Optimal intergenerational risk sharing*

Otto van Hemert†
Universiteit van Amsterdam, Tinbergen Institute

June 6, 2005

Abstract

This paper studies optimal intergenerational transfer policy under stochastic labor income and capital returns. It has implications for Social Security, government tax and debt policy, and DB pension funds. A stylized two-period overlapping-generations model is developed where a central planner implements pay-as-you-go transfers. I allow for autocorrelation in the labor income and skewness in the capital return and calibrate the model parameters to US data. I show that state-contingent transfers facilitate intergenerational risk sharing in a way that is similar to portfolio insurance using put options. That is, the working generation provides downside risk insurance to the old on their savings. In addition, when no riskfree asset is available, these transfers improve utility by substituting for this missing asset. I further find that imposing an incentive constraint for the working generation has little impact when transfers also have this substitution role, but it causes the transfer scheme to collapse to the zero-transfer scheme when a risk free asset is available.

*I would like to thank Frank de Jong, Joost Driessen, Enrico Perotti, Paul Sengmuller, Per Strömberg and seminar participants at the Tinbergen Institute Amsterdam, Fifth RTN workshop on retirement in Toulouse, Universiteit van Amsterdam and Stockholm Institute for Financial Research for helpful comments. I also gratefully acknowledge the hospitality at the Financial Markets Group of the London School of Economics, where part of this research has been conducted. A previous version of this paper circulated under the name "Optimal intergenerational transfers and the associated utility gain".

†Universiteit van Amsterdam, Finance Group, Faculty of Economics and Econometrics, Roeterstraat 11, 1018 WB Amsterdam, The Netherlands. E-mail: hemert@tinbergen.nl.

1 Introduction

It is by now known that a central planner, like the government, can facilitate risk sharing in addition to free market possibilities. It can do so by implementing intergenerational transfers; for example via Social Security. There is surprisingly little literature though on the transfer scheme that is optimal from this intergenerational risk sharing perspective. The papers that do exist on this topic focus on sharing aggregate labor income risk by assuming a constant Pay-As-You-Go (PAYG) tax on the working generations' labor income. However, I will show that risk sharing with an optimally chosen state-contingent PAYG tax involves much larger welfare gains. Moreover, the focus on labor income risk is very restrictive. As I will illustrate using U.S. data, the return on the savings of the retired generations is subject to much greater stochastic fluctuations creating a much larger potential for intergenerational risk sharing.

This paper is the first (to the best of my knowledge) to analyse intergenerational risk sharing taking into account the financial risk of both the workers and the retirees. I study a stylized two-period overlapping-generations model where the young generation faces stochastic labor income and the old generation faces a stochastic return on its savings. The central planner implements transfers from the young to the old (i.e. PAYG transfers) as to maximize the unconditional expected utility for the steady state generation. The size of the transfer depends on both the young generation's realised labor income (affordability) and the old generation's pre-transfer wealth (desirability). The latter is determined by the old's labor income, consumption and transfer paid when they were young and the return on their savings. I focus on a small open economy where the transfers do not influence future marginal productivity of labor and capital.

Another important part of my set-up is the assumed joint stochastic process for the labor income and capital return. I will use data on US GNP and the US stock market to proxy for labor income and capital return. I assume a four state Markov process which allows me to match the mean, standard deviation and skewness in the labor income and capital return as well as the autocorrelation in labor income.

I consider several alternative model specifications to further deepen the understanding of intergenerational risk sharing. I analyse the model with and without an additional, risk free investment asset. Moreover, I investigate to what extent intergenerational risk sharing is possible when voluntary cooperation is required. By imposing an incentive constraint for the young the implementation of the transfer scheme will be enhanced. I consider this a first step towards more advanced modeling of the political voting system.

Academics have shown interest in intergenerational transfers for a long time. Diamond (1965) showed that a decentralized, non-stochastic economy can be dynamically inefficient and that transfers from young to old can be Pareto improving. Only more recently, academics started to stress the ability of a central planner to facilitate intergenerational risk sharing. For an early treatment see e.g. Merton (1983) or Gordon and Varian (1988). To evaluate the welfare impact of intergenerational risk sharing, often Pareto optimality is considered. However, in a stochastic setting this may be defined in different manners, depending on the moment different agents' welfare is evaluated. With interim Pareto optimality agents' welfare is evaluated at birth. This concept is rather weak since it does not take into account insurance possibilities across states of the world one is born in. In this paper I use the concept of ex ante Pareto optimality which does take into account these insurance possibilities. For my model set-up this boils down to maximizing unconditional expected utility for the steady state generation.[1]

The paper closest in spirit to mine is De Menil and Sheshinski (2003). As in this paper they study PAYG transfers in a two-period overlapping generations model, use ex ante optimality for the steady state generation as Pareto concept, and focus on a small open economy. Transfers are restricted to be a constant fraction of the young's labor income. Necessary conditions for the optimal PAYG rate to be nonzero are derived. For selected countries they calibrate their model parameters and numerically solve for the optimal values.

Bohn (2003) provides comparative statics on ex ante Pareto optimality in an economy with stochastic productivity shocks. Wages are more exposed to productivity risk than the return on capital. He argues that in practice the promised transfers from the workers to the retirees are often safe and therefore seem to shift risk in the wrong direction. However, besides productivity risk, capital is subject to stochastic depreciation shocks. As he shows himself, this makes the return on capital more risky than wages. Taking into account the possible role for transfers to share this depreciation risk as well might overturn his conclusion.

Krueger and Kubler (2002) study the impact of introducing a 2% PAYG pension rate. In a small open economy it can be interim Pareto improving, but this is mitigated by a capital crowding-out effect in a general equilibrium setting. For several reasons their results are hard to compare with mine. First they focus on the weaker interim Pareto optimality concept. Second, the transfers are of a very particular form, which is likely to be far from the optimal PAYG rate. Third, they assume no riskless borrowing, but households can sell short the risky asset. This implies that very young households have to cut consumption or bear additional risk to pay for the pension tax.

This paper extends De Menil and Sheshinsky (2003), Bohn (2003), and Krueger and Kubler (2002) along various dimensions. First, while these papers restrict the transfer to be a constant fraction of labor income, this paper allows transfers to depend on all available information. That is, the transfer is conditional on the entire history of the realised labor income and capital

[1]For a further discussion on Pareto optimality concepts see e.g. Demange and Laroque (1999) and Demange (2002).

return. Second, I use a more realistic stochastic setting with skewness in the labor income and capital return and autocorrelation in the labor income. Third, I shed light on the implication of adding a risk free asset to the investment opportunity set. Fourth, I examine to what extent intergenerational transfers can be incentive compatible for the paying generation.

I start by assuming there is only a single asset available; which is risky. State-contingent transfers facilitate intergenerational risk sharing in a way that is similar to portfolio insurance using put options. That is, the working generation provides downside risk insurance to the old on their savings. However, this insurance is imperfect in the sense that the old also bear part of the downside risk. Moreover, the transfer depends not only on the old's desirability of a transfer, but also on the young's affordability. That is, ceteris paribus we see larger transfers when the young received a high labor income. In the absence of a risk free asset the intergenerational transfers serve an additional purpose. The transfer can be considered as being the sum of a zero-mean, state-dependent component and a constant component. The first component provides the downside risk insurance to the old on their savings. The second component effectively substitutes for the missing riskfree asset. For a low degree of risk aversion the added value of the transfer scheme comes from the downside risk sharing, while as for a high degree of risk aversion the substitution of the missing risk free asset is more important. Based on model parameter values calibrated to U.S. data I find that for the downside risk sharing the capital return risk is more important than the labor income risk. Not only the standard deviation of the log capital return matters, but negative skewness also plays a substantial role. Furthermore, I provide evidence that risk is effectively only shared over generations not too far apart in time. I find that cooperation by the young is only an issue for low degrees of risk aversion. For high degrees of risk aversion the substitution of the absent risk free asset provides a major incentive to cooperate even for generations that face a transfer with a less attractive state-dependent component.

Next I assume a risk free asset is present. In this set-up the risk free asset represents a long-term investment (e.g. with a horizon of 20 years) that provides a fixed real return (i.e. inflation corrected). In reality most long-term bonds are nominal and over the long run these are quite risky in real terms.[2] However the market for inflation-indexed bonds might mature and this analysis explores the implications for intergenerational risk sharing. Some of the results change compared to the case when no risk free asset is present. Obviously the substitution role of intergenerational transfers disappears. However, the young still provide imperfect downside risk protection to the old on their savings. This risk sharing causes the optimal risky asset allocation to be greater than it would have been under autarky. Cooperation by the young is now a major issue for all degrees of risk aversion. In fact, imposing an incentive constraint for the young

[2] See e.g. Campbell and Viceira (2002, p.109) for an illustration.

causes the zero transfer scheme to be the only implementable one. This again illustrates that the substitution role is linked to implementability of the fully optimal state-contingent transfer scheme.

The structure of this paper is the following. In Section 2 I present the general model set-up, discuss the characteristics of the assumed joint stochastic process for the labor income and capital return, and explain the solution technique. The calibration of the model parameters is presented in Section 3. I discuss optimal transfer policy without risk free asset in Section 4 and with risk free asset in Section 5. Section 6 concludes, discusses implications for Social Security reform and suggests directions for future research.

2 A two-period overlapping generations model

I first discuss the model set-up. Second, I present details on the assumed stochastic environment. Third, I explain two numerical techniques to solve the model. These two techniques are consistent with each other, but both have their own advantages.

2.1 Set-up of the model

I consider a stylized overlapping generations model where generations live for two periods. In the first period a generation works and is referred to as young. In the second period a generation invests his savings and is referred to as old. I abstract from population growth and intermediate death, so the young and old generation have an equal number of members. The labor income, Y, and the capital return on savings, R, in period t are determined by the state of the world, ω_t, which is assumed to follow a Markov process. Further introducing notation, ω is an element of the finite space Ω. I assume the state of the world is exogenous, which seems a reasonable assumption for a small open economy where foreign capital can flow in and out freely. In addition to the two generations alive in a certain period there is a third player referred to as the central planner. The central planner may transfer wealth between the two generations alive. Following De Menil and Sheshinsky (2004) I assume that institutional requirements are such that only transfers from the young to the old (i.e. PAYG transfers) can be implemented.

