

# Conditional Probability of Default Methodology

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## Abstract

This paper presents the Conditional Probability of Default (CoPoD) methodology for modelling the probabilities of loan defaults (PoDs) by small and medium size enterprises (SMEs) and unlisted firms as functions of identifiable macroeconomic and financial variables. The process of modelling PoDs represents a challenging task, since the time series of PoDs usually contain few observations, thus making ordinary least squares (OLS) estimation imprecise or unfeasible. CoPoD improves the measurement of the impact of macroeconomic variables on PoDs and consequently the measurement of loans' credit risk *through time*, thereby making a twofold contribution. First, econometrically, it recovers estimators that show greater robustness than OLS estimators in finite sample settings under the Mean Square Error criterion. Second, economically, on the basis of economic theory and empirical evidence, CoPoD can incorporate a procedure to select a relevant set of macroeconomic explanatory variables that have an impact on the PoDs. We implement CoPoD with information from Norway and Mexico.

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# 1 Introduction

Over the last two decades, macroeconomic cycles have frequently been associated with cycles in bank lending and asset prices, often followed by episodes of stress in the financial system. There have been numerous episodes in which banks' portfolio credit losses (unexpected losses) have completely or nearly exhausted the banking system's capital.<sup>1</sup> Along these lines, Goodhart, Hofmann and Segoviano (2004) show that, after the 1992 Norwegian crisis, estimates of annual bank portfolio unexpected losses increased on average 47.73% from the levels recorded just before the crisis, while the annual level of empirical frequencies of loan defaults (PoDs) by small and medium size enterprises (SMEs) and unlisted firms increased on average 55.42%. Equally, in Mexico, after the 1994 crisis, estimations of quarterly bank portfolio unexpected losses increased on average 69.61% from the levels recorded just before the crisis, while the quarterly level of PoDs increased on average 18.56%.<sup>2</sup> Therefore, the proper measurement of financial institutions' credit risk should include macroeconomic developments and consequently changes in risk through time.

In this paper, we present a methodology for the modelling of the empirical frequencies of loan defaults (PoDs) by SMEs and unlisted firms as functions of identifiable macroeconomic and financial variables.<sup>3</sup> This allows us to obtain PoDs conditional on the business cycle. We therefore refer to this procedure as the conditional probability of default (CoPoD) methodology. CoPoD not only allows one to measure changes in risk as macroeconomic conditions change, it also improves such measurement from an econometric and economic perspective, thus, improving the measurement of loans' credit risk through time.

In order to model the impact of macroeconomic and financial developments on PoDs, risk managers and regulators have commonly used ordinary least squares (OLS) estimation procedures. When attempting to do so, they usually face a challenging problem, since frequently the number of observations on the time series of PoDs barely exceeds the number of parameters to be estimated. Under these circumstances, the recovered parameters indicating the impact of different macroeconomic and financial variables on PoDs possess large variances and are very sensitive to small changes in the data, thus making the measurement of the impact of macroeconomic developments on loans' credit risk imprecise. We claim that CoPoD improves the measurement of the impact of macroeconomic developments on loans' credit risk by making a twofold contribution. First, econometrically, the proposed methodology, based on the Jaynes (1957) generalized maximum entropy rule (GME), recovers estimators that in the setting of finite samples are superior to OLS estimators under the Mean Square Error (MSE) criterion. Second, economically, on the basis of a hypothesis that is consistent with economic theory and empirical evidence, a procedure is proposed to select the set of explanatory variables that have a significant effect on loans' credit risk.

This hypothesis implies that diverse incentive structures have been created and significant economic structural changes have taken place in countries that have liberalized financial systems. Under such economic frameworks, fluctuations in key macroeconomic and financial variables have the potential to generate endogenous cycles in credit, economic activity and asset prices. These cycles, in turn, appear to involve and indeed may amplify financial imbalances, which can place great stress on the financial system. As a result, an analysis of these variables may be able to provide significant information about systemic vulnerabilities in the economy, which have the potential to increase loans' credit risk. This hypothesis is consistent with theoretical models with credit constraints and a financial accelerator,<sup>4</sup> and with theories that emphasize the importance of the incentive structures created under financial liberalization that can exacerbate the intensity of such cycles. The relevant economic theory includes second-generation models in the currency

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<sup>1</sup>See Caprio and Klingebiel (1996).

<sup>2</sup>Unexpected losses (UL) were computed as the 99.9% value at risk of the portfolio profit and loss distribution. The latter was estimated with a structural portfolio credit risk model. In Norway, UL went from an average of 7.5% of risk weighted assets to an average of 11.08%, while the annual average level of PoDs (for the specific risk-rating class of loans that was analysed in this paper) rose from 7% to 10.88%. In Mexico, UL went from an average of 10.4 % of risk weighted assets to an average of 17.64%, while the annual average level of PoDs (for the specific risk-rating class of loans that was analysed in this paper) went from 13.9 % to 16.48%. It has been reported that the financial system in Mexico was already stressed in 1994, before the outset of the crisis. This might be a possible explanation of why the level of PoDs just before the crisis was already very high, i.e.13.9%. Source, Norges Bank and Comision Nacional Bancaria y de Valores (Financial Regulatory Agency in Mexico).

<sup>3</sup>The implementation of CoPoD may prove to be highly relevant since in most countries, SMEs and unlisted firms represent the backbone of the economy, making a significant contribution to their GDP and to the sustainability of their employment levels. Furthermore, loans granted to SMEs and unlisted companies usually represent an important percentage of the assets held by most commercial banks.

<sup>4</sup>See Kiyotaki and Moore (1997).

crisis literature, which stress the role of self-fulfilling expectations and herding behavior in determining the intensity of the cycles; models that point out that under financial liberalization the scope for risk-taking is increased; and theories that call attention to the creation of perverse mechanisms, such as moral hazard lending and carry trades, that under financial liberalization can exacerbate banking and currency crises.<sup>5</sup> This hypothesis is also consistent with the empirical evidence.<sup>6</sup>

In this paper, we present an empirical implementation of CoPoD with two databases containing information on the empirical frequencies of default experienced by loans given to SMEs and unlisted companies in Norway and Mexico. These databases are provided by Norges Bank (Central Bank of Norway) and by Comision Nacional Bancaria y de Valores (Mexican financial regulatory agency) respectively. For Norway, we have yearly observations from 1988 to 2001. For Mexico, we have quarterly observations from the second quarter of 1995 to the last quarter of 2000. The dataset is one of a few that cover an entire business cycle for both countries under analysis and focus on SMEs and unlisted borrowers;<sup>7</sup> however, the number of observations is still small in statistical terms. This data set is not without its difficulties, as we discuss later in the paper. Nevertheless, despite the limitations of the data, we were able to find regularities in the explanatory power of lagged fluctuations of the ratio of credit to GDP and lagged fluctuations of real asset prices on the PoDs of both countries. These findings are in line with the central hypothesis advanced in this paper, as well as with previous empirical studies. Moreover, this exercise shows that CoPoD is applicable in settings that suffer from information scarcity in both developed and developing economies.<sup>8</sup>

While we restrict our attention to loans, CoPoD can easily be extended to measure the effect of macroeconomic developments on the loss given default (LGD), a variable that is also relevant in the estimation of the profit and loss distribution (PLD) of loan portfolios and that is subject to similar or more stringent data limitations than PoDs.

The outline of the chapter is as follows. In Section 2, we provide the motivation behind the CoPoD. We detail the CoPoDs econometric set up and solve for the CoPoD estimators. We continue reproducing the large and finite sample properties (Golan, Judge and Miller, 1997) of the generalized maximum entropy (GME) rule, which is the theoretical backbone of CoPoD. Based on these properties, those authors show that in a setting of finite samples, GME estimators are weakly superior to OLS estimators under the Mean Square Error (MSE) criterion.<sup>9</sup> In Section 3, we elaborate the main hypothesis setting out the conditions that are likely to generate financial stress in the economy. In Section 4, we present the proposed procedure to select the explanatory variables to include in the model. We start by describing the initial set of macroeconomic and financial variables that according to various theoretical arguments provide information on financial vulnerabilities in the system. We also describe the dependent variables and then we describe the procedure used to select the set of explanatory variables. Next, we present the sets of explanatory variables that, under different specifications, were chosen for Norway and Mexico. In Section 5, using the explanatory variables that were chosen, we implement CoPoD to recover their parameters. In Section 6, we perform a Monte Carlo experiment that shows with an empirical application the quantification of the gain in efficiency of the CoPoD estimators relative to OLS estimators. In Section 7, we offer an analysis of the results and note the consistency of our results with theoretical arguments and empirical evidence.

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<sup>5</sup>See Obstfeld (1995), Calvo (1998) and Flood and Marion (1999) for the first. See Allen and Gale (1998) for the second and Garber and Lall (1996) and Dooley (1997) for the third.

<sup>6</sup>There is a growing literature documenting this empirical evidence. See Heiskanen (1993), Frankel and Rose (1996), Mishkin (1997), Demirgüç-Kunt and Detragiache (1998), Kaminsky and Reinhart (1999), Kaminsky, Lizondo and Reinhart (1998), Goldstein, Kaminsky and Reinhart (2000), Eichengreen and Areta (2000), Reinhart and Tokatlidis (2001), Borio and Lowe (2002), Goodhart, Hofmann and Segoviano (2004).

<sup>7</sup>With few exceptions, e.g. Berger and Udell (1990) and Jimenez and Saurina (2004b), much of the existing literature on credit risk relies on data referring to only one time period or, at best, to short time series. Frequently, the datasets used are biased towards big firms and publicly traded companies.

<sup>8</sup>Time series of PoDs are usually very short in both developed and developing economies. The Bank for International Settlements (BIS) clearly acknowledges such data restrictions. “The state of credit risk management in the banking industry is rather paradoxical. Credit risk is simultaneously the new and old frontier. New, because, until recent years, so little had been done at the conceptual and practical level to address it. The most evident symptom is the extraordinary dearth of data which makes it difficult to obtain reliable estimates: most banks have systematically been throwing this data away, not realizing that it could represent their ultimate comparative advantage. Old, because, since the origins of the industry, credit risk has been by far the most common source of banking distress and failure”. Speech by A. Crockett (2002).

<sup>9</sup>Weakly superior refers to the fact that theoretical results derived by Golan, Judge and Miller (1997), indicate that, under the MSE criterion, in finite sample settings, the GME estimators are superior to OLS estimators. However, asymptotically, both estimators are equivalent.

Finally, our conclusions are summarized in Section 8.

## 2 Conditional probability of default (CoPoD) methodology

### 2.1 CoPoD: rationale

The set-up of our problem begins within the Merton (1974) framework. Merton assumes that the value of the assets of the borrower at time  $t$ , denoted by  $S_t^i$ , follows a geometric Brownian motion,

$$\frac{dS_t^i}{S_t^i} = \mu^i dt + \sigma^i dW_t^i,$$

where  $\mu^i$  is the instantaneous asset return,  $\sigma^i$  is the instantaneous asset volatility and  $W_t^i$  is a standard Brownian Motion.

If it is also assumed that the initial logarithmic asset value is  $\ln[S_t^i] = 0$ , then

$$\ln[S_T^i] \sim N\left[\left(\mu^i - \frac{1}{2}\sigma^{i^2}\right)(T-t), \sigma^i\sqrt{T-t}\right].$$

Therefore, we can represent the standardized logarithmic asset value of this borrower at time  $T$ , as  $s(T) = \frac{\ln[S_T^i] - \left(\mu^i - \frac{1}{2}\sigma^{i^2}\right)(T-t)}{\sigma^i\sqrt{T-t}}$ . As a result,  $s(T) \sim \Phi(0, 1)$ . Moreover, this borrower is assumed to default at some time  $T > t$ , if, at that time, the value of this borrower's assets falls below a pre-specified barrier,  $a_t^i$ , which is usually modelled as a function of the borrower's leverage structure. Therefore, default can be characterized by  $s(T) \leq a_t^i$ . Thus, at time  $t$ , the probability of default at time  $T$  is given by

$$PoD_t = \Phi(a_t^i), \tag{1}$$

where  $\Phi(\cdot)$  is the standard normal cumulative distribution function (*cdf*).

The empirical frequencies of loan defaults by SMEs and unlisted companies classified under a given sectoral activity or risk-rating category, i.e., the  $i_{th}$  category, are grouped in the  $T$ -dimensional vector **PoD**.<sup>10</sup> Each observation in the vector of frequencies of loan defaults **PoD** represents the empirical measure of probability of default for the  $i_{th}$  type of companies at each point in time  $t$ . Since each observation in the vector of PoDs is restricted to lie between 0 and 1, we make the following transformation

$$\mathbf{a}^i = \Phi^{-1}(\mathbf{PoD}),$$

where  $\Phi(\cdot)$  is the inverse standard normal *cdf*. In order to simplify the notation, we will write vector  $\mathbf{a}^i$  as  $\mathbf{a}$ , since our analysis will focus on specific sectoral activities or risk-rating categories of loans and therefore it is not necessary to distinguish this variable by the  $i_{th}$  superscript.

We are interested in modelling the empirical frequency of loan defaults as a function of identifiable macroeconomic and financial developments  $X$ , therefore we can formalize the problem as<sup>11</sup>

$$\mathbf{a} = X\boldsymbol{\beta} + \mathbf{e}, \tag{2}$$

where  $\mathbf{a}$  is a  $T$ -dimensional vector of noisy observations (transformation of the PoDs),  $X$  is a known ( $T \times K$ ) matrix of macroeconomic and financial series and  $\boldsymbol{\beta}$  is a  $K$ -dimensional vector of unknown coefficients that we are interested in estimating. Consequently, we know  $X$ , observe  $\mathbf{a}$  and wish to determine the unknown and unobservable parameter vector  $\boldsymbol{\beta}$ . This is an inverse problem since we must recover  $\boldsymbol{\beta}$  on the basis of only indirect, partial or incomplete information.

A great challenge for the credit risk measurement of loans is the extraordinary lack of available data for modelling. Obtaining any data on the evolution of credit risk through time is extremely difficult. Many

<sup>10</sup>Classifications can also be characterized by sectoral economic activity, geographical domicile, etc. It is assumed that all the firms within a given classification will exhibit homogeneous default behaviour.

<sup>11</sup>An alternative way to look at this problem is to assume that defaults reflect an underlying, continuous credit change indicator (normal equivalent deviate in the language of probit analysis) that has a standard normal distribution. Thus, we can formalize the problem as:  $\mathbf{PoD} = \Phi(X\boldsymbol{\beta} + \mathbf{e})$ . Where the inverse normal CDF transformation converts this equation to a linear problem  $\mathbf{a} = X\boldsymbol{\beta} + \mathbf{e}$ .

banks, even in industrialized countries, have only recently introduced rating and credit risk evaluation systems. In emerging market economies or when analyzing SMEs and unlisted firms in general, such systems have only just started to be implemented. Most of the time, they simply do not exist. Under these circumstances, we may find the problem specified in equation (2) to be *ill-posed*. This is because either (i) the number of unknowns may be larger than the number of data points or (ii) the number of observations barely exceeds the number of parameters to be estimated. In the first case, there are infinitely many solutions that satisfy the basic relationship set up in this equation, so, using traditional procedures, we have no basis for picking out a particular solution vector for  $\beta$  (i.e., the regression coefficients of the X variables are indeterminate). In the second case, the regression coefficients, although determinate, possess large standard errors (in relation to the coefficients themselves), which means that the coefficients cannot be estimated with great precision. Consequently, OLS estimators can be very sensitive to small changes in data,<sup>12</sup> which represents an important problem for risk managers who try to evaluate the impact of specific events on the credit risk of their portfolios.

Given this challenging situation, how can we proceed? *Ill-posed* problems can be addressed in two ways. First, by incorporating additional restrictions such as convenient distributional assumptions, or second, by defining a selection rule to select one of the infinitely many possible solutions. The first approach is adequate only when the assumptions or restrictions are consistent with the data generating process. However, if data restrictions are significant, limitations in data quality and/or quantity may introduce uncertainty about the model and parameter estimates, making both model and parameter risks significant (Koyluoglu, 2003). Therefore, in order to avoid imposing arbitrary assumptions or restrictions, we employ the second approach using an entropy decision rule. The emphasis of this rule is on recovering whatever information is *consistent* with the data (Jaynes, 1957). We undertake an empirical application of the model defined by Judge and Golan (1992). We then present those authors' results regarding the large sample properties and the theoretical distribution approximation of the GME point estimate for finite samples. On the basis of these results, our empirical application's results show that GME solutions exhibit reduced mean squared error (MSE) relative to traditional competitors in finite sample settings. Alternatively, both estimators achieve Cramer-Rao efficiency bounds; however, in small sample settings, OLS estimators are inefficient and GME estimators perform better under the MSE criterion. In Section 6 we present a simulation study that supports this claim.

## 2.2 CoPoD: econometric modelling

Following Judge and Golan (1992), we reformulate the model set in equation (2) as follows. Suppose that we have non-sample information about the unknown parameter and noise components  $\beta$  and  $\mathbf{e}$ . For example, we can have prior beliefs about the signs or ranges of plausible values for each of the unknowns.

Accordingly, the linear inverse model may be written in terms of random variables, and the estimation problem is to recover probability distributions for  $\beta$  and  $\mathbf{e}$  that reconcile the available prior information with the observed sample information. It is important to emphasize that the random variables are merely conceptual devices used to express the prior and sample knowledge in a mutually compatible format.

As a result, we treat each  $\beta_k$  as a discrete random variable with a compact support  $2 \leq M < \infty$  possible outcomes.

