signal on the fundamental value". So we will simply assume it.[7] The following proposition shows that in addition to (13), for (14) we simply need that the correlation between the private signals of traders in different group is not too high.

Proposition 6 *Let (θ, x, x', y) is a jointly normally distributed system of $1+n_1+n_2+1$ dimension. Let us assume that*

$$\frac{\partial E\left(\theta|x'_j,y\right)}{\partial x'_j} > 0.$$

Then

$$\frac{\partial E(E\left(\theta|x'_j,y\right)|x_i,y)}{\partial x_i} < 0$$

if and only if

$$\rho_{x_ix'_j} < \rho_{x_i,y}\rho_{x'_jy}, \qquad (15)$$

where $\rho_{z_1z_2}$ is the linear correlation between the variables in the subscript.

[7] However, it is easy to show that (13) holds if and only if

$$corr(\theta,x_j) > corr(\theta,y)corr(y,x_j).$$

Proof. By the projection theorem $E\left(\theta|x_j',y\right)$ is linear and normally distributed. Therefore $E(E\left(\theta|x_j',y\right)|x_i,y)$ is also linear and

$$\frac{\partial E(E\left(\theta|x_j',y\right)|x_i,y)}{\partial x_i}=\frac{\partial E\left(x_j'|x_i,y\right)}{\partial x_i}\frac{\partial E\left(\theta|x_j',y\right)}{\partial x_j'},$$

which has the same sign as the first term by assumption. Using the projection theorem again

$$\frac{\partial E\left(x_j'|x_i,y\right)}{\partial x_i}=\frac{\sigma_{x_i}}{\sigma_{x_j'}}\frac{\rho_{x_i x_j'}-\rho_{x_i,y}\rho_{x_j' y}}{1-\rho^2_{x_j' y}},$$

where σ_z terms are standard errors. Observing that the first term and the denominator of the second term are always positive, gives the result. ■

It is easy to see that the particular information structure we used in our model satisfied (15) for all parameters as $\rho_{x_i,x_j'}$ was zero. In contrast, in the standard asymmetric information set-up of (12), (15) simplifies to $\frac{\alpha}{\alpha+\gamma}<\alpha\frac{\beta}{(\alpha+\gamma)(\beta+\gamma)}$, which obviously never holds. This illustrates well the rigidity of the standard system.

Condition (15) is very intuitive. It shows that the opposite movement of first and second order expectations driving our results is expected to arise in any situation where the nature of the information of the two groups are different and there is a public signal which is related to both pieces of information. Hence, if we are talking about two similar groups with access to the same type of sources, analyzing data with the same methods, we should not expect our effects to arise. However, when a group of agents forming expectations of the expectations of another group with different characteristics, (15) is very realistic to hold. Here, we may think of fund-managers versus non-professional investors, technical analysts versus fundamentalists, investment banks allowed to participating in an IPO or in a Treasury bill auction versus participants in the secondary market etc.

Until now, we have not say anything about price-formation. It is important for two separate reasons. Firstly, prices convey information, so traders will learn from them and they will try to use them as signals for future traders. Therefore, we should know whether the relationship between private signals and first- and second-order expectations change when prices enter into their information sets. Secondly, we are interested in second-order expectations only to the extent that they translate directly to expectations of the aggregate demand in the next period and – consequently – to expectations of future prices. The answer to the first problem is very model specific. Prices may convey very different type of information in different market-structures. For example, if prices are fully revealing, second-order expectations will be equal to first-order expectations, as the law of iterated expectations will apply. The second aspect is more intuitive. It is clear that the demand of new entrants should depend positively on their expectation of the fundamental value, so second-order expectations of early traders will be closely related to the expected demand of new entrants. The possible problem here is that the weight of the demand of new entrants in the aggregate demand may not be large enough. For example, if we have investors in the first group who trade in both periods, future demand will be related to their own future demand as well. As a trader's expectation of her own future expectation equals to her present expectation by the law of iterated expectations, if both (13) and (14) hold, expected future aggregate demand will be influenced by two factors: the expectation of the own future demand of the trader, which is positively dependant on her private signal, and the expectation of the demand of new entrants, which is negatively dependant on her private signal. Hence, the aggregate effect would be unclear. Our last proposition gives comforting answers for both questions for models where prices are linear in signals. It will state that if (15) holds and the proportion of new entrants is large enough, a first period trader's expectation of the second period price depends negatively on her private signal, even if she uses the information embodied in the first period price optimally and even if first period traders trade in both periods. The proposition also illustrates that if signals satisfy (15), expectations similar to those of the forecasters interviewed by Frankel and Froot (1989), who expected interim prices to increase in contrast to the fundamental value, can be perfectly rational.

Proposition 7 *Let us consider the system of fundamental value, two groups of private signals, a public signal and noise terms*

$$\left(\theta, x_1 ... x_{n_1}, x'_1, ..., x'_{n_2}, y, u_1, u_2\right) \sim N\left(0, \Sigma\right)$$

where private signals of the same group are distributed symmetrically and u_1, u_2 are independent of the private signals. Let first and second period prices be linear functions of signals and noise terms, where private signals of the same group enter symmetrically:

$$\begin{aligned} p_2 &= \frac{1}{n_2 e'_2 + n_1 e_2}\left(b_2 \sum_{i=1}^{n_1} x_i + b'_2 \sum_{j=1}^{n_2} x'_j + c_2 y + f_2 p_1 + u_2\right) \\ p_1 &= \frac{1}{n_1 e_1}\left(b_1 \sum_{i=1}^{n_1} x_i + c_1 y + u_1\right) \end{aligned}$$

where b_i, c_i, e_i $i = 1, 2$ and f_2 are undetermined coefficients, and n_1 and n_2 are the number of traders in the first and second group respectively. If

$$\rho_{x_i x'_j} < \rho_{x_i, y} \rho_{x'_j, y}$$

holds, then there exits an $\overline{n}_2$ that

$$\frac{\partial E\left(p_2 | x_i, y, p_1\right)}{\partial x_i} < 0$$

for all $n_2 > \overline{n}_2$.

Proof. First, observe that

$$\frac{\partial E\left(p_2 | x_i, y, p_1\right)}{\partial x_i} = \frac{1}{n_2 e'_2 + n_1 e_2}\left(b_2 + \left(n_1 - 1\right) b_2 \frac{\partial E\left(x_k | x_i, y, p_1\right)}{\partial x_i} + b'_2 n_2 \frac{\partial E\left(x'_j | x_i, y, p_1\right)}{\partial x_i}\right)$$

by the symmetry assumptions. As neither $\frac{\partial E(x_k|x_i,y,p_1)}{\partial x_i}$ or $\frac{\partial E\left(x'_j|x_i,y,p_1\right)}{\partial x_i}$ depend on n_2, if n_2 is large enough, the sign of $\frac{\partial E\left(x'_j|x_i,y,p_1\right)}{\partial x_i}$ will determine the sign of $\frac{\partial E(p_2|x_i,y,p_1)}{\partial x_i}$. So we have to show that if (15) holds, then $\frac{\partial E\left(x'_j|x_i,y,p_1\right)}{\partial x_i} < 0$. Note, that observing p_1 together with y and x_i is informationaly equivalent to observing

$$h = \frac{1}{n_1 - 1} \sum_{k \neq i} x_k + \frac{u_1}{(n_1 - 1) b_1} = \frac{1}{(n_1 - 1) b_1} \left(n_1 e_1 p_1 - b_1 x_i - n_1 c_1 y \right).$$

