

# Market Liquidity and Funding Liquidity\*

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This version: February 2007

## Abstract

We provide a model that links an asset's market liquidity — i.e., the ease with which it is traded — and traders' funding liquidity — i.e., the ease with which they can obtain funding. Traders provide market liquidity, and their ability to do so depends on their availability of funding. Conversely, traders' funding, i.e., their capital and the margins they are charged, depend on the assets' market liquidity. We show that, under certain conditions, margins are destabilizing and market liquidity and funding liquidity are mutually reinforcing, leading to liquidity spirals. The model explains the empirically documented features that market liquidity (i) can suddenly dry up, (ii) has commonality across securities, (iii) is related to volatility, (iv) is subject to “flight to quality”, and (v) comoves with the market, and it provides new testable predictions.

*Keywords:* Liquidity Risk Management, Liquidity, Liquidation, Systemic Risk, Leverage, Margins, Haircuts, Value-at-Risk, Counterparty Credit Risk

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\*We are grateful for helpful comments from Franklin Allen, Yakov Amihud, David Blair, Bernard Dumas, Denis Gromb, Charles Johns, Christian Julliard, John Kambhu, Markus Konz, Martin Oehmke, Filippos Papanikolaou, Ketan Patel, Guillaume Plantin, Felipe Schwartzman, Jeremy Stein, Dimitri Vayanos, Jiang Wang, and Pierre-Olivier Weill. We also thank seminar participants at the New York Federal Reserve Bank and the New York Stock Exchange, Citigroup, Bank for International Settlement, University of Zürich, INSEAD, Northwestern University, Stockholm Institute for Financial Research, Goldman Sachs, IMF, the World Bank, UCLA, LSE, Warwick University, Bank of England, University of Chicago, Texas A&M, University of Notre Dame, HEC, University of Maryland, University of Michigan, Virginia Tech and conference participants at the American Economic Association Meeting, FMRC conference in honor of Hans Stoll at Vanderbilt, NBER Market Microstructure Meetings, NBER Asset Pricing Meetings, NBER Risks of Financial Institutions conference, the Five Star conference, and American Finance Association Meeting.

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Trading requires capital. When a trader — e.g. a dealer, hedge fund, or investment bank — buys a security, he can use the security as collateral and borrow against it, but he cannot borrow the entire price. The difference between the security’s price and collateral value, denoted as the margin, must be financed with the trader’s own capital. Similarly, shortselling requires capital in the form of a margin; it does not free up capital. Therefore, the total margin on all positions cannot exceed a trader’s capital at any time.

Our model shows that the funding of traders affects — and is affected by — market liquidity in a profound way. When funding liquidity is tight, traders become reluctant to take on positions, especially “capital intensive” positions in high-margin securities. This lowers market liquidity, leading to higher volatility. Further, under certain conditions, low future market liquidity increases the risk of financing a trade, thus increasing margins.

Based on the links between funding and market liquidity, we provide a unified explanation for the main empirical features of market liquidity. In particular, our model implies that market liquidity (i) can suddenly dry up, (ii) has commonality across securities, (iii) is related to volatility, (iv) is subject to “flight to liquidity,” and (v) comoves with the market. The model has several new testable implications that link margins and dealer funding to market liquidity: We predict that (i) a shock to speculators’ capital is a state variable affecting market liquidity and risk premia, (ii) a reduction in capital reduces market liquidity, especially if capital is already low (a non-linear effect) and for high-margin securities, (iii) margins increase in illiquidity if the fundamental value is difficult to determine, and (iv) speculators’ returns are negatively skewed (even if they trade securities without skewness in the fundamentals).

Our model is similar in spirit to Grossman and Miller (1988) with the added feature that speculators face the real-world funding constraint discussed above. In our model, different customers have offsetting demand shocks, but arrive sequentially to the market. This creates a temporary order imbalance. Speculators smooth price fluctuations, thus providing market liquidity. Speculators finance their trades through collateralized borrowing from financiers who set the margins to control their value-at-risk (VaR). We derive the competitive equilib-

rium of the model and explore its liquidity implications. We define market liquidity as the difference between the transaction price and the fundamental value, and funding liquidity as a speculator’s scarcity (or shadow cost) of capital.