After the state of the world in period t is known the following actions are taken by the players. The central planner implements a transfer of size $x_t \geq 0$ from the young to the old. The young generation chooses consumption c_t^y, and consequently saves $Y(\omega_t) - x_t - c_t^y$. The young's optimal savings turn out to be always positive, so we do not have to consider a nonnegativity constraint. The old consume $c_t^o = s_t + x_t$, where

$$s_t = \left[Y(\omega_{t-1}) - x_{t-1} - c_{t-1}^y\right] R(\omega_t) \tag{1}$$

is the old's pre-transfer wealth. Since the old simply consume all wealth available to them, only the young and the central planner have a choice to make.

The exogenous state of the world, ω_t, and the endogenous old's pre-transfer wealth, s_t, together summarize all relevant information on the state of the economy in period t, denoted (ω_t, s_t). The policy functions c^y and x for the young and the central planner respectively specify the actions for all possible states of the economy.

The young in period t maximize expected utility conditional on the realised state of the economy in period t. That is

$$c^y(\omega_t, s_t) = \arg\max_{c^y} E_t\left[u\{c^y\} + \beta u\{(Y(\omega_t) - x(\omega_t, s_t) - c^y) R(\omega_{t+1}) + x(\omega_{t+1}, s_{t+1})\}\right], \quad (2)$$

for all $\omega_t \in \Omega$ and $s_t \geq 0$, where β is the subjective discount factor. I assume u is of power utility form with coefficient of relative risk aversion γ. That is

$$u\{c\} = \left(c^{1-\gamma} - 1\right) / (1-\gamma) \text{ for } \gamma > 1, \quad (3)$$

$$u\{c\} = \log(c) \text{ for } \gamma = 1 \quad (4)$$

The central planner maximizes unconditional expected utility for the steady state generation. That is

$$x = \arg \max_{\substack{x(\omega,s),\\ \omega\in\Omega, s\geq 0}} E\left[u\{c^y\} + \beta u\{s_{t+1} + x(\omega_{t+1}, s_{t+1})\}\right], \text{ where} \quad (5)$$

$$s_{t+1} = \left[Y(\omega_t) - x(\omega_t, s_t) - c^y\right] R(\omega_{t+1})$$

Even though the young's and central planner's objective look very similar, there is a potential conflict of interest. The young might consume too much from the perspective of the central planner because they anticipate the central planner will find it optimal to bail them out once they are old via a large intergenerational transfer. I will assume the central planner can credibly commit to transfers as if the young chose the socially optimal consumption, to which the young will optimally react by actually choosing the socially optimal consumption. In equation (2) this subtle point is reflected by writing $x(\omega_{t+1}, s_{t+1})$ instead of $x(\omega_{t+1}, (Y(\omega_t) - x(\omega_t, s_t) - c^y) R(\omega_{t+1}))$, indicating that the young at time t take $x(\omega_{t+1}, s_{t+1})$ as given for their consumption choice. In words, the young take into account the impact of their action (consumption choice) on pre-transfer wealth, but ignore the possible influence on the receivable transfer when old. Two different motivations can be given for this. First, consumption of a single member of a generation is small compared to the consumption of the whole generation and members cannot coordinate. Second, s_{t+1} merely summarizes the relevant informational content of the infinite realised exogenous states of the world, $(\omega_t, \omega_{t-1}, ...)$. The central planner can alternatively formulate his transfers in terms of this history directly. Then assuming the central planner is the Stackelberg leader in the game with the young also aligns the desired young's consumption.

So equivalently we can just assume the central planner sets both young's consumption and transfers and the optimization problem can be written as

$$\begin{aligned} \bar{V} &= \max_{\substack{c^y(\omega,s),x(\omega,s),\\ \omega\in\Omega,s\geq 0}} E\left[u\left\{c^y\left(\omega_t,s_t\right)\right\}+\beta u\left\{s_{t+1}+x\left(\omega_{t+1},s_{t+1}\right)\right\}\right] \qquad (6) \\ s_{t+1} &= \left[Y\left(\omega_t\right)-x\left(\omega_t,s_t\right)-c^y\left(\omega_t,s_t\right)\right]R\left(\omega_{t+1}\right) \end{aligned}$$

where we introduced $\bar{V}$ as the highest obtainable unconditional expected utility. I will assume that the return on capital is finite. Since only transfers from the young to the old are allowed, this implies that total resources available in period t, Y_t+s_t, is bounded. This in turn implies that expected utility in (5) or (6) is bounded and therefore that the problem is well defined.

Subsequent generations have only at a single point in time both positive wealth. I make the assumption that subsequent generations cannot make financial contracts with each other. This can either be motivated by moral hazard problems making borrowing against human capital impossible or by assuming that a generation is not adult yet before it receives labor income in the model making it impossible to participate in a financial contract. So the focus of this paper is on intergenerational risk sharing that is impossible in the marketplace.

I present the solution to above model in Section 4. I extend the model by introducing a risk free asset in Section 5. In both Sections I investigate implications of having an incentive constraint for the young. That is, the transfer scheme must be such that the young voluntarily choose to cooperate over autarky in every state of the world.

2.2 A four-state Markov process

The joint process for the labor income and capital return is assumed to be Markovian and stationary. This means that I abstract from a trend in the labor income process which I consider reasonable given that the focus is on risk sharing. This results in conservative values for the intergenerational transfers and the associated utility gain since a positive growth rate g effectively implies a simple return g on PAYG transfers. Introducing notation, the world can be in four states with the following unconditional probabilities:

ω^1: $P\left(R=R^H,Y=Y^H\right)=\kappa$
ω^2: $P\left(R=R^H,Y=Y^L\right)=p-\kappa$
ω^3: $P\left(R=R^L,Y=Y^H\right)=q-\kappa$
ω^4: $P\left(R=R^L,Y=Y^L\right)=1-p-q+\kappa$

Notice that the capital return is high ($R=R^H$) with unconditional probability p and low ($R=R^L$) with unconditional probability $1-p$. Labor income is high ($Y=Y^H$) with unconditional probability q and low ($Y = Y^L$) with unconditional probability $1-q$. The parameter κ will be used to capture the correlation between the labor income and capital return.

Now I turn to determining the transition matrix, T, i.e. a $4{*}4$ matrix with as (i,j)-th element the probability of transition from state ω^i to state ω^j. For consistency with unconditional probabilities we must have $(\kappa, p-\kappa, q-\kappa, 1-p-q+\kappa)\,T = (\kappa, p-\kappa, q-\kappa, 1-p-q+\kappa)$. I assume that ω_t has no predicting power for R_{t+1}, implying that the capital return is iid.[3] As a result columns 1 and 2 of T should add up to p times the unity vector, and columns 3 and 4 to $1-p$ times the unity vector. Moreover I assume that Y_t summarizes all relevant information at time t for predicting Y_{t+1}. So R_t contains no relevant information in addition to Y_t on future probabilities. As a result row 1 and 3 as well as row 2 and 4 of T should be equal. Under above assumptions it is straightforward to show that T must be of the following form

$$T = \begin{pmatrix} \kappa & p-\kappa & q-\kappa & 1-p-q+\kappa \\ \kappa & p-\kappa & q-\kappa & 1-p-q+\kappa \\ \kappa & p-\kappa & q-\kappa & 1-p-q+\kappa \\ \kappa & p-\kappa & q-\kappa & 1-p-q+\kappa \end{pmatrix} + \begin{pmatrix} \frac{1}{q} & 0 & 0 & 0 \\ 0 & \frac{1}{1-q} & 0 & 0 \\ 0 & 0 & \frac{1}{q} & 0 \\ 0 & 0 & 0 & \frac{1}{1-q} \end{pmatrix} \begin{pmatrix} \lambda & -\lambda & \mu & -\mu \\ -\lambda & \lambda & -\mu & \mu \\ \lambda & -\lambda & \mu & -\mu \\ -\lambda & \lambda & -\mu & \mu \end{pmatrix} \tag{7}$$

The case $\lambda = \mu = 0$ corresponds to a situation with zero autocorrelation in labor income. The more positive λ and μ are, the more positive autocorrelation in labor income there is. The magnitude of λ relative to μ determines the correlation between R_{t+1} and Y_{t+1} conditional on Y_t. I will assume a constant conditional correlation which will result in values for λ and μ close to each other but not exactly equal.

The input parameters R^H, R^L, p, Y^H, Y^L, q, κ, λ and μ together uniquely define the joint stochastic process for the capital return and labor income. To enhance interpretation of the stochastic process I reparameterize these input parameters in equations (8)-(16) using the first moments of the log capital return and the log labor income. Define $r^H \equiv \log\left(R^H\right)$ and $r^L \equiv \log\left(R^L\right)$. Now the mean, standard deviation and coefficient of skewness are given by

$$\bar{r} = pr^H + (1-p)\,r^L \tag{8}$$

$$\sigma_r = \sqrt{p\,(r^H - \bar{r})^2 + (1-p)\,(r^L - \bar{r})^2} \tag{9}$$

$$cs_r = \left[p\left(r^H - \bar{r}\right)^3 + (1-p)\left(r^L - \bar{r}\right)^3\right]/\sigma_r^3 \tag{10}$$

Similarly define $y^H \equiv \log\left(Y^H\right)$ and $y^L \equiv \log\left(Y^L\right)$. The consumption rates, and the wealth equivalent numbers I will typically report do not depend on the mean labor income. This is a

[3] Empirical support for mean-reversion in stock prices leading to negative autocorrelation in stock returns is weak and limited to 3 to 8 year horizons. See e.g. Fama and French (1988), Poterba and Summers (1988) and Lo (1991).

direct consequence of the chosen power utility form. As a result I can set the mean log labor income at 0 without loss of generality. Using the definitions for the standard deviation and coefficient of skewness we have

$$0 = qy^H + (1-q)\,y^L \tag{11}$$

$$\sigma_y = \sqrt{q\left(y^H\right)^2 + (1-q)\left(y^L\right)^2} \tag{12}$$

$$cs_y = \left[p\left(y^H\right)^3 + (1-q)\left(y^L\right)^3\right]/\sigma_y^3 \tag{13}$$

Matching the correlation between the log capital return and log labor income gives

$$\rho_{r_t y_t} = \frac{\kappa\left(r^H - \bar{r}\right)y^H + (p-\kappa)\left(r^H - \bar{r}\right)y^L + (q-\kappa)\left(r^L - \bar{r}\right)y^H + (1-p-q+\kappa)\left(r^L - \bar{r}\right)y^L}{\sigma_r \sigma_y} \tag{14}$$

Matching the unconditional autocorrelation in the log labor income gives

$$\rho_{y_t y_{t-1}} = \frac{\left[q^2 + \lambda + \mu\right]\left(y^H\right)^2 + 2\left[q(1-q) - \lambda - \mu\right]y^H y^L + \left[(1-q)^2 + \lambda + \mu\right]\left(y^L\right)^2}{\sigma_y^2} \tag{15}$$

Finally, without working out fully here, we impose that the conditional autocorrelation in labor income is constant

$$\rho_{r_t y_t | y_{t-1} = y^H} = \rho_{r_t y_t | y_{t-1} = y^L} \tag{16}$$

I will calibrate $\bar{r}$, σ_r, cs_r, σ_y, cs_y, $\rho_{r_t y_t}$ and $\rho_{y_t y_{t-1}}$ to US data in Section 3.