If  $z_{k1}$  and  $z_{kM}$  are the plausible extreme values (upper and lower bounds) of  $\beta_k$ , we can express  $\beta_k$  as a convex combination of these two points. That is, there exists  $p_k \in [0, 1]$  such that, for  $M = 2$ ,  $\beta_k = p_k z_{k1} + (1 - p_k) z_{kM}$ . We can do this for each element of  $\beta$ , and the parameter space,  $\mathfrak{R}$ , may be represented by a compact hyperrectangle,  $\mathcal{L} \subset \mathbb{R}^K$ . In a more general fashion, let  $z_k$  be a set of  $M$  points that span the  $k$ th dimension of  $\mathcal{L}$ . Given an  $M$ -dimensional vector of positive weights that sum to one,  $p_k \gg 0$ , the  $k$ th parameter can be expressed as a convex combination of points  $z_k$  with weights  $p_k$ . These convex combinations may be expressed in matrix form so that any  $\beta \in \text{int}(\mathcal{L})$ , then, we can rewrite  $\beta = \mathbf{Z}\mathbf{p}$ , where  $\mathbf{Z}$  is a  $(K \times KM)$  matrix and  $\mathbf{p} \gg \mathbf{0}$  is a  $KM$ -dimensional vector of weights.

We can also reformulate the vector of disturbances,  $\mathbf{e}$ , and assume that is a random vector with finite location and space parameters. Accordingly, we represent our uncertainty about the outcome of the error process by treating each  $e_t$  as a finite and discrete random variable with  $2 \leq J < \infty$  possible outcomes. We also suppose that there exists a set of error bounds,  $v_{t1}$  and  $v_{tJ}$ , for each  $e_t$ . With positive probability,

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<sup>12</sup>See Gujarati (1995).

for  $J = 2$ , each disturbance may be written as  $e_t = w_t v_{t1} + (1 - w_t) v_{tJ}$  for  $w_t \in [0, 1]$ . As above,  $J \geq 2$  may be used to express the parameters in a more general fashion. As before, we restrict the weights so as to be strictly positive and to sum to 1 for each  $t$ . The  $T$  unknown disturbances may be written in matrix form as  $\mathbf{e} = \mathbf{V}\mathbf{w}$ , where  $\mathbf{V}$  is a  $(T \times TJ)$  matrix and  $\mathbf{w}$  is a  $TJ$ -dimensional vector of weights.

Using the reparameterized unknowns,  $\boldsymbol{\beta} = \mathbf{Z}\mathbf{p}$  and  $\mathbf{e} = \mathbf{V}\mathbf{w}$ , Judge and Golan(1992) rewrite the General Linear Model (GLM), equation (2) as

$$\mathbf{a} = X\mathbf{Z}\mathbf{p} + \mathbf{V}\mathbf{w}. \quad (3)$$

The model specified in equation (3) incorporates macroeconomic and financial developments and accounts for possible noise in the data. Once we have this new specification of the model, we proceed with the definition of the entropy decision rule that we use to recover the unknown parameters without imposing arbitrary assumptions or restrictions. The objective of the GME rule is to choose the set of relative frequencies,  $\mathbf{p}$  and  $\mathbf{w}$ , that could have been generated in the greatest number of ways consistent with what is known.<sup>13</sup>

Thus, following Judge and Golan (1992), we select  $\mathbf{p}, \mathbf{w} \gg \mathbf{0}$  to maximize

$$E(\mathbf{p}, \mathbf{w}) = - \left[ \sum_{k=1}^K \sum_{m=1}^M p_m^k \ln p_m^k \right] - \left[ \sum_{t=1}^T \sum_{j=1}^J w_j^t \ln w_j^t \right], \quad (4)$$

subject to the  $T$  moment-consistency constraints

$$a_t = \sum_{k=1}^K \sum_{m=1}^M x_{tk} z_m^k p_m^k + \sum_{j=1}^J v_j^t w_j^t, \quad (5)$$

and the  $K$  additivity restrictions corresponding to the probability distributions of each of the  $K$  parameters  $\beta_k$ , and the  $t = 1, \dots, T$  additivity restrictions corresponding to the probability distributions of each of the  $T$  disturbances,  $e_t$

$$\begin{aligned} 1 &= \sum_{m=1}^M p_m^k, \\ 1 &= \sum_{j=1}^J w_j^t. \end{aligned} \quad (6)$$

Once the objective function and the set of restrictions are defined, we set the following Lagrangian function

$$\begin{aligned} L &= - \left[ \sum_{k=1}^K \sum_{m=1}^M p_m^k \ln p_m^k \right] - \left[ \sum_{t=1}^T \sum_{j=1}^J w_j^t \ln w_j^t \right] \\ &+ \sum_{t=1}^T \lambda_t \left[ a_t - \sum_{k=1}^K \sum_{m=1}^M x_{tk} z_m^k p_m^k - \sum_{j=1}^J v_j^t w_j^t \right] \\ &+ \sum_{k=1}^K \theta_k \left[ 1 - \sum_{m=1}^M p_m^k \right] + \sum_{t=1}^T \tau_t \left[ 1 - \sum_{j=1}^J w_j^t \right]. \end{aligned} \quad (7)$$

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<sup>13</sup>Because we are coping with partial-incomplete information, *ill-posed* inverse problems arise. Thus, we would like to use a principle or formalism that provides us with the “best” conclusions possible based on the data available. In looking for such a principle two requirements appear essential (Jaynes, 1984). We know something but we do not know everything or perhaps not enough to proceed in a traditional way and we do not want to claim any more or any less than we know. These are the guidelines in which the principle of maximum entropy (MED) is based. The entropy formalism seeks to make the “best” predictions possible from the information that is available and provides a basis for transforming the later into a distribution of probabilities describing our state of knowledge. The MED is developed in Appendix A.1.



Note that  $\boldsymbol{\lambda} \in \mathbb{R}^T$ ,  $\boldsymbol{\theta} \in \mathbb{R}^K$  and  $\boldsymbol{\tau} \in \mathbb{R}^T$ .

In order to recover the probability vectors  $\mathbf{p}$  and  $\mathbf{w}$ , we maximize the Lagrangian function described in equation (7).

Thus, the entropy solution for each  $\hat{p}_m^k$  and  $\hat{w}_j^t$  respectively, is given by

$$\hat{p}_m^k(\hat{\boldsymbol{\lambda}}) = \frac{\exp\left[-\sum_{t=1}^T \hat{\lambda}_t x_{tk} z_m^k\right]}{\Theta_k(\hat{\boldsymbol{\lambda}})}, \quad (8)$$

where  $\Theta_k(\hat{\boldsymbol{\lambda}}) = \sum_{m=1}^M \left[\exp\left[-\sum_{t=1}^T \hat{\lambda}_t x_{tk} z_m^k\right]\right]$  and,

$$\hat{w}_j^t(\hat{\boldsymbol{\lambda}}) = \frac{\exp\left[-\hat{\lambda}_t v_j^t\right]}{\Psi_t(\hat{\boldsymbol{\lambda}})}, \quad (9)$$

where  $\Psi_t(\hat{\boldsymbol{\lambda}}) = \sum_{j=1}^J \left[\exp\left[-\hat{\lambda}_t v_j^t\right]\right]$ .

Once we recover the optimal probability vector  $\hat{\mathbf{p}}$ , we are in a position to form point estimates of the unknown parameter vector  $\boldsymbol{\beta}$  as follows:

$$\hat{\boldsymbol{\beta}} = \mathbf{Z}\hat{\mathbf{p}}. \quad (10)$$

On the other hand, the optimal probability vector  $\hat{\mathbf{w}}$ , may also be used to form point estimates of the unknown disturbance  $\hat{\mathbf{e}} = \mathbf{V}\hat{\mathbf{w}}$ .

The GME rule provides a rationale for choosing a particular solution vector  $\mathbf{p}$ , which is the density that could have been generated in the greatest number of ways consistent with what is known (without imposing arbitrary distributional assumptions). Because we do not want to assert more of the distribution  $\mathbf{p}$  than is known, we choose the  $\mathbf{p}$  that is closest to the uniform distribution and also consistent with the data provided by the moment constraints.

### 2.3 The unconstrained dual problem

In this section we specify the ‘‘unconstrained dual problem’’. This is a dual formulation of the GME rule that is used to evaluate the large sample properties of the GME estimators.

For arbitrary  $\boldsymbol{\lambda} \in \mathbb{R}^T$ , let  $\mathbf{p}(\boldsymbol{\lambda})$  and  $\mathbf{w}(\boldsymbol{\lambda})$  represent the functional form of the optimal GME probabilities, defined in equations (8) and (9). If we substitute these into equation (7), where the optimal  $\mathbf{p}(\boldsymbol{\lambda})$  and  $\mathbf{w}(\boldsymbol{\lambda})$  satisfy the adding up constraints, we formulate the dual objective as a function of the Lagrange multipliers,  $\boldsymbol{\lambda}$ . This function is defined as

$$\begin{aligned}
L(\boldsymbol{\lambda}) &= - \left[ \sum_{k=1}^K \sum_{m=1}^M p_m^k(\boldsymbol{\lambda}) \ln(p_m^k(\boldsymbol{\lambda})) \right] - \left[ \sum_{t=1}^T \sum_{j=1}^J w_j^t(\boldsymbol{\lambda}) \ln(w_j^t(\boldsymbol{\lambda})) \right] \\
&+ \sum_{t=1}^T \lambda_t \left[ a_t - \sum_{k=1}^K \sum_{m=1}^M x_{tk} z_m^k p_m^k(\boldsymbol{\lambda}) - \sum_{j=1}^J v_j^t w_j^t(\boldsymbol{\lambda}) \right]. \\
&= - \left[ \sum_{k=1}^K \sum_{m=1}^M p_m^k(\boldsymbol{\lambda}) \left\{ \left[ - \sum_{t=1}^T \lambda_t x_{tk} z_m^k \right] - \ln[\Theta(\boldsymbol{\lambda})] \right\} \right] \\
&- \left[ \sum_{t=1}^T \sum_{j=1}^J w_j^t(\boldsymbol{\lambda}) \left\{ [-\lambda_t v_j^t] - \ln[\Psi(\boldsymbol{\lambda})] \right\} \right]. \\
&+ \sum_{t=1}^T \lambda_t \left[ a_t - \sum_{k=1}^K \sum_{m=1}^M x_{tk} z_m^k p_m^k(\boldsymbol{\lambda}) - \sum_{j=1}^J v_j^t w_j^t(\boldsymbol{\lambda}) \right]. \\
&= \sum_{t=1}^T \lambda_t a_t + \sum_{k=1}^K \ln[\Theta_k(\boldsymbol{\lambda})] + \sum_{t=1}^T \ln[\Psi_t(\boldsymbol{\lambda})] \equiv M(\boldsymbol{\lambda}).
\end{aligned} \tag{11}$$

Accordingly, within the maximum likelihood (ML) approach, the dual unconstrained function  $M(\boldsymbol{\lambda})$  can be interpreted as a log-likelihood function. Specifically,  $M(\boldsymbol{\lambda})$ , is the negative expected-loglikelihood function for  $\boldsymbol{\lambda}$ .<sup>14</sup> Therefore, the dual version of the GME problem is to choose  $\boldsymbol{\lambda}$  to minimize  $M(\boldsymbol{\lambda})$ . Minimizing  $M(\boldsymbol{\lambda})$  with respect to  $\boldsymbol{\lambda}$  yields  $\hat{\boldsymbol{\lambda}}$ , then, we can compute  $\hat{\mathbf{p}}$ ,  $\hat{\mathbf{w}}$ ,  $\hat{\boldsymbol{\beta}}$  and  $\hat{\mathbf{e}}$  by substitution. In other words, the value  $\min[M(\boldsymbol{\lambda})]$  equals the value  $\max[E(\mathbf{p}, \mathbf{w})]$ .<sup>15</sup>

The gradient of the dual objective  $M(\boldsymbol{\lambda})$  is the model consistency constraints in equation (5).

$$\nabla_{\boldsymbol{\lambda}} M(\boldsymbol{\lambda}) = \mathbf{a} - \mathbf{XZp}(\boldsymbol{\lambda}) - \mathbf{Vw}(\boldsymbol{\lambda}) = 0. \tag{12}$$

Note that the  $t_{th}$  equation in  $\nabla_{\boldsymbol{\lambda}} M(\boldsymbol{\lambda})$  takes the form

$$a_t - \sum_{k=1}^K \sum_{m=1}^M x_{tk} z_m^k p_m^k - \sum_{j=1}^J v_j^t w_j^t.$$

The second partial derivative of this equation with respect to  $\lambda_s$  is

$$\frac{\partial^2 M}{\partial \lambda_s \partial \lambda_t} = - \sum_{k=1}^K x_{tk} \sum_{m=1}^M z_m^k \frac{\partial p_m^k(\boldsymbol{\lambda})}{\partial \lambda_s} - \sum_{j=1}^J v_j^t \frac{\partial w_j^t(\boldsymbol{\lambda})}{\partial \lambda_s}, \tag{13}$$

where

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<sup>14</sup>To see this, recall that we have expressed our uncertainty about  $\boldsymbol{\beta}$  and  $\mathbf{e}$  by viewing them as random variables on supports  $\mathbf{Z}$  and  $\mathbf{V}$ , respectively. If we view  $\mathbf{p}(\boldsymbol{\lambda})$  and  $\mathbf{w}(\boldsymbol{\lambda})$  as families of parametric probability mass functions for these random variables,

the joint likelihood function for a sample size  $N$ , may be written as  $N^{-1}l(\boldsymbol{\lambda}) = - \left[ \sum_{k=1}^K \sum_{m=1}^M f_m^k \ln p_m^k \right] - \left[ \sum_{t=1}^T \sum_{j=1}^J r_j^t \ln w_j^t \right]$ ,

where  $f_m^k$ ,  $r_j^t$  are the observed frequencies of outcomes  $z_m^k$  and  $v_j^t$  respectively. However, we can form an analog version of the log-likelihood function by replacing the frequencies with the associated probabilities. By substitution, the previous equation becomes  $\mathbf{E}[l(\boldsymbol{\lambda})] = - [\mathbf{p}(\boldsymbol{\lambda})' \ln \mathbf{p}(\boldsymbol{\lambda})] - [\mathbf{w}(\boldsymbol{\lambda})' \ln \mathbf{w}(\boldsymbol{\lambda})]$ , which is the negative *expected* log-likelihood function for  $\boldsymbol{\lambda}$ . To ensure that  $\mathbf{E}[l(\boldsymbol{\lambda})]$  satisfies the properties of the observed sample, we will optimize it (minimize it, since it is the negative *expected* log-likelihood function) subject to the consistency constraints in equation (5). After simplifying the expression in equation (11) and imposing the consistency constraints by substitution, we can see that the constrained expected log-likelihood function is simply  $M(\boldsymbol{\lambda})$ .

<sup>15</sup>The dual unconstrained problem was originally developed by Alhassid et al (1978).

$$\frac{\partial p_m^k(\lambda)}{\partial \lambda_s} = - \left\{ x_{sk} \left[ z_m^k p_m^k - p_m^k \sum_n z_n^k p_n^k \right] \right\}, \quad (14)$$

and

$$\frac{\partial w_j^t(\lambda)}{\partial \lambda_s} = \left\{ \begin{array}{l} - \left\{ v_j^t w_j^t - w_j^t \sum_n v_n^t w_n^t \right\}, \text{ for } s = t \\ 0, \text{ for } s \neq t \end{array} \right\}. \quad (15)$$

Since

$$\sigma_{zk}^2 = \sum_{m=1}^M p_m^k (z_m^k)^2 - \left[ \sum_{m=1}^M p_m^k z_m^k \right]^2, \quad (16)$$

$$\sigma_{vt}^2 = \sum_{j=1}^J w_j^t (v_j^t)^2 - \left[ \sum_{j=1}^J w_j^t v_j^t \right]^2. \quad (17)$$

Therefore, we get

$$\frac{\partial^2 M}{\partial \lambda_s \partial \lambda_t} = \sum_{k=1}^K x_{tk} x_{sk} \sigma_{zk}^2 + \sigma_{vt}^2 \quad (18)$$

In matrix form, this equation becomes the Hessian matrix of  $M(\lambda)$ , which is expressed as

$$\nabla_{\lambda\lambda} M(\lambda) = \mathbf{X} \Sigma_{\mathbf{Z}}(\lambda) \mathbf{X}' + \Sigma_{\mathbf{V}}(\lambda), \quad (19)$$

where  $\Sigma_{\mathbf{Z}}(\lambda)$  and  $\Sigma_{\mathbf{V}}(\lambda)$  are the variance-covariance matrices for the distributions  $\mathbf{p}(\lambda)$  and  $\mathbf{w}(\lambda)$ . For any interior solution,  $(\hat{\mathbf{p}}, \hat{\mathbf{w}})$ , each of these variance terms is strictly positive, consequently,  $\Sigma_{\mathbf{Z}}$  and  $\Sigma_{\mathbf{V}}$  are positive definite matrices.

Note that  $\mathbf{X} \Sigma_{\mathbf{Z}} \mathbf{X}'$  is positive semi-definite when  $T > K$ . Since  $\Sigma_{\mathbf{V}}$  is positive definite, then equation (19) is a positive definite matrix. By the sufficient condition for strict convexity,  $M(\lambda)$  is strictly convex in  $\lambda$  and choosing  $\lambda$  to minimize  $M(\lambda)$  will yield a unique solution,  $\hat{\lambda}$ . See Appendix A.2.