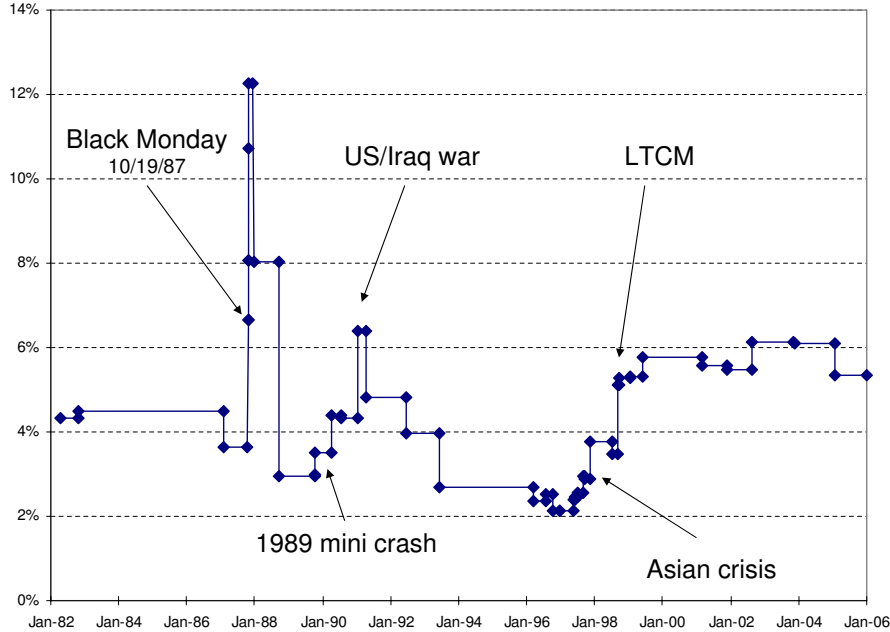


Figure 1: **Margins for S&P500 Futures.** The figure shows margin requirements on S&P500 futures for members of the Chicago Mercantile Exchange as a fraction of the value of the underlying S&P500 index multiplied by the size of the contract. (Initial or maintenance margins are the same for members.) Each dot represents a change in the dollar margin.

We first analyze the properties of margins. We show that margins can increase in illiquidity when margin-setting financiers’ are unsure whether price changes are due to fundamental news or to liquidity shocks and fundamentals have time-varying volatility. This happens when a liquidity shock leads to price volatility, which raises the financier’s expectation about future volatility. Figure 1 shows that margins did increase empirically for S&P 500 futures during the liquidity crises of 1987, 1990, and 1998. We denote margins as “destabilizing” if they can increase in illiquidity, and note that anecdotal evidence from prime brokers suggests that margins often behave this way.

The model also shows that margins can, in contrast, decrease in illiquidity and thus be “stabilizing.” This happens when financiers know that prices diverge due to temporary mar-

ket illiquidity and know that liquidity will be improved shortly as complementary customers arrive. This is because a current price divergence from fundamentals provide a “cushion” against future adverse price moves, making the speculator’s position less risky in this case. In summary, our model predicts that margins depend on market conditions and are more destabilizing in specialized markets in which financiers cannot easily distinguish fundamental shocks from liquidity shocks or predict when a trade converges.

Turning to the implications for market liquidity, we first show that, as long as speculator capital is so abundant that there is no risk of hitting the funding constraint, market liquidity is naturally at its highest level and is insensitive to marginal changes in capital and margins. However, when speculators hit their capital constraints — or risk hitting their capital constraints over the life of a trade — then they reduce their positions and market liquidity declines.

When margins are destabilizing or speculators have large existing positions, there can be multiple equilibria and liquidity can be *fragile*. In one equilibrium, markets are liquid, leading to favorable margin requirements for speculators, which in turn helps speculators make markets liquid. In another equilibrium, markets are illiquid, resulting in larger margin requirements (or speculator losses), thus restricting speculators from providing market liquidity. Importantly, any equilibrium selection has the property that small speculator losses can lead to a discontinuous drop of market liquidity. This “sudden dry-up” or fragility of market liquidity is due to the fact that with high levels of speculator capital, markets must be in a liquid equilibrium, and, if speculator capital is reduced enough, the market must eventually switch to a low-liquidity/high-margin equilibrium.<sup>1</sup> The events following the Russian default in 1998 are a vivid example of fragility of liquidity since a relatively small shock had a large impact. Compared to the total market capitalization of the US stock and bond markets, the losses due to the Russian default were minuscule but, as Figure 1 shows, caused a shiver in world financial markets.

