

Long-term debt and hidden borrowing*

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Abstract

We consider borrowers with the opportunity to raise funds from a competitive banking sector that shares information, and from an alternative hidden lender. The presence of the hidden lender restricts the contracts that can be obtained from the banking sector. In equilibrium some borrowers obtain funds from both the banking sector and the inefficient hidden lender simultaneously. We further show that as the cost of borrowing from the hidden lender increases, total welfare increases. We generalize the model to allow for a partially hidden lender and obtain qualitatively similar results.

Keywords: Long term debt, hidden borrowing, debt contracts, adverse selection

JEL Codes: D82, D14, G21, D86

1 Introduction

Households have many potential sources of credit available, including secured mortgages, installment loans, bank overdrafts, store credit, credit cards, payday loans, borrowing from family, and borrowing from other “informal” sources. Similarly, small and large firms face a number of different financing options ranging from private placements, securitized loans, trade credit, and personal loans to the owner. These alternative forms of financing differ in a number of ways. While some of these differences might be endogenous (such as the

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interest rate, or the term length for repayment), there are also exogenous differences—for instance, with respect to the seniority of the claims and enforcement.

An empirical puzzle is that borrowers appear to borrow from apparently costly lenders while not fully exhausting cheaper sources. Gross and Souleles (2002), for example, report that in a large sample of credit card holders, almost 70% percent of those borrowing on bankcards have positive housing equity. Similarly, small businesses often use uncollateralized trade credit when collateral and collateralized loans are available. Finally, there is both theoretical work and empirical evidence of formal and informal sources of credit coexisting in developing countries (Bell et al (1997), Bose (1998), Jain (1999)). In these cases where some agents simultaneously borrow from both sources, it is unclear whether agents are rationed by the formal sector before they access the informal one.¹

We suggest that an important consideration in understanding this puzzle is that while some lenders share information about borrowers, others do not.² This allows borrowers the chance to conceal liquidity shocks that affect their creditworthiness by borrowing from junior lenders whose loans are hard to observe by senior lenders. Thus, even when seniority is well defined, a senior lender cares about the existence of junior lenders because the possibility that the borrower is using them affects the information obtained through interim repayments.

As an example, missing a lease payment can trigger a renegotiation with the financier and lead to a higher future interest rate. This reflects the financier’s renewed assessment of the borrower’s ability to repay. An effort to renegotiate the loan may well be costly for the borrower, because of the information revealed in the process. This can be interpreted as an endogenous renegotiation cost.

In order to avoid this penalty, the borrower might borrow from elsewhere to conceal the bad news that she has suffered a liquidity shock. In turn, this makes missing a payment even worse news as it reflects a liquidity shock so large that it is prohibitively costly to conceal. The presence of a hidden lender increases this informational penalty and leads

¹While some of this literature posits a tradeoff between an informal sector, with better information about borrowers’ abilities to repay, and a formal sector with a lower cost of capital, the information available to the informal lender does not play any role in our model.

²Note that other explanations have been posited to explain this apparent puzzle; for example, Laibson et al. (2001) calibrate a model of life-cycle borrowing with time inconsistent preferences, and Haliassos and Reiter (2003) discuss a model of separate mental accounts. The results of this paper need not contradict such explanations but can be seen as complementary to them. While our results (as any models which assume fully rational consumers) fail to explain the coexistence of credit card debt and liquid assets, they can be seen as suggesting some endogenous illiquidity of certain assets.

to higher renegotiation costs. In the model that we present, the resulting overall cost of renegotiation is sufficiently high so that the financier would repossess the asset or foreclose following a missed payment.

We illustrate these ideas more formally in a two-period model, where heterogeneous agents can access two sources of funds: a competitive banking sector that shares information, and an opaque lending sector. Banks are senior claimants and seek to obtain information regarding borrowers through interim payments. While, most of our discussion views banks as providing flexible long-term (two period) financing, one could also interpret the banking sector as providing a sequence of short-term loans.

Our principal results are that in the absence of the opaque sector realized contracts are complex menus. Higher levels of interim payments lead to lower final payments. This is not only to take into account that less is owed, but also because a higher interim payment reflects that the borrower is less of a credit risk.³ However, with a viable alternative hidden lender, a borrower might be tempted to borrow from that source in order to disguise her type. This possibility is anticipated by the original lender in the banking sector. In the model the unique equilibrium results in only a single level of interim payment observed in the banking sector. Further, some agents borrow from the opaque sector to make this payment. Thus, in equilibrium, these agents are simultaneously borrowing from both the banking and the opaque sectors.

We consider how the welfare of consumers and the transparent sector vary with the cost of borrowing from the opaque source. In particular, a lower cost of borrowing benefits consumers for a fixed level of borrowing, but it also encourages a greater number of inefficient types to continue to borrow rather than terminate the debt contract and would lead the contract in the transparent sector to change. Overall welfare falls.⁴

A key element of the model is that a lender may not perfectly observe all the loans that a borrower may hold. Empirically, this is certainly the case. For example, although information sharing takes place through credit bureaus, there are many lenders who choose neither to pay for access to credit bureaus nor to provide information to them. Trade credit, informal, black market lending and payday lenders are clear examples. Further examples

³This result mirrors the observation in Allen (1985) that long-term contracts allow interim payments to provide information.

⁴This result is to some extent an application of the general theory of the second best, discussed by Lipsey and Lancaster (1956). This states that in the presence of some market imperfections that cannot be eliminated, there is no guarantee that a move towards eliminating other market imperfections will make the market more efficient and indeed it may make the market less efficient.

include consumer credit, store credit and other sources that do not participate in formal information-gathering credit bureaus, both in developing countries and elsewhere currently and historically. Even when a lender has access to a credit bureau, the costs associated with accessing and processing the relevant information may lead lenders to obtain and use this information only in particular circumstances. Such circumstances would include the loan approval stage, missed payments, and renegotiation. Otherwise there is unlikely to be continual monitoring. In this paper, we simply take it for granted that some types of borrowing are not commonly observed by all lenders.⁵

The banking sector cannot write contracts that make payments depend on the agent's borrowing from the hidden lender. This is a natural consequence of the assumption that the banking sector cannot observe borrowing from the hidden lender. This paper is therefore related to a growing literature on non-exclusive contracts and on hidden savings, which includes Allen (1985), Cole and Kocherlakota (2001), Bisin, and Guaitoli (2004), and Doepke and Townsend (2004). Our model differs from those in a number of respects and in its motivation. In particular, we consider different lending sectors that vary in the information that they have, we allow for adverse selection rather than presenting a pure moral hazard model (Bizer and DeMarzo (1992) and Arnott and Stiglitz (1991) for example), and we model agents to be risk-neutral with limited liability, rather than risk-averse and seeking to smooth consumption or buy insurance—an important focus of this literature.

A key aspect of our analysis is varying the cost of borrowing from the hidden source.⁶ Allen (1985) and others focus on the case where this cost is equal to the social planner's rate. Innes (1990), in order to generate monotonicity in repayment schedules, considers the case where money can be repaid immediately so that essentially the cost of borrowing is zero.

Section 2 of this paper introduces the model and elaborates the key assumptions. In Section 3, we solve for the equilibrium and characterize the principal results; in particular, we discuss comparative statics with respect to the cost of borrowing from the opaque sector and implications for welfare. We briefly discuss an extension to the case where borrowing

⁵For further information on consumer credit reporting in the US and further references both on the theory and development of credit bureaus and reporting institution, the interested reader is referred to Hunt (2002).

⁶The general model of Doepke and Townsend (2004), as illustrated in their example in Section 7.1, allows for this more general interest rate; however, as in Cole and Kocherlakota (2001) and Ljungqvist and Sargent (2003), they consider hidden saving and insurance rather than hidden borrowing and focus on numerical rather than analytical solutions.

from the opaque sector can be observed with some probability in Section 4. The final section concludes.

2 The Model

Although the underlying economic mechanisms have wider applicability, we focus the model on the particular example of a start-up firm that is raising funds for an investment project. We first present the basic set-up, timing, and structure of the model. We then introduce additional assumptions that rule out uninteresting cases and simplify the analysis of the model.

2.1 Set-up

We introduce a two-period model to consider the interaction between alternative sources of borrowing, a transparent banking sector, and an opaque hidden lending sector. In the transparent sector, credit is provided by a continuum of agents that we call banks. Banks are risk-neutral deep pockets, and there is competition among them. Banks share information, and so the borrowing position of any agent with a bank is perfectly observable and verifiable among all banks. We normalize the gross riskless market interest rate of this formal sector to one. The principal assumptions on the banking sector can be summarized as follows:

Assumption 1: The total amount of loanable funds in the banking sector exceeds demand.

Assumption 2: A borrower can repay her outstanding balance and switch to another bank at any point in time.

Assumption 3: Banks perfectly share the information about the borrower's outstanding loans.

These assumptions guarantee both that banks do not make a profit on average and that conditional on the information known at any point in time, every contract offered must break even. In short, there can be no observable cross-subsidies between borrowers. If a set of borrowers knew and were able to prove to a third party that they are subsidizing other borrowers, they would switch to another bank, leaving their previous bank with only subsidized borrowers and so losses.⁷ Note that we assume that banks are committed to

⁷Note that the assumption of perfect competition within the banking sector is not crucial for our results. If there were small costs in switching banks, for example, in the absence of an opaque sector, banks will offer contracts that are contingent on type (although not fully contingent). The introduction of a hidden sector would still lead to less contingent contracts and reduce welfare.

sharing information. In particular this implies that they cannot simply replicate the hidden lender as they have no means to hide such contracts from other banks.

In addition to the transparent banking sector we introduce an alternative opaque lending sector that lends at a flat repayment rate r ($r > 1$), which is exogenously given.⁸ A key characteristic of this alternative borrowing source is that it does not share information with the rest of the financial system. That is, the borrowing position of any agent in the opaque sector is not observable by banks. Further, we model the opaque sector as a junior lender. This is certainly consistent with an interpretation as trade credit or credit cards.⁹ In our model lenders exogenously belong to either the banking sector or the opaque sector. Pagano and Japelli (1993) discuss determinants of belonging to either group as an endogenous decision.

Demand for funds come from borrowers who require these funds for an investment project, and who are heterogeneous in the quality of their projects. They are risk neutral and maximize total consumption across periods. We assume that they have all the bargaining power and propose contracts before knowing their types. This assumption may not be crucial for the qualitative insights; however, it is convenient in characterizing a unique equilibrium outcome.

The timing of the model is as follows:

At $t = 0$ each borrower does not know her type. In order to raise D units of funding necessary to invest in the project, the borrower proposes a schedule of first and associated second period repayments $\{p, q(p)\}$ to a bank.