2.3 Solution technique

In this subsection I present two techniques to solve for optimal consumption and transfers. The first solution technique uses an iterative procedure and will be referred to as the iterative solution technique. It is similar to Deaton and Laroque's (1992) technique, but an adjustment is needed to make it applicable to our steady state setting. Define expected utility for the young at time t while being in state of the economy (ω_t, s_t) by $V_t(\omega_t, s_t)$. The central planner's objective is to maximize the unconditional expected value of V_t, which we denoted $\bar{V}$. This means the central planner puts equal weight on the expected utility of all generations. First define the following state-dependent value function $W(\omega, s)$

$$W(\omega_t, s_t) = \max_{\substack{c^y(\omega,s), x(\omega,s), \\ \omega \in \Omega, s \geq 0}} E_t\left[\left(V(\omega_t, s_t) - \bar{V}\right) + \left(V(\omega_{t+1}, s_{t+1}) - \bar{V}\right) + ...\right] \tag{17}$$

I take conditional expected utilities in deviation from the unconditional expected utility to make $W(\omega_t, s_t)$ bounded for all (ω_t, s_t). Now the following equation holds

$$W(\omega, s) = \max_{\substack{c^y(\omega,s), x(\omega,s), \\ \omega \in \Omega, s \geq 0}} E\left[u\{c^y\} + \beta u\{s + x\} - \bar{V} + W\left(\omega^+, s^+\right)\right], \tag{18}$$

where ω^+ and s^+ are the successors of ω and s respectively. Using (18) we can solve for the policy functions c^y and x using an iterative procedure. Specifically, choose a grid on s. Remember that Ω was already assumed to be finite. Determine a new estimate for W on the left-hand side by using values from the previous iteration step for W on the right-hand side and determining new estimates for c^y and x. Continue until both c^y and x converge. For this we do not need to know $\bar{V}$, but W will be determined up to a constant error only. This procedure produces the optimal policy functions c^y and x, but not immediately the optimal value for $\bar{V}$. To determine this you need to simulate paths for the state of the economy (ω, s) using the derived optimal policy functions.

A second solution technique uses the fact that s_t merely summarizes the relevant informational content of the infinite history of realised exogenous states of the world, $(\omega_{t-1}, \omega_{t-2}, ...)$ in addition to ω_t. Conditioning the young's consumption and the central planner's transfer at time t first on ω_t only, then on ω_t and ω_{t-1}, etc., leads to a converging sequence of policy functions and utility levels. I will refer to this solution technique as the direct solution technique since it immediately provides an estimate of the associated unconditional expected utility, $\bar{V}$. Denote the set of different K-histories by H^K and the number of elements in Ω by M. Now H^K has M^K elements, i.e. there are M^K different K-histories. Conditioning on the last K states of the world only means optimally choosing the young's consumption and the central planner's transfer for all different K-histories:

$$\begin{aligned} \bar{V}^K &= \max_{\substack{c^{y,K}(h),\\ x^K(h),\\ h\in H^K}} \sum_{h\in H^K} \sum_{h^+\in H^K} P\left(h_t = h, h_{t+1} = h^+\right)\left(u\left(c^{y,K}(h)\right) + \beta u\left(s_{t+1} + x^K\left(h^+\right)\right)\right) \quad (19) \\ s_{t+1} &= \left[Y(h) - x^K(h) - c^{y,K}(h)\right] R\left(h^+\right) \end{aligned}$$

In Sections 4 and 5 we will see that for the chosen model parameter values the sequence $\bar{V}^1$, $\bar{V}^2$,$\bar{V}^3$, ... converges rapidly in the sense that values do not change after two to four steps in the reported precision.

In Sections 4 and 5 I will choose a grid for the iterative solution technique and a K for the direct solution technique. For the choices made the optimal policy functions of the two techniques are very close to each other and the associated unconditional expected utility levels are equal in the reported precision. The reason to use both techniques is that they both have their unique advantages. The iterative solution technique provides policy functions for all states of the economy (ω, s), including out of equilibrium states. This greatly enhances the interpretation and presentation of the solution. The direct solution technique is much faster because it can use gradient and Hessian information. It is able to shed light on the question to what extent historical realised states of the world should influence the optimal consumption and transfer today. Put differently, it makes clear over how many generations labor income and capital return shocks are effectively spread. Moreover, it is capable handling an extended version of the model with an incentive constraint for the young. This is practically impossible with the iterative solution technique because no gradient information can be used.

3 Calibration

I interpret one period in the model to correspond to 20 years in real life. I estimate the mean, standard deviation and coefficient of skewness for the log capital return using data on stocks. I use a broad index comprising all NYSE, AMEX and NASDAQ firms over 1927 to 2003.[4] Moreover I have data on the short nominal interest rate and the Consumer Price Index. Following Fama and French (2002), amongst others, I believe that the equity premium is lower than the realized premium when measured over the given sample period. While the realised excess (simple, yearly) return in the data is 8.25%, I set the equity premium at 4.00%. I therefore deduct 4.25% from the realised real equity return to create a corrected series for the real equity return. As discussed in Section 2, I assume independent identically distributed capital returns. I generate a large series of 20-year log returns with a bootstrap procedure. Specifically, each 20-year log return is the sum of 20 randomly chosen yearly log returns. For comparison I also report figures for yearly log stock returns. The yearly mean log stock return is 2.34%, and the mean 20-year log stock return is 47%. Since log stock returns add up this is just 20 times the yearly mean log stock return. The standard deviation for the yearly log stock return is 22%, while as for the 20-year log stock return it is $\sqrt{20}$ times bigger at 97%. For the coefficient of skewness I find a large negative -0.71 for the yearly log stock returns. Due to the manifestation of the Central Limit Theorem it is a much smaller -0.14 for the 20-year log stock return.

I calibrate the labor income process to US GNP per capita data. To this end I simulate from Diebold and Senhadji's (1996) estimated best-fitted trend-stationary model for the log annual US GNP per capita using data from 1869 to 1993.[5]

$$y_t = -1.07 + 0.0031t + 1.33y_{t-1} - 0.51y_{t-2} + \varepsilon_t \tag{20}$$

where ε_t is iid with zero mean and standard deviation $\sigma_\varepsilon = 0.043$. I will assume ε_t is normal. Since I focus on stationary processes, I abstract from the deterministic trend. Moreover, the results I report will not depend on the unconditional mean log labor income. This is a direct

[4] The author would like to thank Kenneth R. French for making this data available at his website.

[5] In fact, they use two different assumptions regarding the pre-1929 data. The assumptions originate from Romer (1989) and Balke and Gordon (1989). I will report and use figures based on data with Romer's assumption only. With Balke and Gordon's assumption I find very similar estimates for my model parameters.

consequence of the well-known separability of wealth in a power utility framework. Using Monte Carlo simulation I generate a large series of 20-year (detrended) log labor income, and for comparison also a large series of 1-year (detrended) log labor income. For the autocorrelation in the 1-year log labor income I find a considerable 0.88, but for the 20-year log labor income it is only 0.04. The standard deviation for the 1-year log labor income is 0.11, and for the 20-years it is 0.05. Notice that for labor income not the logs but the levels add up over time. This means that the standard deviation of the labor income level when measured relative to the mean decreases in period length and consequently that the standard deviation of the log labor income decreases in period length.[6] Without loss of generality (in the power utility setting) I will normalise the labor income level to one. In fact, the standard deviation of the normalised labor income level approximately equals 0.11 for a one year period and 0.05 for a 20-year period as well. I find a coefficient of skewness of 0.00 for the 1-year log labor income and −0.02 for the 20-year log labor income.

Bohn (2003) provides estimates for the correlation between the log capital return and the log labor income ranging from 0.16 to 0.69. In the base case I choose 0.3, but I will provide comparative statics for different values. See Table I (Panel A) for an overview of the estimated moments of the joint log process for capital return and labor income. Table I (Panel B) provides the implied model parameter values.

Table I. Calibrated model parameters

Panel A provides values for the first moments of the joint stochastic process for the log capital return and the log labor income. Panel B provides the implied values for the four-state Markov process as presented in Section 2.

Panel A		Panel B	
$\bar{r}$	0.47	R^H	3.88
σ_r	0.96	R^L	0.57
cs_r	−0.16	p	0.54
$\bar{y}$	0.00	Y^H	1.05
σ_y	0.05	Y^L	0.95
cs_y	−0.02	q	0.51
ρ_{ry}	0.30	κ	0.35
ρ_{rr^-}	0.00	λ	$5.4 * 10^{-3}$
ρ_{yy^-}	0.04	μ	$4.6 * 10^{-3}$

[6] For iid labor income the decrease would be a factor $\sqrt{20} \approx 0.45$, but due to the high positive autocorrelation it is less.

The subjective discount factor is set at $\beta = 0.8$. In Section 5 I assume there is a risk free asset with a gross 20-year return of $R_f = 1.13$, which is based on an average yearly 0.62% real return on three-months U.S.Treasury Bills over the 1927 to 2003 period.

4 Optimal transfer policy without a risk free asset

In this section I solve the model presented in section 2 when no risk free asset is present. I use both the iterative and the direct solution technique.