It is easy to see that $\sigma_h^2 > \sigma_{x_i}^2$. By the projection theorem

$$\frac{\partial E\left(x'_j|x_i, y, p_1\right)}{\partial x_i} = \frac{\partial E\left(x'_j|x_i, y, h\right)}{\partial x_i} = \left(\sigma_h^2 - cov\left(x_i, x_k\right)\right) \sigma_y^2 \sigma_{x_i} \sigma_{x'_j} \frac{\rho_{x_i x'_j} - \rho_{y x_i} \rho_{y x'_j}}{D},$$

where D is the determinant of the variance-covariance matrix of x_i, y, p_1, so it must be positive. As $\sigma_h^2 > \sigma_{x_i}^2 > cov\left(x_i, x_k\right)$, the proposition holds. ■

5 Concluding remarks

The view that the market may fail to correct mispricing because informed traders have to calculate with interim price fluctuations due to fad-following noise traders – i.e. they have to face noise trader risk – is more and more popular in the finance literature. In this paper we showed that the presence of noise traders is not a necessary part of this argument. We argued instead that *rational trader risk* might result in the same effect. If the information structure is rich enough, there can be situations where traders informed of the mispricing early are unwilling to trade on their information because they have to calculate with the actions of rational new entrants in the future with different information and different beliefs. We also showed that rational trader risk might cause inefficiency and severe mispricing.

We consider the main strength of our model being that despite of its simplicity, it can replicate several phenomena which are usually associated with irrational behaviour. Besides the limits to informed trading and inefficiency, it may also result in opposite short-term and long-term expectations similar to those of the forecasters in Frankel and Froot's (1988) example, and we showed examples with positively sloped demand curves in equilibrium which can be interpreted as positive feed-back trading.

In a related paper, Kondor (2004), we show how our informational requirements can be generalized for contexts with general distributions and we explore some consequences in non-financial applications. For further research we are considering to incorporate our information structure in a dynamic general asset-pricing framework with no parametric assumptions on the utility function or the distribution of signals. We are convinced that our story will remain robust to any reasonable model as long as the number of informed traders is finite. We also plan to analyse the normative consequences of our information structure in a future work. At this point, it is worth to mention the possible implications to the decision maker's problem who can decide to reveal a public signal or keep it in secret. It was clear already in our wallet-game example, that the presence of public signal is crucial. If the teacher had not announced her guess in advance, the second-order expectation of the students would not have been negatively related to their private signal and all of our results would have disappeared. Hence, in a market where second- and higher-order expectations matter, we expect non-trivial consequences of public announcements. The efficient mechanism for selling objects in information structures similar to ours is also an interesting question for the future.

However, probably the most important task is to identify markets and periods where the information structure is similar to the one in our set-up and test our model. Following the lines of our microstructure model, one should concentrate on markets where there is a group of traders with specific information arriving early to the market with the intention to buy assets for resale. Such possible applications include IPO-s with a preferred customer group, treasury bill auctions where only particular financial institution may participate and the 24-hour FX market where traders in different time zones enter the market in different time. After all, the question of real-world relevance of rational trader risk is an empirical question.

Appendix

A.1 Deriving the system of equations determining equilibrium

Let us start with the second period. First we have to specify $E(\theta|p_1, x_j', y)$ and $var^{-1}(\theta|p_1, x_j', y)$. Note, that observing y and p_1 is equivalent to observing

$$g = \frac{e_1}{b_1}p_1 - \frac{a_1}{b_1} - \frac{c_1}{b_1}y = \theta_1 + \frac{1}{n_1}\sum \varepsilon_j + \frac{1}{b_1 n_1}u_1$$

where we used the market clearing condition

$$n_1 b_1 \theta_1 + b_1 \sum \varepsilon_i + n_1 c_1 y + u_1 = n_1 e_1 p_1. \tag{16}$$

The variance of this signal is

$$\begin{aligned} var(g) &= \frac{1}{\gamma} + \frac{1}{n_1\alpha} + \frac{1}{b_1^2 n_1^2 \delta} = \frac{1}{\gamma} + \frac{1}{\tau_g} \\ \tau_g &= \frac{b_1^2 \delta_1 \alpha_1 n_1^2}{\left(\alpha_1 + n_1 \delta_1 b_1^2\right)}. \end{aligned}$$

Similarly, observing p_2 together with y, p_1 and x_i is informationaly equivalent to observing

$$\begin{aligned} h_{i2} &= \theta_2 + \frac{1}{n_2 - 1}\sum_{k \neq j} \varepsilon_k + \frac{u_2}{(n_2-1)\, b_2} = \frac{1}{(n_2-1)\, b_2}\left(n_2 e_2 p_2 - n_2 c_2 y - n_2 f_2 p_1 - b_2 x_j'\right) \\ var\left(h_{i2}\right) &= \frac{1}{(n_2-1)\,\alpha_2} + \frac{1}{b_2^2 (n_2-1)^2 \delta_2} = \frac{1}{\gamma} + \frac{1}{\tau_2} \end{aligned}$$

where we used the market clearing condition

$$n_2 b_2 \theta_2 + b_2 \sum \varepsilon_i + n_2 c_2 y + n_2 f_2 p_1 + u_2 = n_2 e_2 p_2. \tag{17}$$

By the projection theorem

$$\begin{aligned}
E(\theta|x_j', y, h_{i2}, g) &= \qquad (18)\\
&= \frac{y\beta\left(2\gamma+\alpha_2+\tau_2+\tau_g\right)+g\tau_g\left(\gamma+\alpha_2+\tau_2\right)+\alpha_2 x_j'\left(\gamma+\tau_g\right)+\tau_2 h_{2i}\left(\gamma+\tau_g\right)}{\left(2\beta\gamma+\beta\alpha_2+\beta\tau_2+\gamma\alpha_2+\gamma\tau_2+\beta\tau_g+\gamma\tau_g+\alpha_2\tau_g+\tau_2\tau_g+\gamma^2\right)} = \\
&= \frac{\begin{pmatrix} \frac{e_2}{b_2}\frac{n_2\tau_2(\gamma+\tau_g)}{n_2-1}p_2 + x_j'\left(\gamma\alpha_2+\alpha_2\tau_g-\frac{\tau_2(\gamma+\tau_g)}{n_2-1}\right)+ \\ +p_1\left(\frac{e_1}{b_1}\tau_g\left(\gamma+\alpha_2+\tau_2\right)-\frac{1}{b_2}\frac{f_2 n_2\tau_2(\gamma+\tau_g)}{n_2-1}\right)+ \\ y\left(\beta\left(2\gamma+\alpha_2+\tau_2+\tau_g\right)-\frac{1}{b_1}c_1\tau_g\left(\gamma+\alpha_2+\tau_2\right)-\frac{1}{b_2}c_2 n_2\frac{\tau_2(\gamma+\tau_g)}{n_2-1}\right)\end{pmatrix}}{\left(2\beta\gamma+\beta\alpha_2+\beta\tau_2+\gamma\alpha_2+\gamma\tau_2+\beta\tau_g+\gamma\tau_g+\alpha_2\tau_g+\tau_2\tau_g+\gamma^2\right)}
\end{aligned}$$

and

$$var(\theta|x_j', y, h_{i2}, g) = \frac{\left(2\gamma+\alpha_2+\tau_2+\tau_g\right)}{\left(2\beta\gamma+\beta\alpha_2+\beta\tau_2+\gamma\alpha_2+\gamma\tau_2+\beta\tau_g+\gamma\tau_g+\alpha_2\tau_g+\tau_2\tau_g+\gamma^2\right)}. \tag{19}$$