At $t = \frac{1}{2}$, each borrower learns the type of her project, which is parametrized by $\alpha \sim U[0, 1]$. At this point, the borrower can either liquidate the project and fully repay the loan or continue with the project.¹⁰

At $t = 1$ agents realize a cash flow α that corresponds to their type. They can choose to borrow d from the informal lending source. The informal lender is junior to the bank loan, and banks do not observe d . Agents can use these funds to either consume or repay p to the bank. Agents consume anything left over, so residual income cannot be used to

⁸This can be endogenized easily, as we discuss in the paragraph preceding Section 3.1.

⁹Other types of hidden lending, including informal lending or borrowing from family, are more ambiguous with respect to seniority.

¹⁰We model this option to stop the project as a costless liquidation in a very early stage; but supposing that the agent was able to recover a sufficiently large salvage value at an early stage would generate similar qualitative results.

repay future debts.¹¹

At $t = 2$ the project is successful and delivers $B + \alpha$ with probability ν . Otherwise the project fails and delivers only α . In both cases, seniority of debt is such that the borrower repays banks first and then repays opaque lenders up to rd . The borrower consumes all the remaining funds.¹²

The parameter α represents the creditworthiness of the borrower, since the expected final cash flow of the project is positively correlated with its interim cash flow. Note that overall, a project of type α generates $-D + \alpha + \nu(B + \alpha) + (1 - \nu)\alpha = -D + \nu B + 2\alpha$. We define $z = D - \nu B$ as a measure of the fraction of potential projects that are inefficient. High values of z denote a high proportion of inefficient projects. In particular, $z = 2$ implies that no projects should be funded, while $z = 0$ implies that all projects are efficient and should be funded. With intermediate values of z , only projects with $\alpha \geq \frac{z}{2}$ are efficient.

Figure ?? summarizes both the borrower's actions and the payoffs required and generated by the investment project.

This concludes the set-up of the model that contains the basic elements required to consider the implications of hidden borrowing for the structure of formal lending contracts. In particular, agents must be heterogeneous, there must be two periods in order that interim payoffs play a role, and there must be some possibility of default for the agent's type to have any meaningful consequences.

2.2 Further Assumptions

In this section, we add three auxiliary assumptions that help to simplify the analysis. The first assumption ensures that contracts are renegotiation-proof. Beyond the interpretation of a succession of short-term contract, this allows the model to be seen naturally as characterizing a long-term bank contract rather than a contract that is surely renegotiated. The second assumption is a transversality condition that precludes unlimited borrowing. The final assumption imposes parametric restrictions that rule out uninteresting cases.

Specifically we begin by making the following assumption, which ensures that banks in the transparent sector break even both at the *ex-ante* and interim stages, as stated in

¹¹In fact consuming everything is optimal in the model even when the consumption good is costlessly storable.

¹²Note that α is neither observable nor verifiable in Period 1 and it need not be in Period 2 in case of a successful project outcome. We do assume however, that in case the project fails, triggering liquidation and investigation, it becomes verifiable. Introducing a small verification cost in Period 2, in the spirit of the costly state verification literature (Gale and Hellwig (1985)) would not affect the qualitative results.

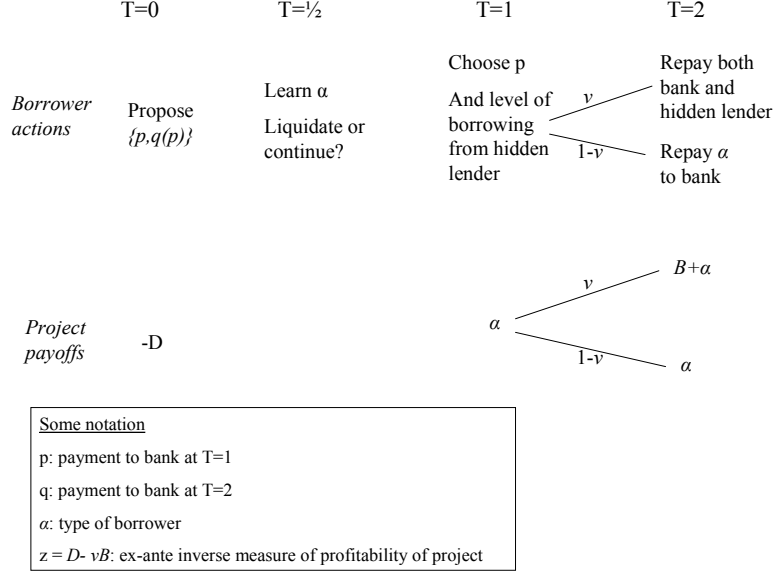


Figure 1: Borrower actions and project payoffs

Lemma 1.¹³

Assumption 4: Some infinitesimal proportion of borrowers already know their own types at $t = 0$.

Lemma 1 : *The set of contracts $p, q(p)$ is renegotiation proof and breaks even at all future possible stages.*

Proof. Competition between banks ensures that banks break even at the *ex-ante* stage.

Assumption 4 guarantees that the initial schedule of contracts is already contingent on all the future public information about the borrower's type so borrowers will effectively not switch (or renegotiate on the threat of switching) to another bank. This follows since if, contingent on some possible future information, a contract contained cross subsidies then among the small subset of agents that already know their type, subsidizing agents would

¹³The renegotiation-proof condition is effectively equivalent to an exclusivity-proof contract—that is, a contract that guarantees that at any point in time the borrower does not want to switch to another bank (see Rampini and Bisin 2004). In the absence of Assumption 4, more general contracts could arise in period 0, but renegotiation would lead to the outcomes characterized by the model.

propose alternative contracts to other banks while subsidized agents would stay.¹⁴ Thus, those banks offering contracts which allowed for cross subsidies would suffer losses. ■

As a transversality condition we make the following assumption:

Assumption 5: An agent cannot borrow from the opaque lender if the nominal value of the loan is higher than the highest possible residual income.

This assumption can be understood as a “no fraud” condition. For example, it might be appropriate if agents can be punished beyond limited liability if it were found (perhaps with some probability) that they did not intend to repay in any possible state of the world. This is a sensible borrowing limit, since most legal systems allow for punishment above limited liability (i.e., prison or personal liability) whenever a borrower takes a loan that she does not intend to repay even in the best possible situation. Alternatively if one thinks of the alternative lender as an informal lender in development economies, as friends and family, or as a black market, one could think that while limited liability may still hold, it would not apply to agents that use the system without any intention of repayment. Assumption 5 ensures that borrowing from the hidden source in order to consume will not occur. Borrowing from the hidden sector to consume and repay in the good state is inefficient given that $r > \frac{1}{\nu}$. Borrowing to consume would only be worthwhile if the agent intends to default for sure and assumption 5 precludes this possibility.

Finally, we make parametric restrictions that preclude some trivial and uninteresting cases.

Assumption 6: $D > 2$ and $2 > z = D - \nu B > 0$

The first restriction ensures that no borrower can repay for sure; the second restrictions ensure that all types of borrowers will default to a different extent if the project is unsuccessful (so from the point of view of lenders they really are different types) and in particular, some projects are efficient and some are not.

3 Equilibrium

The feasible strategies for the borrower are any offer of schedules $\{p, q(p)\}$ at $t = 0$. Furthermore the borrower has to decide whether to pursue the project at $t = \frac{1}{2}$ or liquidate. Finally the borrower has to decide which payment from the schedule to pay, funding any shortfall through borrowing from the hidden source. The bank has to choose whether to

¹⁴While one might consider that such offers are off-equilibrium offers and so might be rendered meaningless by a modeler with judicious use of off-equilibrium beliefs. We focus on equilibria robust to the intuitive criterion.

agree to the proposed menu of contracts.

In order to characterize the equilibrium, we can draw on the revelation principle and think of the borrower's choice of repayment from the schedule $\{p, q(p)\}$ as a function of her type—that is, we could think of offering a schedule $\{p(\alpha), q(\alpha)\}$. As discussed above, in Lemma 1, any meaningful contract on the schedule—that is any contract that is ever taken up in equilibrium—will break even at all the stages of the contract and so will not contain any observable cross subsidies. Since there is competition among banks and given that borrowers make offers, the equilibrium schedule will maximize the *ex-ante* welfare of consumers. Finally, associated with each of the payments that are ever made in equilibrium, incentive compatibility must be satisfied (that is, once a borrower has learned her type α , she prefers to pay $p(\alpha)$ rather than any other $p(\alpha')$).

The equilibrium configuration crucially depends on whether the interest rate at which the informal sector lends r is above or below the threshold $\frac{2-\nu}{\nu}$. We separate these two cases in the discussion that follows.

Throughout, the exogenous interest rate r can be thought of as a measure of the degree of inefficiency of the opaque sector. The break-even rate for r is $\frac{1}{\nu}$, and this would be the endogenous rate for the opaque sector if there were no other frictions or inefficiencies. In this case $r = \frac{1}{\nu}$, because regardless of the amount borrowed, the opaque lender will always be repaid if the good state is realized and will always face default in the bad state.¹⁵ However, whether we think of the opaque lender as trade credit, a credit card, or informal lender, it is reasonable to believe that the interest rate charged could be above this break-even rate. Causes of an apparently high interest rate include that the opaque lender is not as specialized as the bank in lending money and that informal credit is also used for purposes other than concealing liquidity shocks. We therefore allow for the possibility that $r > \frac{1}{\nu}$.

3.1 Very inefficient informal sector

In this section we explore the implications of a very inefficient opaque sector. In particular we explore the resulting equilibrium when $r > \frac{2-\nu}{\nu}$. We begin by characterizing an equilibrium where there is full separation among those types that borrow—that is, each different type repays the formal sector a different interim payment and there is no borrowing from

¹⁵This follows from the seniority of bank debt, the size of the project, and Assumption 5. Note that this is independent of the type of the project, and this is precisely the reason why the information held by the hidden lender is irrelevant to our analysis.

the opaque sector. We then go on to briefly discuss other equilibria and argue that they are somewhat unreasonable.

Proposition 1 *When the opaque sector lends at a sufficiently high interest rate ($r > \frac{2-\nu}{\nu}$), then there exists a fully separating equilibrium where consumers offer the contract schedule $\{p(\alpha), q(\alpha)\}$ with the interim payment equal to the first period cashflow $p(\alpha) = \alpha$ and the corresponding final payment $q(\alpha) = \frac{D-\alpha-(1-\nu)\alpha}{\nu}$. All types $\alpha < \frac{z}{2}$ liquidate at $t = \frac{1}{2}$.*

Proof. The banks' equilibrium beliefs are consistent with the borrower behaviour—that is, banks believe that a type that pays $p = \alpha$ is an α -type (note that some types will simply prefer to liquidate at the $t = \frac{1}{2}$ stage).

This fully contingent contract has to fulfill the break-even and incentive compatibility conditions.