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<sup>1</sup>Fragility can also be caused by asymmetric information on the amount of trading by portfolio insurance traders (Genotte and Leland (1990)), and by losses on existing positions (Chowdhry and Nanda (1998)).

Further, when markets are illiquid, market liquidity is highly sensitive to further changes in funding conditions. This is due to two liquidity spirals: first, a “*margin spiral*” emerges if margins are increasing in market illiquidity because a reduction in speculator wealth lowers market liquidity, leading to higher margins, tightening speculators’ funding constraint further, and so on. For instance, Figure 1 shows how margins gradually escalated within a few days after Black Monday in 1987. Second, a “*loss spiral*” arises if speculators hold a large initial position that is negatively correlated with customers’ demand shock. In this case, a funding shock increases market illiquidity, leading to speculator losses on their initial position, forcing speculators to sell more, causing a further price drop, and so on.<sup>2</sup> These liquidity spirals reinforce each other, implying a larger total effect than the sum of their separate effects. Paradoxically, liquidity spirals imply that a larger shock to the customers’ demand for immediacy leads to a reduction in the provision of immediacy during such stress times. Consistent with our predictions, Mitchell, Pulvino, and Pedersen (2007) find significant liquidity-driven divergence of prices from fundamentals in the convertible bond markets after capital shocks to the main liquidity providers, namely convertible arbitrage hedge funds.

Our model also provides a natural explanation for the *commonality* of liquidity across assets since shocks to speculators’ funding constraint affect all securities. This may help explain why market liquidity is correlated across stocks (Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2001) and Huberman and Halka (2001)), and across stocks and bonds (Chordia, Sarkar, and Subrahmanyam (2005)). In support of the idea that commonality is driven at least in part by our funding-liquidity mechanism, Chordia, Sarkar, and Subrahmanyam (2005) find that “money flows ... account for part of the commonality in stock and bond market liquidity.” Moreover, their finding that “during crisis periods, monetary expansions are associated with increased liquidity” is consistent with our model’s prediction that the effects are largest when traders are near their constraint. Coughenour and Saad

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<sup>2</sup>The loss spiral is related to the multipliers that arise in Grossman (1988), Kiyotaki and Moore (1997), Shleifer and Vishny (1997), Chowdhry and Nanda (1998), Xiong (2001), Kyle and Xiong (2001), Gromb and Vayanos (2002), Morris and Shin (2004), Plantin, Sapra, and Shin (2005) and others. To our knowledge, our paper is the first to model the margin spiral and the interaction between the two multipliers.

(2004) provide further evidence of the funding-liquidity mechanism by showing that the co-movement in liquidity among stocks handled by the same NYSE specialist firm is higher than for other stocks, commonality is higher for specialists with less capital, and decreases after a merger of specialists.

Next, our model predicts that market liquidity declines as fundamental volatility increases, which is consistent with the empirical findings of Benston and Hagerman (1974) and Amihud and Mendelson (1989).<sup>3</sup> The model implies that the liquidity differential between high-volatility and low-volatility securities increases as speculator capital deteriorates — a phenomenon sometimes referred to as “flight to quality” or “flight to liquidity.” According to our model, this happens because a reduction in speculator capital induces traders to provide liquidity mostly in securities that do not use much capital (low volatility stocks since they have lower margins). Hence, illiquid securities are predicted to have more liquidity risk. Recently, Hendershott, Moulton, and Seasholes (2006) test these predictions using inventory positions of NYSE specialists as a proxy for funding liquidity. Their findings support our hypotheses that market liquidity of high volatility stocks is more sensitive to changes in inventory shocks and that this is more pronounced at times of low funding liquidity. Moreover, Pastor and Stambaugh (2003) and Acharya and Pedersen (2005) document empirical evidence consistent with flight to liquidity and the pricing of this liquidity risk.