4.1 The optimal transfer scheme

Figure I shows the optimal consumption as function of the old's pre-transfer wealth for different states of the world and a coefficient of relative risk aversion (γ) of four. Panel A plots consumption for the young and Panel B for the old. Lines correspond to the solution using the iterative solution technique. Points correspond to the direct solution technique with $K = 3$.

I first discuss the lines in Figure I. The old generation may end up with low pre-transfer wealth for various reasons. It may have received low labor income or paid a high PAYG transfer when it was young. It may also have received a low return on savings. As we can see in Panel B, the young provide imperfect downside risk insurance to the old. Since $c_t^o = s_t + x_t$, the distance to the diagonal gives the transfer from the young to the old. In particular, we do not see perfect downside risk insurance in the sense that the transfer equals the payoff of a protective put on the pre-transfer wealth. Such perfect insurance would have resulted in a constant c_t^o over the lower range of s_t. The reason is that risk sharing with future generations is effectively limited to generations not too far apart in time. Therefore, to maximize unconditional expected utility it is optimal to let the old bear part of the risk on their invested capital themselves. To see this point notice that for e.g. generation $t-2$ to pay generation t an amount z, generation $t-1$ has to act as intermediary. More precisely, generation $t-1$ would have to prepay z when young, to be compensated by receiving the opportunity cost $R_t z$ when old. So this transfer reduces generation $t-2$ his wealth when young by $R_t z$. In contrast, a transfer from generation $t-1$ to generation $t-2$ would reduce generation $t-1$ his wealth when young just by z. In general, the further two generations are apart in time, the more risk is born by the younger, paying, generation. I will further illustrate this point when discussing Figure II.

Another interesting point to notice is that the transfer does not only depend on the old's pre-transfer wealth (i.e. desirability of a transfer), but also on the young's labor income (affordability of a transfer). Ceteris paribus we see that the transfer is lower when the young received low labor income. Also notice from Panel B that for large s_t there is a zero transfer. This is a direct consequence of restricting transfers to go from the young to the old (PAYG), and therefore making sharing the upside potential of the old's pre-transfer wealth impossible. In Panel A we see the intuitive result that the young consume less when they received low labor income or when they transferred much wealth to the old.

Next I discuss the points in Figure I, which correspond to the direct solution technique with $K = 3$. We have that c_t^y and x_t depend on the last three states of the world, i.e. $(\omega_t, \omega_{t-1}, \omega_{t-2})$. Therefore $s_t = \left(Y(\omega_{t-1}) - c_{t-1}^y - x_{t-1}\right) R(\omega_t)$ and $c_t^o = s_t + x_t$ depend on the past four states of the world, i.e. $(\omega_t, \omega_{t-1}, \omega_{t-2}, \omega_{t-3})$. Figure I plots the solution for all $4^4 = 256$ different 4-histories. I use different symbols for the four possible most recent states of the world, ω_t. Generally we see that results are very much in line with the results obtained using the iterative solution technique. Theoretically it is optimal for the central planner to condition his transfer on either the endogenous s_t together with ω_t or alternatively on infinite history of the state of the world. Figure I provides a first illustration that conditioning on the past few states of the world only provides a very good approximation of the fully optimal transfer scheme. The points provide some information on the distribution of the endogenous s_t under the transfer scheme, even though the probability of occurrence differs across the 4-histories. First, after a low return for the old there is always a strictly positive transfer. Perhaps more surprising at first sight is that the transfer is often also strictly positive after a high return for the old. The reason is that in the absence of a risk free asset even constant transfers facilitate intergenerational risk sharing by substituting for the absent risk free asset. In autarky a $\gamma = 4$ generation with a unit gross return risk free asset in the investment opportunity set would invest a positive amount in this risk free asset. Second, we see transfers up to around 0.4 when $R_t = R^L$. This is a large value considering the labor income for the young is either 0.95 or 1.05.

Figure II shows the wealth equivalence as a function of γ for transfers conditional on different subsets of available information. Wealth equivalence, W^{eq}, is defined as the proportional increase in the young's labor income under autarky required to make the young equally well off as under the optimal transfer scheme:

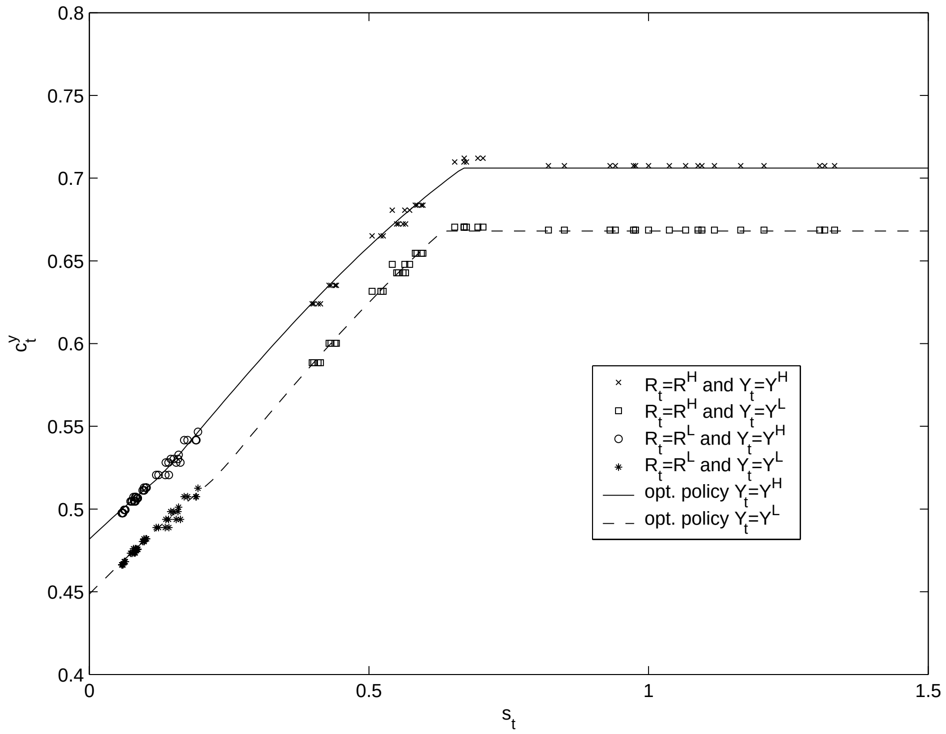
$$E\left[u\left(c^y\right) + \beta u\left(c^o\right) | Y = Y(\omega)\right] = E\left[u\left(c^y_{autarky}\right) + \beta u\left(c^o_{autarky}\right) | Y = W^{eq} Y(\omega)\right] \qquad (21)$$

The upper line corresponds to the situation where the central planner conditions the transfer on the past three realised states of the world ($K = 3$). Graphs for $K > 3$ are indistinguishable from the $K = 3$ graph, so the upper line can be considered as showing the wealth equivalence associated with the fully optimal transfer scheme. The wealth equivalence steeply increases with the coefficient of relative risk aversion γ. For the log investor the gain is only 3%, while as for the more risk averse $\gamma = 7$ investor it is 38%. The second-from-above, dashed, line shows the wealth equivalence from conditioning on the past two realised states of the world. The third-form-above, dot-dashed, line corresponds to the wealth equivalence when conditioning on the single

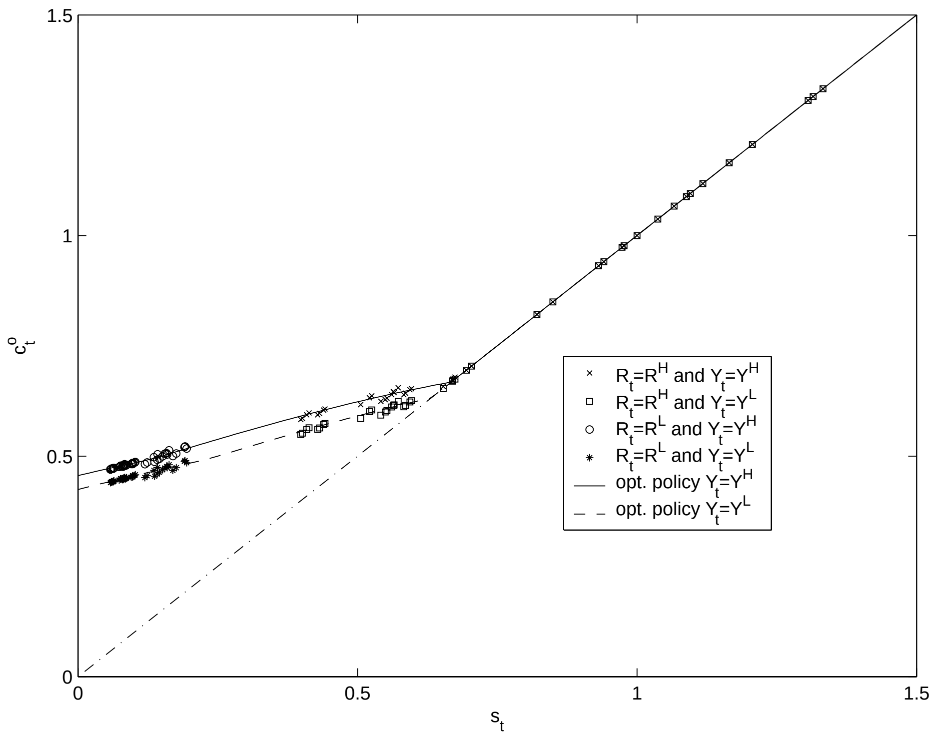
Figure I: Consumption as function of the old's pre-transfer wealth for different states of the world ($\gamma = 4$)

Lines correspond to the solution using the iterative solution technique. Points correspond to the direct solution technique with $K = 3$.

Panel A: consumption for the young



Panel B: consumption for the old



past realised state of the world, ω_t, only. We see that conditioning on ω_t only captures already the most of the wealth equivalent gain. Adding ω_{t-1} to the information conditioning set increases the wealth equivalence somewhat. Further adding ω_{t-2} has almost negligible effect on the wealth equivalence. This again illustrates that effectively risk sharing is restricted to generations not too far apart in time. The third-from-below, solid, line shows the wealth equivalence when only information on the old's capital return, R_t, is taken into account. It is close to the line for ω_t indicating that it is mainly the risk in the capital return that provides valuable risk sharing possibilities. The second-from-below, dashed, line and the lowest, dot-dashed, line correspond to the case where Y_t respectively no conditioning information is used by the central planner. The lines are almost indistinguishable from each other and well above 1. This indicates that the role of transfers to substitute for a risk free asset is important, but that there is little additional gain from the risk sharing contingent on the realised labor income.