## 2.4 Large sample properties

In order to compare the performance of the GME estimators with competing estimators it is useful to look at their large sample properties.<sup>16</sup> Golan, Judge and Miller (1997) make use of the dual formulation of the problem to evaluate the large sample properties of the GME estimators. The authors show that the asymptotic distribution of GME estimators can be derived by finding the distribution of  $\widehat{\lambda}_{\mathbf{T}}$ . Given that  $\widehat{\beta}_{\mathbf{T}} = \mathbf{Z} \mathbf{p}(\widehat{\lambda}_{\mathbf{T}})$  is a continuous function of  $\widehat{\lambda}_{\mathbf{T}}$ , they use the  $\delta$ -method (Spanos, 1986) to approximate the distribution of  $\widehat{\beta}_{\mathbf{T}}$ . These authors claim that under the conditions presented in Appendix A.3, the GME estimators converge in probability to the following distribution

$$\sqrt{T} \left( \widehat{\beta}_{\mathbf{T}} - \beta_0 \right) \implies \mathbf{N} \left[ \mathbf{0}, \mathbf{Q}^{-1} \Sigma^* \mathbf{Q}^{-1} \right], \quad (20)$$

where  $\mathbf{Q}$  and  $\Sigma^*$  are matrices defined as in Appendix A.3.

This distribution is identical to the limiting distribution of the standard OLS estimator. Intuitively, the GME solution is asymptotically equivalent to the OLS estimator because the first-order conditions (normal equations) are identical in the limit of  $T$ . Alternatively, both estimators achieve Cramer-Rao efficiency bounds, however, on the basis of results presented in the following section, it is claimed that, in small sample settings, OLS estimators are inefficient and GME estimators perform better under the MSE criteria.

<sup>16</sup>In this section, we reproduce the large sample properties results derived by Golan, Judge and Miller (1997). For proofs and detailed explanation, we refer the reader to the original article.

## 2.5 Finite sample properties

Golan, Judge and Miller (1997) use the asymptotic normality property of the GME estimator, equation (20), to approximate the distribution of the GME point estimate for finite samples. On the basis of the limiting distribution of  $\widehat{\boldsymbol{\beta}}_{\mathbf{T}}$ , they specify the finite sample approximation as

$$\widehat{\boldsymbol{\beta}}_{\mathbf{T}} \sim \mathbf{N} \left[ \boldsymbol{\beta}, \boldsymbol{\Sigma}_{\mathbf{Z}}(\widehat{\boldsymbol{\lambda}}_{\mathbf{T}}) \mathbf{X}' \mathbf{X} \mathbf{C}^{-1} \mathbf{D} \mathbf{C}'^{-1} \mathbf{X}' \mathbf{X} \boldsymbol{\Sigma}_{\mathbf{Z}}(\widehat{\boldsymbol{\lambda}}_{\mathbf{T}}) \right], \quad (21)$$

where

$$\begin{aligned} \mathbf{C} &= \mathbf{X}' \mathbf{X} \boldsymbol{\Sigma}_{\mathbf{Z}}(\widehat{\boldsymbol{\lambda}}_{\mathbf{T}}) \mathbf{X}' \mathbf{X} + \boldsymbol{\Sigma}_{\mathbf{V}}(\widehat{\boldsymbol{\lambda}}_{\mathbf{T}}), \\ \mathbf{D} &= \mathbf{X}' \boldsymbol{\Sigma}_{\mathbf{e}} \mathbf{X}, \end{aligned} \quad (22)$$

where  $\text{Var}(\mathbf{e}) = \boldsymbol{\Sigma}_{\mathbf{e}}$ . Note that if the GME problem is specified as a pure inverse problem (i.e., the structure of the error term in equation (3) is not included), the  $\boldsymbol{\Sigma}_{\mathbf{V}}$  terms disappear. It is interesting to see that, in this case, if  $\boldsymbol{\Sigma}_{\mathbf{Z}}$  is full rank, the variance-covariance matrix of  $\widehat{\boldsymbol{\beta}}_{\mathbf{T}}$  is  $(\mathbf{X}' \mathbf{X})^{-1} \mathbf{X}' \boldsymbol{\Sigma}_{\mathbf{e}} \mathbf{X} (\mathbf{X}' \mathbf{X})^{-1}$ , which is identical to the variance-covariance structure of the OLS estimator.

In general, the presence of  $\boldsymbol{\Sigma}_{\mathbf{V}}$  in the inverted terms  $\mathbf{C}$  reduces the variance of  $\widehat{\boldsymbol{\beta}}_{\mathbf{T}}$  under the noise specification in equation (3). To make this point clear, consider a special case in which  $\boldsymbol{\Sigma}_{\mathbf{e}} = \sigma^2 \mathbf{I}_T$  (the Gauss-Markov setting) and  $\mathbf{X}$  is orthogonal. In this case, the approximate variance-covariance matrix for  $\widehat{\boldsymbol{\beta}}_{\mathbf{T}}$  is

$$\sigma^2 \boldsymbol{\Sigma}_{\mathbf{Z}} (\boldsymbol{\Sigma}_{\mathbf{Z}} + \boldsymbol{\Sigma}_{\mathbf{V}})^{-2} \boldsymbol{\Sigma}_{\mathbf{Z}},$$

that is a diagonal matrix in which the  $k_{th}$  element is

$$\sigma^2 \left( \frac{(\sigma_{ZK})^2}{(\sigma_{ZK})^2 + (\sigma_{VK})^2} \right)^2.$$

It is clear that the approximate variance of  $\widehat{\boldsymbol{\beta}}_{\mathbf{T}}$  is smaller than the variance of the OLS-LS estimator, which is  $\sigma^2 \mathbf{I}_K$ .

Since  $\boldsymbol{\Sigma}_{\mathbf{Z}}$  and  $\boldsymbol{\Sigma}_{\mathbf{V}}$  are functions of  $\boldsymbol{\lambda}$ , equation (11), the approximate variance of  $\widehat{\boldsymbol{\beta}}_{\mathbf{T}}$  depends on  $\widehat{\boldsymbol{\lambda}}_{\mathbf{T}}$ . Maximizing the parameter and error entropies yields GME estimators  $\widehat{\boldsymbol{\beta}}_{\mathbf{T}}$  that have minimum variance. In order to see this, note that the denominator of the approximate variance-covariance matrix contains the terms  $\boldsymbol{\Sigma}_{\mathbf{Z}} + \boldsymbol{\Sigma}_{\mathbf{V}}$ , which is the sum of the variance-covariance matrices for distributions  $\widehat{\mathbf{p}}$  and  $\widehat{\mathbf{w}}$ . Thus the maximum entropy solution corresponds with the maximum denominator for the approximate variance of  $\widehat{\boldsymbol{\beta}}_{\mathbf{T}}$ , which minimizes the approximate variance of  $\widehat{\boldsymbol{\beta}}_{\mathbf{T}}$ .<sup>17</sup>

Finally, Golan, Judge and Miller (1997) claim that, although the finite sample GME solution is almost certainly biased, the GME consistency constraints must be satisfied and its bias cannot become very large. Therefore, given the properties of limited bias and minimum variance, GME solutions exhibit reduced MSE relative to OLS estimators in finite sample settings. In particular, Monte Carlo experiments presented in Section 6 reveal that the GME estimators exhibit a proper finite sample behavior since they show reduced MSE relative to OLS estimators.

## 2.6 CoPoD: how does it differ from other methodologies?

In recent years, a number of econometric approaches that try to predict episodes of financial instability in the financial system, as a whole, have emerged in the literature. These approaches are known as "early warning systems". Some of the best known are the "signals methodology" by Kaminsky, Lizondo and Reinhart (1998) and multivariate probit or logit regression models à la Frankel and Rose (1996). While

<sup>17</sup>For example, for  $M=2$ , it is possible to rewrite  $S(\widehat{\mathbf{p}})$  from equation (A.3.1) in terms of  $\widehat{p}_k$  and  $(1-\widehat{p}_k)$ . It then follows that maximizing  $S(\widehat{\mathbf{p}}_k)$  to choose the GME solutions also maximizes the  $\sigma_{ZK}$ , the variance of the distribution on  $\mathbf{z}_k$ . Using the same arguments with  $S(\widehat{\mathbf{w}})$ , we get to maximize the  $\sigma_{VK}$ , the variance of the distribution on  $\mathbf{v}_k$ . Therefore, the maximum entropy solution corresponds with the maximum denominator for the approximate variance of  $\widehat{\boldsymbol{\beta}}_{\mathbf{T}}$ , which minimizes  $\widehat{\boldsymbol{\beta}}_{\mathbf{T}}$  approximate variance.

the selection of the explanatory variables used in these methodologies is also based on theoretical models of currency and banking crises, CoPoD differs from these models in both the objective of measurement and the econometric techniques applied.

With respect to the objective of measurement, it is the empirical frequencies of loan default (PoDs) by SMEs and unlisted companies that represent the dependent variable in our study. Consequently, rather than trying to assess the probability of episodes of financial instability in the financial system as a whole, as the early warning systems do, the core objective behind the development of CoPoD is to improve the measurement of the impact of key macroeconomic and financial developments on the likelihood of default of specific types of loans within an economy.

With respect to the econometric techniques, the "signals methodology" is based on the assumption that the economy behaves differently on the eve of financial crises and that this aberrant behavior has a recurrent systematic pattern. Therefore, the "signals methodology" monitors a large set of indicators (variables) that signal that a crisis is likely whenever they cross a certain threshold. The procedure used by these models to specify significant variables is not amenable to the usual statistical tests of significance. Moreover, the "signals methodology" requires the specification of thresholds and forces the modeler to be quite specific about the timing of early warnings. These requirements may prove difficult to fulfil, if one wanted to apply a similar methodology to analyze the credit risk of loans, given the data restrictions that credit risk modelers face. This methodology also imposes some restrictions (e.g. that indicators send a signal only when they reach a threshold) that may omit valuable information.

Multivariate probit or logit regression models define the dependent variable to take the value of one, if a period is classified as a crisis, and a value of zero if there is no crisis. When such a regression is fitted on a pooled set of country data, the statistical significance of the estimated regression coefficients reveals which indicators are "significant" and which are not, and the predicted value of the dependent variable should identify in which periods countries have a higher or lower probability of a crisis. Since credit risk modelers are usually interested in identifying significant variables affecting the credit risk of specific assets (i.e., loans in this case), similar methodologies to analyze the impact of different macroeconomic and financial variables on the credit risk of specific types of loans have commonly been applied. In fact, probit or logit models that use information aggregated at the loan level (as we do) and that try to model PoDs as functions of macroeconomic and financial variables have been developed by the financial industry. Examples developed by the private sector include Wilson (1997a,b) and Kim (1999) and by regulators Boss (2002) and Jimenez and Saurina (2004b). However, these methodologies use OLS estimation procedures to recover the parameters of their explanatory variables. The use of OLS estimation represents an important limitation for these models, as often the number of observations in the sample barely exceeds the number of parameters to be estimated. As already mentioned, under these circumstances the parameters recovered with OLS procedures are inefficient and very sensitive to small changes in the data. When implementing CoPoD, the procedure used to select the set of explanatory variables involves the use of multivariate OLS regressions, with the specific objective of making a final selection of the variables which, besides being consistent with theoretical arguments and empirical evidence, provide the best fit. However, once the set of explanatory variables has been chosen, we recover their parameters with CoPoD, improving their efficiency. Thus, we can improve the measurement of the impact of different macroeconomic and financial developments on the credit risk of specific types of loans.

Note also that our methodology differs from bank failure prediction models that try to assess the probability of bank failures based on banks' information at the balance sheet level. Altman et al (1981), Looney et al (1989), Fissel et al (1996) and Kolari et al (2001) are examples of this literature. These models rely on data referring to only one date or, at best, to a short time period and do not incorporate general macroeconomic and financial system variables. So, they cannot measure the evolution of risk through time. Moreover, since they look at aggregate (balance sheet level) data, they cannot identify the specific types of assets (loans or other financial instruments) in banks' portfolios that could increase/decrease the likelihood of bank failures, nor can they measure the diversification effects brought about by different types of loans (assets).

## 3 Economic framework

### 3.1 Empirical evidence

It has been claimed that financial liberalization provides a more efficient allocation of investment in both physical and human capital by improving the efficiency of the banking system and easing financing constraints. In turn, this fosters long run economic growth.<sup>18</sup> Leahy et al (2001) present empirical evidence that supports this hypothesis. These findings have underpinned the financial liberalization that has been taking place across the globe since the 1970s.

In recent years, however, both developed and developing economies have experienced recurrent and sometimes violent boom-bust cycles in credit growth, economic activity and asset price changes (particularly in real estate). These often ended in systemic crises of the banking sector.<sup>19</sup> This has been shown by the experience of Scandinavia in the early 1990's, Japan in the 1990's, Mexico in 1994 and East Asia in 1997-98. Consequently, in this paper, we pose the following questions:

**1. Does financial liberalization change economic structures in a way that makes banking crises more likely?**

A central finding of the large and growing literature on the causes of banking crises is that financial liberalization significantly increases their probability. Demirgüç-Kunt and Detragiache (1998) and Kaminsky and Reinhart (1999) show that financial liberalization helps to explain the occurrence of banking crises in large samples of developing and developed countries.

**2. Does a common pattern exist in the development of key macroeconomic and financial variables that could be the product of structural economic changes provoked by financial liberalization?**

A few studies have tried to derive stylized facts for the development of macroeconomic and financial variables in the wake of financial liberalization.<sup>20</sup> Anecdotal evidence of boom-bust cycles in bank lending, economic activity and asset prices after financial liberalization have been documented separately for Scandinavia and East Asia by Drees and Pazarbasioglu (1998) and Collyns and Senhadji (2002) respectively. In order to assess more broadly whether, after financial liberalization, economic structures have been altered and whether there is a common pattern in the development of key macroeconomic and financial variables after financial liberalization, Goodhart, Hofmann and Segoviano (2004) consider the development of real GDP, bank lending, property prices and share prices in the wake of financial liberalization for a sample of 16 OECD countries.<sup>21</sup>

Their results, reproduced in Figure 1, reveal that financial liberalization is generally followed by a boom-bust cycle in economic activity, bank lending and asset prices. Individual country data suggest that all these countries experienced a cycle after financial liberalization, although with substantial variation in the timing of its occurrence. The evidence provided by these results appears to indicate that financial liberalization does alter economic structures and that common patterns (cycles) in key variables can be observed.<sup>22</sup>

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<sup>18</sup>See Levine (1997) for theoretical arguments.

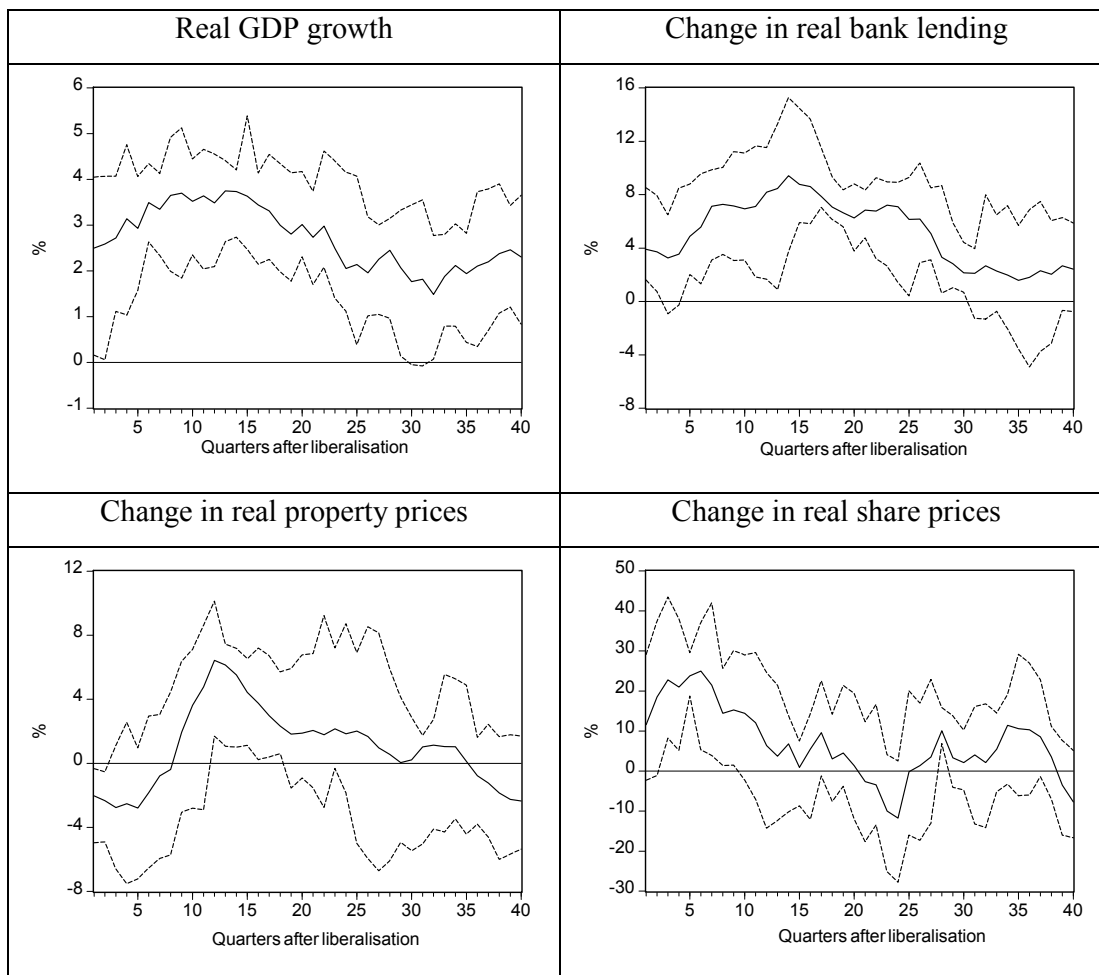
<sup>19</sup>See Bordo et al (2001) for an account of currency, banking and twin crises.