After some straightforward, but tedious algebra, substituting (18) and (19) into (8) and equating the coefficients of b_2, e_2, f_2 with those in (6) result in the following equations

$$\begin{aligned}
0 &= F\left(b_2\right) = \rho b_2\left(n_2-1\right)\alpha_2\left(2\gamma\alpha_1+\alpha_1\alpha_2+\delta_1 n_1 b_1^2\left(2\gamma+\alpha_2+n_1\alpha_1\right)\right)- \qquad (20)\\
&\quad -\left(n_2-2\right)\alpha_2^2\left(\gamma\alpha_1+\delta_1 n_1 b_1^2\left(\gamma+n_1\alpha_1\right)\right)+ \\
&\quad +\rho b_2^3\left(n_2-1\right)\delta_2\left(n_2-1\right)\left(2\gamma\alpha_1+n_2\alpha_1\alpha_2+\delta_1 n_1 b_1^2\left(2\gamma+n_1\alpha_1+n_2\alpha_2\right)\right)+ \\
&\quad +b_2^2\delta_2\alpha_2 n_2\left(n_2-1\right)\left(\gamma\alpha_1+\delta_1 n_1 b_1^2\left(\gamma+n_1\alpha_1\right)\right)
\end{aligned}$$

$$e_2 = e_2\left(b_2\left(b_1\right), b_1\right) = \frac{k_1 b_1^2 + k_0}{l_1 b_1^2 + l_0} \tag{21}$$

$$f_2 = f_2\left(b_2\left(b_1\right), b_1, e_1\right) = \frac{m_1 e_1 b_1}{l_1 b_1^2 + l_0} \tag{22}$$

where coefficients k_1, k_0, l_1, l_0, m_1 are the functions of b_2 only, and they are always positive:

$$\begin{aligned}
k_1 &= \left(n_2-2\right)\begin{pmatrix} \delta_2 n_1\left(n_2-1\right)\delta_1\left(\beta\alpha_2 n_2+\gamma\alpha_2 n_2+n_2 n_1\alpha_1\alpha_2+\gamma^2+n_1\alpha_1\beta+2\beta\gamma+n_1\alpha_1\gamma\right)b_2^2+ \\ +n_1\delta_1\alpha_2\left(\beta\alpha_2+\gamma\alpha_2+n_1\alpha_1\alpha_2+\gamma^2+n_1\alpha_1\beta+2\beta\gamma+n_1\alpha_1\gamma\right)\end{pmatrix} \\
k_0 &= \left(n_2-2\right)\delta_2\left(n_2-1\right)\alpha_1\left(\gamma\alpha_2 n_2+\beta\alpha_2 n_2+2\beta\gamma+\gamma^2\right)b_2^2+\left(\gamma\alpha_2+\gamma^2+2\beta\gamma+\beta\alpha_2\right)\alpha_1\alpha_2 \\
l_1 &= \begin{pmatrix} \rho\delta_2 n_1\left(n_2-1\right)^2\delta_1\left(\alpha_2 n_2+2\gamma+n_1\alpha_1\right)b_2^2+b_2 n_2 n_1\delta_1\left(n_1\alpha_1+\gamma\right)\left(n_2-1\right)^2\alpha_2\delta_2 \\ +\rho n_1\delta_1\alpha_2\left(\alpha_2+2\gamma+n_1\alpha_1\right)\left(n_2-1\right)\end{pmatrix} \\
l_0 &= \rho\delta_2\left(n_2-1\right)^2\alpha_1\left(\alpha_2 n_2+2\gamma\right)b_2^2+\gamma\left(n_2-1\right)^2\alpha_2\delta_2 b_2\alpha_1 n_2+\left(n_2-1\right)\rho\left(2\gamma+\alpha_2\right)\alpha_1\alpha_2 \\
m_1 &= \left(n_2-2\right)\delta_1\alpha_1 n_1^2\left(\delta_2\left(n_2-1\right)\left(\alpha_2 n_2+\gamma\right)b_2^2+\left(\gamma+\alpha_2\right)\alpha_2\right).
\end{aligned}$$

Furthermore, the equation for c_2 will be

$$c_2 = (n_2 - 2) \frac{\left(2\beta\gamma + \beta\alpha_2 + \beta\tau_2 + \beta\tau_g - \frac{1}{b_1} c_1 \left(\gamma\tau_g + \alpha_2\tau_g + \tau_2\tau_g\right)\right)}{\left((n_2 - 1)\,\rho\,(2\gamma + \alpha_2 + \tau_2 + \tau_g) + \frac{1}{b_2} n_2 \left(\gamma\tau_2 + \tau_2\tau_g\right)\right)}. \tag{23}$$

where τ_2 and τ_g are functions of b_2 defined above, and they are also always positive.

Repeating the same steps for the first period, calculating the conditional expectation of θ and p_2 conditional on the information set of a trader i and forming the weighted average, s, results in

$$E(s|x_i, p_1, y) = k_y y + k_{p_1} p_1 + k_{x_i} x_i$$

where

$$k_y = \frac{\left((1-\mu)\left(\beta(2\gamma+\alpha_1+\tau_1) - \frac{n_1 c_1 \gamma\tau_1}{(n_1-1)b_1}\right) + \mu\frac{1}{e_2}\left(b_2\left(\beta(\alpha_1+\tau_1+\gamma) + \frac{\tau_1\beta}{(n_1-1)b_1} n_1 c_1\right) + c_2\left(\alpha_1\beta+\gamma\alpha_1+\tau_1\beta+\gamma\tau_1+2\gamma\beta+\gamma^2\right)\right)\right)}{(2\beta\gamma+\beta\alpha_1+\beta\tau_1+\gamma\alpha_1+\gamma\tau_1+\gamma^2)}$$

$$k_{p_1} = \frac{\left((1-\mu)\gamma\tau_1 \frac{1}{(n_1-1)b_1} n_1 e_1 + \mu\frac{1}{e_2}\left(b_2\left(-\frac{\tau_1\beta}{(n_1-1)b_1} n_1 e_1\right) + f_2\left(\alpha_1\beta+\gamma\alpha_1+\tau_1\beta+\gamma\tau_1+2\gamma\beta+\gamma^2\right)\right)\right)}{(2\beta\gamma+\beta\alpha_1+\beta\tau_1+\gamma\alpha_1+\gamma\tau_1+\gamma^2)}$$

$$k_{x_j} = \frac{\left((1-\mu)\left(\gamma\alpha_1 - \frac{\gamma\tau_1}{(n_1-1)}\right) + \mu\frac{1}{e_2} b_2\left(-\alpha_1\beta + \frac{\tau_1\beta}{(n_1-1)}\right)\right)}{(2\beta\gamma+\beta\alpha_1+\beta\tau_1+\gamma\alpha_1+\gamma\tau_1+\gamma^2)}$$

and the variance is

$$var\,(s|y, p_1, x_i) =$$

$$= \mu^2 \frac{b_2^2 n_2 \delta_2 + \alpha_2}{e_2^2 \alpha_2 n_2^2 \delta_2} + \frac{(\alpha_1 + \tau_1 + 2\gamma)(1-\mu)^2 + 2\frac{b_2}{e_2}(\alpha_1 + \tau_1 + \gamma)\,\mu\,(1-\mu) + \frac{b_2^2}{e_2^2}(\alpha_1 + \tau_1 + \beta + \gamma)\,\mu^2}{(\alpha_1\beta + \gamma\alpha_1 + \tau_1\beta + \gamma\tau_1 + 2\gamma\beta + \gamma^2)}.$$