Table II presents the optimal transfer for $\gamma = 4$ when the transfer depends on ω_t, R_t, Y_t and when it is a fixed amount. That is, it shows the optimal contract associated with the wealth equivalence of the lower four lines in Figure II at $\gamma = 4$.

Table II. Transfers conditional on different subsets of available information in different states of the world for $\gamma = 4$

	$x_t = x(\omega_t)$	$x_t = x(R_t)$	$x_t = x(Y_t)$	$x_t = x$
$R_t = R^H, Y_t = Y^H$	0.00	0.00	0.37	0.38
$R_t = R^H, Y_t = Y^L$	0.00	0.00	0.38	0.38
$R_t = R^L, Y_t = Y^H$	0.36	0.33	0.37	0.38
$R_t = R^L, Y_t = Y^L$	0.32	0.33	0.38	0.38

As can be seen in the last two columns of Table II, the role of intergenerational transfers to substitute for an absent risk free asset comprises of a transfer of approximately 0.38. The transfer is slightly lower (0.37) after a high labor income when it depends on the past labor income.[7] This is due to the positive correlation between the capital return and the labor income. When the transfer is conditional on the past return we see positive transfers after a low return. This is consistent with the imperfect downside risk protection for the old by the young. When the transfer is conditional on the past state of the world, both the desirability of a transfer by the old and the affordability by the young is taken into account. The highest transfer occurs after a low capital return and a high labor income. Finally notice that transfers in the last two columns

[7] The assumption of a constant PAYG tax rate, like in De Menil and Sheshinsky (2004), is a special case of $x_t = x(Y_t)$.

Figure II: Wealth equivalence as function of γ for transfers conditional on different subsets of available information

Results for the direct solution technique

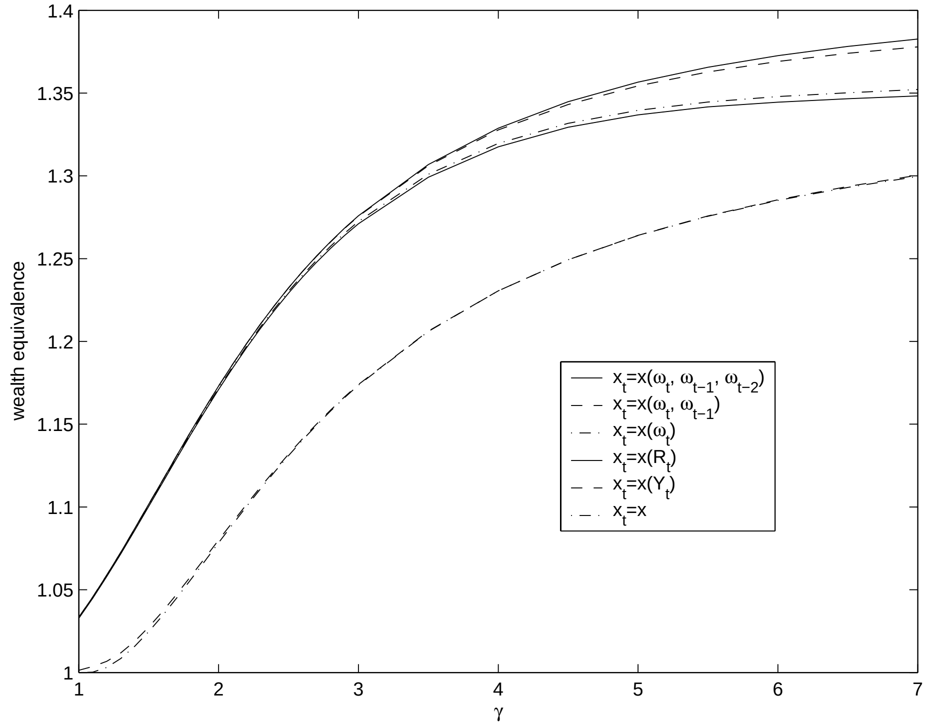
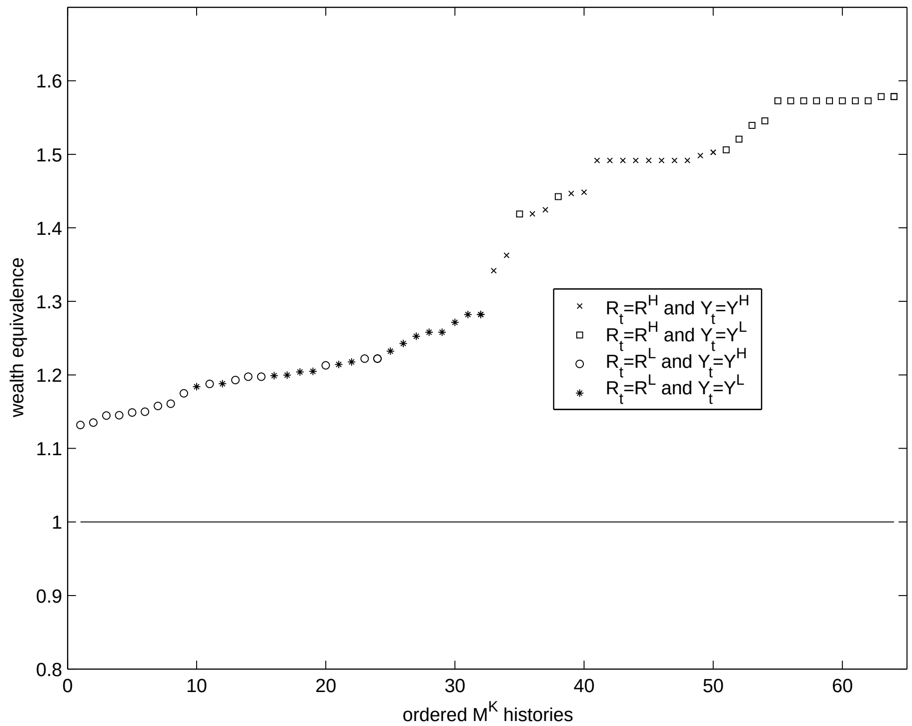


Figure III: Wealth equivalence for different 3-histories of the state of the world ($\gamma = 4$)

Result for the direct solution technique with $K = 3$



are higher than in the first two. The reason is that risk is shared less well resulting in a lower optimal investment in the risky asset and therefore a higher optimal transfer (which basically substitutes for a missing risk free asset).

Figure III presents the wealth equivalence for different 3-histories of the state of the world for $\gamma = 4$. Wealth equivalence is defined similar to (21), but with expectations taken conditional on the available information at time t. That is, for all 3-histories $(\omega_t, \omega_{t-1}, \omega_{t-2})$, $4^3 = 64$ in total, the wealth equivalence is determined for the young generation at time t. We see that the transfer scheme is most welfare improving for the young after a high return for the old. In this case they have to transfer little wealth to the old while having some downside risk protection on their savings when they are old themselves. This particularly holds when in addition the young received low labor income, represented by the squares in Figure III. The transfer scheme is less welfare improving for the young when the old realised a low return on savings and a large transfer is called for. Still we see that the wealth equivalence is greater than 1 irrespective of the circumstances when young. I will show in Subsection 4.3 that this holds for all but very low degrees of risk aversion.

4.2 Determinants of the utility gain

The second column of Table III shows the wealth equivalent gain when the input parameter given in the first column is changed for a $\gamma = 4$ investor. Results are obtained using either the iterative solution technique or the direct solution technique for $K \geq 3$. The third column presents the change in the wealth equivalent gain relative to the wealth equivalent gain in the base case, being 32.88%. First focus on Panel A. Parameter values 10% larger than in the base case are considered. As is clear for from the table, the standard deviation on the capital return is an important determinant of the wealth equivalent gain. The higher it is, the bigger the potential for intergenerational risk sharing, the larger the wealth equivalent gain. The mean capital return is also important. The larger $\bar{r}$, the smaller the role of the central planner to substitute for a risk free asset with unit gross return. The risk in labor income is too small in my calibration to play an important role. The subjective discount factor plays only a minor role as well when utility is evaluated relative to autarky with the same β. This is comforting given that choosing a value for β is not trivial.

Table III. Wealth equivalent gain with alternative parameter values for $\gamma = 4$
Results based on either the iterative solution technique or the direct solution technique with $K \geq 3$. The wealth equivalent gain in the base case equals 32.88%.

Panel A: 10% **increase in** $\bar{r}$, σ_r, σ_y **and** β

10% increase in	wealth equivalent gain	% change wrt base case
mean log risky ret. $\bar{r}$	30.64%	−6.81%
st. dev. log risky ret. σ_r	40.56%	23.36%
st. dev. log labor inc. σ_y	32.83%	−0.15%
subjective discount rate β	33.28%	1.22%

Panel B: zero value for cs_r, cs_y, ρ_{yy^-} **and** ρ_{ry}

zero value for	wealth equivalent gain	% change wrt base case
coef. skewness log risky ret. cs_r	28.61%	−12.99%
coef. skewness log labor inc. cs_y	32.88%	0.00%
autocorr. log labor inc. ρ_{yy^-}	32.90%	0.06%
corr. log risky ret. and log labor inc. ρ_{ry}	33.96%	3.28%

In Panel B I show the input parameters for which I consider a zero value the most interesting alternative specification. We see that in particular the skewness of the capital return is important. The wealth equivalent gain in the presence of negative skewness in the capital return is considerably higher than without skewness. This is intuitive considering the PAYG transfers provides a means to share downside risk on the old's saving and the concavity of the utility function u. The skewness and autocorrelation in labor income are of negligible importance. This is a direct consequence of the labor income being relatively little risky in the calibration. Finally, notice that changing the correlation between the log capital return and the log labor income from 0.3 in the base case to 0 increases the wealth equivalent gain. In general the smaller this correlation the larger are the intergenerational risk sharing possibilities. However, again because labor income is not very risky in my calibration the increase in the wealth equivalent gain is only 3.28%.