Market-making firms are often net long the market. For instance, Ibbotson (1999) reports that security brokers and speculators have median market betas in excess of one. Therefore, capital constraints are more likely to be hit during market downturns, and this, together with the mechanism outlined in our model, helps to explain why sudden liquidity dry-ups occur more often when markets decline and why liquidity co-moves more during downturns. Following our model’s prediction, Hameed, Kang, and Viswanathan (2005) document that co-movements in liquidity indeed are higher during large negative market moves.

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<sup>3</sup>The link between volatility and liquidity is shared by the models of Stoll (1978), Grossman and Miller (1988), and others. What sets our theory apart is that this link is connected with margin constraints. This leads to testable differences since, according to our model, the link is stronger when speculators are poorly financed, and high-volatility securities are more affected by speculator wealth shocks — our explanation of flight to quality.

Finally, the risk alone that the funding constraint becomes binding limits speculators' provision of market liquidity. Our analysis shows that speculators' optimal (funding) risk management policy is to maintain a "safety buffer." This affects initial prices, which increase in the covariance of future prices with future shadow costs of capital (i.e., with future funding illiquidity).

Our paper is related to several literatures.<sup>4</sup> Most directly related are the models with margin-constrained traders: Grossman and Vila (1992) and Liu and Longstaff (2004) derive optimal strategies in a partial equilibrium with a single security; Chowdhry and Nanda (1998) focus on fragility due to dealer losses; and Gromb and Vayanos (2002) derive a general equilibrium with one security and study welfare and liquidity provision. Our paper contributes to the literature by considering the simultaneous effect of margin constraints on multiple securities and by examining the nature of those margin constraints. Stated simply, the existing theoretical literature uses a fixed or decreasing margin constraint — say \$5,000 per contract — and studies what happens when trading losses cause agents to hit this constraint, whereas we study how market conditions lead to changes in the margin requirement itself — e.g., an increase from \$5,000 to \$15,000 per futures contract as happened in October 1987 — and the resulting feedback effects between margins and market conditions.

We proceed as follows. First, we describe the real-world funding constraints for the main liquidity providers, namely market makers, banks, and hedge funds (Section 1). We then describe the model (Section 2) and derive our four main new results: (i) margins increase with market illiquidity when financiers cannot distinguish fundamental shocks from liquidity shocks and fundamentals have time-varying volatility (Section 3); (ii) this makes margins destabilizing, leading to sudden liquidity dry ups and margin spirals (Section 4); (iii) liquidity

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<sup>4</sup>Market liquidity is the focus of market microstructure (Stoll (1978), Ho and Stoll (1981, 1983), Kyle (1985), Glosten and Milgrom (1985), Grossman and Miller (1988)), and is related to the limits of arbitrage (DeLong, Shleifer, Summers, and Waldmann (1990), Shleifer and Vishny (1997), Abreu and Brunnermeier (2002)). Funding liquidity is examined in corporate finance (Shleifer and Vishny (1992), Holmström and Tirole (1998,2001)) and banking (Diamond and Dybvig (1983), Allen and Gale (1998, 2004, 2005)). Funding and collateral constraints are also studied in macroeconomics (Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Lustig (2004)), and general equilibrium with incomplete markets (Geanakoplos (1997, 2003)). Finally recent papers consider illiquidity with constrained traders (Attari, Mello, and Ruckes (2005), Bernardo and Welch (2004), Brunnermeier and Pedersen (2005), Eisfeldt (2004), Morris and Shin (2004), and Weill (2004)).

crises simultaneously affect many securities, mostly risky high-margin securities, resulting in commonality of liquidity and flight to quality (Section 5); and (iv) liquidity risk matters even before speculators hit their capital constraints (Section 6). Finally, we outline how our model's new testable predictions may be helpful for a novel line of empirical work that links measures of speculators' funding conditions to measures of market liquidity (Section 7).

## 1 Margins, Haircuts and Capital Constraints

A central element of our paper is the capital constraints that the main providers of market liquidity face. To set the stage for our model, we review these institutional features for securities firms such as hedge funds, banks' proprietary trading desks, and market makers. (Readers mainly interested in our theory can skip to Section 2.)