<sup>20</sup>Reinhart and Tokatlidis (2001) derived stylised facts for the long-run effects of liberalisation for a large sample of developed and developing countries.

<sup>21</sup>Australia, Belgium, Denmark, Finland, France, Ireland, Italy, Japan, Korea, New Zealand, Norway, Spain, Sweden, Switzerland, the UK and the US.

<sup>22</sup>On average, real GDP growth starts to rise immediately after liberalisation and peaks after about three years. Then real growth gradually declines and falls below its initial value after about five years. Real lending growth starts to rise about one year after the date of liberalisation and peaks after about three years. Subsequently, the growth rate of real lending declines and falls below its initial value after about seven years. Property prices starts to rise one year after liberalisation. The increase in property prices peaks after about three years and then gradually declines. After about six years, property prices start to fall. Real share prices appear to be rising at a brisk pace already at the time of liberalisation. After liberalisation, the increase in share prices further accelerates and peaks after about six quarters. About five years after liberalisation, share prices start to fall. Thus the sample appears to support the notion that episodes of financial liberalisation are followed by pronounced boom-bust cycles. See Goodhart, Hofmann and Segoviano (2004).

Figure 1: Post-liberalisation Cycles



Note: Solid lines are sample averages, dotted lines are upper and lower quartiles.  
 Source: Goodhart, Hofmann and Segoviano (2004).

**3. Therefore, the critical question is the following: Do fluctuations in key macroeconomic and financial variables provide significant information about vulnerabilities in the financial system that can explicitly increase the credit risk of banks?**

A robust finding that emerges from the literature on leading indicators of banking crises is that rapid domestic credit growth increases the credit risk of banks. This has been documented by Pill and Pradhan (1995), Kaminsky and Reinhart (1999) and Eichengreen and Areta (2000). Mishkin (1997) documents how in Mexico, bank credit to private non-financial enterprises went from a level of around 10 percent of GDP in the late 1980s to 40 percent of GDP in 1994. The stock market rose significantly during the early 1990s, up until the 1994 Mexican crisis, triggered by the Colosio assassination and the uprising in the Mexican state of Chiapas. Subsequently, the prices of stocks and other assets fell and banking and foreign exchange crises occurred. Heiskanen (1993) documents similar events in Norway, where the ratio of bank loans to nominal GDP went from 40 percent in 1984 to 68 percent in 1988. Similarly, Vale (2004) reports that between December 1984 and September 1986 the Norwegian real 12-month growth in bank loans stayed above 20%. Asset prices soared, while consumption also increased significantly. The collapse in oil prices in late 1985, together with macroeconomic conditions which were binding during that time, triggered the

Norwegian crisis, causing the most severe banking crisis and recession since the war.<sup>23</sup>

However, in most studies, asset price development analyses have mainly focused on equity prices, rather than on real estate values, or they do not take account of the combination of events and the interactions between credit, asset prices (real estate), the real economy and their implications for financial risk. Borio and Lowe (2002) take a step forward in this direction.<sup>24</sup> They analyze a sample of 34 developed and middle-income developing economies and provide evidence that the combination of events, in particular the simultaneous occurrence of rapid credit growth and rapid increases in asset prices (rather than either one alone), appear to be common factors that augment the probability of episodes of financial instability in the banking system.

## 3.2 Hypothesis

We hypothesize that financial liberalization provokes structural changes in the economy and creates incentive structures that have the potential to convert fluctuations in key macroeconomic and financial variables into endogenous events that define cycles in credit, economic activity and asset prices. These cycles, in turn, appear to involve and indeed may amplify financial imbalances which, when unwound, can place great stress on the financial system. This hypothesis implies that cycles in the financial system are amplified due to the fact that at the upturn of the cycle, banks sharply increase lending as prices of assets held as collateral increase, and the state of confidence in the system is positive. It is also during this stage that the seeds of financial imbalances are sown and the financial vulnerability (risk) of the economy increases, as also do levels of leverage of the banking system. As a result, when vulnerabilities in the economy are high, sudden changes in the system's state of confidence have the potential to interact and become endogenous, self-fulfilling fluctuations that define (and possibly exacerbate) the downturn of the cycle. During this stage, the previously sown risk is harvested and stress in the financial system rises. Alternative theories (as we argue below) also indicate that incentive structures created under financial liberalization play an important role in market participants' willingness to take on risks and therefore in the intensity of the cycles.

## 3.3 Underlying economic theory

Our hypothesis is fully consistent with theoretical models that include credit constraints and a financial accelerator, as in Bernanke and Gertler (1989) and Kyotaki and Moore (1997). Financial liberalization relaxes borrowing constraints faced by the private sector, and therefore has similar effects to a positive, permanent productivity shock to the economy. In models with credit-constrained borrowers, a positive productivity shock provokes a boom-bust cycle in lending (credit growth), economic activity and asset prices. The reasoning is that such a shock increases the value of collateralizable assets. As the borrowing capacity of entrepreneurs depends on the value of their collateralizable assets, an appreciation in the value of the assets increases the level of lending in the economy. This in turn fuels further economic activity and asset price growth, which again increases borrowing capacity, and so on. This goes on until the rise in capital stock becomes so large that profit margins crumble. Then the process reverses itself. The result is a credit cycle à la Kiyotaki and Moore (1997). Thus the evolution of a simultaneous boom-bust cycle in credit growth and asset prices in systems that have gone through the process of financial liberalization confirms the theory.

Incentive structures created under financial liberalization appear to intensify the cycles in the macro-economy and in the financial system through different channels. For example, by increasing competitive pressures among financial intermediaries, seeking to increase market share, financial liberalization has increased the scope for risk taking, herd behavior and leverage. Along these lines, it has been argued that financial liberalization appears to have strengthened the effects of the financial accelerator mechanism.<sup>25</sup>

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<sup>23</sup>A detailed study of the Norwegian banking crisis has been published by the Norges Bank (2004).

<sup>24</sup>Borio and Lowe (2002), building on the "signals methodology" pioneered by Kaminsky, Lizondo and Reinhart (1999) take account of the combination of events, look at cumulative processes and incorporate in their study the development of prices of real estate assets.

<sup>25</sup>In order to test this hypothesis empirically, Goodhart, Hofmann and Segoviano (2004) performed rolling regressions for a reduced form credit growth equation, where they regressed the change in real bank lending on its own lag, the lagged change in property prices, the lagged change in real GDP and the lagged change in the short-term real interest rate. The rolling



As the liberalization of banking systems has usually been accompanied by liberalizations of capital and stock markets, it has become easier for the largest and safest borrowers from banks to raise funds on the capital and stock market.<sup>26</sup> Therefore, in an effort to recover lost business, banks have increased lending to SMEs, unlisted firms and individuals. Smaller size borrowers are, in general, riskier and the cost of acquiring information on large numbers of idiosyncratic borrowers is greater, so banks have placed increasing weight on collateral as a basis for lending. As a result, changes in the value of collateralizable assets, predominantly real estate, are likely to enhance the effects of the financial accelerator and therefore have provoked more pronounced cycles.<sup>27</sup>

Allen and Gale (1998) provide an alternative theory as to how incentive structures under financial liberalization have increased the scope for risk-taking and have exacerbated cycles in asset prices. They argue that many investors in real state and stock markets obtain their investment funds from external sources. If the ultimate providers of funds are unable to observe the characteristics of the investment, and there exists limited liability for investors, a classic risk-shifting problem is caused. Risk-shifting increases assets' investment returns and causes investors to bid up asset prices. A crucial determinant of asset prices is the amount of credit that is provided for speculative investment. Financial liberalization, by expanding the volume of credit for speculative investments, can interact with the agency problem and lead to booms in asset prices.<sup>28</sup> Therefore delegated investment and risk management have the potential to enhance risk-taking behavior.

Dooley (1997), on the other hand, argues that financial liberalization can exacerbate banking crises when there is implicit insurance provided by the government. In these cases, foreign investors first acquire insured claims on residents. When government reserves are exactly matched by its contingent insurance liabilities, expected yield on domestic liabilities falls below world rates. Then foreign investors sell the insured assets to the government, exhausting its reserves.<sup>29</sup> Financial liberalization, by providing foreigners with increased access to domestic liabilities and by increasing implicit insurance (because liberalization is often backed by official creditor governments or by international agencies), exacerbates this effect. Therefore, to the extent that implicit guarantees lead banks to engage in moral hazard lending, the implicit guarantees represent a hidden government deficit that can exacerbate banking crises. This phenomenon has also been reported in the guise of carry trades by Garber and Lall (1996).<sup>30</sup>

Self-fulfilling expectations and herding behavior in international capital markets have also been accused of playing an important role in the intensity of the cycles by second-generation models in the currency crisis literature. References for this literature include Obstfeld (1995), Flood and Marion (1999).<sup>31</sup>

Peer-group performance measures or index tracking can also encourage herding and short-termism among institutional investors, with the potential to create self-fulfilling fluctuations in leverage and asset prices. For example, a bank manager who systematically loses market share and who under-performs his

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regression results clearly support the view that bank lending has become more sensitive to property price movements after financial liberalisation.

<sup>26</sup>See Kaminsky and Schmukler (2003) for a cross-country chronology of the banking sector and stock market liberalisations.

<sup>27</sup>Recently, it has also been observed that the value of collateral assets exerts a powerful stimulus to consumption spending, since the latter has been increasingly financed by borrowing against capital gains on homes. This has occurred in step with rising real estate prices. Empirical experience has shown that this powerful stimulus can be drastically reduced or reversed when house prices stop increasing, even where prices have not fallen, pushing the economy into recession. This has already been experienced in the Netherlands. The rate of Dutch house price inflation slowed from 20% in 2000 to zero by 2003. This appeared to be the perfect soft landing since prices did not fall. Yet consumer spending dropped by 12% in 2003, the biggest fall in any developed country in a decade, pushing the economy into a recession. This type of effect may be observed in the future, as the levels of indebtedness against capital gains of real estate are at record levels in countries like the United States, Britain and Australia.

<sup>28</sup>McKinnon and Pill (1996, 1997) suggest an alternative theory of financial crises. They claim that government guarantees are the fundamental cause in crises; because the government guarantees deposits, banks are not subject to the usual discipline of the market. This allows banks to engage in speculative investment, which bids up asset prices and creates a boom on them, which eventually busts.

<sup>29</sup>For this reason, Dooley stresses the importance of looking at total levels of debt in the economy, versus only public debt.

<sup>30</sup>Garber and Lall (1996) estimate that Mexican banks held \$16 billion worth of Tesobono (Mexican treasury bonds) swaps at the time of the peso devaluation in 1994. Initial devaluation led to a price fall of 15%, and margin calls of \$2.4 billion, almost half of the \$5 billion reserves lost by the Mexican Central Bank one day after the devaluation.

<sup>31</sup>Empirical evidence is consistent with these models. Kaminsky and Reinhart (1999), Reinhart and Tokatlidis (2001) have reported that crises are typically preceded by a multitude of weak and deteriorating economic fundamentals which have caused speculative attacks as market sentiment shifts and, possibly, herd behaviour takes over. This is in contrast to first-generation models that focus on poor fundamentals as the cause of the crises.

competitors, in terms of earnings growth, increases his probability of being sacked. Thus, managers have a strong incentive to behave in the same way as their peers, which at an aggregate level enhances boom-bust cycles in lending (Rajan, 1994).

Pressures to meet short-term earnings targets, for instance, or incentive structures that reward staff at intermediaries according to volume of business rather than risk-adjusted return can lead to underestimation of long term risk and imprudent leveraging. This effect has been reported by Saunders et al (1990) and Gorton and Rosen (1995).

As argued above, macroeconomic cycles appear to be intensified by different incentive structures created under financial liberalization. On the other hand, bank regulation, in the form of capital adequacy requirements, is itself inherently procyclical; it bites in downturns, but fails to provide restraint in booms. The more “risk-sensitive” the regulation, as Basel II is intended to be,<sup>32</sup> the greater the scope for procyclicality to become a problem, particularly in light of the changing nature of macroeconomic cycles. Therefore, an issue that deserves close attention is that the incentive structures created under financial liberalization could interact with the incentive structures embedded in Basel II proposals, thus enhancing even further the intensity of macroeconomic cycles and the procyclicality of the financial system. The simulation exercises performed in Segoviano and Lowe (2002), Goodhart, Hofmann and Segoviano (2004) and Goodhart and Segoviano (2004) suggest that the new Basel II accord, which deliberately aims at significantly increasing the risk sensitiveness of capital requirements, may in fact considerably accentuate the procyclicality of the regulatory system. The authors present evidence that suggests that, in the past, required increases of capital ratios in downturns have been brought about by cutting back lending rather than by raising capital. The new capital accord may therefore lead to an amplification of business cycle fluctuations, especially in downturns.<sup>33</sup>

On the basis of the theoretical arguments and empirical evidence presented in this section, we shall define an initial set of macroeconomic and financial variables to analyze in order to select the set of explanatory variables to include in our model. This is because we believe that key macroeconomic and financial variables exhibit regularities when macroeconomic imbalances are being created and when vulnerabilities in the financial system are increasing. Thus information on systemic vulnerabilities should be attainable from the analysis of these variables.

## 4 Procedure to select the explanatory variables

In this section, we propose a procedure for selecting the set of explanatory variables used for the implementation of CoPoD. This involves two steps. In the first step, we select an initial set of macroeconomic and financial variables that, according to theory and empirical evidence, affect credit risk. In the second step, we explore the variables’ information content by computing their fluctuations both with respect to a long-term trend, and with respect to the previous series observation. We call the first “gaps” and the latter “growth rates”.<sup>34</sup> Having obtained the gaps and growth rates, we run multivariate OLS regressions to identify the specifications that are consistent with economic theory and empirical evidence and that show the best goodness of fit.

This exercise should be seen as a procedure to identify alternative specifications (containing relevant variables) that affect the credit risk of specific types of loans in the countries under analysis. *By no means* do we want our results to be interpreted as an *attempt to define a fixed specification (set of explanatory variables)* that explains credit risk in the financial systems of the countries under analysis.<sup>35</sup> In this section, we also describe the dependent variable under study.

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<sup>32</sup>See Basle Committee on Banking Supervision (2002).

<sup>33</sup>Goodhart, Hofmann and Segoviano (2004) analyze the “credit crunch” experience in the USA recession of 1990/91, when required bank capital adequacy ratios were being raised in the aftermath of the first Basel Accord in 1988.

<sup>34</sup>See section 4.3.1 for definitions of gaps and growth rates.

<sup>35</sup>This distinction is important, since the default frequency of loans classified under different risk rating-categories (ratings) can be affected by different macroeconomic and financial variables, even if the borrowers to whom these loans are granted operate in the same country. Moreover, even if loans classified under different ratings were affected by the same macroeconomic and/or financial variables, the degree of the impact of these variables (coefficients) might be different between loans with different ratings.

## 4.1 The initial set of macroeconomic and financial variables

In order to select the explanatory variables to include in our model, we initially analyzed a set of macroeconomic and financial variables that have been emphasized in the theoretical and empirical literature about crisis periods, and that are available for Norway and Mexico.

Based on the arguments presented in the previous section, variables associated with financial liberalization merit scrutiny. Real aggregate credit in the economy, the ratio of credit to GDP, M2 balances, real interest rates and the ratio of M2 to foreign exchange reserves were considered. Pill and Pradhan (1995), Kaminsky and Reinhart (1999) and Eichengreen and Areta (2000) and Borio and Lowe (2002) have reported that real aggregate credit in the economy and the ratio of credit to GDP are important indicators of banking problems. McKinnon and Pill (1996) have reported rapid increases in monetary aggregates linked to banking crises. Galbis (1993) reports that real interest rates have increased after financial liberalization. The ratio of M2 to foreign exchange reserves captures the extent to which the liabilities of the banking system are backed by international reserves. In the event of a currency crisis, individuals may rush to convert their domestic currency deposits into foreign currency, so this ratio seems to capture the ability of the central bank to meet those demands (Calvo and Mendoza, 1996). Currency crises may take place after a period of large inflows of foreign short-term capital. Such inflows, usually driven by the combined effect of capital account liberalization and high domestic interest rates, result in an expansion of domestic credit (Khamis, 1996). When foreign interest rates rise, domestic ones fall, or when confidence in the economy shifts, foreign investors quickly withdraw their funds and the domestic banking system may become illiquid (Calvo, Leiderman and Reinhart, 1993).

We included the current account balance since this variable indicates the amount of foreign investment needed in the economy and is therefore a variable that could signal the vulnerability of the economy to shifts in investors' confidence. Consumption and investment were also included since these variables can indicate the uses of funding in the economy and therefore can shape the expectations of investors in terms of the capacity of the economy to produce growth opportunities in the future. Foreign Direct Investment was also included as a measure of the vulnerability of the economy to foreign capital. (Sturm, Berger and Haan 2004).<sup>36</sup> To capture adverse macroeconomic shocks that hurt banks by increasing the share of non-performing loans, we considered changes in real GDP. An index of equity prices, an index of residential property prices and an aggregate asset price index<sup>37</sup> were also included and justified by the findings of Borio and Lowe (2002) and Goodhart, Hofmann and Segoviano (2004), as discussed in Section 3.<sup>38</sup>

The realized volatility of short-term interest rates was considered since this variable affects banks' balance sheets adversely if shifts in interest rates force banks to increase the interest rates paid to depositors. If the asset side of the balance sheets of banks consists of long-term loans at fixed interest rates, the rate of return on assets cannot be adjusted quickly enough and banks will suffer reduced profits or bear losses. Volatility in interest rates is likely to hurt bank balance sheets, even if it can be passed on to borrowers, as volatile rates and uncertainty affect cash-flow planning and high lending rates result in a larger fraction of non-performing loans (Mishkin, 1997).<sup>39</sup> The difference between long and short nominal rates was included as a variable that indicates market expectations on growth in the economy.