The resulting equations for the coefficients are

$$\begin{aligned} 0 &= G(b_1) = \\ &= \left(\begin{array}{c} (n_1-1)^2\left(1 - \mu\frac{f_2}{e_2}\right) o_1 b_1^3 + (n_1 - 1)\, o_2 b_1^2 + \\ + (n_1-1)^2\left(1 - \mu\frac{f_2}{e_2}\right) o_0 b_1 - \left((n_1 - 2) - \mu\frac{f_2}{e_2}(n_1 - 1)\right) \alpha_1 o_2 \frac{1}{\delta_1}\frac{1}{n_1} \end{array}\right) \end{aligned} \tag{24}$$

$$\begin{aligned} e_1 &= e_1\left(\frac{b_2}{e_2}(b_1), b_1\right) = \left(\frac{n_1 - 2}{n_1 - 1} - \mu\frac{f_2}{e_2}\right)\frac{v_1 b_1^2 + v_0}{o_1 b_1^2 + o_2 b_1 + o_0} = \\ &= \frac{\frac{n_1-2}{n_1-1}\left(v_1 b_1^2 + v_0\right)\left(k_1 b_1^2 + k_0\right)}{\left(k_1 b_1^2 + k_0\right)\left(o_1 b_1^2 + o_2 b_1 + o_0\right) + \left(v_1 b_1^2 + v_0\right)\mu m_1 b_1} \end{aligned} \tag{25}$$

with

$$
\begin{aligned}
o_1 &= \rho\delta_1(n_1-1)\left(\begin{array}{c}(1-\mu)^2\frac{b_2^2n_2\delta_2+\alpha_2}{e_2^2\alpha_2n_2^2\delta_2}\left(2\beta\gamma+\beta n_1\alpha_1+\gamma n_1\alpha_1+\gamma^2\right)+\\ +(2\gamma+n_1\alpha_1)(1-\mu)^2+2\frac{b_2}{e_2}(\gamma+n_1\alpha_1)\mu(1-\mu)+\frac{b_2^2}{e_2^2}(\beta+\gamma+n_1\alpha_1)\mu^2\end{array}\right)\\
o_0 &= \rho\alpha_1\left(\begin{array}{c}\mu^2\frac{b_2^2n_2\delta_2+\alpha_2}{e_2^2\alpha_2n_2^2\delta_2}\left(2\beta\gamma+\alpha_1\beta+\alpha_1\gamma+\gamma^2\right)+(\alpha_1+2\gamma)(1-\mu)^2+\\ +2\frac{b_2}{e_2}(\alpha_1+\gamma)\mu(1-\mu)+\frac{b_2^2}{e_2^2}(\alpha_1+\beta+\gamma)\mu^2\end{array}\right)\\
o_2 &= (n_1-1)\alpha_1\delta_1n_1\frac{(1-\mu)\gamma e_2-\mu b_2\beta}{e_2}\\
v_1 &= \delta_1(n_1-1)\left(n_1\alpha_1\beta+n_1\alpha_1\gamma+2\beta\gamma+\gamma^2\right)\\
v_0 &= \left(2\beta\gamma+\alpha_1\beta+\alpha_1\gamma+\gamma^2\right)\alpha_1.
\end{aligned}
$$

Observe, that o_1, o_2, o_0 are depending on $\frac{b_2}{e_2}$only and o_1, o_0, v_1, v_0 are positive whenever $\frac{b_2}{e_2}$ is positive, and the sign of o_2 is the same as the sign of $(1-\mu)\gamma e_2 - \mu b_2\beta$. Furthermore,

$$
c_1 = \frac{\frac{((n_1-2)e_2-\mu f_2(n_1-1))\left(\mu_c\beta(2\gamma+\alpha_1+\tau_1)+\mu\frac{1}{e_2}b_2\beta(\alpha_1+\tau_1+\gamma)+\mu\frac{1}{e_2}c_2\left(\alpha_1\beta+\gamma\alpha_1+\tau_1\beta+\gamma\tau_1+2\gamma\beta+\gamma^2\right)\right)}{\rho(n_1-1)(e_2-\mu f_2)}}{\left(\begin{array}{c}\left((\alpha_1+\tau_1+2\gamma)\mu_c^2+2\frac{b_2(\alpha_1+\tau_1+\gamma)\mu\mu_c}{e_2}+\frac{b_2^2(\alpha_1+\tau_1+\beta+\gamma)\mu^2}{e_2^2}\right)\\ +\mu^2\frac{\left(b_2^2n_2\delta_2+\alpha_2\right)\left(2\beta\gamma+\beta\alpha_1+\beta\tau_1+\gamma\alpha_1+\gamma\tau_1+\gamma^2\right)}{e_2^2\alpha_2n_2^2\delta_2}\end{array}\right)+\frac{\tau_1n_1(\mu_ce_2\gamma-\beta\mu b_2)}{\rho(n_1-1)b_1e_2}}. \tag{26}
$$

The slope of the residual supply curves in each period comes from the market clearing conditions (16) and (17):

$$
\begin{aligned}
\lambda_1 &= \frac{1}{(n_1-1)e_1}\\
\lambda_2 &= \frac{1}{(n_2-1)e_2}
\end{aligned}
$$

, which gives the second order conditions

$$
2\frac{1}{(n_1-1)e_1}+\rho var(s|y,p_1,x_1) > 0 \tag{27}
$$

$$
2\frac{1}{(n_2-1)e_2}+\rho var(\theta|x_i,y,h_{i2},g) > 0. \tag{28}
$$

Hence, any set of coefficients which satisfies equations (20)-(26) together with the two second order conditions (27) and (28) will give equilibria.

A.2 Existence and uniqueness and coefficients in equilibrium

For the proof of proposition 1 and that of proposition 3, we will need the following two lemmas.

Lemma 1 *If b_1 satisfies (24) and the second order condition (27), then*

1. *either $b_1 > 0$ and $\left(\frac{n_1-2}{n_1-1}-\mu\frac{f_2}{e_2}\right) > 0$ and $o_2, e_1 > 0$ or*

2. $b_1 < 0$ *and* $\left(\frac{n_1-2}{n_1-1} - \mu\frac{f_2}{e_2}\right) > 0$, $o_2 < 0$ *and* $e_1 > 0$ *or*

3. $b_1 < 0$ *and* $\left(\frac{n_1-2}{n_1-1} - \mu\frac{f_2}{e_2}\right) < 0$, $o_2 < 0$ *and* $e_1 < 0$.