Break-even condition: Given that the first payment $p = \alpha$ reveals the type of the agent as α , the break-even condition for the second payment is $D = \alpha + \nu q + (1 - \nu)\alpha$. This determines that the break-even second payment $q = \frac{D-p-(1-\nu)p}{\nu}$.

We analyze the incentive compatibility condition by considering two deviations: imitating a lower type and imitating a higher type.

Incentive compatibility condition 1: The contract needs to guarantee that no agent wants to imitate a lower quality agent. Suppose that an agent of quality α claims to be a lower quality agent $\alpha' < \alpha$ by paying a first payment $p = \alpha'$; in that case, her total utility would be $(\alpha - \alpha') + \nu(B - \frac{D-\alpha'-(1-\nu)\alpha'}{\nu})$. Note that $(\alpha - \alpha')$ is the additional consumption at $t = 1$ from reporting a lower type, while $(B - \frac{D-\alpha'-(1-\nu)\alpha'}{\nu})$ is the net consumption in the good state (which occurs with probability ν) after repaying $q(\alpha')$. Instead, by revealing her own type she would get $\nu(B - \frac{D-\alpha-(1-\nu)\alpha}{\nu})$. The difference between these two terms is $-(1 - \nu)(\alpha - \alpha') < 0$, and so it cannot be optimal to claim to be an agent of a lower type.

Incentive compatibility condition 2: The contract also needs to guarantee that no agent wants to imitate a higher quality agent by borrowing from the hidden source and paying a first payment $p > \alpha$. Suppose for contradiction that an agent claims to be a higher quality agent by paying a first payment $p = \alpha'' > \alpha$ and borrowing $\alpha'' - \alpha$ from the hidden source to fund this payment. The total utility of the agent would be $\nu(B - \frac{D-\alpha''-(1-\nu)\alpha''}{\nu}) - r(\alpha'' - \alpha)$ instead of $\nu(B - \frac{D-\alpha-(1-\nu)\alpha}{\nu})$. The difference between the two is $(2 - \nu - \nu r)(\alpha'' - \alpha)$ which is negative if and only if $r > \frac{2-\nu}{\nu}$, so this is the necessary and sufficient condition for this incentive compatibility condition to hold.

Notice that off-equilibrium beliefs only apply to $p > 1$, and even assigning the most optimistic beliefs to such offers (that is $\alpha = 1$) agents prefer their equilibrium contracts. ■

Lemma 2 *The above equilibrium achieves first best.*

Proof. In the first best a borrower should be funded if and only if she generates sufficient expected revenues—that is, if and only if $\alpha + v(B + \alpha) + (1 - \nu)\alpha \geq 0$. This is precisely the marginal borrower in the equilibrium described above. ■

Corollary 1 *In the absence of an opaque sector, the first best can be achieved.*

Proof. The absence of an opaque sector is equivalent to $r \rightarrow \infty$, and so the results above apply. ■

Formally, beyond the equilibrium described in Proposition 1, there are many other equilibria. First, there are some that are essentially observationally equivalent in the sense that many other redundant $(p, q(p))$ contracts could be included in the offered schedule that are never taken up and that have no effect on outcomes (for example contracts with very high p 's and q 's). Henceforth we ignore such equilibria. A more substantive source of multiplicity of equilibria arises from the private information on the part of a (small) proportion of borrowers. As is common in these sorts of games, this opens the possibility of equilibria where there is no borrowing; for example, supported by the beliefs that the only offers are from those borrowers who know that their own types are $\alpha = 0$ (and such beliefs are never challenged because offers are off-equilibrium). We note that such equilibria exist but could be refined away assuming trembles or other equilibrium refinements. Most importantly, we find them unreasonable, and rather than dwell on such technicalities, we focus instead on more efficient equilibria—indeed we highlight above an equilibrium that achieves first best.

To summarise this section, when the hidden lender is sufficiently inefficient, borrowers do not attempt to conceal low interim cashflows and do not borrow from the hidden lender. The resulting bank contract is flexible and allows for different first payments with corresponding final payments. An alternative interpretation for the schedule of possible payments is to suppose instead that only one contract from the schedule is initially agreed upon, but that the contract is renegotiated following the cashflow realization in the interim period. The flexibility, under this interpretation, would therefore reflect low (endogenous) costs of renegotiation.

3.2 Relatively Efficient Informal Sector

In the previous section, we supposed that the opaque sector was so inefficient, or equivalently that the cost of borrowing from the opaque sector was so high, that it had no effect on outcomes and on the contracts taken up in the transparent sector. In this section, we explore the equilibrium outcome when the opaque sector is more efficient, that is when $r < \frac{2-\nu}{\nu}$. Note in particular, that this regime includes the case where there are no frictions in the opaque sector and $r = \frac{1}{\nu}$.

In the proof of Proposition 1, we argued that in the case where types were fully separating in their payments and paid exactly their period 1 incomes, then no type (at this interim stage) would want to imitate a higher type if and only if $r \geq \frac{2-\nu}{\nu}$. In particular, this implies that the outcomes described in Proposition 1 can no longer be an equilibrium. Instead, there will be some pooling among different types of agents with regard to their interim payments.

As described below, the outcome will be full pooling in the sense that all types that borrow from the transparent sector will choose the same contract from the schedule. In equilibrium only one level of repayment to the transparent sector will be observed. Rather than the menu of contracts actually taken up in the previous section, borrowing from the transparent sector will entail the same payment p at $t = 1$ for all types who have not liquidated and the same remaining debt q due at $t = 2$ (which will be fully repaid in the good state and only partially repaid—depending on type—in the bad state).

Before getting to the result, we first introduce a couple of preliminary results: a “continuity of pools” lemma and a result on the weak monotonicity of payments with type. These results rely primarily on the incentive-compatibility constraints of borrowers and, in particular, require no assumptions about the distribution of types.

Lemma 3 (*Continuity of p*) *For every three borrowers with types α , β , and γ such that $\alpha > \beta > \gamma$ where $p(\alpha) = p(\gamma)$, it must be the case that $p(\alpha) = p(\beta) = p(\gamma)$.*

Proof. See Appendix. ■

Lemma 4 (*Monotonicity of p*) *For every type $\alpha > \beta$ that does not liquidate, $p(\alpha) \geq p(\beta)$.*

Proof. See Appendix. ■

We proceed to a central result of the paper: that with r sufficiently low, the only bank contract taken up by borrowers is a single contract that requires the same first payment for all borrowing types.

Proposition 2 *When the lending rate from the hidden sector is sufficiently low ($r \leq \frac{2-\nu}{\nu}$), then all borrowers who do not liquidate pay the same interim payment $p(\alpha) = p$ and owe the same amount, q , to the bank in period 2.*

The proof, which appears in the appendix, has a simple structure: We conjecture that there must be at least two types that make different interim payments and find a contradiction. We focus on the highest two payments (and by Lemma 4 these will correspond to the highest differing types). We find that borrowers, at the ex-ante stage where the contracts are determined, would rather that the top two pools be combined as a single pool, in order to maximize their anticipated surplus. Since they propose these contracts, this will be the equilibrium outcome. An induction argument for a finite number of pools will imply that one overall pool appears as the equilibrium contract.¹⁶ There are a number of different cases that must be considered (depending on the level of p and the size of the pools), but working through each of them is relatively straightforward. It is worth noting that the assumption that types are uniformly distributed induces linearity into the model which is a driving force towards the corner solutions that lead to the stark result of a single pool as the equilibrium outcome.¹⁷

Proposition 2 states that the equilibrium contract has only one possible first payment p and a second payment q that makes the bank break even on average, given the pool of agents that do not liquidate. In particular, the proposition shows that there are no ways by which higher types can efficiently separate from lower types. This implies some cross-subsidies from higher to lower types of agents and therefore involves inefficient liquidation decisions. This fully characterizes the structure of the equilibrium; however, to gain further insight and in particular to analyze welfare, we proceed by precisely calculating the values of p , q , and the equilibrium liquidation policy.

3.2.1 Equilibrium payments and welfare

In this section, we characterize the liquidation policy. We then define welfare. In equilibrium, since borrowers propose contracts before knowing their types, total welfare will be maximized. We conclude by characterizing this maximized value.

We begin by introducing some further notation to discuss the liquidation policy. Specif-

¹⁶Note that in the case where $r > \frac{2-\nu}{\nu}$ an induction argument would be inappropriate because there could be an infinite number of pools.

¹⁷Other distributional assumptions may lead to a finite number of pools but never to full separation. The uniform distribution helps in keeping the analysis simple.

ically, let l denote the type that is “just indifferent” between liquidating and continuing the project with the (p, q) contract. Under perfect information, $l = \frac{z}{2}$: however, limited liability and the cross subsidies between agents inside the pool (from higher quality to lower quality ones) will imply that $l < \frac{z}{2}$. This reflects an important externality in our model. Whenever there is some pooling between agents, there will be cross subsidies from agents of higher quality to the agents of lower quality. This generates an inefficient liquidation policy, as some inefficient projects are not liquidated due to this implicit subsidy.

In order to characterize l , we begin by arguing that in this equilibrium, there is no loss in supposing that the first-period payment p will be chosen so that $l \leq p$.

Lemma 5 *In equilibrium when $r < \frac{2-\nu}{\nu}$, then without loss of generality we can focus on $l \leq p$.*

Proof. Suppose that $l \geq p$

The conditions that determine l are the indifference of the marginal agent who decides to liquidate. This can be expressed as $0 = \nu(B + l - q) + (l - p)$.

A further condition is the break even condition of the bank $D = p + \nu q + (1 - \nu)\frac{1+l}{2}$. This can be rewritten again as $q = \frac{D-p-(1-\nu)\frac{1+l}{2}}{\nu}$.

Using these two conditions, we obtain that $l = \frac{2z-1+\nu}{3+\nu}$.¹⁸ In particular, therefore, so long as $l \geq p$, p is undetermined, since it will also have no effect on overall welfare. Then, without loss of generality we can think about $l = p$. ■

We proceed by focusing on the case $l \leq p$. First note that in the case that $l = 0$, it is trivial that the optimal choice of p is $p = 0$, and overall welfare in this case is $W = 1 - z$ (this is simply the average surplus generated by a project, given that all types of projects will be pursued).

Alternatively, it may be optimal to choose an interior l . In this case we can characterize l by noting that a couple of conditions must be satisfied. First, by definition, an agent of type l must be indifferent between liquidating or continuing with the project; that is,

$$0 = \nu(B + l - q - r(p - l)). \quad (1)$$

¹⁸ In particular this means that conditional on $l > p$ the type of the marginal liquidating agent l is constant and smaller than $\frac{z}{2}$. This is intuitive since for a type α , with $\alpha > p$ then a slightly lower or higher p makes no difference—for this α -type borrower with $\alpha > p$, changes in p merely transfer utility 1-for-1 between periods 1 and 2. Thus conditional on $l > p$, it is unsurprising that the marginal indifferent l is independent of p .