Another case of rate of return mismatch occurs when banks borrow in a foreign currency and lend in a domestic currency. In this case, an unexpected depreciation of the domestic currency threatens bank profitability and eventually, solvency. Banks that raise funds abroad might choose to issue domestic loans denominated in foreign currency, thus eliminating currency mismatches. In this case, foreign exchange risk is shifted onto borrowers, and an unexpected depreciation would still affect bank profitability negatively through an increase in non-performing loans. We have therefore included the nominal foreign exchange

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<sup>36</sup>Sturm, Berger and Haan (2004) find that the ratio of investment to GDP is robustly related to the probability that a country receives IMF credit. A low ratio of investment to GDP may indicate limited access to international capital markets. Knight and Santaella (1997), Vreeland (1999) also provide support for this view.

<sup>37</sup>The aggregate asset price index combines prices of three asset classes, equity, residential property and commercial property. It weights the components by estimates of the shares of the asset classes in private sector wealth. The methodology is described in detailed in Borio et al (1994). We thank Borio and Lowe for providing us with the aggregate asset price index series.

<sup>38</sup>Note that, as already mentioned in Section 3, an alternative explanation of the causes and effects of increases in asset prices is provided by the literature on the agency problem of excessive risk-taking associated with limited liability. See Allen and Gale (1999).

<sup>39</sup>See also the Global Financial Stability Report, IMF, September 2003 for empirical evidence between Financial Market Volatility and Financial System Instability.

rate. Foreign currency loans were a source of banking problems in the Nordic countries in the early 1990's (Drees and Pazarbasioglu, 1998) and in Mexico (Mishkin, 1997). A real foreign exchange rate index was also included.<sup>40</sup> A summary of the variables that were analyzed, the code that we used to identify them and their source is in Table 1.<sup>41</sup>

Table 1: Initial set of macroeconomic and financial variables

Code	Variable
<b>REPROPRI</b>	Residential Property Prices*
<b>INCOCD</b>	Real Estate Price Index*
<b>EQPRI</b>	Equity Price Index**
<b>SHAPRI</b>	Share Price Index**
<b>AGGPRI</b>	Aggregate Asset Price Index*
<b>NEER</b>	Nominal Fx**
<b>M2</b>	M2 Monetary Aggregate**
<b>REER</b>	Real Fx**
<b>RESER</b>	International Reserves**
<b>REINT</b>	Real Interest Rates**
<b>SHORTINT</b>	Short Interest Rates**
<b>LONGINT</b>	Long Interest Rates**
<b>GDPREAL</b>	Real GDP**
<b>CRED</b>	Real Credit Aggregate**
<b>CONS</b>	Real Consumption Aggregate**
<b>CA</b>	Current Account Balance**
<b>FDI</b>	Foreign Direct Investment**
<b>INV</b>	Real Investment Aggregate**
<b>CREDOVGDP</b>	Ratio of Credit to GDP***
<b>INVOVGDP</b>	Ratio of Investment to GDP***
<b>CONOVGDP</b>	Ratio of Consumption to GDP***
<b>CUACCOVINV</b>	Ratio of Current Account to Investment***
<b>M2OVRES</b>	Ratio of M2 to International Reserves***
<b>LOMISH</b>	Long minus Short Interest Rates***
<b>INREVO1</b>	Realized Volatility of Interest Rates***

\*National Sources as per detailed documentation and BIS calculations based on national data.

\*\*IMF international financial statistics.

\*\*\*Author's calculations based on national data.

## 4.2 The dependent variables

The empirical frequencies of loan default (PoDs) by SMEs and unlisted firms are the dependent variables under study. The motivation for developing CoPoD is to improve the measurement of the impact of macroeconomic developments on loans' credit risk, and, as a result, to improve the measurement of loans' credit risk through time. This is in contrast to the attempt to assess the probability of episodes of financial instability in the financial system as a whole, as the early warning systems do. Note that since the likelihood of default of specific types of loans is affected by the state of the business cycle, we select a set of explanatory

<sup>40</sup>An increase in the real exchange rate index implies depreciation.

<sup>41</sup>Of course, this is not an exhaustive list of potential variables. In particular political variables can also be linked to the timing of the crises. Variables capturing the effectiveness of the legal system have also been found to be significant in explaining banking sector problems. Variables reflecting exogenous events can also explain specific crises. None of these are considered. For the effect of political variables see Mishra (1997). For the effect of legal structures see Arkelof and Romer (1993).

variables that affect the credit risk of these types of loans, on the basis of theoretical arguments behind crisis models.

In our dataset, loans given to SMEs and unlisted firms are classified and aggregated according to their risk-rating categories. For a given risk-rating category, the empirical frequencies of loan defaults that are recorded during each period of time are accommodated in a vector. Therefore, each element of this vector reflects the average default behavior of borrowers classified under a given risk-rating category at different points in time.

These databases were provided by Norges Bank and by Comision Nacional Bancaria y de Valores (CNBV).<sup>42</sup> In Norway, risk-rating classification is done using Norges Bank’s risk rating model.<sup>43</sup> Banks operating in Mexico, based on a rating system set out by the regulatory authority, determine ratings internally and then report them to CNBV.<sup>44</sup> For Norway we have yearly observations from 1988 to 2001. For Mexico, we have quarterly observations from the second quarter of 1995 to the last quarter of 2000.

Classification and aggregation of loans can be done according to other loan characteristics, such as sectoral activity of borrowers, geographical location of borrowers, type of collateral backing up the loan etc. Unfortunately, we do not have such information. This restriction in aggregation involves a trade off. On the down side, under the risk-rating category aggregation, the number of observations in the time series of PoDs is usually very small. This type of aggregation does not allow us to explore the behavior of credit risk from different perspectives, e.g. the default behavior of borrowers according to their sectoral activity or type of collateral. Yet, despite these shortcomings, the dataset is one of few that, by aggregating via risk-rating category, allows us to study the impact of macroeconomic cycles on banks’ capital requirements. Moreover, the data cover an entire business cycle in both of the analyzed countries and focuses on SMEs and unlisted borrowers. These characteristics allow us to explore a number of important issues.

### 4.3 Guideline for refinement in the selection of explanatory variables

#### 4.3.1 Statistical treatment of variables

Our aim here is to analyze combinations of fluctuations in different macroeconomic and financial variables as possible causes of changes in credit risk. When we compute fluctuations in these variables, they are calculated using only information that would have been available to the analyst up to the time when the analysis was done.<sup>45</sup>

These fluctuations are computed with respect to two types of “reference values”: a long-term trend and the previous observation. When computing movements with respect to a long-term trend, we are interested in capturing the explanatory power of cumulative processes, rather than growth rates over just one period. The reasoning behind this approach is that vulnerabilities may build up over an extended period, rather than in a single period. We refer to these movements with respect to long-term trends as “gaps”. In order to estimate the long-term trend, we use a “Dynamic” Hodrick-Prescott (HP) filter using information from 1970. For a detailed description of this procedure, please refer to Appendix A.5.

Some criticism might be made of the way that any filter weighs the data. Taking this into consideration, we also computed and tested the explanatory power of fluctuations with respect to the previous observation. We refer to these as “growth rates”.

Both types of reference values represent different ways of using the information available to the model builder. We do not consider such values as “fundamental values”. This distinction highlights a key issue, especially in the case of price variables, since we do not try to identify asset price bubbles. An asset price bubble can be characterized by a significant over-pricing of an asset from its “fundamental value”. There is no attempt in this paper to assess “fundamental values” and measure price deviations from them. For the purposes of this exercise, the more relevant issue is to assess the combination of events that has the potential to increase banks’ credit risk. Consequently, we would like to steer the discussion away from the market efficiency debate.

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<sup>42</sup>The Central Bank of Norway and the Mexican Financial Regulatory Agency, respectively.

<sup>43</sup>For details of this model, refer to Bernhardsen (2001) and Eklund et al (2001).

<sup>44</sup>The Mexican rating methodology is described in: <http://www.cnbv.gob.mx>

<sup>45</sup>We refer to this set of information as “ex-ante” information.

### 4.3.2 Multivariate OLS regressions for selection of variables

Using equation (2), we run multivariate OLS regressions, exploring different combinations of variable “gaps” and variable “growth rates” with different lags.<sup>46</sup> For Norway, we used up to 8 lags, since the frequency of the data was annual, whereas for Mexico we used up to 30 lags, since the frequency of the data was quarterly.<sup>47</sup> Each lag was treated as a different explanatory variable.

Since the time series of the dependent variables contain very few observations, we tried to be as parsimonious as possible. As a result, we started specifying regression systems with the fewest possible variables and explored how far these could take us. We continued increasing the set of explanatory variables used in the specifications, keeping in mind the trade off with degrees of freedom when increasing regressors. Therefore, we restricted specifications to contain 2 and 4 explanatory variables for Norway and 3 and 6 explanatory variables for Mexico.<sup>48</sup>

Once we defined the number of explanatory variables to be included in each specification, we computed OLS multivariate regressions for all the possible combinations with the defined number of explanatory variables for each specification.

### 4.4 Selected explanatory variables

Model specifications were selected based on the consistency of the explanatory variables with theoretical arguments and empirical evidence and on the specifications’ goodness of fit, indicated by the Adjusted R-squared and Akaike criteria. Specifications that were inconsistent with theoretical arguments were ruled out. Under these criteria, Table 2 and Table 3 show the selected specifications for Norway and Mexico respectively, using gaps and under three different specifications.

#### Results for Norway:

Table 2: OLS results for Norway: gaps

OLS results for Norway: gaps						
Variables	Specification 1		Specification 2		Specification 3	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
Constant	3.664095	0.0000	3.649223	0.0000	3.637719	0.0000
GapCREDOVGDP(-4)	0.554376	0.0059	0.547557	0.0012	0.287520	0.0050
GapAGGPRINDX(-3)	0.208837	0.0225	0.264793	0.0014		
INREVO1(-1)			812.556400	0.0195	923.145100	0.0000
LOMISH(-2)			0.009309	0.1000		
GapAGGPRINDX(-1)					-0.584903	0.0000
GapM2OVRES					0.292388	0.0000
R-squared	0.6683		0.7874		0.9465	
Adjusted R-squared	0.6080		0.6929		0.9227	
Akaike criterion	-3.7125		-3.8718		-5.2510	
Schwarz criterion	-3.5756		-3.6436		-5.0228	
F-statistic	11.0798		8.3345		39.7886	
Prob (F-statistic)	0.0023		0.0043		0.0000	

Dependent Variable: aNor. Method: OLS. Sample: 1988-2001. Included observations: 14.

Newey-West HAC Standard Errors & Covariance (lag truncation=2).

**Specification 1**, contains the results for Norway, with two explanatory variables. These are the gap in the ratio of credit to GDP (GapCREDOVGDP) and the gap in the aggregate asset price index

<sup>46</sup> See Section 7 for further explanation of lags.

<sup>47</sup> We assumed that longer lags were not consistent with economic theory.

<sup>48</sup> The reason was that, for Norway, the number of observations (14) is smaller than for Mexico (23).

(GapAGGPRINDX). These variables are lagged four and three periods respectively. As we can see, the parameter estimates of these variables are statistically highly significant, with p-values of just over .59% and 2.2%. Given that we are only using two explanatory variables, the goodness of fit (as indicated by the adjusted R-squared) is quite high. As we will argue in Section 7, the sign of the coefficients and the timing of the lags are consistent with theoretical arguments and empirical evidence.

In **Specification 2**, we present results augmenting the number of explanatory variables to include the interest rate volatility (INREVO1) and the difference between long-term minus short-term interest rates (LOMISH). These variables are lagged one year and two years, respectively, and their parameters are significant at the 1.9% and 10% significance level. In this specification, the goodness of fit improves, as indicated by the lower Akaike criterion and higher adjusted R-squared. Parameter estimates of the gap in credit to GDP and the gap in the aggregate asset price index remain statistically significant at the same lags as in the previous specification, and are consistent with empirical evidence.

In **Specification 3**, the results for the parameters of GapCREDOVGDP and INREVO1 are similar to the results reported in Specification 2. However, when we explore the explanatory power of the gap in the aggregate asset price index (GapAGGPRINDX) lagged only one year, this variable had a negative and highly significant parameter (with an extremely small p-value). As we will argue in Section 7, this result is consistent with our hypothesis. We also explore the explanatory power of the gap in the ratio of M2 over international reserves. This variable has a highly significant parameter. In this specification, the goodness of fit improves even further, as indicated by an even lower Akaike criterion and higher adjusted R-squared.

### Results for Mexico:

Table 3: OLS results for Mexico: gaps

OLS results for Mexico: gaps						
Variables	Specification 1		Specification 2		Specification 3	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
Constant	2.840080	0.0000	2.641826	0.0000	2.510054	0.0000
GapCREDOVGDP(-24)	3.632902	0.0014	4.879514	0.0000	3.066573	0.0117
GapINCOCDRE(-20)	14.548380	0.0000	6.154473	0.0070		
GapSHAPRIRE(-15)	1.901396	0.0006				
GapSHAPRIRE(-16)			1.053357	0.0001		
INREVO1(-10)			648.5374	0.0444	1204.7040	0.0194
GapM2OVRES(-6)			0.521614	0.0024	0.656527	0.0045
GapINVOVGDP(-8)			-6.206859	0.0000	-6.355086	0.0001
GapINCOCDRE(-11)					-7.124320	0.0119
R-squared	0.6311		0.9281		0.8751	
Adjusted R-squared	0.5728		0.9012		0.8383	
Akaike criterion	1.2061		-0.1687		0.2974	
Schwarz criterion	1.4036		0.1769		0.5936	
F-statistic	10.8345		34.4375		23.8110	
Prob (F-statistic)	0.0002		0.0000		0.0000	

Dependent Variable: aMex, Method: OLS. Sample: 1995:2 2000:4. Included observations: 23.

Newey-West HAC Standard Errors & Covariance (lag truncation=2).

We performed a similar exercise with the Mexican dataset. In this case, there was no information of an aggregate asset price index, as in the case of Norway. Therefore, instead, we used a series of real estate prices (INCOCDRE) and a series of stock prices (SHAPRIRE). On the other hand, the time series for

Mexico contained more observations since its frequency was quarterly. Results for Mexico for the three selected specifications using gaps are presented in Table 4.3. These are as follows:

**Specification 1**, contains the results for Mexico with three explanatory variables. These are the gap in the ratio of credit to GDP (GapCREDOVGDP), the gap in the real estate price index (GapINCOCDRE) and the gap of the real share price (GapSHAPRIRE). These variables are lagged twenty-four, twenty and fifteen periods, respectively. As can be seen, the parameters of these variables are statistically highly significant with p-values of less than 1%. Given that we are only using three explanatory variables, the goodness of fit is quite high. As we will argue in Section 7, the sign of the coefficients and the timing of the lags are consistent with theoretical arguments and empirical evidence.

In **Specification 2**, we increase the number of explanatory variables and include the interest rate volatility (INREVO1), the gap of the ratio of M2 over reserves (GapHCDM2OVRES) and the gap of the ratio of investment over GDP (GapINVOVGDP). These variables are lagged ten, six and eight periods (in this case, quarters), respectively. Under this specification, the goodness of fit improves, as is indicated by the lower Akaike criterion and higher adjusted R-squared. The coefficients of the gaps in the credit to GDP, the real estate price index and real share price remain highly statistically significant with p-values of less than 1% and at similar lags as in the previous specification. The coefficients of the interest rate volatility, the gap of the ratio of M2 over reserves and the gap of investment over GDP are all consistent with theoretical arguments and empirical evidence, as we will argue in Section 7, and are statistically highly significant.

In **Specification 3**, results for the parameters of GapCREDOVGDP, INREVO1, GapM2OVRES and GapINVOVGDP are similar to the results reported in Specification 2. However, when we explore the explanatory power of the gap of the aggregate real estate price index (GapINCOCDRE) lagged eleven periods, this variable had a negative and highly significant coefficient. This result is consistent with our hypothesis, as we will argue in Section 7.

### Results with growth rates

We performed similar exercises with growth rates. These results are presented in Appendix A.6, where Table 9 and Table 10 show the results for Norway and Mexico respectively. The explanatory variables that we present in these specifications were very similar to those in the specifications with gaps and, in general, although not in all cases, they were statistically significant. The goodness of fit for each regression equation was adequate (although lower than in the exercise with gaps).

Nonetheless, we propose the use of gaps rather than growth rates because, as already mentioned, when using gaps we are interested in capturing the explanatory power of cumulative processes, rather than growth rates over just one period. In addition, after obtaining the results with growth rates, two issues made them less useful. First, lags in the explanatory variables were generally longer (therefore, for some variables, the timing of the signals is no longer consistent with empirical evidence). Second, when we tested different lags in the explanatory variables, their coefficients became highly unstable.

### Final remarks

Up to now, our attention has focused on selecting the set of explanatory variables to include in the model. For this purpose, we have used multivariate OLS regressions. However, as can be observed, the time series of PoDs for both Norway and Mexico are very small in statistical terms. This is a problem that is commonly faced by credit risk modelers, since usually data series are very restricted. Under these circumstances, it is well known that, if OLS estimation procedures are used, one is likely to encounter the following consequences:<sup>49</sup>

1. Although (Best Linear Unbiased Estimators) BLUE, the OLS estimators have large variances, making precise estimation difficult.

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<sup>49</sup>See Gujarati (1995).