4.3 Adding an incentive constraint for the young

Up to now the only restriction on transfers was that they are in the direction from the young to the old, i.e. $x \geq 0$. In this Subsection I investigate the additional restriction that the young are at least as well off in expected utility terms under the transfer scheme than under autarky for all states of the economy. Notice that the old are always at least as well off in expected utility terms because $x \geq 0$. In fact, it is straightforward to show that transfer schemes that satisfy the two restrictions are exactly those that are sustainable as a subgame-perfect equilibrium.[8] By imposing an incentive constraint for the young the implementation of the transfer scheme will be enhanced. I consider this a first step towards more advanced modeling of the political voting system. To summarize, the transfer scheme should be such that the young voluntarily decide to cooperate knowing their realised labor income and the old's realised capital return. This is quite a large information set to use for the decision and will therefore result in a conservative estimate for the wealth equivalent gain.[9]

As was clear from Figure III, cooperation is not an issue for $\gamma = 4$. In fact for my parameter values it is only an issue for investors with low risk aversion. Table IV shows the wealth equivalent gain for when the only restriction on transfer is $x \geq 0$ and for when we have the additional incentive constraint. For $\gamma = 2.5$ and higher cooperation is satisfied in the first-best solution and the wealth equivalent gains are therefore equal. On the other hand, for the log investor the whole transfer scheme collapses in the sense that only the zero transfer scheme satisfies both the $x \geq 0$ and the cooperation restriction. To understand why especially for low γ cooperation is an issue, recall that in the absence of a risk free asset the intergenerational transfers serve dual purposes. The transfer can be considered as being the sum of a zero-mean, state-dependent component and a constant component. The first provides downside risk insurance to the old on their savings and the second substitutes for the missing risk free asset. A young generation might anticipate that the state-dependent component of the transfer is a bad deal given his information, but still rationally choose to cooperate due to the substitution component. However, for low risk aversion the substitution component is not very important. See e.g. the lowest line in Figure II, which shows that the wealth equivalence with a fixed transfer is close to one for low γ. This causes the first-best state-contingent risk sharing to be unsustainable when cooperation in the defined manner is required.

Table IV. Wealth equivalent gain with and without additional incentive constraint for the young

γ	weq gain	weq gain (cooperation)
1	3.34%	0.00%
1.5	10.61%	9.06%
2.0	17.31%	17.30%
2.5	23.23%	23.23%

[8] See e.g. Rangel (2003) for a treatment of subgame-perfect equilibria with intergenerational transfers in a non-stochastic setting.

[9] Ideally one would like to have a model with more than two periods. However, this involves such an increase in the computational burden that it can only be handled by simplyfying the stochastic environment considerably.

$\gamma = 1.5$ is an intermediate case. As can be deducted from Table IV, 89% of the wealth equivalent gain under first-best solution can be attained with the cooperation restriction. Figure IV shows the wealth equivalence for the different 3-histories of the state of the world for the $\gamma = 1.5$ investor without (Panel A) and with (Panel B) the additional incentive constraint for the young. Notice in Panel A that the transfer scheme is a bad deal for young generations when the old realised a low return on capital. Under the first-best transfer scheme they transfer a substantial amount to the old. They might not receive such a large transfer themselves when they are old, because they might realise a high capital return themselves. In Panel B we see that exactly the generations in this situation when young are made just as well off, i.e. a wealth equivalence of 1. This generally does not mean however that the transfer is zero for them. They just get a better deal compared to Panel A to make them indifferent with respect to cooperation.

5 Introduction of a risk free asset

In Section 4 we have seen that in the absence of a risk free asset the intergenerational transfers serve dual purposes. They provide downside risk insurance to the old on their savings and they substitute for the absent risk free asset. In this Section I investigate optimal intergenerational risk sharing when a risk free asset (in real terms) is present. As shown by Campbell and Viceira (2002) nominal bonds are quite risky at such horizons and inflation-indexed assets might not be available at a reasonable price. However, the markets for inflation-indexed products might further develop in the future and this analysis explores the implications for intergenerational risk sharing.

As in Section 2 I assume that the central planner can credibly commit to transfers in terms of the history of exogenous states of the world. This means that there is no conflict of interest between the young and the central planner regarding the young's consumption and asset allocation choice. So equivalently the central planner sets not only transfers, but also the young's consumption and asset allocation. Denoting the fraction allocated to the risky asset by α, (6) becomes

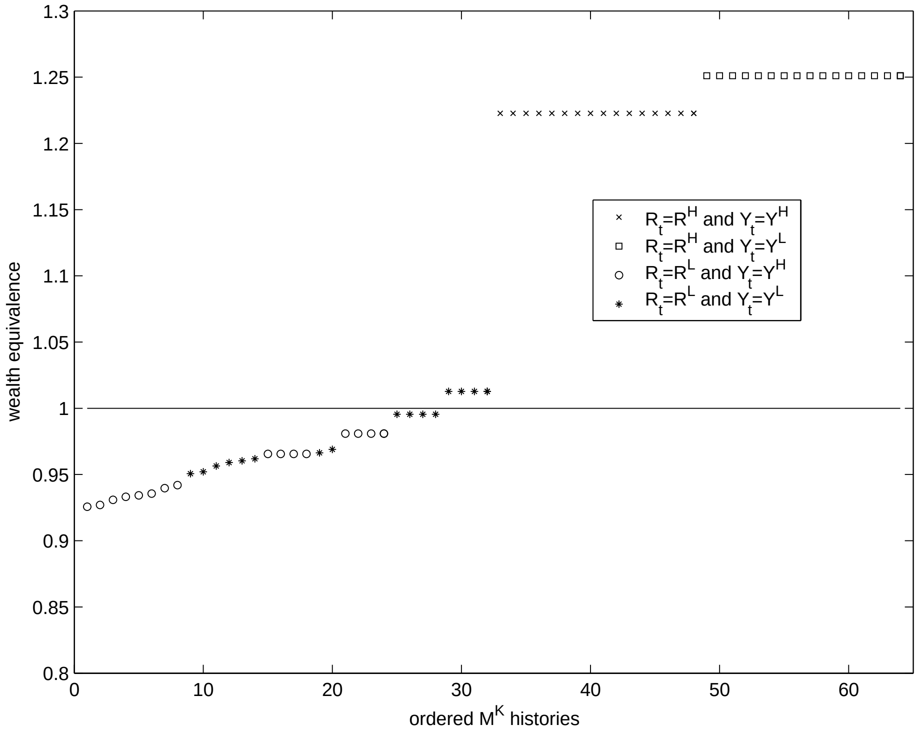
$$\begin{aligned} \bar{V} &= \max_{\substack{c^y(\omega,s),x(\omega,s),\alpha(\omega,s)\\ \omega\in\Omega,s\geq 0}} E\left[u\left\{c^y\left(\omega_t,s_t\right)\right\}+\beta u\left\{s_{t+1}+x\left(\omega_{t+1},s_{t+1}\right)\right\}\right] \qquad (22)\\ s_{t+1} &= \left[Y\left(\omega_t\right)-x\left(\omega_t,s_t\right)-c^y\left(\omega_t,s_t\right)\right]\left[\alpha\left(\omega_t,s_t\right)R\left(\omega_{t+1}\right)+\left(1-\alpha\left(\omega_t,s_t\right)\right)R_f\right] \end{aligned}$$

I will assume no short-sale possibilities for either asset, so we have $\alpha \in [0, 1]$.

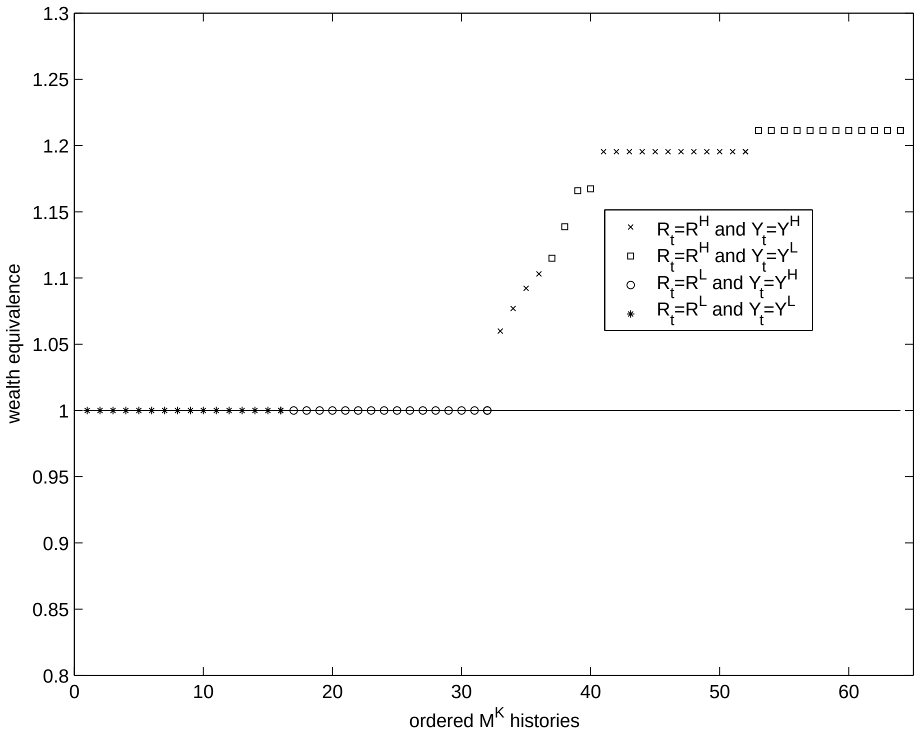
Figure IV: Wealth equivalence for different 3-histories of the state of the world ($\gamma = 1.5$)

Results for the direct solution technique with $K = 3$.

Panel A: main model



Panel B: requirement that young voluntarily choose to cooperate



5.1 The optimal transfer scheme

Figure V shows the model solution for $\gamma = 4$ in a similar manner as Figure I did for the case where a risk free asset was absent. Panel A and B show consumption for the young and old respectively as a function of the old's pre-transfer wealth. Panel C shows the fraction invested in the risky asset as function of the old's pre-transfer wealth. As in Figure I, points correspond to the direct solution technique and the solid and dashed lines correspond to the iterative solution technique. Also depicted in Panel C as dot-dashed line is the autarkian risky asset allocation which by the scale independency of power utility is state independent.