In addition, banks need to break even on average, and so

$$D = p + \nu q + (1 - \nu) \frac{1 + l}{2}. \quad (2)$$

Note that the indifference condition (1) implies that $B + \alpha > q$ for every $\alpha > l$, and so it is appropriate to write the break-even condition as above, being sure that the loan will be fully repaid if the contract is successful for every borrowing type. Substituting for q from (2) into (1), we obtain the following expression for l :

$$l = \frac{2z + \nu + 2p(r\nu - 1) - 1}{\nu + 2r\nu + 1}. \quad (3)$$

We characterize the equilibrium p , under the assumption that both the optimal p and l are interior. Having done so, it is easy to verify conditions under which this is indeed the case and then go on to consider outcomes when these conditions fail.

Continuing under the assumption that l is interior, we consider the first order condition, and we maximize total welfare in order to find the contract offered in the optimal equilibrium (other equilibria exist but as discussed at the end of Section 3.1, we focus attention on the most efficient equilibrium).

We begin with the expression of total welfare.

$$\begin{aligned} W &= \int_l^1 (2x + \nu B - D) dx - (\nu r - 1) \int_l^p (p - x) dx \\ &= lz - z - l^2 + 1 - \frac{1}{2}(\nu r - 1)(p - l)^2. \end{aligned} \quad (4)$$

The first term represents the net (positive or negative) welfare from each project financed, while the second term is the welfare loss out of inefficient borrowing. Note that the above condition (in the upper limit of the second integral) supposes that $p < 1$, which it will be easy to verify is true in equilibrium.

The first order condition that characterizes the optimal p is:

$$\frac{dW}{dp} = (z - 2l) \frac{dl}{dp} - (\nu r - 1)(p - l) \left(1 - \frac{dl}{dp}\right) = 0, \quad (5)$$

where, by taking the derivative of l , as defined in equation (3), with respect to p :

$$\frac{dl}{dp} = \frac{2(r\nu - 1)}{2r\nu + 1 + \nu}. \quad (6)$$

Note that this derivative is strictly positive since $r\nu > 1$. One might expect this to be the case, as the interim payment increases, concealing a low type becomes more costly and so more projects might be liquidated at $t = 1/2$.

Substituting expression (6) into (5), the expression that implicitly defines the optimal first payment p is:

$$\frac{dW}{dp} = (z - 2l)\frac{2(r\nu - 1)}{2r\nu + 1 + \nu} - (\nu r - 1)(p - l)\left(\frac{3 + \nu}{2r\nu + 1 + \nu}\right) = 0. \quad (7)$$

Simple algebraic manipulation yields the following equilibrium expression for the optimal p :

$$p = \frac{2z - l + \nu l}{3 + \nu}. \quad (8)$$

We solve simultaneously for p and l from this equation and equation (3) to obtain:

$$l = \frac{2z + 2\nu + 2z\nu + 4rz\nu + \nu^2 - 3}{6\nu + 8r\nu + \nu^2 + 1} \quad (9)$$

$$p = \frac{4z\nu - 2\nu + 4rz\nu + \nu^2 + 1}{6\nu + 8r\nu + \nu^2 + 1} \quad (10)$$

and substituting these expressions into equation (4) we can calculate a value W_I for welfare. The notation W_I is intended to highlight that this is the welfare under the optimal interior solution when it is feasible. However this need not be the global optimum since choosing $p = 0 = l$ and generating an expected surplus of $1 - z$ is always feasible.

The equilibrium expression for p , Equation (10), together with the break even-condition, Equation (2), and the expression for l , Equation (9), determine the equilibrium value for the second payment q .

Lemma 6 *Both l and p are interior when $z \geq \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)}$ and $W_I \geq 1 - z$.*

Proof. Note that p is linear in z . It is sufficient therefore to consider the two extremes $z = 0$ and $z = 2$. For $z = 2$, $p = 1$. For $z = 0$, $p = \frac{(1-\nu)^2}{6\nu+8r\nu+\nu^2+1}$ which is greater than 0 and less than 1. Furthermore for $z = 2$, $l = 1$ and $l > 0$ as long as $z > \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)}$.

For values of z below $\frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)}$ the optimal contract is to set $p = 0$, which leads to $l = 0$.

Thus $z \geq \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)}$ is required for an interior l and p to be feasible. An interior l would generate more surplus and so would be the equilibrium outcome when the welfare generated is higher than the next best alternative—choosing $l = 0$ and $p = 0$, or equivalently:

$$W_I \geq 1 - z. \quad \blacksquare$$

Note that for $z = 2$ both $z \geq \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)}$ and $W_I \geq 1 - z$ hold as strict inequalities, and so in particular for large enough z , both l and p will be interior.

Equations (9) and (10) show that in the parameter range where l is interior then both l and p are linear and increasing in z . That they should be increasing is quite intuitive. As z goes up, projects become less and less attractive, so the optimal first payment p goes up to increase the liquidation threshold l . On the other hand, as z goes down, more projects should be funded so p goes down. Further, as z increases the parameters are more likely to be such that the optimal choice of l is interior and in particular this is always the case when $z = 2$.¹⁹

3.3 Equilibrium summary

There are three equilibrium regimes. When the hidden lender is relatively inefficient ($r > \frac{2-\nu}{\nu}$) there is full separation, where each type of borrower who does not liquidate pays an interim payment equal to the interim cashflow, $p = \alpha$, and a corresponding second period payment that accurately assesses the credit-worthiness of the borrower, $q = \frac{D-\alpha-(1-\nu)\alpha}{\nu}$. Firms get financed as long as $\alpha > \frac{z}{2}$; equivalently, projects are financed if and only if they are efficient. In this region bank contracts have interest rates between period 1 and period 2 that are contingent on interim payments. These allow the bank to perfectly elicit information about the agent's type. If one interprets the long-term contingent contract as a short-term loan that is renegotiated at $t = 1$, this would be a situation where the endogenous costs of renegotiation are low. That is, small changes in p lead to small changes in q .

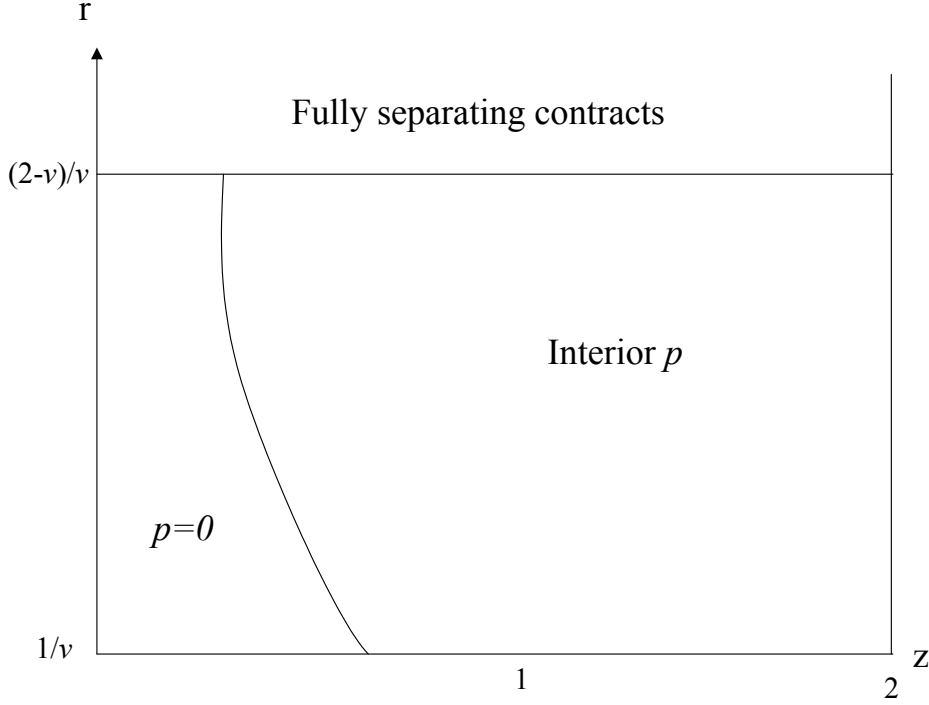
When the informal sector is relatively efficient ($r < \frac{2-\nu}{\nu}$), pooling cannot be avoided, and in equilibrium only one contract is taken up from the banking sector. The existence of cross subsidies from higher types to lower ones induces too little liquidation. There are two cases to consider.

¹⁹See Lemma 9 in the appendix.

If the average project is relatively profitable ($z < \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)}$), the welfare loss from funding inefficient projects is smaller than is the loss from forcing some borrowers to borrow from the opaque sector. Therefore it is optimal to fund all projects, and in this case there is no interim payment ($p = 0$ and $q = \frac{2D-1+\nu}{2\nu}$).

Finally, if the average project is relatively unprofitable ($z > \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)}$ and $W_I > 1-z$), the equilibrium will see some types of projects liquidated ($l = \frac{2z+2\nu+2z\nu+4rz\nu+\nu^2-3}{6\nu+8r\nu+\nu^2+1}$). All types that do not liquidate will make an interim payment to the bank and a corresponding second period payment that takes into account the information implied by the equilibrium liquidation policy ($p = \frac{4z\nu-2\nu+4rz\nu+\nu^2+1}{6\nu+8r\nu+\nu^2+1}$ and $q = \frac{D-p-(1-\nu)\frac{l+1}{2}}{\nu}$). In this case some agents borrow simultaneously from both sectors. Both sources of inefficiency operate in this regime—namely, some inefficient projects are conducted, and there is some costly borrowing from the inefficient opaque lender. The bank contract is determined by optimally trading off these two sources of inefficiency. This situation can also be interpreted as a case where renegotiation is (endogenously) costly. Any deviation from p would lead to liquidation.

All three regions are non-trivial as illustrated in the diagram below, which illustrates these three equilibrium regions for a general ν .



3.4 Comparative statics on welfare

First note that when $r > \frac{2-\nu}{\nu}$ welfare is first best and independent of r within this range. In the case where the optimal contract involves $p = 0$, welfare is equal to $(1 - z)$ and, again, within this range it is independent of r , raising r to a level where either an interior p is optimal (which requires $W_I > 1 - z$) or the full separation equilibrium is attained (and the first best level of welfare is achieved) trivially raises welfare.

The most interesting analysis is for parameters in the region with a single interior interim payment to the bank—that is, where $r \leq \frac{2-\nu}{\nu}$. Raising r so that the equilibrium shifts to the fully separating case, which is first best, trivially raises welfare. We now consider how welfare varies with r within this region.

Having obtained explicit characterizations of l and p in terms of the exogenous parameters of the model and noting that welfare in this region is given by Equation (4), we

consider the comparative statics of welfare. It is of particular interest, to consider how welfare changes (and the channels through which it changes) as r , the exogenous rate of interest in the opaque sector, varies.