2. Because of the large variances mentioned above, the confidence intervals tend to be much wider, leading to the acceptance of the “zero null hypothesis” (e.g., the true population coefficient is zero) more readily. Equivalently, the t-ratio of one or more coefficients tends to be statistically insignificant.

3. Although the t-ratio of one or more coefficients is statistically insignificant, the overall measure of goodness of fit can be very high.

4. The OLS estimators and their standard errors can be sensitive to small changes in the data.

The second point implies that the sample data may be compatible with a diverse set of hypotheses. Hence, the probability of accepting a false hypothesis (type II error) increases. However, the coefficients corresponding to the set of explanatory variables that were selected in each specification, despite the possible increases in type II errors, were highly significant. Moreover, these coefficients had signs consistent with theoretical considerations. These facts give us some *assurance with respect to the selected variables* in each specification. However, large variances and high sensitivity of OLS estimators represent important problems for risk managers who try to evaluate the impact of specific macroeconomic and financial events on the credit risk of their portfolios. In order to diminish the negative effects of these problems, we propose to recover the parameters of the selected explanatory variables with CoPoD. This is the objective of the following section.

## 5 CoPoD: empirical implementation

In this section, we describe the procedure to implement CoPoD and present the results that were obtained.

### 5.1 Implementation procedure

Once we selected the set of explanatory variables to be used for Norway and Mexico, which are indicated in Tables 2 and 3 respectively, we were in a position to define the  $X$  ( $T \times K$ ) matrices of explanatory variables to be used in each specification.

Recall that in Section 2, when we specified equation (3), each  $\beta_k$  was treated as a discrete random variable with  $2 \leq M < \infty$  possible outcomes,  $z_{k1}, \dots, z_{kM}$ . Also, we expressed  $\beta_k$  as a convex combination of points  $z_k$  with weights  $p_k$ , (we restricted the weights to be strictly positive and to sum to 1 for each  $k$ ).

The restrictions imposed on the parameter space through  $\mathbf{Z}$  reflect prior knowledge about the unknown parameters. However, such knowledge is not always available, and a variety of possible bounds on  $\beta$  may be explored.

Golan Judge and Miller (1997) propose the use of wide bounds, if knowledge is minimal and one wants to ensure that  $\mathbf{Z}$  contains  $\beta$ . Lastly, those authors recommend to use  $M = 5$  and  $J = 5$ . Based on different sampling experiments, they report that it appears that the greatest improvement in precision comes from using  $M = 5$  and  $J = 5$  for each  $\beta_k$  and  $e_t$  respectively.<sup>50</sup>

Taking in consideration Golan Judge and Miller (1997) findings, in order to obtain the CoPoD estimators, we undertook the following steps:

1. For each specification, with the sample at hand, we estimated the vector of coefficients  $\beta_{OLS}$ .
2. We assumed the values of  $\beta_{OLS}$  to be the “true parameter values” and performed a bootstrap (Horowitz, 2001) with 10,000 trials to simulate the distributions of  $\beta_{OLS}$ .
3. Once that we obtained these distributions, we calculated the standard errors,  $\sigma$ , for each coefficient, and used these standard errors to define the bounds of  $\mathbf{Z}$ , using a three-sigma rule.<sup>51</sup>

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<sup>50</sup>The authors measure the precision of each estimator under the squared error loss,  $SEL = \|\beta - \hat{\beta}\|^2$ . They take the average  $SEL$  ( $MSEL$ ) as an estimate of the empirical precision risk for each competing method of information recovery. Accordingly, they compute the precision risk as the average sum of squared errors,  $SSE = \|y - XB\|^2$ .

<sup>51</sup>Chebyshev’s inequality may be used as a conservative means of specifying bounds. For any random variable,  $\mathbf{x}$ , such that  $E(\mathbf{x}) = 0$  and  $Var(\mathbf{x}) = \sigma^2$ , the inequality provides  $\Pr[|x| < v\sigma] \geq v^{-2}$  for arbitrary  $v > 0$ . An example is the familiar  $3\sigma$  rule that excludes at most one-ninth of the mass for  $v = 3$ . For a recent discussion of probability bounds and the  $3\sigma$  rule, refer to Pukelsheim (1994).

4. Then, each  $\beta_k$  was expressed as the convex combination:

$$\beta_k = z_{k1}p_{k1} + z_{k2}p_{k2} + z_{k3}p_{k3} + z_{k4}p_{k4} + z_{k5}p_{k5}. \quad (23)$$

5. Since the bounds were defined with respect to the three-sigma rule we set  $z_{k1} = -3\sigma$  and  $z_{k5} = 3\sigma$ . Equivalently, the point  $z_{k3}$  was set equal to the mean  $\beta_{OLS}$  and the points  $z_{k2}$  and  $z_{k4}$  were set equidistant between the mean and the bounds, e.g.  $z_{k2} = \frac{z_{k1} + z_{k3}}{2}$  and  $z_{k4} = \frac{z_{k3} + z_{k5}}{2}$ .

Accordingly, in the same manner, we treated each error term  $e_t$  as a finite and discrete random variable with  $2 \leq J < \infty$ , possible outcomes,  $v_{t1}$  and  $v_{tJ}$ . We also expressed each  $e_t$  as a convex combination of points  $v_t$  with weights  $w_t$ . As before, we restricted the weights to be strictly positive and to sum to 1 for each  $t$ . Given our ignorance regarding the error distribution, in order to determine the error bounds of  $\mathbf{V}$ , we followed Golan Judge and Miller (1997) who propose calculation of the sample scale parameter and use this with the three-sigma rule.

## 5.2 Recovered coefficients

With the elements presented in the previous section, we were in a position to reformulate equation (2) as indicated in equation (3). Once we reformulated the problem, we proceeded to recover the probability vectors  $\mathbf{p}$  and  $\mathbf{w}$  using the Lagrangian specified in equation (7). With the recovered probability vectors, we formed point estimates of the unknown parameter vector  $\hat{\beta}$  as indicated in equation (10). The recovered coefficients for the selected explanatory variables for Norway and Mexico, using gaps under the different specifications, are presented in Tables 4 and 5.

Table 4: CoPoD (GME) results for Norway: gaps

CoPoD (GME) results for Norway: gaps			
Variables	Specification 1	Specification 2	Specification 3
	Coefficient	Coefficient	Coefficient
Constant	3.664919	3.648975	3.637959
GapCREDOVGDP(-4)	0.558448	0.546334	0.290302
GapAGGPRINDX(-3)	0.204428	0.263379	
INREVO1(-1)		826.461660	916.893640
LOMISH(-2)		0.009365	
GapAGGPRINDX(-1)			-0.579636
GapM2OVRES			0.290901

Dependent Variable: aNor. Method: CoPoD. Sample: 1988-2001. Included observations: 14.

Note that when we used CoPoD to recover the coefficients of the chosen variables for each specification, the signs of these coefficients remained consistent with the results reported in Section 4.4. These results are consistent with theoretical arguments and empirical evidence, as will be discussed in Section 7.

In a similar exercise, we recovered the coefficients of the selected explanatory variables, using growth rates. The results are provided in Appendix A.6, where Tables 11 and 12 show the results for Norway and Mexico, respectively.

The finite sample properties presented in Section 2.5 indicate that the coefficients recovered with CoPoD should have smaller variances than the OLS coefficients. In the following section, we quantify the efficiency gain of CoPoD estimators.

## 6 Monte Carlo experiment

Based on the theoretical results presented in Section 2.5, Golan, Judge and Miller (1997) claim that in finite sample settings, the GME (i.e. CoPoD estimators) exhibit reduced mean squared error (MSE) due

Table 5: CoPoD (GME) results for Mexico: gaps

CoPoD (GME) results for Mexico: gaps			
Variables	Specification 1	Specification 2	Specification 3
	Coefficient	Coefficient	Coefficient
Constant	2.858823	2.639446	2.508866
GapCREDOVGDP(-24)	3.952335	4.883196	2.944946
GapINCOCDRE(-20)	15.849527	6.122219	
GapSHAPRIRE(-15)	1.970538		
GapSHAPRIRE(-16)		1.047109	
INREVO1(-10)		642.310330	1228.236600
GapM2OVRES(-6)		0.525521	0.636409
GapINVOVGDP(-8)		-6.247341	-6.255630
GapINCOCDRE(-11)			-7.422670

Dependent Variable: aMex, Method: CoPoD. Sample: 1995:2 2000:4. Included observations: 23.

to their properties of limited bias and minimum variance. Although the exact finite-sample properties of the CoPoD and OLS estimators are unknown, the simulated distribution of the recovered parameters may provide some useful information. For this reason, in order to quantify the relative performance CoPoD estimators, we performed a Monte Carlo exercise by which we simulated the distributions of the OLS and CoPoD estimators. Then, we compared these distributions under the Mean Squared Error (MSE) criterion.

In order to perform the Monte Carlo exercise, we chose **specification 1 for Norway**. When we carried out this exercise, we went through the following steps:

1. From the chosen specification, we used the matrix of explanatory variables,  $X$  ( $T \times K$ ), and the vector of observations  $\mathbf{a}$  ( $T \times 1$ ) to compute the vector of coefficients  $\beta_{OLS}$ .
2. With the matrix  $X$  ( $T \times K$ ), the vector of observations  $\mathbf{a}$  ( $T \times 1$ ) and assuming the values of  $\beta_{OLS}$  to be the "true parameter values", we obtained the vector of residuals as:  $\mathbf{R} = \mathbf{a} - X\beta_{OLS}$ .
3. With these elements we performed a Bootstrap (Horowitz, 2001), drawing 10,000 random trials. Each trial consisted on drawing random realizations from the matrix  $X$  and the vector  $\mathbf{R}$ . With these elements and the  $\beta_{OLS}$  that were considered to be the "true parameter values", we computed simulated values of  $\mathbf{a}$ . With the simulated values of  $\mathbf{a}$  and the random realizations of  $X$ , we recovered the OLS estimators,  $\hat{\beta}_{OLS}$  and CoPoD estimators,  $\hat{\beta}_{CoPoD}$ . We repeated this process 10,000 times to build the distributions of  $\hat{\beta}_{OLS}$  and  $\hat{\beta}_{CoPoD}$ .
4. With these distributions, we proceeded to compute the MSE for each parameter  $\beta_{OLS}$  and  $\beta_{CoPoD}$  as

$$MSE [\hat{\beta}] = E \left[ \left( \hat{\beta} - \beta \right)^2 \right].$$

### Results of the Monte Carlo experiment

In Table 6, we present in the first line, the vector  $\beta$ , containing the assumed "true parameter values"<sup>52</sup>. For each of the distributions of the recovered CoPoD and OLS parameters, we present from the second to the sixth lines, their Mean, their Bias, their Bias-Squared, their Variance and MSE respectively.

In order to simplify the analysis, we present in Table 7 a summary of the results.

<sup>52</sup>Note that the values in this vector are equal to the values of  $\beta_{OLS}$  reported in Table 2, Specification 1.

Table 6: MSE components for OLS and CoPoD estimators

Component	$\beta_{1OLS}$	$\beta_{2OLS}$	$\beta_{3OLS}$	$\beta_{1CoPoD}$	$\beta_{2CoPoD}$	$\beta_{3CoPoD}$
Assumed $\beta$	3.664095	0.554376	0.208837	3.664095	0.554376	0.208837
Mean Estimated	3.665974	0.568877	0.200651	3.666028	0.567699	0.194892
Bias	0.001879	0.014501	-0.008186	0.001933	0.013323	-0.013945
Bias-Squared	0.000004	0.000210	0.000067	0.000004	0.000177	0.000194
Variance	0.000086	0.036383	0.009252	0.000032	0.024066	0.004830
MSE	0.000089	0.036593	0.009319	0.000035	0.024243	0.005024

The first line shows the difference between the Bias-Squared of the distributions of  $\hat{\beta}_{OLS}$  and the distributions of  $\hat{\beta}_{CoPoD}$ .

The second line presents the difference between the Variance of the distributions of  $\hat{\beta}_{OLS}$  and the distributions of  $\hat{\beta}_{CoPoD}$ .

Equally, the third line shows the difference between MSE of the distributions of  $\hat{\beta}_{OLS}$  and the distributions of  $\hat{\beta}_{CoPoD}$ .

A positive number indicates that the MSE component corresponding to the distribution of  $\hat{\beta}_{OLS}$  is greater than the MSE component corresponding to the distribution  $\hat{\beta}_{CoPoD}$ .

Equivalently, the fourth, fifth and sixth lines illustrate the differences shown in lines one to three as a percentage of the MSE of the respective CoPoD parameter distribution, i.e.

$$\begin{aligned} \text{Bias-Squared Difference (Percentage)} &= \frac{\text{Bias-Squared } \hat{\beta}_{OLS} - \text{Bias-Squared } \hat{\beta}_{CoPoD}}{\text{MSE } \hat{\beta}_{CoPoD}} \\ \text{Variance Difference (Percentage)} &= \frac{\text{Variance } \hat{\beta}_{OLS} - \text{Variance } \hat{\beta}_{CoPoD}}{\text{MSE } \hat{\beta}_{CoPoD}} \\ \text{and MSE Difference (Percentage)} &= \frac{\text{MSE } \hat{\beta}_{OLS} - \text{MSE } \hat{\beta}_{CoPoD}}{\text{MSE } \hat{\beta}_{CoPoD}}. \end{aligned}$$

Table 7: Summary statistics: MSE results

Summary Statistics	$\beta_1$	$\beta_2$	$\beta_3$
Bias-Squared Difference	0.000000	0.000033	-0.000127
Variance Difference	0.000054	0.012317	0.004423
MSE Difference	0.000054	0.012350	0.004295
Bias-Squared Difference (Percentage)	-0.583168%	0.135217%	-2.536907%
Variance Difference (Percentage)	152.220232%	50.806308%	88.029769%
MSE Difference (Percentage)	151.637064%	50.941525%	85.492862%

From the results, presented in Tables 6 and 7, it is possible to observe the following:

1. The Bias Squared for distributions of  $\hat{\beta}_{1CoPoD}$  and  $\hat{\beta}_{2CoPoD}$  are marginally bigger than for the distributions of  $\hat{\beta}_{1OLS}$  and  $\hat{\beta}_{2OLS}$  respectively.
2. In Table 6, it can be observed that *the distributions of  $\hat{\beta}_{CoPoD}$  always show smaller variances than the distributions of  $\hat{\beta}_{OLS}$* . This can also be seen in the positive difference and the percentage difference shown in Table 7.

3. The reduction in variance of the distributions of  $\hat{\beta}_{CoPoD}$  is relatively large in comparison to the increase in bias of these distributions. *Therefore, overall, the MSE for the distributions of  $\hat{\beta}_{CoPoD}$  is smaller than for the distributions of  $\hat{\beta}_{OLS}$  in all the cases.*

The results of this Monte Carlo exercise are consistent with the finite sample properties derived by Golan, Judge and Miller (1997) presented in Section 3.5, which indicate: First, although the GME solution is almost certainly biased, the consistency constraints must be satisfied and the bias cannot become very large. Second, given the properties of limited bias and reduced variance, GME solutions exhibit reduced mean squared error (MSE) relative to OLS estimators in finite sample settings.

## 7 Analysis of the results

Before we analyze our results, it may be useful to define the concept of “crisis inside lag” and summarize the empirical evidence provided by previous studies.

### 7.1 Crisis inside lag

Consider the behavior of the “default cycle” with respect to the outbreak of the crises.

Vale (2004) reports that the first Norwegian failure after the 1930s occurred in the autumn of 1988. In the years 1988 to 1990, 13 banks failed. However, on account of the small size of these banks, the situation facing the banking sector did not yet constitute a systemic crisis. With two exceptions, this first cycle of bank problems was solved by merging the failed bank with a larger, solvent bank. Nonetheless, the situation took on systemic dimensions by 1990, when the largest banks’ portfolios deteriorated. The scale of defaults reached a peak in 1992. Loan losses started to decrease in 1993. This effect is consistent with the time series of the empirical default frequencies (PoDs) that we analyzed for Norway.

In Mexico, a currency crisis occurred in December 1994. However, the time series of PoDs reveal that Mexico experienced the highest number of empirical defaults in the quarter ending September 1996.

From this information, we can define the crisis “inside lag” for Norway as the time that elapsed from the outbreak of the “systemic crisis” to the time that defaults reached their highest frequency in 1992 (two years). Equally, for Mexico, we can define the crisis “inside lag” as the time from the outbreak of the “currency crisis” to the time that defaults reached their highest frequency in September 1996 (one year and nine months).

These findings are in line with Kaminsky and Reinhart (1999)<sup>53</sup> who find that the peak of the banking crisis most often comes after the currency crash,<sup>54</sup> suggesting that existing problems in the financial sector were aggravated or new ones created by the high interest rates that were required to defend the exchange rate peg or by the foreign exchange exposure of banks after a currency collapse. These adverse feedback mechanisms are in line with those suggested by Mishkin (1997) and can be amplified by banks’ inadequate hedging of foreign exchange risk, as the Asian crisis indicated, or via the financial accelerator mechanism in the presence of the collapsing value of collateral, as discussed in Goodhart, Hofmann and Segoviano (2004).

It takes time for these adverse feedback mechanisms to feed through the economy and to be reflected in non-performing loans, i.e. a company is not likely to default at the onset of a crisis. So, once financial problems begin, it takes time before a company finally defaults. More recently, independent research results by Jimenez and Saurina (2004a) report similar “inside lags” for the “default cycle” in Spain.

PoDs might also include informational “noise” components. For example, banks’ desire to hide their problems due to their reputation or to regulatory constraints (capitalization requirements) might delay the registry of defaults. Governments might also have political or economic incentives to delay the registration of defaults. In fact, this might have been a factor that could have influenced the “inside lag” in Mexico.<sup>55</sup> Additionally, the information used in this study is information reported to the central bank and the financial

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<sup>53</sup>Kaminsky and Reinhart (1999) analyse 76 currency crises and 26 banking crises, in 20 countries (developed and developing) for the period from 1970 to the mid 1990’s.