Qualitatively both the young and old's consumption look similar to consumption in the absence of a risk free asset. Again the young provide imperfect downside risk insurance to the old. Again the central planner takes into account both the desirability for the old (i.e. value for s_t) and the affordability for the young (i.e. value for Y_t). As before, in Panel B of Figure V, the transfer is given by the vertical distance to the diagonal. For a given value for the old's pre-transfer wealth, s_t, the size of the transfer is very similar to the size in the absence of a risk free asset as depicted in Panel B of Figure I. However, with a risk free asset low values for s_t occur less often and therefore nonzero transfers occur less often and transfers are smaller conditional on being nonzero. So with a risk free asset the old are provided very similar downside risk protection by the young, but they need it less often.

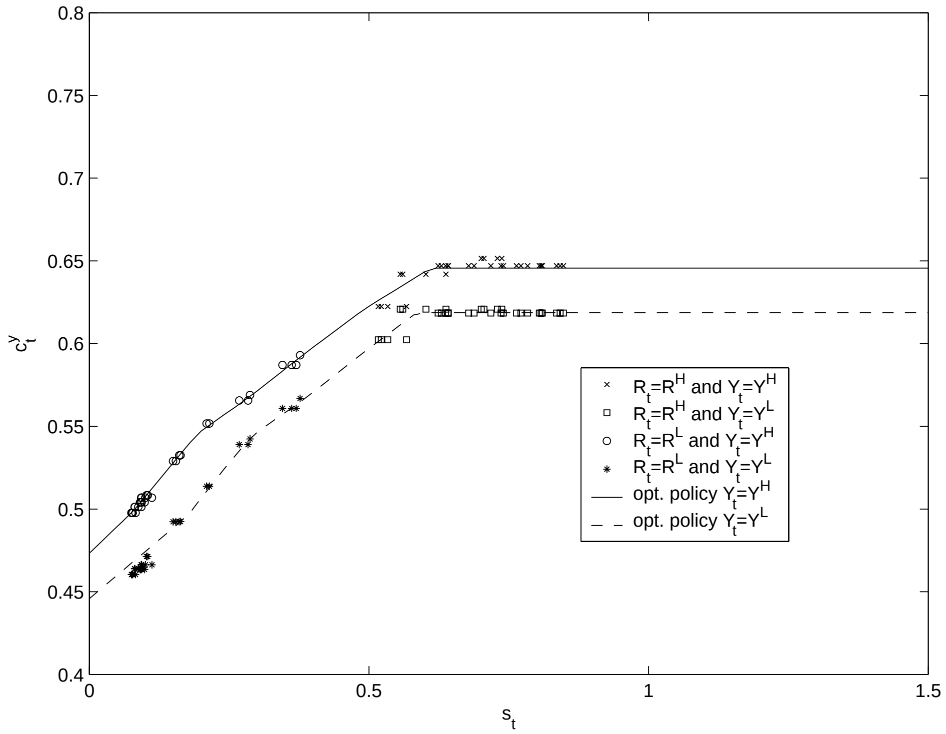
Now focus on Figure V, Panel C. First consider the young's optimal risky asset allocation when they transferred zero wealth to the old. This is for s_t more than approximately 0.7. Risk taking is larger than under autarky because the young will receive some downside risk protection when they are old. It is particular large when the young received low labor income since downside risk protection is means tested. This also explains why risk taking is even larger when s_t is less than 0.7 and the young transferred a nonzero amount to the old.

Figure VI plots wealth equivalence as a function of γ for different conditioning information sets. The upper line corresponds to the situation where the central planner conditions the transfer on the past three realised states of the world ($K = 3$). Graphs for $K > 3$ are indistinguishable from the $K = 3$ graph, so the upper line can be considered as showing the wealth equivalence associated with the fully optimal transfer scheme. In contrast to Figure II, which is the counterpart of Figure VI when no risk free asset is present, wealth equivalence is hump-shaped in γ. For low γ, the non-negativity constraint on the risk free asset, i.e. $\alpha \leq 1$, is binding in most states of the world. This means that in this range for γ the wealth equivalence is increasing in γ because the benefits from sharing risk are higher for a higher degree of risk aversion. For yet higher γ the non-negativity constraint on the risk free asset is not binding and the risky asset allocation decreases in γ. This means that the risk sharing potential decreases

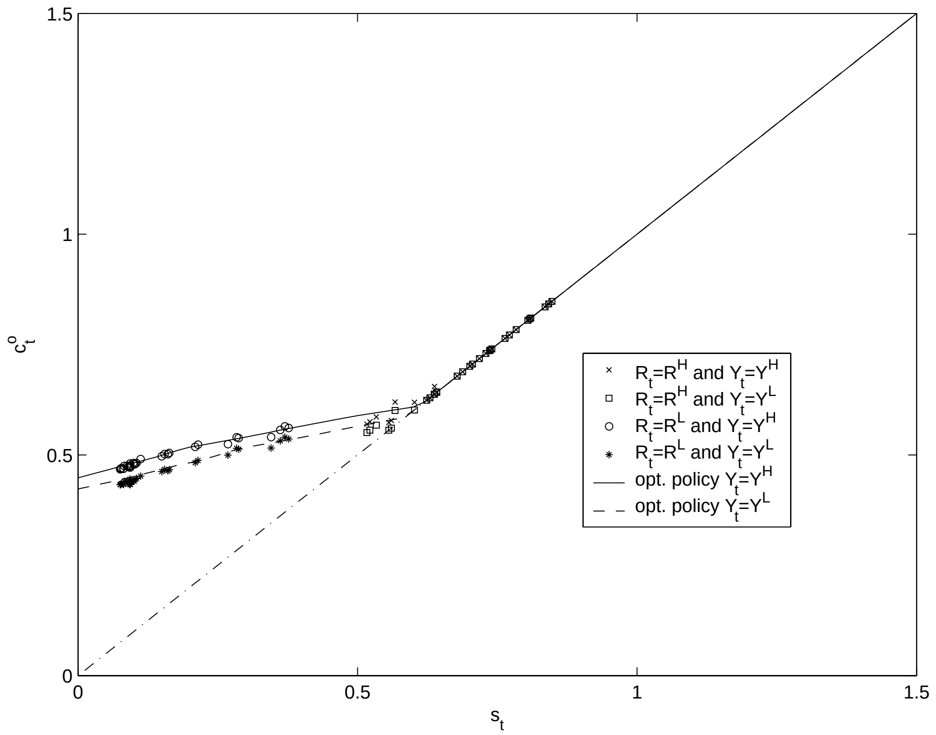
Figure V: Consumption as function of the old's pre-transfer wealth for different states of the world ($\gamma = 4$)

Lines correspond to the solution using the iterative solution technique. Points correspond to direct solution technique with $K = 3$.

Panel A: consumption for the young



Panel B: consumption for the old



Panel C: allocation for the young

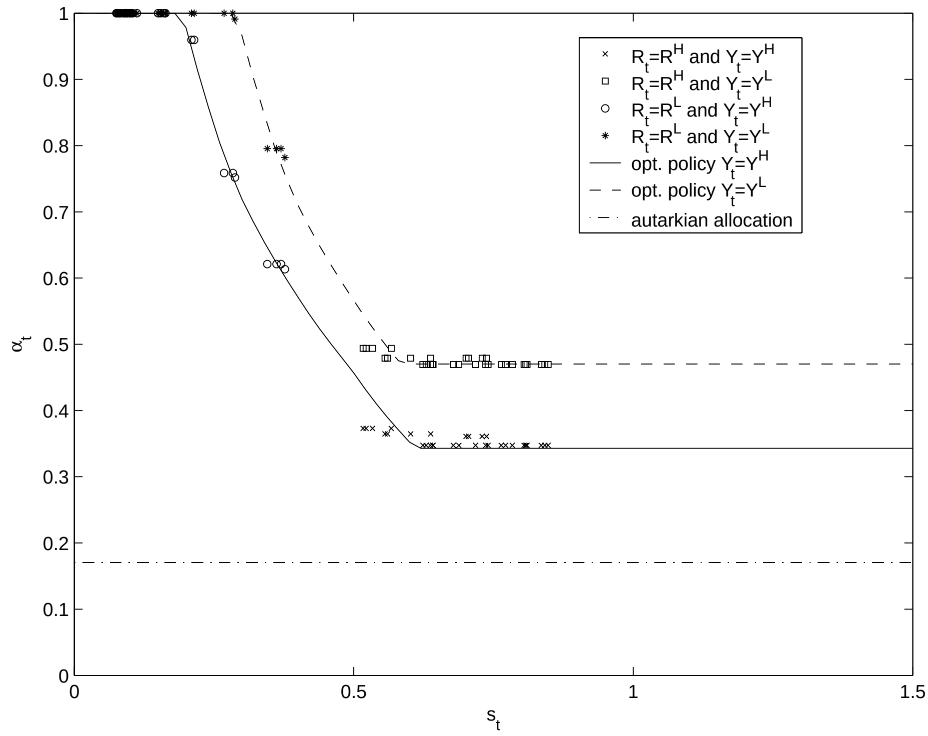
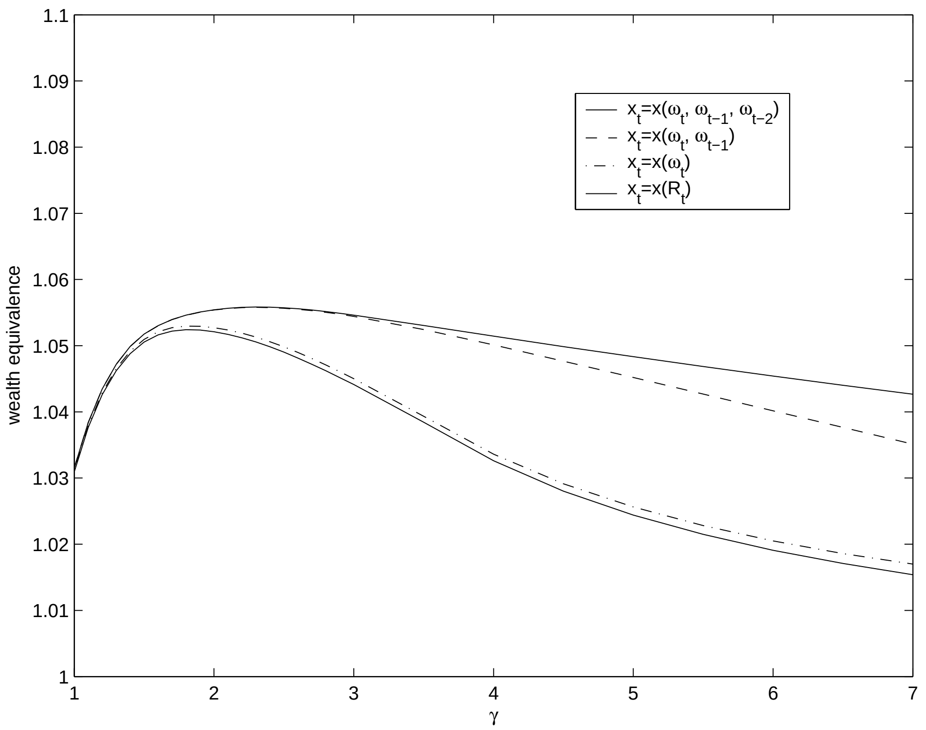


Figure VI: Wealth equivalence as function of γ for transfers conditional on different subsets of available information

Results for the direct solution technique with $K = 3$



in γ and therefore the wealth equivalence decreases in γ.