First note that in the range $r < \frac{2-\nu}{\nu}$ and for $z > \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)}$ and $W_I \geq 1 - z$.

$$\frac{dl}{dr} = \frac{4\nu(3+\nu)(1-\nu)(2-z)}{(6\nu+8r\nu+\nu^2+1)^2} > 0. \quad (11)$$

In particular, this suggests that one source of inefficiency is reduced, since as r increases l rises and so fewer inefficient projects are conducted.

Note also that

$$\frac{dp}{dr} = -\frac{4\nu(1-\nu)^2(2-z)}{(6\nu+8r\nu+\nu^2+1)^2} < 0. \quad (12)$$

As the interim payment falls, and since the lowest type borrowing rises, then the amount of borrowing from the opaque sector falls; however, since the cost of borrowing from the opaque sector rises, the welfare consequences may be ambiguous.²⁰ By examining welfare directly we can see that the first of these two effects always dominates, specifically:

Lemma 7 Proposition 3 *Welfare is non-decreasing in the hidden lender's rate ($\frac{dW}{dr} \geq 0$) and strictly increasing when the lender's rate is sufficiently low ($r < \frac{2-\nu}{\nu}$) and the proportion of inefficient projects is high ($z > \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)}$ and $W_I > 1 - z$).*

Proof. See Appendix. ■

4 Partially Hidden Borrowing

We modify the model slightly to allow for a partially hidden lender. We introduce the possibility that the banking sector observes the level of hidden borrowing of the lender with some probability $(1-h)$. With probability h , borrowing from the non-banking sector remains hidden. A rationale for this modelling assumption is that the banking sector investigates each of its borrowers with some probability $(1-h)$ and once a borrower is investigated its borrowing position with all possible alternative lenders is perfectly known

²⁰Note that the welfare as defined in Equation (4) does not take into account surplus gained by the alternative sector. Including this surplus into the welfare calculation would suggest that the *only* source of inefficiency would be inefficient liquidation and so only the first effect would apply and the qualitative results would apply—welfare increases in r . The analysis here would still be of interest, as Equation (4) captures consumer surplus.

by the whole banking sector. On the contrary, if a particular borrower is not investigated, the banking sector cannot observe any borrowing outside the pool of competitive creditors and is aware of the possibility of some additional lending.

If a borrower is investigated, we assume that the borrower is aware of it, and that she has the opportunity to repay the opaque sector immediately. Early repayment entails a cost sd , where d is the amount borrowed from the opaque sector.

If an agent is investigated and repays early, then we know that full separation holds. By the proof of Proposition 1, if the full separation contract is offered, then there are no incentives to imitate downwards. With observable payoffs there is no feasible way to imitate upwards as banks would take into account any inefficient borrowing and discount that to calculate the agent's true type. Given that the incentive compatibility constraints for the fully revealing equilibrium hold and that it achieves first best, this is the only equilibrium once an agent has been investigated.

We make the following assumption to guarantee that early repayment is the optimal strategy of the borrower once the alternative lender becomes transparent:

Assumption 7 $s < r - \frac{1}{\nu}$.

Lemma 8 *Once the hidden borrowing is observed, early repayment is the optimal strategy for the agent.*

Proof. Borrowing from a hidden source gives no concealment benefit, so the only benefit from that borrowing comes from either investing or consuming those funds. Investing them at the gross market interest rate of 1 or consuming them gives a (negative) expected utility of $1 - \nu r$. This loss has to be compared with the cost of early repayment $-\nu s$. Early repayment is therefore the optimal strategy as long as $-\nu s > 1 - \nu r$, which is guaranteed by Assumption 7. ■

The model with probabilistic observability of the hidden borrowing is therefore like a switching model in which, with probability $(1 - h)$ full separation is achieved for sure and with probability h the model looks like that of the previous sections. In this latter case the only difference is that, from the point of view of the borrower, the costs and benefits of the hidden borrowing need to be recalculated, since with probability $(1 - h)$ hidden borrowing is useless and entails a cost s .

In fact, once the alternative borrowing remains hidden, the rest of the model with probabilistic observation of the hidden borrowing can be fully solved by realizing that the cost of borrowing from the hidden source is now $\frac{hr + (1-h)s}{h}$ instead of just r . Borrowing one

unit from the hidden source costs r with probability h and costs s with probability $(1 - h)$. It only produces some concealment benefit to the borrower with probability h , so the whole cost has to be re-scaled by h .

We write $r_h = \frac{hr + (1-h)s}{h}$ as the effective interest rate when borrowing from the opaque sector remains hidden with probability h , the rate of interest is r when borrowing remains hidden, and the cost of early repayment when the banking sector observes the borrowing is s . With this notation, we obtain the following results, which are similar to those in the fully hidden case:

Proposition 4 *When the opaque sector lends at a sufficiently high effective interest rate ($r_h > \frac{2-\nu}{\nu}$), then there exists an equilibrium where consumers offer the schedule $\{p(\alpha), q(\alpha)\}$ with $p(\alpha, d) = \alpha$ and $q(\alpha) = \frac{D-\alpha-(1-\nu)\alpha}{\nu}$ and there is no borrowing from the hidden sector. When the opaque sector lends at a sufficiently low effective interest rate ($r_h \leq \frac{2-\nu}{\nu}$), all types that do not liquidate make the same interim payment p and owe the same amount, q , to the bank in period 2.*

Proof. The proof is almost identical to the ones in Propositions 1 and 2 except that now borrowing from the hidden source entails higher costs, and so further details are omitted.

■

The functional form of the welfare equation and the incentive compatibility conditions are similar to those of the basic model, so similar results to those of Section 3.2.1 hold. In particular, if $r_h \leq \frac{2-\nu}{\nu}$ the only possible equilibrium is one of full pooling, and if the optimal solution is interior then the optimal first payment is:

$$p = \frac{4z\nu - 2\nu + 4zr_h\nu + \nu^2 + 1}{6\nu + 8r_h\nu + \nu^2 + 1} \quad (13)$$

and the type of borrower who is just indifferent between liquidating the project and continuing it is:

$$l = \frac{2z + 2\nu + 2z\nu + 4r_h + \nu^2 - 3}{6\nu + 8r_h\nu + \nu^2 + 1}. \quad (14)$$

Total welfare in this regime can be expressed as:

$$W = h \left[\int_l^1 (2x + vB - D)dx - (\nu r - 1) \int_l^p (p - x)dx \right] + (1 - h) \left[\int_{\frac{z}{2}}^1 (2x + vB - D)dx - \nu s \int_l^p (p - x)dx \right]. \quad (15)$$

The first term corresponds to the welfare when the hidden sector remains hidden, while the second one is related to when it becomes observable. The above expression can be rearranged as

$$W = h \left[\int_l^1 (2x + vB - D)dx - (\nu r_h - 1) \int_l^p (p - x)dx \right] + (1 - h) \left[\int_{\frac{z}{2}}^1 (2x + vB - D)dx \right]. \quad (16)$$

Note that the expression in the first bracket is identical to the expression in Section 3 with a change of the social cost of borrowing from r to r_h and that the second bracket is constant in p and l .

The welfare implications of the changes in the probability of the hidden sector becoming transparent $(1 - h)$ are as follows: A higher $(1 - h)$ implies higher welfare in a couple of ways. First is the automatic switching from the pooling equilibrium to the first-best full separation equilibrium whenever the banking sector observes the hidden lending. Second, increasing $(1 - h)$ increases r_h , and so the results on welfare increasing in r from Section 3 apply. Similarly an increase in s raises r_h and so also raises welfare.

Note that our analysis is related to the literature on the interactions between direct screening of lenders through active investigation and the indirect screening that can be achieved by offering them a menu of contracts, as in Manove et al. (2001). While in most models these are seen as substitutes, in our model they are complements. That is, an increase in $(1 - h)$ leads to more information about some borrowers directly and also to a more informative equilibrium with respect to the other borrowers (who may have loans from the alternative sector that remain hidden).²¹

²¹Even though so far we have considered h as an exogenous parameter, endogenizing it seems relatively straightforward. We could allow banks to choose their monitoring effort h at a cost. Higher transparency (lower h) would be more costly and competition among banks should equalize the marginal cost of additional monitoring (reducing h) with its marginal gain in terms of welfare in equilibrium.

5 Conclusions

In this paper we have presented a model in which a banking sector and an alternative opaque source of lending coexist. The results show that if the alternative source of borrowing is sufficiently inefficient, banking contracts will achieve first-best. The optimal contract gives incentives to borrowers to reveal their intermediate cash flows perfectly by rewarding higher interim payments with lower future interest rates. However if the alternative source of borrowing is relatively efficient, then the fully contingent contract is not sustainable as agents may want to conceal their types by borrowing from the hidden source and repaying a larger part of their loans early. Here, the optimal contract is not contingent on the interim payments of the loans. There is only one possible first payment and associated second payment. The contract fails to achieve first-best for two reasons. First, a number of inefficient projects are funded, and, second, some borrowers access the inefficient alternative sector. Note that for some parameters this first payment would be zero and all repayment would be in the final period. Thus the model can also be seen as characterising the timing of debt repayments.

It is worth restating that the banks, even though they may be more efficient, cannot simply replicate the loans provided by the hidden sector. This follows since it is assumed that a bank is committed to share all information about its loans to other banks. In principle, one might think that the bank could offer two types of credit and commit not to act on the information revealed. However, even if the commitment was credible, this arrangement would lead to cross-subsidies among borrowers that would be observable by other banks, and so the bank offering this arrangement would suffer from cream-skimming and suffer losses.

We show that overall welfare increases if the cost of borrowing from the opaque sector is higher. This result is in contrast to some conventional wisdom in discussions of developing economies, which focuses on the role that the informal sector may play in alleviating the financing constraints. The informal sector lends to firms and households when the formal sector is rationing them.²² However in our model, as the informal sector gets more efficient, the banking sector has to offer a less contingent contract, and total welfare falls. This approach, considering the interaction between formal and informal sectors from

²²Jain (1999), who also discusses related literature, provides an explanation for the observation of borrowers in both sectors based on a trade-off between the formal sector's lower opportunity cost of funds and the informal sector's better information. Instead we assume that the formal sector is unambiguously more efficient and the informal sector may or may not have an informational advantage.

an informational point of view, provides a counter-argument to traditional literature by showing how an efficient informal sector may reduce welfare.

Relaxing the assumption that the alternative sector is not entirely opaque makes it less appealing for a borrower to use the costly alternative sector to disguise her type as this may turn out to be ineffective. In this case, the qualitative results outlined above carry through in this richer environment and moreover, welfare is decreasing in the opacity of the informal sector. These results suggest that as the informational transparency of the financial sector as a whole improves, banks are able to offer more sophisticated financial instruments.