<sup>54</sup>Kaminsky and Reinhart (1999) use as the measure of severity of the crisis, the bailout cost as a percentage of GDP.

<sup>55</sup>See: “Programa de Capitalizacion y Compra de Cartera (PCCC)” at: [www.ipab.org.mx](http://www.ipab.org.mx)

regulator at the “loan level”, aggregated by “risk rating class”. This degree of aggregation does not allow us to identify specific patterns that might be contained in “inside lags”. For example, the economic activity of borrowers that defaulted earlier/later, after the crisis, the type of loans and collateral that they had, or even, as a result of political/economic reasons, whether there was any sector(s) that received benefits from the authorities or was allowed to delay default. Despite the limitation of the data, and the differences between the economies of both countries, it is interesting that the two inside lags were quite similar in both countries.

## 7.2 Empirical evidence for the timing of the explanatory variables

With respect to the empirical evidence for the timing of the signals, Demirgüç-Kunt and Detragiache (1998) find that variables that contribute to systemic banking sector fragility may be in place (signalling) for two years (on average), before problems become manifest.<sup>56</sup> Kaminsky and Reinhart (1999), Goldstein, Kaminsky and Reinhart (2000) show that, on average, the indicators they selected send the first signal of a crisis anywhere between a year and 18 months before the outbreak of a crisis.<sup>57</sup> Borio and Lowe (2002) report that the performance of their indicators improves considerably when the lead-time of the indicators is lengthened to three years.<sup>58</sup>

## 7.3 Econometrically: what have we achieved?

When implementing CoPoD, the procedure used to select the set of explanatory variables involves the use of multivariate OLS regressions with the specific objective of making a further refinement of the initial set of macroeconomic and financial variables that was considered. This is shown in the results reported in Section 4.4. However, since we are dealing with very small samples, OLS estimation procedures produce coefficients with large variances and high sensitivity to changes in the data. Therefore, once the set of explanatory variables for each specification is chosen, in Section 5.2, we recover their coefficients with CoPoD, with the objective of improving their efficiency. The Monte Carlo experiment results presented in Section 6 endorse the theoretical results presented in Section 2.5, which claim that in finite sample settings, CoPoD estimators exhibit reduced variances. Thus, from the econometric point of view, we have some assurance with respect to the selected variables for each specification (results reported in Section 4.4), and with respect to the statistical properties of the coefficients of such variables in finite sample settings (results reported in Sections 5.2 and 6).

## 7.4 Economically: how consistent are our results with theoretical arguments and empirical evidence?

When selecting the explanatory variables, we gave great weight to the consistency of their coefficients with theoretical arguments and empirical evidence. In this section, we focus on analyzing our results with respect to those arguments.

The results obtained and discussed in this section are consistent with our earlier hypothesis. As already mentioned in Section 3, this hypothesis implies that during the upturn of the cycle, banks sharply increase lending as the prices of the assets that are held as collateral increase, and the state of confidence in the system is positive. It is also during this stage that the seeds for financial imbalances are sown and the financial vulnerability of the economy increases, as the levels of leverage of the banking system rise. When financial imbalances unwind and market sentiment shifts, (speculative attacks may occur and, possibly, herding behavior takes over), the prices of assets provided as collateral weaken, perhaps sharply. Sudden changes in the system’s state of confidence have the potential to interact and become endogenous, self-fulfilling fluctuations that possibly exacerbate the downturn of the cycle. During this stage, stress in the financial system builds up, as evidenced by the sharp increase in the empirical frequency of loan defaults

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<sup>56</sup>They analyse, among other indicators, the credit to GDP ratio, M2 over reserves ratio, credit growth and a law and order variable.

<sup>57</sup>These papers show that credit to GDP signals on average twelve months before the outbreak of a crisis and stock prices signal on average fourteen months before the outbreak of a crisis.

<sup>58</sup>Borio and Lowe (2002) examine combinations of indicators including credit and asset prices. They analyze cumulative processes and use only ex-ante information.

(PoDs) that is observed during the crisis “inside lag”. Thus, during the downturn of the cycle, a negative relationship between PoDs (which are increasing) and asset prices (which are falling) is observed.

### Results for Norway:

**Table 4** shows the recovered coefficients with CoPoD for Norway:

**Specification 1** shows the coefficients for the gap in the ratio of credit to GDP (GapCREDOVGDP) lagged four periods, and the gap in the aggregate asset price index (GapAGGPRINDX) lagged three periods. If we take off the Norwegian crisis “inside lag” (two years), gaps in the ratio of credit to GDP and in the aggregate asset price index would have been signalling an increase in PoDs for two years and one year, respectively before the outbreak of the crisis. These results are consistent with the empirical evidence for the timing of such signals, as we pointed out above. These results are also consistent with theoretical credit cycles à la Kiyotaki and Moore, as well as the results reported by Kaminsky and Reinhart (1999), Eichengreen and Areta (2000), Borio and Lowe (2002) and Goodhart, Hofmann and Segoviano (2004).

**Specification 2**, was augmented to include the coefficients for the interest rate volatility variable (INREVO1) lagged one year, and the difference between long-term minus short-term interest rates (LOMISH) lagged two years. Parameter estimates of the Gap in Credit to GDP and the Gap in the aggregate asset price index keep their signs at the same lags as in the previous specification, and hence are consistent with the empirical evidence. These results indicate that increases in interest rate volatility appear to explain increases in PoDs within the “inside lag” window; confirming that interest rate volatility does affect adversely banks’ balance sheets as we discussed in Section 4.1. Furthermore, since increases in interest rate volatility have been reported at the outbreak of crises, these results appear to be consistent with the empirical evidence for the timing of the signals.<sup>59</sup> LOMISH is a proxy for the expectations of growth in the economy. We would expect that during the two years prior to the crisis (which is what the model signals), expectations of growth would be high and values of LOMISH would increase during the boom cycle. However, we would also expect that once the crisis breaks out, expectations of growth would decline and values of LOMISH would decrease during the downturn of the cycle. On the other hand, it takes the PoDs two years from the outbreak of the crisis until they reach their maximum frequency (inside lag). After two years, PoDs start decreasing. Therefore, a positive relationship between LOMISH (lagged two years) and the behavior of empirical defaults is consistent with our hypothesis.

In **Specification 3**, the results for the parameters of GapCREDOVGDP and INREVO1 are similar to the results reported in Specification 2. However, when we recovered the coefficient of the gap in the aggregate asset price index (GapAGGPRINDX) lagged only one year, this variable had a negative coefficient. This result is consistent with our hypothesis.<sup>60</sup> Once the economy goes into a downturn of asset prices (within the “inside lag”), we would also expect a sharp increase in PoDs, as the value of the assets held as collateral decreases. Therefore, we would anticipate that the relationship between PoDs and GapAGGPRINDX (lagged one period) would be negative. We also explore the explanatory power of the gap in the ratio of M2 over international reserves. As we explained in Section 4.1, since M2 could be interpreted as a proxy of current liabilities for the government and international reserves as a proxy of current assets, the ratio of M2 over reserves is a proxy that indicates the extent to which the liabilities of the banking system are backed by international reserves (Calvo and Mendoza, 1996). In the event of a crisis, individuals may rush to convert their domestic currency deposits into foreign currency (Jorion, 2002). So we would expect that the gap of the ratio of M2 over reserves would increase at the outbreak of the crisis, because reserves will fall further than deposits. Therefore, as PoDs increase, GapM2OVRES also increases.

### Results for Mexico:

**Table 5** shows the recovered coefficients with CoPoD for Mexico.

<sup>59</sup>See the Global Financial Stability Report, IMF, September 2003.

<sup>60</sup>Once more, these results are consistent with theoretical credit cycles à la Kiyotaki and Moore and the results reported by Kaminsky and Reinhart (1999), Eichengreen and Areta (2000), Borio and Lowe (2002) and Goodhart, Hofmann and Segoviano (2004).

**Specification 1** shows the recovered coefficients for the gap in the ratio of credit to GDP (GapCREDOVGDP), the gap in the real estate price index (GapINCOCDRE) and the gap in the real share price (GapSHAPRIRE). These variables are lagged twenty-four, twenty and fifteen periods, respectively.<sup>61</sup> Note again that if we take off the Mexican crisis “inside lag” (eleven quarters), GapCREDOVGDP would have been explaining an increase in PoDs three years and one quarter before the outbreak of the crisis. GapINCOCDRE would have been signalling increases in PoDs two years and one quarter before the crisis and GapSHAPRIRE would have been explaining an increase in PoDs one year before the outbreak of the crisis. These results are consistent with the empirical and theoretical evidence for the timing of the signals, as we pointed out in the previous section.

**Specification 2** was augmented to include the interest rate volatility (INREVO1), the gap of the ratio of M2 over reserves (GapHCDM2OVRES) and the gap of the ratio of investment over GDP (GapINVOVGDP). These variables are lagged ten, six and eight periods, respectively. The recovered coefficients of the Gaps in the credit to GDP, the real estate price index and real share price keep their signs at similar lags as in the previous specification. These results indicate that increases in interest rate volatility appear to explain increases in PoDs within the “inside lag” window, which confirms that interest rate volatility does affect adversely banks’ balance sheets, as documented by Mishkin (1997). Furthermore, since increases in interest rate volatility have been reported at the outbreak of crises, these results appear to be consistent with empirical evidence for the timing of the signals. As we explained before, GapM2OVRES is a proxy that indicates to what extent the liabilities of the banking system are backed by international reserves (Calvo and Mendoza, 1996). Therefore, a positive coefficient in this variable within the crisis “inside lag” is consistent with the expectation that GapM2OVRES would increase as individuals rush to convert their domestic currency deposits into foreign currency at the outbreak of the crisis. In emerging market economies, investment virtually disappears at the outbreak of a crisis. This has been a phenomenon that has been regarded as a “drag” from the crisis and that could cause recovery from a crisis to take longer.<sup>62</sup> The negative coefficient of GapINVOVGDP lagged eight periods (within the crisis “inside lag” window) is consistent with this empirical fact. Therefore, as GapINVOVGDP decreases, PoDs increase.

In **Specification 3**, the results for the coefficients of GapCREDOVGDP, INREVO1, GapM2OVRES and GapINVOVGDP are similar to the results reported in Specification 2. However, when we recovered the coefficient of the gap of the aggregate real estate price index (GapINCOCDRE) lagged eleven periods, this variable had a negative coefficient. This result is consistent with our hypothesis. Once the economy goes into a downturn of asset prices (within the “inside lag”), we would also expect PoDs to start rising, as the value of the assets held as collateral decreases. Therefore, we would anticipate that the relationship between PoDs and GapINCOCDRE (lagged eleven periods) would be negative.

In summary, despite the shortcomings of the data, the different characteristics of the Norwegian and Mexican economies and the particularities of their respective crises, we were able to find regularities in the explanatory power of fluctuations in credit to GDP and fluctuations in asset prices on the empirical frequency of loan defaults in both countries. These results indicate that cumulative processes (gaps) provide better information than growth rates. The signs of the coefficients are consistent with theoretical arguments and empirical evidence. There is also consistency with respect to the timing when fluctuations in these variables signal changes in the financial risk of the system.

## 8 Conclusions

We propose the Conditional Probability of Default Methodology for modelling the probabilities of loan defaults by SMEs and unlisted firms as functions of identifiable macroeconomic and financial variables.

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<sup>61</sup>Note that although the vector of the Mexican dependent variable only contains 23 observations (from the second quarter of 1995 to the fourth quarter of 2004) the matrix of independent variables contains 126 observations (quarterly information from the first quarter of 1970 to the fourth quarter of 2004); therefore, it was possible to lag the independent variables up to twenty four quarters. The lags of the independent variables are consistent with other empirical studies. See Section 7.

<sup>62</sup>Sturm, Berger and Haan (2004) find that the ratio of Investment to GDP is robustly related to the probability that a country receives IMF credit.



Thus, CoPoD allows for the estimation of PoDs conditional on the business cycle, making it possible to measure the evolution of risk through time. The latter is achieved under the strong data restrictions binding the credit risk modelling of SMEs and unlisted firms.

CoPoD makes a twofold contribution. From the econometric point of view, CoPoD produces estimators that, in the setting of finite samples, are superior to OLS estimators under the mean square error criterion.<sup>63</sup> From an economic point of view, based on theoretical arguments and empirical evidence, CoPoD involves a procedure to select a set of explanatory variables that have a significant effect on loans' credit risk.

We present an empirical implementation of CoPoD with databases containing information on the PoDs of loans given to SMEs and unlisted companies in Norway and Mexico. The dataset is one of few that cover an entire business cycle and focus on SMEs and unlisted borrowers; however, the number of observations is still small in statistical terms. Despite the limitations of the data and regardless of the different characteristics of the Norwegian and Mexican economies and the particularities of their respective crises, these results indicate that gaps (cumulative processes) provide better information than growth rates. Results show that increases of credit to GDP and asset prices have a significant explanatory power on the PoDs in both countries. It is also observed that when periods of combined strong increases in credit and real asset prices occur, there is an enhanced likelihood of stress in the financial system occurring (reflected by increased PoDs), some two to four years ahead.

Since PoDs are explained by lagged values of relevant explanatory variables, it is possible to obtain *ex-ante* measures of probabilities of loan defaults given a set of realised or simulated (in the case of stress testing) values of macroeconomic explanatory variables. Therefore, the implementation of this methodology opens the possibility of being able to assess the impact of macroeconomic shocks on PoDs before such shocks have an effect in empirical defaults. *Ex-ante* measurements of loan defaults can also be used to evaluate the impact of macroeconomic shocks on banks' economic capital before such shocks are reflected upon unexpected losses. This can be achieved if the *ex-ante* PoDs, i.e. forecasted PoDs, are used in conjunction with the Consistent Information Multivariate Density Optimizing (CIMDO) methodology, which we propose to recover portfolio multivariate distributions.<sup>64</sup>

The joint implementation of the CoPoD and the CIMDO is specially useful for stress testing purposes. CoPoD allows the modeller to quantify the effects of macroeconomic shocks on PoDs (specified with macroeconomic scenarios). Stressed PoDs can then be used to obtain the loan portfolio's multivariate distribution using the CIMDO methodology. With this distribution, it becomes possible to estimate the change of economic capital that would be necessary to withstand a given macroeconomic shock.

Taking into consideration the dearth of data for credit risk measurement, these methodologies were designed with the objective of improving credit risk measurement through time. We aim to provide a set of tools that is useful for the timely recognition of risks as macroeconomic conditions change. If more timely recognition leads to minimizing the negative effects of such risks, this will potentially enhance financial institutions' competitive advantages and systems' financial stability.

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<sup>63</sup>Asymptotically, the CoPoD estimators are equivalent to Ordinary Least Squares estimators. Therefore, the estimators' efficiency gains produced by CoPoD will tend to disappear as the sample size increases. Thus CoPoD seems to be especially useful in settings where information is restricted. In settings where information is not restricted, CoPoD will produce equivalent estimators than OLS; however, the latter may be easier to be implemented.

<sup>64</sup>The CIMDO methodology is presented in a companion paper, Segoviano, 2005.

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# Appendix

## A.1 The generalized maximum entropy rule

Using the entropy concept developed in the *XIXth* century by Boltzman and continued lately by Maxwell, Gibbs, Bernoulli, Laplace, Shannon (1948) developed the “entropy of the distribution of probabilities” to measure the uncertainty of a collection of events.

In developing this approach, Shannon (1948) supposed that an experiment with  $N$  trials (repetitions) was carried out. This experiment had  $K$  possible outcomes (states). He assumed that  $N_1, N_2, \dots, N_K$  represented the number of times that each outcome occurs in the experiment of length  $N$ , where  $\sum N_k = N$ ,  $N_k \geq 0$ , and  $k = 1, 2, \dots, K$ .

In this setting there are  $N$  trials and each trial has  $K$  possible outcomes; therefore, there are  $K^N$  conceivable outcomes in the sequence of  $N$  trials. Of these, a particular set of frequencies

$$p_k = \frac{N_k}{N} \quad \text{or} \quad N_k = Np_k \quad \text{for} \quad k = 1, 2, \dots, K,$$

can be realized in a given number of ways as measured by the multiplicity factor (possible permutations). Thus, the number of ways that a particular set of  $N_k$  is realized, can be represented by the multinomial coefficient

$$W = \frac{N!}{Np_1!Np_2!\dots Np_k!} = \frac{N!}{\prod_k N_k!},$$

or its monotonic function

$$\ln W = \ln N! - \sum_{k=1}^K \ln N_k! \tag{A.1.1}$$

Given (??), Stirling’s approximation  $\ln x! \approx x \ln x - x$  as  $0 < x \rightarrow \infty$  is used to approximate each component on the right hand side of (??). Then, for large  $N$ ,

$$\ln W \approx N \ln N - N - \sum_{k=1}^K \ln N_k \ln N_k + \sum_{k=1}^K N_k$$

since  $\sum_{k=1}^K N_k = N$ , we get

$$\ln W \approx N \ln N - \sum_{k=1}^K N_k \ln N_k. \tag{A.1.2}$$

The ratio  $\frac{N_k}{N}$  represents the frequency of the occurrence of the possible  $K$  outcomes in a sequence of length  $N$  and  $\frac{N_k}{N} \rightarrow p_k$  as  $N \rightarrow \infty$ . Consequently (A.1.2) yields,

$$\begin{aligned} \ln W &\approx N \ln N - \sum_{k=1}^K Np_k \ln (Np_k) \\ &= N \ln N - \sum_{k=1}^K N_k \ln N - N \sum_{k=1}^K p_k \ln p_k \\ &= -N \sum_{k=1}^K p_k \ln p_k. \end{aligned}$$

Finally,

$$\begin{aligned} N^{-1} \ln W &\approx - \sum_{k=1}^K p_k \ln p_k \\ H(p) &= - \sum_{k=1}^K p_k \ln p_k \end{aligned} \tag{A.1.3}$$



Which is the Shannon entropy measure, where  $p_k \ln p_k = 0$  for  $p_k = 0$ .