The second-from-above, dashed, line and the third-from-above, dot-dashed line, correspond to the situation where the transfer is conditional on the past two respectively one state of the world. The two lines are quite far apart indicating that in the presence of a risk free asset a relatively large part of the gains are effectively due to risk sharing between non-subsequent generations. As in Figure II, conditioning on the past return only implies values close to those realised when conditioning on the past state of the world. This again illustrates that mainly capital risk is shared intergenerationally for my parameter values.

Finally notice in Figure VI that I did not add the wealth equivalence for $x_t = x(Y_t)$ and $x_t = x$, as I did in Figure II. The reason is that they coincide with the horizontal axis. When a risk free asset with gross return greater or equal to one is present, the substitution role for intergenerational transfers disappears.

5.2 Determinants of the utility gain

In the second column of Table V I present the wealth equivalent gain when the input parameter given in the first column is changed for a $\gamma = 4$ investor. Results are obtained using either the iterative solution technique or the direct solution technique for $K \geq 3$. The third column presents the change in the wealth equivalent gain relative to the wealth equivalent gain in the base case, being 5.14%. First focus on Panel A. Parameter values 10% larger than in the base case are considered. Again the mean capital return is important. In contrast to Table III where the substitution role of transfers figured prominently, now the larger $\bar{r}$, the larger the allocation to the risky asset, the larger the role of the central planner to facilitate intergenerational risk sharing by providing downside risk insurance. Also the standard deviation on the capital return affects the wealth equivalent gain considerably. The higher it is, the lower the risky asset allocation but also the larger the potential of intergenerational risk sharing for a given allocation. Per saldo we see that the larger the standard deviation on the capital return, the larger the wealth equivalent gain. Again the risk in labor income is too small in my calibration to play an important role and also the subjective discount factor β plays only a minor role. The effect of an increase in the risk free rate r_f is opposite to that of an increase in $\bar{r}$. The higher it is, the smaller the allocation to the risky asset, the smaller the wealth equivalent gain.

Table V. Wealth equivalent gain with alternative parameter values for $\gamma = 4$
Results based on either the iterative solution technique or the direct solution technique with $K \geq 3$. The wealth equivalent gain in the base case equals 5.14%.

Panel A: 10% **increase in $\bar{r}$, σ_r, σ_y, β and $r_f = \log(R_f)$**

10% increase in	wealth equivalent gain	% change wrt base case
mean log risky ret. $\bar{r}$	5.53%	7.59%
st. dev. log risky ret. σ_r	5.38%	4.67%
st. dev. log labor inc. σ_y	5.14%	0.00%
subjective discount rate β	5.18%	0.78%
log risk free rate $r_f = \log(R_f)$	4.79%	−6.81%

Panel B: zero value for cs_r, cs_y, ρ_{yy^-} and ρ_{ry}

zero value for	wealth equivalent gain	% change wrt base case
coef. skewness log risky ret. cs_r	4.84%	−5.84%
coef. skewness log labor inc. cs_y	5.14%	0.00%
autocorr. log labor inc. ρ_{yy^-}	5.16%	−0.39%
corr. log risky ret. and log labor inc. ρ_{ry}	5.72%	11.28%

In Panel B I show the input parameter for which I consider a zero value as alternative specification the most interesting. The skewness of the capital return is important. As in the case without risk free asset, the wealth equivalent gain in the presence of negative skewness in the capital return is considerably higher than without skewness. The skewness and autocorrelation in labor income are of negligible importance. Finally, notice that changing the correlation between the log capital return and the log labor income from 0.3 in the base case to 0 increases the wealth equivalent gain by 11.28%.

5.3 Adding an incentive constraint for the young

Again I discuss the impact of adding an incentive constraint for the young. That is, the transfer scheme must be such that the young generation would voluntarily choose cooperation over autarky. In the absence of a risk free asset we saw that a young generation might anticipate that the transfer called for by the downside risk insurance component of the transfer scheme is a bad deal given the state of the economy, but still rationally chooses to cooperate due to the substitution component. Only for very low risk aversion the substitution role was too unimportant to sustain the first-best transfer scheme in an incentive compatible setting.

In the presence of a risk free asset the incentive constraint has a larger impact. In fact, the transfer scheme collapses to the zero-transfer scheme for all γ. So for downside risk insurance to be sustainable in an incentive compatible setting of the kind introduced, it is crucial whether intergenerational transfers also play a role as substitute for a missing risk free asset.

6 Concluding remarks

I used a stylized two-period overlapping-generations model to investigate intergenerational risk sharing. I calibrated the model parameters to U.S. data. Some important features of the optimal intergenerational transfer scheme were identified. The young generation provides imperfect downside risk insurance to the old on their savings. The size of the transfers and the associated utility gain depend heavily on the capital return risk and not so much on the labor income risk. An implication for Social Security design is that to exploit intergenerational risk sharing to the fullest the transfer and therefore the Social Security payment should not so much depend on the labor income of the workers, but should optimally take into account the return the retirees have made on their retirement savings (for example measured as the return on a world index).

Furthermore, whether a risk free asset is present or not is found to be crucial for two reasons. First, when not present the transfers substitute for the missing risk free asset. Second, when not present cooperation by the young is only a minor issue. When present, imposing an incentive constraint for the young leads to the collapse of the transfer scheme to the zero scheme. That is, for the fully optimal state-contingent transfer scheme to be sustainable in an incentive compatible setting of the kind introduced, it is crucial that the transfers also play the role of substitute for a missing risk free asset. A natural and important question that arises is to what extent a risk free asset is present in the free market. For this it is important to stress that in this set-up this represents a long-term investment (say 20 years) that provides a fixed real return (i.e. inflation corrected). Nominal bonds do not qualify for such an asset and inflation-indexed bond markets are just recently starting to develop. Further maturing of the inflation-indexed bond markets will depend on the inflationary policy of the government. A practical implication is that inflationary policy and Social Security policy are related and should therefore be considered simultaneously. I consider the impact of the political system on the sustainability of transfer schemes an important topic for future research. The analysis on the impact of an incentive constraint for the young in this paper is a first step in this direction.

The model developed in this paper could be extended to investigate some related interesting issues. Depending on the particular extension one of the two presented solution techniques might be more suitable than the other. The iterative solution technique is suitable to analyse a closed economy where transfers influence future labor income and capital return. It is also suitable to investigate optimal out-of-equilibrium transfers, which is relevant for initiating the transfer scheme. On the other hand the direct solution technique might be better for handling issues related with cooperation, like the impact of investor heterogeneity on the optimal incentive compatible transfer scheme.

7 Literature

Balke, Nathan S. and Robert J. Gordon (1989), The estimation of prewar gross national product: Methodology and new evidence, *Journal of Political Economy*, 97(1), 38-92.

Bohn, Henning (2003), Intergenerational risk sharing and fiscal policy, Working Paper.

Campbell, John Y., and Luis M. Viceira (2002), *Strategic asset allocation: portfolio choice for long-term investors*, Oxford University Press Inc., New York.

Deaton, Angus and Guy Laroque (1992), On the behavior of commodity prices, *Review of Economic Studies*, 59(1), 1-23.

Demange, Gabrielle, and Guy Laroque (1999), Social Security and demographic shocks, *Econometrica*, 67 (3), 527-542.

Demange, Gabrielle (2002), On optimality in intergenerational risk sharing, *Economic Theory*, 20, 1-27.

De Menil, Georges, and Eytan Sheshinski (2004), The optimal balance of intergenerational transfers and funded pensions in the presence of risk, Working Paper.

Diamond, Peter A. (1965), National debt in a neoclassical growth model, *American Economic Review*, 55, 1126-1150.

Diebold, Francis X. and Abdelhak S. Senhadji (1996), The uncertain unit root in real GNP: Comment, *American Economic Review*, 86(5), 1291-1298.

Fama, Eugene F. and Kenneth R. French (1988), Permanent and temporary components of stock prices, *Journal of Political Economy*, 96(2), 246-273.

Fama, Eugene F., and Kenneth R. French (2002), The equity premium, *Journal of Finance*, 57(2), 637-659.

Gordon, Roger H., and Hal R. Varian (1988), Intergenerational risk sharing, *Journal of Public Economics*, 37, 185-202.

Krueger, Dirk, and Felix Kubler (2002), Pareto improving Social Security reform when financial markets are incomplete!?, NBER Working Paper 9410.

Lo, Andrew W. (1991), Long-term memory in stock market prices, *Econometrica*, 59(5), 1279-1313.

Merton, Robert C. (1983), On the role of Social Security as a means for efficient risk sharing in an economy where human capital is not tradable, in *Financial Aspects of the United States Pension System* (Z. Bodie and J. Shoven, Eds.), Chicago: University of Chicago Press for the NBER.

Poterba, James M., and Lawrence H. Summers (1988), Mean reversion in stock prices: evidence and implications, *Journal of Financial Economics*, 22, 27-59.

Rangel, Antonio (2003), Forward and backward intergenerational goods: why is Social Security good for the environment?, *American Economic Review*, 93(3), 813-834.

Romer, Christina D. (1989), The prewar business cycle reconsidered: new estimates of gross national product, *Journal of Political Economy*, 97(1), 1-37.