While we presented a model of start-up financing for an investment project, the central mechanisms and in particular the interaction of different sources of borrowing and the implications for contractual form have wide applicability. For example, over the last twenty years the consumer sector in the UK has seen the development of more flexible mortgages integrated with checking accounts and credit lines. Intuitions developed through this paper suggest that this may be partially a consequence of financial institutions having better means of processing and transferring information about consumer credit histories.

Our results highlight that one of the possible reasons for long-term debt contracts being inflexible with respect to interim payments is that the information that long-term lenders would extract from these interim payments would be corrupted by additional borrowing from hidden sources of funds. They also suggest an explanation for simultaneous borrowing from different sources even when there is a clear pecking order among them and there is available borrowing from the cheaper one (for example mortgages and credit card borrowing). Finally, we also show that the existence of an alternative opaque source of borrowing may be welfare diminishing because it may distort the set of contracts that the competitive lending sector may offer.

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6 Appendix

Proof of lemmas 3 and 4

Proof. Suppose that borrowers face the choice between two generic contracts a and b and without loss of generality, we label them so that $p_a > p_b$. There are a number of cases to consider:

- (i) $\alpha > p_a > \beta > p_b > \gamma$
- (ii) $p_a > p_b > \alpha > \beta > \gamma$
- (iii) $\alpha > \beta > \gamma > p_a > p_b$
- (iv) $\alpha > p_a > p_b > \beta > \gamma$
- (v) $\alpha > \beta > p_a > p_b > \gamma$

We examine each in turn:

- (i) $\alpha > p_a > \beta > p_b > \gamma$

Agent α will go for contract a whenever

$$\alpha - p_a + \nu(B + \alpha - p_a - q_a - r(p_a - \alpha)1_{p_a > \alpha}) \geq \alpha - p_b + \nu(B + \alpha - p_b - q_b - r(p_b - \alpha)1_{p_b > \alpha})$$

since $\alpha > p_a > p_b$ then $1_{p_a > \alpha} = 1_{p_b > \alpha} = 0$ and the above expression can be rewritten

as

$$\nu(q_b - q_a) \geq (p_a - p_b)(1 + \nu) \quad (17)$$

The inequality is independent of α so all the agents with type above p_a will make the same choice.

For an agent of type γ she would the a contract over the b contract whenever

$$\gamma - p_a + \nu(B + \gamma - p_a - q_a - r(p_a - \gamma)1_{p_a > \gamma}) \geq \gamma - p_b + \nu(B + \gamma - p_b - q_b - r(p_b - \gamma)1_{p_b > \gamma})$$

Given that $1_{p_a > \gamma} = 1_{p_b > \gamma} = 1$ we can rewrite the above expression as

$$\nu(q_b - q_a) \geq (p_a - p_b)(1 + \nu + r\nu) \quad (18)$$

Finally an agent of type β will go for contract a over contract b whenever

$$\beta - p_a + \nu(B + \beta - p_a - q_a - r(p_a - \beta)1_{p_a > \beta}) \geq \beta - p_b + \nu(B + \beta - p_b - q_b - r(p_b - \beta)1_{p_b > \beta})$$

Given that $1_{p_a > \beta} = 1$ and $1_{p_b > \beta} = 0$,

$$\nu(q_b - q_a) \geq (p_a - p_b)(1 + \nu) + r\nu(p_a - \beta) \quad (19)$$

Given that $(p_a - p_b)(1 + \nu) + r\nu(p_a - \beta) > (p_a - p_b)(1 + \nu) + r\nu(p_a - p_b)$

and so since γ prefers contract a to contract b , an agent of type β must also prefer

contract a to contract b .

$$(ii) \ p_a > p_b > \alpha > \beta > \gamma$$

In this case the condition for an α type agent to prefer the a contract over the b contract is

$$\nu(q_b - q_a) \geq (p_a - p_b)(1 + \nu + r\nu)$$

which is the identical condition for the β type agent

$$(iii) \ \alpha > \beta > \gamma > p_a > p_b$$

The condition which ensures that the β type agent prefers the a contract over the b contract is identical to the condition for the α -type agent and for the γ -type agent and so will hold true in this case.

$$(iv) \ \alpha > p_a > p_b > \beta > \gamma$$

The condition which ensures that the β type agent prefers the a contract over the b contract is identical to the condition for the γ -type agent and so will hold true in this case.

$$(v) \ \alpha > \beta > p_a > p_b > \gamma$$

The condition which ensures that the β type agent prefers the a contract over the b contract is identical to the condition for the α -type agent and so will hold true in this case.

In each of the cases therefore if the preferred contract for an α agent is the b contract, it is also the preferred one for agents β and γ . Similarly, the opposite is also true, if the γ agent prefers contract a then so do the β and the α agent. This proves Lemma 4. Furthermore if the preferred contract for both an α agent and a γ agent is a then this is also the preferred contract for any β -type agent where $a > \beta > \gamma$. This proves Lemma 3.

■

Proof of Proposition 2

Proof. We prove by contradiction. Suppose that this result is false, then there must be at least two types that pay different amounts. We focus on the highest two payments (and by Lemma 4 these will correspond to the highest differing types). We will find that in equilibrium, the top two pools would rather be combined as a single pool. Then an induction argument for a finite number of pools will imply that one overall pool appears as the equilibrium contract.

We continue by considering the top two pools of types that do not liquidate.

First note that if any type α strictly prefers not to liquidate then all types $\beta > \alpha$ would prefer to mimic α than to liquidate. Thus in restricting attention to the highest two payments $p_1 < p_2$ and associated types (and by Lemma 4 we know that higher types are associated with higher payments) then we can be sure that there are some $\alpha_1 \leq \alpha_2$ such

that types $(\alpha_1, \alpha_2]$ pay p_1 in the first period (with the associated q_1) and $(\alpha_2, 1]$ pay p_2 in the first period (with the associated q_2 in the second period).

The proof is somewhat involved but the structure is as follows. First we highlight a number of possible cases. In each case we seek to determine the optimal choice of α_2 (and associated p_1 and p_2) given that there are two pools in the range $(\alpha_1, 1]$ while keeping α_1 indifferent (and so all other types below may also remain with their existing contracts and there are no changes to equilibrium or welfare consequences from types below α_1).²³

By definition $p_2 > p_1$. There are a number of cases to consider:

- I $p_2 > 1$ and $p_1 > \alpha_2$
- II $p_2 > 1$ and $\alpha_2 > p_1 > \alpha_1$
- III $p_2 > 1$ and $\alpha_1 > p_1$
- IV $1 > p_2 > \alpha_2$ and $p_1 > \alpha_2$
- V $1 > p_2 > \alpha_2$ and $\alpha_2 > p_1 > \alpha_1$
- VI $1 > p_2 > \alpha_2$ and $\alpha_1 > p_1$

We focus on each in turn and show that the optimum outcome in all cases pushes α_2 into a corner.

Recall also that we have $q_1 = \frac{D-p_1-(1-\nu)\frac{\alpha_1+\alpha_2}{2}}{\nu}$ and $q_2 = \frac{D-p_2-(1-\nu)\frac{1+\alpha_2}{2}}{\nu}$

CASE I $p_2 > 1$ and $p_1 > \alpha_2$

The IC condition for α_2 is $\nu(B + \alpha_2 - q_2 - r(p_2 - \alpha_2)) = \nu(B + \alpha_2 - q_1 - r(p_1 - \alpha_2))$

which yields $q_1 - q_2 = rp_2 - rp_1$

and substituting in for q_1 and q_2 and simplifying yields $p_2 = \frac{1-\nu}{\nu r-1} \frac{1-\alpha_1}{2} + p_1$

in particular note that $\frac{dp_2}{d\alpha_2} = \frac{dp_1}{d\alpha_2}$.

The participation constraint for α_1 is $\nu(B + \alpha_1 - q_1 - r(p_1 - \alpha_1)) = k$

With $p_1 = \frac{1}{\nu r-1}(-k + \nu B - D + (1-\nu)\frac{\alpha_1+\alpha_2}{2} + \nu(1+r)\alpha_1)$

and note in particular that $\frac{dp_2}{d\alpha_2} = \frac{dp_1}{d\alpha_2} = \frac{1-\nu}{2(\nu r-1)}$

Welfare

Consider overall welfare

$$W = \int_{\alpha_1}^1 (\nu B - D + 2x)dx - (\nu r - 1) \int_{\alpha_1}^{\alpha_2} (p_1 - x)dx - (\nu r - 1) \int_{\alpha_2}^1 (p_2 - x)dx + cst$$

Where the constant terms arises from the welfare of those types $\alpha < \alpha_1$

$$W = (\nu B - D)(1 - \alpha_1) + 1 - \alpha_1^2 + (\nu r - 1) \left[-p_1(\alpha_2 - \alpha_1) + \frac{1}{2}(\alpha_2^2 - \alpha_1^2) - p_2(1 - \alpha_2) + \frac{1}{2}(1 - \alpha_2^2) \right] +$$

cst

$$\text{so } \frac{dW}{d\alpha_2} = (\nu r - 1)(p_2 - p_1 - \frac{1-\nu}{2(\nu r-1)}(1 - \alpha_1))$$

²³While noting that sufficiently bizarre off-equilibrium beliefs could justify a wide range of equilibria, we focus on the most efficient equilibria (which would also be the one preferred by the borrowers).

This expression cannot be easily signed but note that it is independent of α_2 and so without loss of generality we can maximize welfare taking $\alpha_2 = p_1$ and case II applies.

CASE II $p_2 > 1$ and $\alpha_2 > p_1 > \alpha_1$

The IC condition for α_2 is $\nu(B + \alpha_2 - q_2 - r(p_2 - \alpha_2)) = (\alpha_2 - p_1) + \nu(B + \alpha_2 - q_1)$ substituting in the expressions for q_1 and q_2 gives

$$\nu(B + \alpha_2 - \frac{D - p_2 - (1-\nu)\frac{1+\alpha_2}{2}}{\nu} - r(p_2 - \alpha_2)) = (\alpha_2 - p_1) + \nu(B + \alpha_2 - \frac{D - p_1 - (1-\nu)\frac{\alpha_1 + \alpha_2}{2}}{\nu})$$

which yields $p_2 = \alpha_2 + \frac{(1-\nu)(1-\alpha_1)}{2(r\nu-1)}$. In particular, note that $\frac{dp_2}{d\alpha_2} = 1$.

The participation constraint for α_1 is $\nu(B + \alpha_1 - q_1 - r(p_1 - \alpha_1)) = k$.