Furthermore, $\left(\frac{n_1-2}{n_1-1} - \mu\frac{f_2}{e_2}\right)$ *always have the same sign as* $\left(1 - \mu\frac{f_2}{e_2}\right)$.

Proof. By (25) and (27) the second order condition is

$$2\frac{\left(o_1 b_1^2 \pm o_2 b_1 + o_0\right)}{(n_1-1)\left(\frac{n_1-2}{n_1-1} - \mu\frac{f_2}{e_2}\right)} + o_1 b_1^2 + o_0 > 0.$$

Hence, the second order condition multiplied by $(n_1-1)\, b_1 \left(\frac{n_1-2}{n_1-1} - \mu\frac{f_2}{e_2}\right)$ and $2G(b_1)$ subtracted gives

$$\left(n_1 - 2 - \frac{\mu}{e_2} f_2 (n_1-1)\right)\left(\frac{2}{n_1} o_2 \frac{\alpha_1}{\delta_1} - b_1 (n_1-1)\, o_0 - b_1^3 (n_1-1)\, o_1\right)$$

which must have the same as the sign of $b_1 \left(\frac{n_1-2}{n_1-1} - \mu\frac{f_2}{e_2}\right)$. This results in four possible combinations of signs of b_1, $\left(\frac{n_1-2}{n_1-1} - \mu\frac{f_2}{e_2}\right)$, o_2 and e_1: three of them are in the lemma, and there is an additional one, where $b_1 > 0$, $\left(\frac{n_1-2}{n_1-1} - \mu\frac{f_2}{e_2}\right) < 0$ and $o_2 > 0$. But the last one is not possible, because if $o_2 > 0$ and $b_1 \geq 0$ then

$$\begin{aligned}
\left(\frac{n_1-2}{n_1-1} - \mu\frac{f_2}{e_2}\right) &= \frac{n_1-2}{n_1-1} - \mu\frac{m_1 e_1 b_1}{k_1 b_1^2 + k_0} = \\
&= \frac{n_1-1}{n_1-2}\left(1 - \frac{\left(v_1 b_1^2 + v_0\right)}{\frac{\left(k_1 b_1^2 + k_0\right)}{\mu m_{1 b_1}}\left(o_1 b_1^2 + o_2 b_1 + o_0\right) + \left(v_1 b_1^2 + v_0\right)}\right) > 0.
\end{aligned}$$

Furthermore, $\left(\frac{n_1-2}{n_1-1} - \mu\frac{f_2}{e_2}\right)$ has the same sign as $\left(\frac{n_1-2}{n_1-1} - \mu\frac{f_2}{e_2}\right)$ straightforwardly, in cases 1 and 2 as $\left(1 - \mu\frac{f_2}{e_2}\right) > \left(\frac{n_1-2}{n_1-1} - \mu\frac{f_2}{e_2}\right)$. We have to show only that in case 3 $\left(1 - \mu\frac{f_2}{e_2}\right) > 0 > \left(\frac{n_1-2}{n_1-1} - \mu\frac{f_2}{e_2}\right)$ is not possible. But this comes from the observation of equation (24), as there is no negative root of the equation when $\left(1 - \mu\frac{f_2}{e_2}\right) > 0 > \left(\frac{n_1-2}{n_1-1} - \mu\frac{f_2}{e_2}\right)$ and $o_2 < 0$. ■

Lemma 2 *For any given* b_1^* *and any parameter values,(20), (21) and (28) gives a unique* b_2 *and* e_2 *and both of them will be positive and the second order condition of the second period will be satisfied.*

Proof. First observe that for any b_1,(20) has exactly one positive root and (21) uniquely determines a positive e_2 for any given b_1 and positive b_2. Simple substitution shows that (20) and (21) can be written as

$$\begin{aligned}
&0 = F(b_2, b_1^*) = \\
&= (n-1)\left(\rho\,(2\gamma + \tau_g + n\alpha)\,\delta\,(n-1)\, b_2^3 + n\,(\gamma + \tau_g)\,\alpha\delta b_2^2 + 2\rho\,(\alpha + 2\gamma + \tau_g)\,\alpha b_2\right) - 2\alpha^2 (\gamma + \tau_g)(n-2)
\end{aligned}$$

$$e_2 = \frac{\left(2\beta\gamma + \beta\alpha_2 + \beta\tau_2 + \gamma\alpha_2 + \gamma\tau_2 + \beta\tau_g + \gamma\tau_g + \alpha_2\tau_g + \tau_2\tau_g + \gamma^2\right)}{\rho\,(2\gamma + \alpha_2 + \tau_2 + \tau_g) + \frac{1}{b_2} n_2 \frac{\gamma\tau_2 + \tau_2\tau_g}{n_2 - 1}} \frac{(n_2-2)}{(n_2-1)}.$$

where τ_g is depending on b_1^*. Hence, (28) is equivalent to

$$(n-2)\,\rho\,(\alpha+2\gamma+\tau_g+\tau_2)+2\rho\,(\alpha+2\gamma+\tau_g+\tau_2)+\frac{2\,(\gamma+\tau_g)\,\tau_2}{(n-1)\,b_2}n>0$$

or – after substituting out τ_2 and rearranging – it is also equivalent to

$$\left(2\rho\alpha\,(\alpha+2\gamma+\tau_g)+2\alpha\delta b_2\,(\gamma+\tau_g)\,(n-1)+\rho b_2^2\delta\,(n-1)\,(n\alpha+2\gamma+\tau_g)\right)>0.$$

Now, for a b_2^* which satisfies $0=F(b_2^*,b_1^*)$ and the second order condition, if we multiply the second order condition with b_2^* and subtract $F(b_2^*,b_1^*)$ the result has to be the same sign as b_2^*. But these steps result in

$$2\alpha^2\,(\gamma+\tau_g)\,(n-2)+b_2^2\delta\alpha\,(n-1)\,(n-2)\,(\gamma+\tau_g)>0$$

which is always positive. Consequently, only a positive b_2 can be both the solution of (20) and can satisfy the second order condition. Additionally, (28) shows that any b_2 which gives a positive e_2 satisfies the second order condition. Together with the observation that for any b_1 there is exactly one positive root which satisfies (20), these results show that any b_1^* gives a unique b_2, e_2 which satisfy the second order condition of the second period and both of them are positive. ■

Proof of Proposition 1. When $\mu=0$, $(1-\mu)\,\gamma e_2-\mu b_2\beta=\gamma e_2$, so o_2 is positive. Furthermore, (24) simplifies to

$$F\,(b_1)=\rho\,(n_1-1)^2\,\delta_1\,(2\gamma+\alpha_1 n_1)\,b_1^3+(n_1-1)\,\alpha_1 b_1^2\delta_1 n_1\gamma+\rho\,(n_1-1)\,\alpha_1\,(\alpha_1+2\gamma)\,b_1-(n_1-2)\,\alpha_1^2\gamma$$

which is independent of b_2. Simple observation shows that this equation has exactly one positive root, b_1^+. Lemma 1 shows that if $o_2>0$, negative roots of $F\,(b_1)$ do not satisfy the second order condition, hence b_1^+ is our only remaining candidate for the equilibrium. But if $b_1, o_2>0$, (25) gives a unique e_1 and it will be positive. Hence, b_1^+ satisfies the second order condition. Regarding the second period, b_1^+ can serve as b_1^* in Lemma 2, so b_2, e_2 is uniquely given. Then the rest of the coefficients are determined in a straightforward way.