Jaynes (1957) proposed making use of the entropy concept to choose an unknown distribution of probabilities when only partial information is available. He proposed maximizing the function presented in equation (24), subject to the limited available data, this in order to obtain the probability vector  $\mathbf{p}$  that can be realized in the greatest number of ways consistent with the known data.

The rationale provided by Jaynes (1957) for choosing a particular solution, i.e. probability vector  $\mathbf{p}$  from partial information is known as the principle of maximum entropy or generalized maximum entropy rule (GME). Let

$$L = - \sum_k^K p_k \ln p_k + \sum_{t=1}^T \lambda_t \left[ y_t - \sum_k^K p_k f_t(x_k) \right] + \mu \left[ 1 - \sum_k^K p_k \right], \quad (\text{A.1.4})$$

be the Lagrange function. Then, the problem of maximum entropy is to maximize  $L$ .

In this function, the information contained in the data has been formalized in  $1 \leq t \leq T$  moment-consistency constraints of the form  $\sum_{k=1}^K p_k f_t(x_k) = y_t$ . These moment-consistency-constraints are formulated with  $T$  functions  $\{f_1(x), f_2(x), \dots, f_T(x)\}$  representing the information contained in the data and with a set of observations (averages or aggregates)  $\{y_1(x), y_2(x), \dots, y_T(x)\}$  that are consistent with the distribution of probabilities  $\{p_1, p_2, \dots, p_k\}$ . Note that the problem is *under-identified* if  $T < K$ .

In this function, the additivity restriction  $\sum_{k=1}^K p_k = 1$  has to be fulfilled as well, since  $\mathbf{p}$  represents a probability distribution. Note also that  $\lambda_t$  represents the Lagrange multiplier of each of the  $1 \leq t \leq T$  moment-consistency constraints and  $\mu$  represents the Lagrange multiplier of the probability additivity constraint.

Using the method of Lagrange multipliers, the maximum entropy solution is given by

$$\hat{p}_k = \frac{1}{\sum_{k=1}^K \exp \left[ - \sum_{t=1}^T \hat{\lambda}_t f_t(x_k) \right]} \exp \left[ - \sum_{t=1}^T \hat{\lambda}_t f_t(x_k) \right]. \quad (\text{A.1.5})$$

## A.2 Uniqueness of the maximum entropy solution

Note that the additivity restrictions in equation (6) are composed of  $K$  unit simplices of dimension  $M \geq 2$  and  $T$  unit simplices of dimension  $J \geq 2$ . By denoting the individual simplices as  $\mathcal{U}_M$  and  $\mathcal{U}_J$ , respectively, the additivity constraint set can be written as the Cartesian product of these sets,  $\mathcal{C} = \mathcal{U}_M^K \times \mathcal{U}_J^T$ .

Clearly,  $\mathcal{C}$  is a non-empty and compact set because each of the components simplices is non-empty and compact. Further, we only consider the interior of the additivity constraint set,  $\text{int}(\mathcal{C})$ , which contains all  $\mathbf{p}, \mathbf{w} \gg \mathbf{0}$ . The model constraint set in equation (5) further restricts  $\mathcal{C}$  to the probability distributions that are *consistent* with the data. The fully restricted constraint set can be written as  $\mathcal{C}^* = \{(\mathbf{p}, \mathbf{w}) \in \text{int}(\mathcal{C}) : \mathbf{a} - \mathbf{xZp} - \mathbf{Vw}\}$ .

To verify the uniqueness of the solution, note that the Hessian matrix of the objective function in equation (4) is

$$\nabla_{(p,w)}(p',w')E(\mathbf{p}, \mathbf{w}) = \begin{bmatrix} -\mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{W}^{-1} \end{bmatrix},$$

where  $\mathbf{P}^{-1}$  is a ( $\mathbf{KM} \times \mathbf{KM}$ ) diagonal matrix with elements  $-(p_m^k)^{-1}$  and  $\mathbf{W}^{-1}$  is a ( $\mathbf{TJ} \times \mathbf{TJ}$ ) diagonal matrix with elements  $-(w_j^t)^{-1}$ .

This matrix is negative definite for  $\mathbf{p}, \mathbf{w} \gg \mathbf{0}$ , which satisfies the sufficient conditions for strict concavity. Therefore, there is a unique global maximum for the problem if,  $\mathcal{C}^* \neq \emptyset$ .

### A.3 Large sample properties regularity conditions

Golan, Judge and Miller (1997) make use of the dual formulation of the problem to evaluate the large sample properties of the GME estimators. The authors show that the asymptotic distribution of GME estimators can be derived by finding the distribution of  $\widehat{\boldsymbol{\lambda}}_{\mathbf{T}}$ . Given that  $\widehat{\boldsymbol{\beta}}_{\mathbf{T}} = \mathbf{Z}\mathbf{p}(\widehat{\boldsymbol{\lambda}}_{\mathbf{T}})$  is a continuous function of  $\widehat{\boldsymbol{\lambda}}_{\mathbf{T}}$ , they use the  $\delta$ -method (Spanos, 1986) to approximate the distribution of  $\widehat{\boldsymbol{\beta}}_{\mathbf{T}}$ . The authors examine the properties of a generalized entropy formulation that employs convergent functions of the sample information. In particular, they consider the vector of weighted averages formed by dividing  $\mathbf{X}'\mathbf{a} = \mathbf{X}'\mathbf{X}\boldsymbol{\beta} + \mathbf{X}'\mathbf{e}$  by the sample size  $T^{-1}$ . They claim that under the following regularity conditions, these weighted averages converge in probability.

1. There exists a finite, positive definite matrix  $\mathbf{Q}$  such that

$$\lim_{T \rightarrow \infty} \left( \frac{\mathbf{X}'\mathbf{X}}{T} \right) = \mathbf{Q}.$$

2.  $E(\mathbf{e}) = \mathbf{0}$ ,  $Var(\mathbf{e}) = \sum_{\mathbf{e}}$ , and  $F(\mathbf{e})$  satisfies the Lindeberg condition (Billingsley 1986, equation 27.8)

$$T^{-1} \sum_{t=1}^T \int_{\ell} \|\mathbf{e}\|^2 dF(\mathbf{e}) \rightarrow 0,$$

where  $\ell = \{\mathbf{e} : \|\mathbf{e}\| > \varepsilon\sqrt{T}\}$  for  $\varepsilon > 0$ .

3. The variance-covariance matrix of  $\varepsilon = \frac{\mathbf{X}'\mathbf{e}}{\sqrt{T}}$  converges to a finite, positive definite matrix

$$\lim_{T \rightarrow \infty} \left( \frac{\mathbf{X}'\sum_{\mathbf{e}}\mathbf{X}}{T} \right) = \boldsymbol{\Sigma}^*.$$

4.  $\boldsymbol{\beta}_0 \in \text{int}(\mathcal{L})$ .

5.  $\mathbf{V} = O(T^{-1})$ .

### A.4 Information measures

The amount of information captured by the GME model can be measured by using a normalized entropy (information measure). This statistic (Golan, 1988) measures the importance of the contribution of each data constraint ( $t=1,2,\dots,t$ ) in reducing uncertainty. As we have already mentioned, the maximum level of entropy uncertainty results when the information-moment constraints are not enforced and the distribution of probabilities is maximally dispersed, and thus uniformly distributed (each possible outcome is equally probable and therefore the distribution is maximally uninformative). As we add each piece of effective data, a departure from the uniform distribution results and implies a reduction of uncertainty. The proportion of the remaining total uncertainty is measured by the normalized entropy

$$\begin{aligned} S(\widehat{\mathbf{p}}) &= \left( \frac{- \left[ \sum_{k=1}^K \sum_{m=1}^M \widehat{p}_m^k \ln(\widehat{p}_m^k) \right]}{K \ln(M)} \right), \\ S(\widehat{\mathbf{w}}) &= \left( \frac{- \left[ \sum_{t=1}^T \sum_{j=1}^J \widehat{w}_j^t \ln(\widehat{w}_j^t) \right]}{T \ln(J)} \right), \end{aligned} \tag{A.3.1}$$

where  $S(\widehat{\mathbf{p}})$  and  $S(\widehat{\mathbf{w}}) \in [0, 1]$ . Values  $S(\widehat{\mathbf{p}}) = \mathbf{0}$  and  $S(\widehat{\mathbf{w}}) = \mathbf{0}$  imply no uncertainty. This is the case when the distribution is maximally informative in that  $\mathbf{p}$  and  $\mathbf{w}$  degenerate on particular values, e.g.  $\widehat{p}_m^k = 1$  for some  $km$  and  $\widehat{p}_n^k = 0$  for all  $kn \neq km$  and  $\widehat{w}_j^t = 1$  for some  $tj$  and  $\widehat{w}_n^t = 0$  for all  $tn \neq tj$ . Alternatively,  $S(\widehat{\mathbf{p}}) = \mathbf{1}$  and  $S(\widehat{\mathbf{w}}) = \mathbf{1}$  implies perfect uncertainty. Note that whereas the variance of a discrete distribution measures the concentration of mass about the mean,  $S(\widehat{\mathbf{p}})$  and  $S(\widehat{\mathbf{w}})$  measure the concentration of mass over the support of the  $\mathbf{p}$  and  $\mathbf{w}$  distributions.

An analog measure, the *information index* (Soofi, 1992) can be defined as

$$\begin{aligned}\mathbf{R}_I(\hat{\mathbf{p}}) &= 1 - S(\hat{\mathbf{p}}), \\ \mathbf{R}_I(\hat{\mathbf{w}}) &= 1 - S(\hat{\mathbf{w}}).\end{aligned}\tag{A.3.2}$$

We can interpret  $\mathbf{R}_I(\hat{\mathbf{p}})$  and  $\mathbf{R}_I(\hat{\mathbf{w}})$  as statistics to measure the reduction in uncertainty.

## A.5 Procedure to obtain long-term trends

As we stated in Section 4.3.1, for each analyzed variable, we estimate “gaps” with respect to long-term trends. In order to compute the latter, we use “dynamic” Hodrick-Prescott (HP) filters using information from 1970. This procedure is illustrated as follows:

Table 8: Gaps with a dynamic HP filter

Period	Contemporaneous	Lag1 (-1)
$t_1$	$\text{GapVar}_1 = \text{Var}_1\text{-HTDVar}_1^1$	$\text{GapVar}(-1)_1 = \text{Var}_0\text{-HTDVar}_0^1$
$t_2$	$\text{GapVar}_2 = \text{Var}_2\text{-HTDVar}_2^2$	$\text{GapVar}(-1)_2 = \text{Var}_1\text{-HTDVar}_1^2$
$t_3$	$\text{GapVar}_3 = \text{Var}_3\text{-HTDVar}_3^3$	$\text{GapVar}(-1)_3 = \text{Var}_2\text{-HTDVar}_2^3$
$t_4$	$\text{GapVar}_4 = \text{Var}_4\text{-HTDVar}_4^4$	$\text{GapVar}(-1)_4 = \text{Var}_3\text{-HTDVar}_3^4$
$t_5$	$\text{GapVar}_5 = \text{Var}_5\text{-HTDVar}_5^5$	$\text{GapVar}(-1)_5 = \text{Var}_4\text{-HTDVar}_4^5$

Var refers to the variable and HTDVar is the HP trend component of the analyzed variable.

Subscripts refer to the time of the observation.

Superscripts refer to the information set included.

For example, let us assume that we were currently at  $t_3$  and we wanted to estimate the gap for a given variable at time  $t_3$  (contemporaneous gap). First, using a HP filter, we would include information from 1970 up until  $t_3$  to compute the trend component (superscript in the HP trend component, e.g.,  $\text{HTDVariable}^3$ ). Second, we would obtain the difference between the value of the variable at  $t_3$  (subscript in the variable component, e.g.,  $\text{Variable}_3$ ) and the value of the trend component at  $t_3$  (subscript in the trend component, e.g.,  $\text{HTDVariable}_3$ ); therefore,  $\text{GapVariable}_3 = \text{Variable}_3 - \text{HTDVariable}_3^3$ . Note that we only use information up to the period that we analyze, e.g.  $t_3$ , because at  $t_3$  an analyst would only have information up to this date. In order to estimate Lags for the gaps we followed a similar procedure. If we were at  $t_3$  and we wanted to estimate the credit gap, lagged one period. First, using an HP filter, we would include information from 1970 up until  $t_3$  to compute the trend component (superscript in the HP trend component, e.g.,  $\text{HTDVariable}^3$ ). However, now in the second step, we would obtain the difference between the value of the variable at  $t_2$  (lagged one period, e.g., subscript in the variable component, e.g.,  $\text{Variable}_2$ ) and the value of the trend component at  $t_2$  (lagged one period, e.g., subscript in the trend component, e.g.,  $\text{HTDVariable}_2$ ); therefore,  $\text{GapVariable}(-1)_3 = \text{Variable}_2 - \text{HTDVariable}_2^3$ . This procedure was repeated for all the included lags.

## A.6 Results with growth rates

Table 9: OLS results for Norway: growth rates

OLS results for Norway: growth rates						
Variables	Specification 1		Specification 2		Specification 3	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
Constant	3.663943	0.0000	3.639994	0.0000	3.658267	0.0000
GrCREDOVGDP(-7)	0.475878	0.0116	0.415443	0.0000	0.275960	0.0243
GrAGGPRINDX(-6)	0.168454	0.1306	0.254088	0.0010		
INREVO1			1468.7580	0.0005	959.3723	0.0072
GrM2OVRES(-3)			0.113979	0.0015	0.119739	0.0260
GrAGGPRINDX(-1)					-0.203191	0.0543
R-squared	0.4177		0.8586		0.7461	
Adjusted R-squared	0.3118		0.7958		0.6332	
Akaike criterion	-3.1498		-4.2796		-3.6940	
Schwarz criterion	-3.0129		-4.0513		-3.4658	
F-statistic	3.9449		13.6627		6.6100	
Prob (F-statistic)	0.0511		0.0007		0.0091	

Dependent Variable: aNor. Method: OLS. Sample: 1988-2001. Included observations: 14.  
Newey-West HAC Standard Errors & Covariance (lag truncation=2).

Table 10: OLS results for Mexico: growth rates

OLS results for Mexico: growth rates						
Variables	Specification 1		Specification 2		Specification 3	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
Constant	2.736912	0.0000	2.634441	0.0000	2.481142	0.0000
GrCREDOVGDP(-26)	3.506881	0.0059	3.189803	0.0046		
GrINCOCDRE(-25)	11.360630	0.0003	8.793358	0.0001		
INREVO1(-10)	3360.2340	0.0000	3176.2660	0.0000	4494.4460	0.0000
LOMISH(-11)			-0.039998	0.0000		
GrM2OVRES(-14)			0.444616	0.0001		
GrINVOVGDP(-14)			-2.760626	0.0003		
GrCREDOVGDP(-25)					4.268453	0.0031
GrINCOCDRE(-6)					-7.002888	0.0561
GrSHAPRIRE(-8)					-958600	0.0457
R-squared	0.7061		0.9270		0.5881	
Adjusted R-squared	0.6597		0.8996		0.4966	
Akaike criterion	0.9789		-0.1530		1.4032	
Schwarz criterion	1.1764		0.1925		1.6501	
F-statistic	15.2133		33.8596		6.4257	
Prob (F-statistic)	0.0000		0.0000		0.0021	

Dependent Variable: aMex, Method: OLS. Sample: 1995:2 2000:4. Included observations: 23.  
Newey-West HAC Standard Errors & Covariance (lag truncation=2).

Table 11: CoPoD (GME) results for Norway: growth rates

<b>CoPoD (GME) results for Norway: growth rates</b>			
Variables	<b>Specification 1</b>	<b>Specification 2</b>	<b>Specification 3</b>
	Coefficient	Coefficient	Coefficient
Constant	3.662104	3.639671	3.659073
GrCREDOVGDP(-7)	0.464223	0.419596	0.285415
GrAGGPRINDX(-6)	0.174416	0.255577	
INREVO1		1463.9580	925.73395
GrM2OVRES(-3)		0.113147	0.121864
GrAGGPRINDX(-1)			-0.205878

Dependent Variable: aNor. Method: CoPoD. Sample: 1988-2001. Included observations: 14.

Table 12: CoPoD results for Mexico: growth rates

<b>CoPoD (GME) results for Mexico: growth rates</b>			
Variables	<b>Specification 1</b>	<b>Specification 2</b>	<b>Specification 3</b>
	Coefficient	Coefficient	Coefficient
Constant	2.782681	2.635378	2.498623
GrCREDOVGDP(-26)	3.609360	3.144585	
GrINCOCDRE(-25)	12.679111	9.010519	
INREVO1(-10)	3218.808500	3153.52060	4417.79550
LOMISH(-11)		-0.030181	
GrM2OVRES(-14)		0.441605	
GrINVOVGDP(-14)		-2.707960	
GrCREDOVGDP(-25)			4.089978
GrINCOCDRE(-6)			-6.235022
GrSHAPRIRE(-8)			-0.946989

Dependent Variable: aMex, Method: CoPoD. Sample: 1995:2 2000:4. Included observations: 23.