Substituting in the expression for q_1

$$\nu(B + \alpha_1 - \frac{D - p_1 - (1-\nu)\frac{\alpha_1 + \alpha_2}{2}}{\nu} - r(p_1 - \alpha_1)) = k$$

$$\text{yields } p_1 = \frac{1}{\nu r - 1}(-k + \nu B - D + (1 - \nu)\frac{\alpha_1 + \alpha_2}{2} + \nu(1 + r)\alpha_1)$$

and note in particular that $\frac{dp_1}{d\alpha_2} = \frac{1-\nu}{2(\nu r - 1)}$

Welfare

Consider overall welfare

$$W = \int_{\alpha_1}^1 (\nu B - D + 2x)dx - (\nu r - 1) \int_{\alpha_1}^{p_1} (p_1 - x)dx - (\nu r - 1) \int_{\alpha_2}^1 (p_2 - x)dx + cst$$

$$W = (\nu B - D)(1 - \alpha_1) + 1 - \alpha_1^2 + (\nu r - 1) \left[-p_1(p_1 - \alpha_1) + \frac{1}{2}(p_1^2 - \alpha_1^2) - p_2(1 - \alpha_2) + \frac{1}{2}(1 - \alpha_2^2) \right] + cst.$$

and in particular

$$\frac{dW}{d\alpha_2} = (\nu r - 1) \left(-\frac{1-\nu}{2(\nu r - 1)}(p_1 - \alpha_1) - p_1 \frac{1-\nu}{2(\nu r - 1)} + p_1 \frac{1-\nu}{2(\nu r - 1)} - (1 - \alpha_2) + p_2 - \alpha_2 \right)$$

$$\frac{dW}{d\alpha_2} = (\nu r - 1) \left(p_2 - 1 - \frac{1-\nu}{2(\nu r - 1)}(p_1 - \alpha_1) \right)$$

$$\frac{d^2W}{d\alpha_2^2} = (\nu r - 1) \left(1 - \left(\frac{1-\nu}{2(\nu r - 1)} \right)^2 \right)$$

The FOC $\frac{dW}{d\alpha_2} = 0$ defines a maximum so long as $\frac{d^2W}{d\alpha_2^2} < 0$ this is true when

$$1 < \left(\frac{1-\nu}{2(\nu r - 1)} \right)^2 \text{ or equivalently } r < \frac{3-\nu}{2\nu}.$$

And so when $r > \frac{3-\nu}{2\nu}$ then $1 > \frac{1-\nu}{2(\nu r - 1)}$ and so setting $\frac{dW}{d\alpha_2} = 0$ defines a minimum and so the maximum is at a corner.

In the case when $r < \frac{3-\nu}{2\nu}$ the FOC defines a maximum rather than a minimum. We argue that this case is vacuous.

Recall that $p_1 < \alpha_2 < 1$. We also know that α_1 satisfies $p_1 = \frac{1}{\nu r - 1}(-k + \nu B - D + (1 - \nu)\frac{\alpha_1 + \alpha_2}{2} + \nu(1 + r)\alpha_1)$

$$\text{so } p_1 - \frac{1}{\nu r - 1}(-k + \nu B - D) - \frac{1-\nu}{(\nu r - 1)}\frac{\alpha_2}{2} = \alpha_1 \frac{2\nu r + 1 + \nu}{2(\nu r - 1)}.$$

Further $p_1 < \alpha_2$ and $(\nu B - D) < 0$ so $\alpha_1 \frac{2\nu r + 1 + \nu}{2(\nu r - 1)} < \alpha_2 - \frac{1-\nu}{(\nu r - 1)}\frac{\alpha_2}{2}$ and so $\alpha_1 < \left(\frac{2(\nu r - 1) - 1 + \nu}{1 + \nu + 2\nu r} \right) \alpha_2$. Given that $2(\nu r - 1) - 1 + \nu < 0$ when $r < \frac{3-\nu}{2\nu}$ then $\alpha_1 < 0$ which is impossible.

CASE III $p_2 > 1$ and $\alpha_1 > p_1$

The IC condition for α_2 is $\nu(B + \alpha_2 - q_2 - r(p_2 - \alpha_2)) = (\alpha_2 - p_1) + \nu(B + \alpha_2 - q_1)$

which yields $p_2 = \alpha_2 + \frac{(1-\nu)(1-\alpha_1)}{2(r\nu-1)}$

The participation constraint for α_1 is

$$\alpha_1 - p_1 + \nu(B + \alpha_1 - q_1) = k$$

$$\alpha_1 + \nu B - D + (1 - \nu)\frac{\alpha_1 + \alpha_2}{2} = k$$

here p_1 is unconstrained

$$\alpha_2 = \frac{k + D - B\nu - \alpha_1 - \frac{1}{2}\alpha_1(-\nu+1)}{-\frac{1}{2}\nu + \frac{1}{2}}$$

$$\text{which yields } \alpha_2 = \frac{2}{1-\nu}(k + D - \nu B) - \alpha_1 \frac{3-\nu}{2}$$

So here α_2 is not a choice variable and so also p_2 cannot be freely chosen, the only choice variable is p_1 .

Welfare

Consider overall welfare

$$W = \int_{\alpha_1}^1 (\nu B - D + 2x)dx - (\nu r - 1) \int_{\alpha_2}^1 (p_2 - x)dx + cst$$

It's clear that from the welfare perspective p_1 is irrelevant and so p_1 does not arise, without loss of generality therefore we can take $p_1 = \alpha_1$ and we're back in case II.

Case IV $1 > p_2 > \alpha_2$ and $p_1 > \alpha_2$

The IC condition for α_2 is $\nu(B + \alpha_2 - q_2 - r(p_2 - \alpha_2)) = \nu(B + \alpha_2 - q_2 - r(p_2 - \alpha_2))$

$$p_2 = \frac{1-\nu}{\nu r-1} \frac{1-\alpha_1}{2} + p_1 \text{ in particular note that } \frac{dp_2}{d\alpha_2} = \frac{dp_1}{d\alpha_2}.$$

The participation constraint for α_1 is $\nu(B + \alpha_1 - q_1 - r(p_1 - \alpha_1)) = k$

substituting in the expression for q_1

$$\nu(B + \alpha_1 - \frac{D - p_1 - (1-\nu)\frac{\alpha_1 + \alpha_2}{2}}{\nu} - r(p_1 - \alpha_1)) = k$$

$$\text{yields } p_1 = \frac{1}{\nu r-1}(-k + \nu B - D + (1 - \nu)\frac{\alpha_1 + \alpha_2}{2} + \nu(1 + r)\alpha_1)$$

$$\text{and note in particular that } \frac{dp_1}{d\alpha_2} = \frac{1-\nu}{2(\nu r-1)}$$

Welfare

Consider overall welfare

$$W = \int_{\alpha_1}^1 (\nu B - D + 2x)dx - (\nu r - 1) \int_{\alpha_1}^{\alpha_2} (p_1 - x)dx - (\nu r - 1) \int_{\alpha_2}^{p_2} (p_2 - x)dx + cst$$

$$W = (\nu B - D)(1 - \alpha_1) + 1 - \alpha_1^2 - (\nu r - 1)(p_1(\alpha_2 - \alpha_1) + \frac{1}{2}(p_2^2 - \alpha_1^2) - p_2(p_2 - \alpha_2)) + cst$$

$$\frac{dW}{d\alpha_2} = -(\nu r - 1)p_1 - \frac{1-\nu}{2(\nu r-1)}\alpha_1 + (\nu r - 1)p_2(2 - \frac{1-\nu}{2(\nu r-1)})$$

Note that this is independent of α_2 and so without loss of generality we can maximize α_2 by setting $\alpha_2 = \min\{p_1, p_2\}$ which would be covered by case V (Note that in case $p_2 = \alpha_2$ the α_2 type strictly prefers the higher contract).

Case V $1 > p_2 > \alpha_2$ and $\alpha_2 > p_1 > \alpha_1$

The IC condition for α_2 is

$$\nu(B + \alpha_2 - q_2 - r(p_2 - \alpha_2)) = (\alpha_2 - p_1) + \nu(B + \alpha_2 - q_1)$$

substituting in the expressions for q_1 and q_2 gives

$$\nu(B + \alpha_2 - \frac{D - p_2 - (1-\nu)\frac{1+\alpha_2}{2}}{\nu} - r(p_2 - \alpha_2)) = (\alpha_2 - p_1) + \nu(B + \alpha_2 - \frac{D - p_1 - (1-\nu)\frac{\alpha_1 + \alpha_2}{2}}{\nu})$$

$$\text{which yields } p_2 = \alpha_2 + \frac{(1-\nu)(1-\alpha_1)}{2(r\nu-1)}$$

in particular note that $\frac{dp_2}{d\alpha_2} = 1$

The participation constraint for α_1 is

$$\nu(B + \alpha_1 - q_1 - r(p_1 - \alpha_1)) = k$$

substituting in the expression for q_1

$$\nu(B + \alpha_1 - \frac{D - p_1 - (1-\nu)\frac{\alpha_1 + \alpha_2}{2}}{\nu} - r(p_1 - \alpha_1)) = k$$

$$\text{yields } p_1 = \frac{1}{\nu r - 1}(-k + \nu B - D + (1-\nu)\frac{\alpha_1 + \alpha_2}{2} + \nu(1+r)\alpha_1)$$

and note in particular that $\frac{dp_1}{d\alpha_2} = \frac{1-\nu}{2(\nu r - 1)}$.

Welfare

Consider overall welfare

$$W = \int_{\alpha_1}^1 (\nu B - D + 2x)dx - (\nu r - 1) \int_{\alpha_1}^{p_1} (p_1 - x)dx - (\nu r - 1) \int_{\alpha_2}^{p_2} (p_2 - x)dx + cst$$

$$W = (\nu B - D)(1 - \alpha_1) + 1 - \alpha_1^2 - (\nu r - 1)(p_1(p_1 - \alpha_1) + \frac{1}{2}(p_1^2 - \alpha_1^2) - p_2(p_2 - \alpha_2) + \frac{1}{2}(p_2^2 - \alpha_2^2)) + cst.$$

and in particular

$$\frac{dW}{d\alpha_2} = \frac{1-\nu}{2(\nu r - 1)}(-(\nu r - 1)(p_1 - \alpha_1) - (\nu r - 1)p_1 + (\nu r - 1)p_1) - (\nu r - 1)(p_2 - \alpha_2) + (\nu r - 1)(p_2 - \alpha_2)$$

$$\frac{dW}{d\alpha_2} = -(\nu r - 1)\frac{1-\nu}{2(\nu r - 1)}(p_1 - \alpha_1)$$

so pushed to a corner where $\alpha_2 = p_1$.