When $\delta_1\to 0$, then $f_2\to 0$ as $m_1\to 0$, and (20) and (21) are independent from b_1 in the limit. Furthermore, (24) and (25) will be

$$\begin{aligned} 0 &= F\,(b_1)= \\ &= \left((n_1-1)\,o_0 b_1-(n_1-2)\,\alpha_1^2\frac{(1-\mu)\,\gamma e_2-\mu b_2\beta}{e_2}\right) \end{aligned}$$

$$e_1=e_1\left(\frac{b_2}{e_2}\right)=\frac{n_1-2}{n_1-1}\frac{v_0}{o_0}$$

with

$$
\begin{aligned}
o_0 &= \rho\alpha_1 \left(\begin{array}{c} \mu^2 \frac{b_2^2 n_2 \delta_2 + \alpha_2}{e_2^2 \alpha_2 n_2^2 \delta_2} \left(2\beta\gamma + \alpha_1\beta + \alpha_1\gamma + \gamma^2\right) + (\alpha_1 + 2\gamma)(1-\mu)^2 + \\ +2\frac{b_2}{e_2}(\alpha_1+\gamma)\mu(1-\mu) + \frac{b_2^2}{e_2^2}(\alpha_1+\beta+\gamma)\mu^2 \end{array} \right) \\
v_0 &= \left(2\beta\gamma + \alpha_1\beta + \alpha_1\gamma + \gamma^2\right)\alpha_1.
\end{aligned}
$$

Hence b_1 is uniquely determined by

$$
b_1(b_2) = \frac{(n_1-2)\,\alpha_1^2 \frac{(1-\mu)\gamma e_2 - \mu b_2 \beta}{e_2}}{(n_1-1)\,o_0}
$$

which has the same sign as $(1-\mu)\gamma e_2 - \mu b_2 \beta$, and it determines a positive e_1, which means the second order conditions are satisfied. Finally, c_1 and c_2 comes from a straightforward substitution of the values of e_2, $f_2 = 0$, b_2, b_1 into (26) and (23). ■

Proof of Proposition 2. In Lemma 2, let us choose $b_1^* = 0$. Then $b_2(0)$ and $e_2(0)$ are pined down and both of them are positive and the second order condition of the second period is satisfied. Hence, we can define a $\mu^* \in (0,1)$ by

$$
\frac{(1-\mu^*)}{\mu^*} = \frac{b_2\beta}{\gamma e_2}
$$

as the left hand side is monotone decreasing and takes values on the whole positive half of the real line. This μ^* leads to $o_2 = 0$, and with $o_2 = 0$, $b_1 = 0$ is a root of (24) for any b_2. Furthermore, $f_2 = 0$ when $b_1 = 0$, so e_1 is positive which satisfies the second order condition of the first period.. Substituting this values into (26) and (23), we end up with an equilibrium, where $b_1 = 0$. ■

Proof of Proposition 3. Lemma 2 shows that in equilibrium, b_2 and e_2 are always positive. Lemma 1 shows that when $o_2 > 0$, which is when $\frac{\gamma e_2}{(\gamma e_2 + \beta b_2)} > \mu$, $b_1 > 0$ and when $\frac{\gamma e_2}{(\gamma e_2 + \beta b_2)} < \mu$ then $b_1 < 0$. In the proof of proposition 2 we showed that when $\mu = \frac{\gamma e_2}{(\gamma e_2 + \beta b_2)}$, $b_1 = 0$. The possible sings of f_2, e_1, c_1 and c_2 comes from the simple discussion of the restrictions in Lemma 1 and equations (26),(23), (25) and (22), so it is omitted. ■

A.3 Market inefficiency and mispricing

Proof of Proposition 4. Our first period measure is

$$
1 - \frac{var(\theta|y, p_1)}{var(\theta|y)} = 1 - \frac{1}{2}\frac{(2\beta+\gamma)(2\gamma+\tau_g)}{(2\beta\gamma + \beta\tau_g + \gamma\tau_g + \gamma^2)}
$$

where τ_g is the variance of the transformed price signal, g, defined above. Hence,

$$
\frac{\partial \frac{var(\theta|y,g)}{var(\theta|y)}}{\partial b_1} = -\frac{(2\beta+\gamma)\gamma^2}{(2\beta\gamma + \beta\tau_g + \gamma\tau_g + \gamma^2)} \frac{\delta_1 \alpha_1^2 n_1^2 b_1}{\left(\alpha_1 + b_1^2 n_1 \delta_1\right)^2}
$$

which proves the first statement of the proposition. For the second period, observe that observing p_2, y and p_1 and x_i is informationaly equivalent with observing

$$\begin{aligned} g_2 &= \theta_2 + \frac{1}{n_2}\sum_i \varepsilon_i + \frac{u_2}{n_2 b_2} = \frac{1}{(n_2-1)\,b_2}\left(n_2 e_2 p_2 - n_2 c_2 y - n_2 f_2 p_1\right) \\ var\left(g_2\right) &= \frac{1}{\gamma} + \frac{1}{\tau_{g_2}} = \frac{1}{\gamma} + \frac{\left(\alpha_2 + n_2\delta_2 b_2^2\right)}{n_2^2\alpha_2\delta_2 b_2^2} \end{aligned}$$

and

$$var\left(\theta|y,p_1,p_2\right) = var(\theta|y,g_2,g) = \frac{(2\gamma+\tau_g+\tau_{2g})}{(2\beta\gamma+\beta\tau_g+\gamma\tau_g+\gamma^2+\beta\tau_{2g}+\gamma\tau_{2g}+\tau_g\tau_{2g})}.$$

substituting it to our second period measure and differentiation with respect to b_1 and b_2 gives the second statement of the theorem. ■

Proof of Proposition 5. By the projection theorem and our results on the coefficients, it is simple to show that the coefficient of $\frac{1}{n_1}\sum x_i$ in the first period linear equation of mispricing defined by

$$E(p_1|y, \frac{1}{n_1}\sum x_i) - E(\theta|y, \frac{1}{n_1}\sum x_i)$$

will be

$$\begin{aligned} &\frac{b_1}{e_1} - \frac{\gamma n_1\alpha_1}{(2\beta\gamma+\beta n_1\alpha_1+\gamma n_1\alpha_1+\gamma^2)} = \\ &= \frac{\frac{(\mu_c\gamma e_2-\mu b_2\beta)}{(e_2-\mu f_2)}\left(\alpha_1+\tau_1\right)\left(2\beta\gamma+\beta n_1\alpha_1+\gamma n_1\alpha_1+\gamma^2\right) - \gamma n_1\alpha_1\left(2\beta\gamma+\beta\alpha_1+\beta\tau_1+\gamma\alpha_1+\gamma\tau_1+\gamma^2\right)}{\left((2\beta\gamma+\beta\alpha_1+\beta\tau_1+\gamma\alpha_1+\gamma\tau_1+\gamma^2)\right)(2\beta\gamma+\beta n_1\alpha_1+\gamma n_1\alpha_1+\gamma^2)}. \end{aligned}$$

We know from Lemma 1 that $\frac{(\mu_c\gamma e_2-\mu b_2\beta)}{(e_2-\mu f_2)}$ must be negative in the large μ case. Hence, in case 3.a and in case 2 of Proposition 3, the expression is negative. At $\mu = 0$ also

$$\begin{aligned} &\frac{b_1}{e_1} - \frac{\gamma n_1\alpha_1}{(2\beta\gamma+\beta n_1\alpha_1+\gamma n_1\alpha_1+\gamma^2)} = \\ &\qquad = \frac{-\gamma^2\left(2\beta+\gamma\right)\frac{(n_1-1)\alpha_1^2}{\left(\alpha_1-b_1^2\delta_1+b_1^2 n_1\delta_1\right)}}{\left((2\beta\gamma+\beta\alpha_1+\beta\tau_1+\gamma\alpha_1+\gamma\tau_1+\gamma^2)\right)(2\beta\gamma+\beta n_1\alpha_1+\gamma n_1\alpha_1+\gamma^2)} < 0. \end{aligned}$$

In perfect competition, we have to change the demand functions (8) and (9) by setting $\lambda_1 = \lambda_2 = 0$. If we derive the new equations determining the coefficients, it turns out that the form of our measures of mispricing does not change, only we have to calculate them with the new equilibrium $b_1, c_1, e_1, e_2, b_2, \tau_1$ values. If $\delta_1 \to 0$, $\tau_1, f_2 \to 0$, hence our measure goes to

$$\alpha_1\frac{\frac{((1-\mu)e_2\gamma-\beta\mu b_2)}{e_2}\left(2\beta\gamma+\beta n_1\alpha_1+\gamma n_1\alpha_1+\gamma^2\right) - \gamma n_1\left(\beta\alpha_1+\gamma\alpha_1+2\beta\gamma+\gamma^2\right)}{(2\beta\gamma+\beta n_1\alpha_1+\gamma n_1\alpha_1+\gamma^2)\,(\beta\alpha_1+\gamma\alpha_1+2\beta\gamma+\gamma^2)}$$

in the case of the private signal, and to

$$\frac{c_1}{e_1} - \frac{(\beta(2\gamma+n_1\alpha_1))}{(2\beta\gamma+\beta n_1\alpha_1+\gamma n_1\alpha_1+\gamma^2)} =$$

$$= \frac{\left((1-\mu)\beta(2\gamma+\alpha_1)+\mu\frac{1}{e_2}b_2\beta(\alpha_1+\gamma)+\mu\frac{1}{e_2}\frac{2\beta\gamma+\beta\alpha_2+\beta\tau_2}{\left(\rho(2\gamma+\alpha_2+\tau_2)+\frac{1}{b_2}n_2\frac{\gamma\tau_2}{n_2-1}\right)}\left(\alpha_1\beta+\gamma\alpha_1+2\gamma\beta+\gamma^2\right)\right)}{(2\beta\gamma+\beta\alpha_1+\gamma\alpha_1+\gamma^2)} - \frac{(\beta(2\gamma+n_1\alpha_1))}{(2\beta\gamma+\beta n_1\alpha_1+\gamma n_1\alpha_1+\gamma^2)}$$

in the case of the public signal. Differentiating with respect to μ gives the last statements of the proposition. ■

Figures

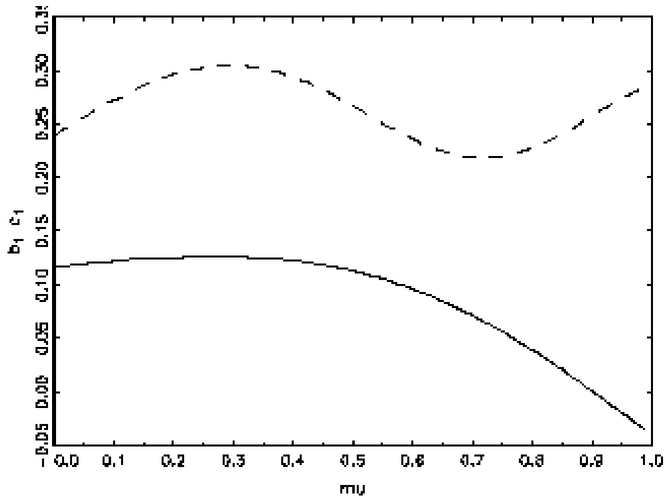

Figure 1: Coefficients of the private signal (solid) and the public signal (dashed) in the first period as μ changes. ($n_1 = 3$, $n_2 = 3$, $\alpha_1 = 2$, $\alpha_2 = 1$, $\delta_1 = 2$, $\delta_2 = 1$, $\gamma = 1$)

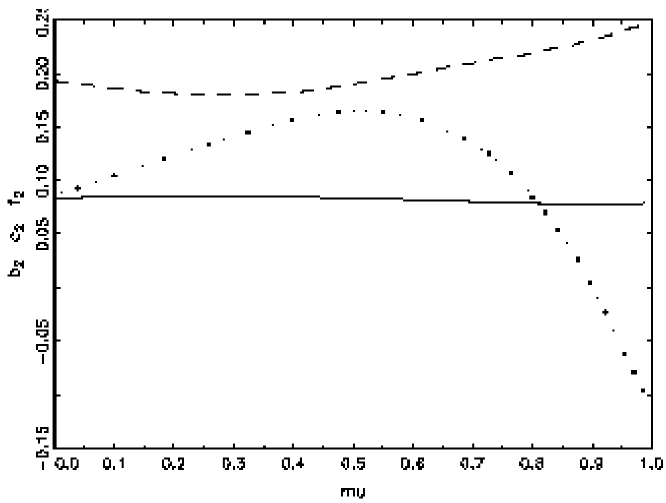

Figure 2: Coefficients of the private signal (solid), the public signal (dashed) and the first period price (dotted) in the second period as μ changes. ($n_1 = 3$, $n_2 = 3$, $\alpha_1 = 2$, $\alpha_2 = 1$, $\delta_1 = 2$, $\delta_2 = 1$, $\gamma = 1$)

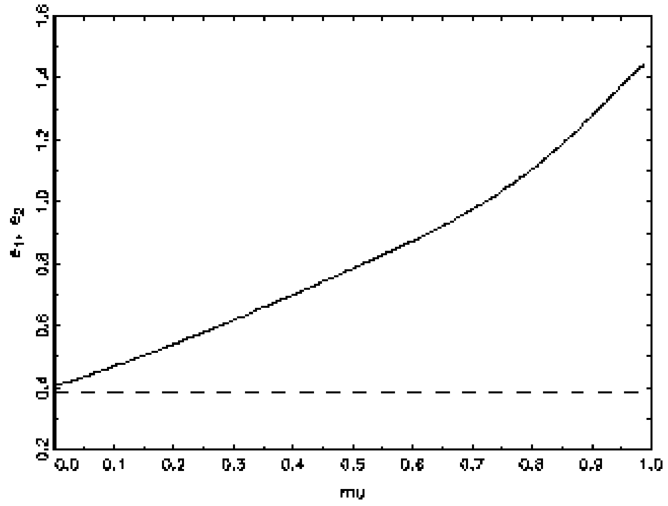

Figure 3: Slope of the demand curve in the first period (solid) and in the second period (dashed) as μ changes. ($n_1 = 3$, $n_2 = 3$, $\alpha_1 = 2$, $\alpha_2 = 1$, $\delta_1 = 2$, $\delta_2 = 1$, $\gamma = 1$)