Case VI $1 > p_2 > \alpha_2$ and $\alpha_1 > p_1$

The IC condition for α_2 is $\nu(B + \alpha_2 - q_2 - r(p_2 - \alpha_2)) = (\alpha_2 - p_1) + \nu(B + \alpha_2 - q_1)$

$$\text{which yields } p_2 = \alpha_2 + \frac{(1-\nu)(1-\alpha_1)}{2(r\nu-1)}$$

The participation constraint for α_1 is

$$\alpha_1 - p_1 + \nu(B + \alpha_1 - q_1) = k$$

$$\alpha_1 + \nu B - D + (1-\nu)\frac{\alpha_1 + \alpha_2}{2} = k$$

and so here p_1 is unconstrained but

$$\alpha_2 = \frac{k + D - B\nu - \alpha_1 - \frac{1}{2}\alpha_1(-\nu+1)}{-\frac{1}{2}\nu + \frac{1}{2}}$$

$$\text{which yields } \alpha_2 = \frac{2}{1-\nu}(k + D - \nu B) - \alpha_1 \frac{3-\nu}{2}$$

So here α_2 is not free and so also p_2 is not freely determined, the only free variable is

p_1 .

Welfare

Consider overall welfare

$$W = \int_{\alpha_1}^1 (\nu B - D + 2x)dx - \int_{\alpha_2}^1 (p_2 - x)dx + cst$$

It's clear that from the welfare perspective p_1 is irrelevant since p_1 does not arise, without loss of generality therefore we can take $p_1 = \alpha_1$ and we're back in case V

Checking corners

Finally, it remains to consider what it means for α_2 to be pushed into a corner in each of these cases and whether it necessarily leads to a contradiction.

There are a number of possibilities, first $\alpha_2 = p_1$ and $1 > \alpha_2 > \alpha_1$ with either $p_2 > 1$ or $1 > p_2 > \alpha_2$. We show that this cannot be the case unless either $\alpha_2 > 1$ or $\alpha_2 = \alpha_1$.

In either case the IC for α_2 is

$$p_2 = \alpha_2 + \frac{1-\nu}{2(\nu r-1)}(1-\alpha_1)$$

and the IC for α_1 is

$$\nu(B + \alpha_1 - q_1 - r(p_1 - \alpha_1)) = k$$

substituting in the expression for q_1 and that $p_1 = \alpha_2$ yields

$$\nu(B + \alpha_1 - \frac{D-p_1-(1-\nu)\frac{\alpha_1+\alpha_2}{2}}{\nu} - r(\alpha_2 - \alpha_1)) = k$$

simplifying we obtain

$$\alpha_2 = \frac{1}{\nu+2r\nu-3} (2B\nu - 2D - 2k + \alpha_1 + \nu\alpha_1 + 2r\nu\alpha_1)$$

The case is not degenerate, that is in equilibrium these top two pools do not collapse into one, as long as α_2 is interior which it needs to be. In particular, it must be that both $\alpha_2 > \alpha_1$ and $1 > \alpha_2$. We begin by considering the first of these two conditions

$\alpha_2 > \alpha_1$ if and only if

$$\frac{1}{\nu+2r\nu-3} (2B\nu - 2D - 2k + \alpha_1 + \nu\alpha_1 + 2r\nu\alpha_1) > \alpha_1$$

Which happens if and only if $\alpha_1 > \frac{1}{2}(1+k)$

Now consider the second of the two conditions that are required for the top two pools to survive and not degenerate into a single pool, namely that $\alpha_2 < 1$. This requires that

$$\frac{1}{\nu+2r\nu-3} (2B\nu - 2D - 2k + \alpha_1 + \nu\alpha_1 + 2r\nu\alpha_1) < 1$$

which is true if and only if

$$2B\nu - 2D - 2k + \alpha_1 + \nu\alpha_1 + 2r\nu\alpha_1 < \nu + 2r\nu - 3$$

Which happens if and only if $\alpha_1 < \frac{\nu+2r\nu-1+2k}{(1+\nu+2r\nu)}$

and so in particular, using that $\alpha_1 > \frac{1}{2}(1+k)$, we require

$$\frac{\nu+2r\nu-1+2k}{(1+\nu+2r\nu)} > \frac{1}{2}(1+k)$$

so $2(\nu + 2r\nu - 1 + 2k) - (1 + \nu + 2r\nu)(1 + k) > 0$

Which happens if and only if $k > 1$ which is impossible—the utility for the best possible type when recognized as the best possible type is for type 1 recognized as such who gets a profit of $\nu B - D + 1 + 1 = 1$ and so it cannot be that k , which is the profit for the α_1 type, is greater than 1.

It remains to consider the cases when either $p_2 > 1 > \alpha_2 > \alpha_1 = p_1$ (and here pushed to a corner means α_2 goes to 1 or to α_1 either way there is just one pool rather than two) or $1 > p_2 > \alpha_2 > \alpha_1 = p_1$. In this final case being pushed to a corner means that α_2 goes to α_1 and so there is a single pool and the requisite contradiction, otherwise α_2 is pushed to p_2 then $1 > p_2 = \alpha_2 > \alpha_1 = p_1$.

In this last case, the IC condition for α_2 is

$$\nu(B + \alpha_2 - q_2) = (\alpha_2 - \alpha_1) + \nu(B + \alpha_2 - q_1) \quad (20)$$

Substituting for q_1 and q_2 we obtain

$$\frac{1}{2}\nu\alpha_1 - \frac{1}{2}\alpha_1 - \frac{1}{2}\nu + \frac{1}{2} = 0 \quad (21)$$

which requires $\alpha_1 = 1$.

Note that it is possible to have full separation with $\alpha_1 = 1$ only in the case that the top pool is infinitesimally thin—the full separation case, but with a finite number of pools such an outcome is ruled out.

This completes the proof. ■

Lemma 9 *The condition $W_I \geq 1 - z$ is more likely to hold the larger is z .*

Proof. First note $W_I \geq 1 - z$ if and only if

$$A = (2z + 2\nu + 2z\nu + 4rz\nu + \nu^2 - 3)(6\nu + 8r\nu + \nu^2 + 1)2z - 2(2z + 2\nu + 2z\nu + 4rz\nu + \nu^2 - 3)^2 - (\nu r - 1)(4z\nu - 2\nu + 4rz\nu + \nu^2 + 1 - (2z + 2\nu + 2z\nu + 4rz\nu + \nu^2 - 3))^2 \geq 0$$

$$A = 2z - 8\nu - 16r\nu + 8z\nu + 16rz\nu + 20\nu^2 - 8\nu^3 - 2\nu^4 + 32r\nu^2 - 16r\nu^3 - 20z\nu^2 + 4z^2\nu + 8z\nu^3 + 2z\nu^4 - 32rz\nu^2 + 4rz^2\nu + 16rz\nu^3 + 24z^2\nu^2 + 4z^2\nu^3 + 56rz^2\nu^2 + 4rz^2\nu^3 + 32r^2z^2\nu^2 - 2$$

Taking the derivative with respect to z yields

$$\frac{dA}{dz} = 8\nu(1 - \nu) + 16r\nu(1 - \nu)^2 + 8z\nu + 8rz\nu - 12\nu^2 + 8\nu^3 + 2\nu^4 + 48z\nu^2 + 8z\nu^3 + 112rz\nu^2 + 8rz\nu^3 + 64r^2z\nu^2 + 2$$

note that this latter is linear in z and in r and takes minimal value when $z = 0$ then

$$\frac{dA}{dz} = 8\nu - 20\nu^2 + 8\nu^3 + 2\nu^4 + 16r\nu(1 - \nu)^2 + 2$$

which is linear in r and takes its minimal value when $r = \frac{1}{\nu}$

then $\frac{dA}{dz} = 8\nu^3 - 4\nu^2 - 24\nu + 2\nu^4 + 18$

which it can be easily verified is non-negative in the range ν is in $(0, 1)$. ■

Proof of Proposition 3

Proof. We begin by considering the parameter range $r < \frac{2-\nu}{\nu}$, $z > \frac{(3+\nu)(1-\nu)}{2(1+\nu+r\nu)}$ and $W_I > 1 - z$. In this range $W = W_I$.

Note that $p - l = \frac{2(1-\nu)(2-z)}{6\nu+8r\nu+\nu^2+1}$ and so $\frac{d(p-l)}{dr} = -\frac{16\nu(1-\nu)(2-z)}{(6\nu+8r\nu+\nu^2+1)^2}$. Next, by taking the derivative of W with respect to r from Equation 4, we obtain:

$$\frac{dW}{dr} = \frac{dl}{dr}(1 - 2l) - \frac{1}{2}\nu(p - l)^2 - (\nu r - 1)(p - l)\frac{d(p - l)}{dr}. \quad (22)$$

Substituting in the expressions for $\frac{dl}{dr}$ and $\frac{d(p-l)}{dr}$ and simplifying

$$\frac{dW}{dr} = \frac{4\nu + 16\nu^2 - 40\nu^3 + 16\nu^4 + 4\nu^5 + 32r\nu^2 - 64r\nu^3 + 32r\nu^4}{2(6\nu + 8r\nu + \nu^2 + 1)^3}(2 - z). \quad (23)$$

The denominator of this expression is positive and $(2 - z) > 0$ and so $\frac{dW}{dr}$ has the same sign as the numerator of the fraction

$$\begin{aligned} \text{sign}\left(\frac{dW}{dr}\right) &= \text{sign}(4\nu + 16\nu^2 - 40\nu^3 + 16\nu^4 + 4\nu^5 + 32r\nu^2 - 64r\nu^3 + 32r\nu^4) \\ &= \text{sign}(1 + 4\nu - 10\nu^2 + 4\nu^3 + \nu^4 + 8r\nu - 16r\nu^2 + 8r\nu^3) \end{aligned} \quad (24)$$

where the second equality holds since $4\nu > 0$.

It follows that $\frac{dW}{dr} > 0$ if and only if $1 + 4\nu - 10\nu^2 + 4\nu^3 + \nu^4 + 8r\nu - 16r\nu^2 + 8r\nu^3 > 0$, which is true if and only if:

$$\frac{1 + 4\nu - 10\nu^2 + 4\nu^3 + \nu^4}{16\nu^2 - 8\nu - 8\nu^3} > r. \quad (25)$$

Note that $\frac{2-\nu}{\nu} > r \geq \frac{1}{\nu}$ and so $\frac{dW}{dr} > 0$ requires

$$\frac{1 + 4\nu - 10\nu^2 + 4\nu^3 + \nu^4}{16\nu^2 - 8\nu - 8\nu^3} > \frac{1}{\nu}, \quad (26)$$

or equivalently,

$$4\nu^3 - 2\nu^2 - 12\nu + \nu^4 + 9 > 0 \quad (27)$$

which is always true for ν in the range $(0, 1)$.

Outside of the parameter range $r < \frac{2-\nu}{\nu}$, $z > \frac{(3+\nu)(1-\nu)}{2(1+\nu+r\nu)}$ and $W_I > 1 - z$, $\frac{dW}{dr} = 0$ trivially since W is independent of r . ■