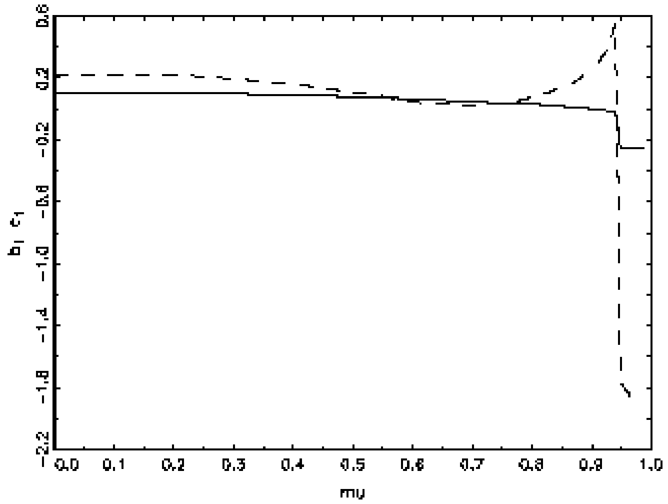

Figure 4: Coefficients of the private signal (solid) and the public signal (dashed) in the first period as μ changes. ($n_1 = 3$, $n_2 = 3$, $\alpha_1 = 2$, $\alpha_2 = 1$, $\delta_1 = 5$, $\delta_2 = 1$, $\gamma = 1$)

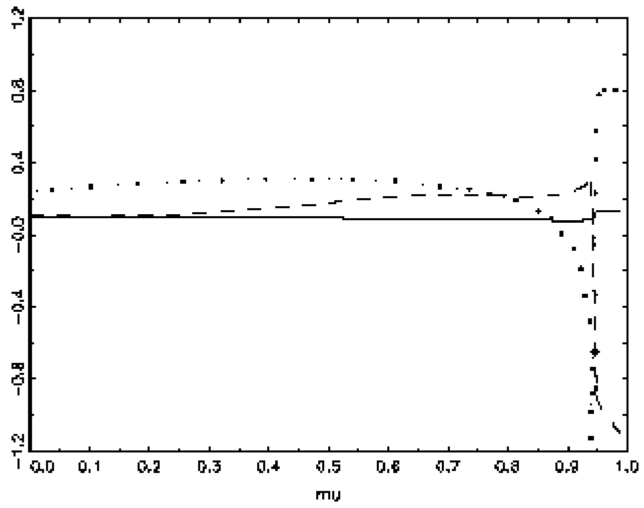

Figure 5: Coefficients of the private signal (solid), the public signal (dashed) and the first period price (dotted) in the second period as μ changes. ($n_1 = 3$, $n_2 = 3$, $\alpha_1 = 2$, $\alpha_2 = 1$, $\delta_1 = 5$, $\delta_2 = 1$, $\gamma = 1$)

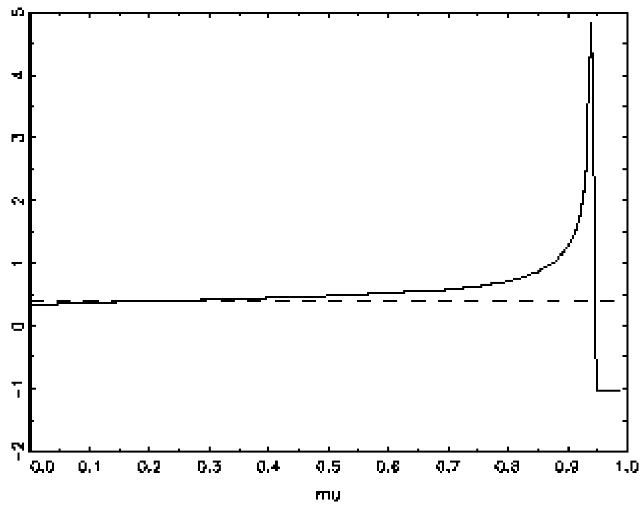

Figure 6: Slope of the demand curve in the first period (solid) and in the second period (dashed) as μ changes. ($n_1 = 3$, $n_2 = 3$, $\alpha_1 = 2$, $\alpha_2 = 1$, $\delta_1 = 5$, $\delta_2 = 1$, $\gamma = 1$)

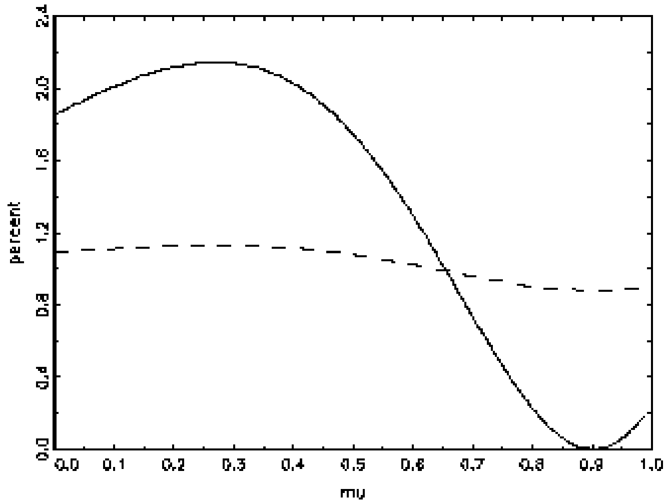

Figure 7: Information content in current prices in the first period (solid) and in the second period (dashed) as μ changes. ($n_1 = 3$, $n_2 = 3$, $\alpha_1 = 2$, $\alpha_2 = 1$, $\delta_1 = 2$, $\delta_2 = 1$, $\gamma = 1$)

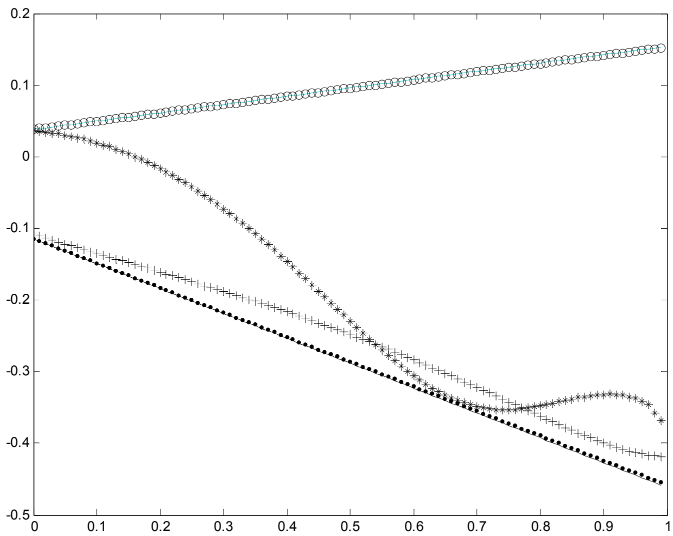

Figure 8: Over- or underreaction of the private signal (crosses) and the public signal (asterisk) in the first period in the 3.a case ($n_1 = 3$, $n_2 = 3$, $\alpha_1 = 2$, $\alpha_2 = 1$, $\delta_1 = 1$, $\delta_2 = 1$, $\gamma = 1$), and the $\delta_1 \to 0$ case (circles – public, dotts– private) as μ changes. The imperfect and perfect competition cases are indistinguisable.

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