### 1.1 Funding Requirements for Hedge Funds

We first consider the funding issues faced by hedge funds since they have relatively simple balance sheets and face little regulation. A hedge fund's capital consists of its equity capital supplied by the partners, and possible long-term debt financing that can be relied upon during a potential funding crisis. Since a hedge fund is a partnership, the equity is not locked into the firm indefinitely, as in a corporation. The investors (that is, the partners) can withdraw their capital at certain times, but — to ensure funding — the withdrawal is subject to initial lock-up periods and general redemption notice periods before specific redemption dates (typically at least a month, often several months or even years). A hedge fund usually does not issue long-term unsecured bonds, but some (large) hedge funds manage to obtain debt financing in the form of medium-term bank loans or in the form of a guaranteed line of credit.<sup>5</sup> Recently, some hedge funds have even raised capital by issuing bonds or permanent equity (e.g., see *The Economist* 1/27/2007, page 75).

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<sup>5</sup>A line of credit may have a "material adverse change" clause or other covenants subject to discretionary interpretation of the lender. Such covenants imply that the line of credit may not be a reliable source of funding during a crisis.



Hedge funds' main source of leverage is collateralized borrowing financed by the hedge fund's prime broker(s). The prime brokerage business is opaque since the terms of the financing are subject to negotiation and are hidden to outsiders. We describe stylized financing terms and, later, we discuss caveats.

If a hedge fund buys at time  $t$  a long position of  $x_t^j > 0$  shares of a security  $j$  at price  $p_t^j$ , it has to come up with  $x_t^j p_t^j$  dollars. The security can, however, be used as collateral for a new loan of, say,  $l_t^j$  dollars. The difference between the price of the security and the collateral value is denoted as the margin requirement  $m_t^{j+} = p_t^j - l_t^j$ . Hence, this position uses  $x_t^j m_t^{j+}$  dollars of the fund's capital. The collateralized funding implies that the capital use depends on margins, not notional amounts.

The margins on fixed income securities and over-the-counter (OTC) derivatives are set through a negotiation between the hedge fund and the prime broker that finances the trade. The margins are typically set so as to make the loan almost risk free for the broker, that is, such that it covers the largest possible adverse price move with a certain degree of confidence (i.e., it covers the Value-at-Risk).<sup>6</sup>

If the hedge fund wants to sell short a security,  $x^j < 0$ , then the fund asks one of its brokers to locate a security that can be borrowed, and then the fund sells the borrowed security. Duffie, Gârleanu, and Pedersen (2002) describe in detail the institutional arrangements of shorting. The broker keeps the proceeds of the short sale to be able to repurchase the security if the hedge fund fails and, additionally, requires that the hedge fund posts a margin  $m_t^{j-}$  that covers the largest possible adverse price move with a certain degree of confidence.

In the U.S., margins on equities are subject to Regulation T, which stipulates that non-brokers/dealers must have an initial margin (downpayment) of 50% of the market value of the underlying stock, both for new long and short positions. Hedge funds can, however, circumvent Regulation T by, for instance, organizing the transaction as a total return swap,

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<sup>6</sup>An explicit equation for the margin is given by (6) in Section 2. Often brokers also take into account the delay between the time a failure by the hedge fund is noticed, and the time the security is actually sold. Hence, the margin of a one-day collateralized loan depends on the estimated risk of holding the asset over a time period that is often set as five to ten days.

which is a derivative that is functionally equivalent to buying the stock.

The margin on exchange traded futures (or options) is set by the exchange. The principle for setting the margin for futures or options is the same as that described above. The margin is set such as to make the exchange almost immune to the default risk of the counterparty, and hence riskier contracts have larger margins.

A hedge fund must finance all of its positions, that is, the sum of all the margin requirements on long and short positions cannot exceed the hedge fund's capital. In our model, this is captured by the following key equation, which must be satisfied at any time  $t$ :

$$\sum_j \left( x_t^{j+} m_t^{j+} + x_t^{j-} m_t^{j-} \right) \leq W_t, \quad (1)$$

Here,  $x_t^{j+} \geq 0$  is the size of a long position and  $x_t^{j-} \geq 0$  is the size of a short position, so that the actual position is  $x_t^j = x_t^{j+} - x_t^{j-}$ .

At the end of the financing period, time  $t + 1$ , the position is “marked-to-market,” which means that the hedge fund receives any gains (or pays any losses) that have occurred between  $t$  and  $t + 1$ , that is, the fund receives  $x_t^j (p_{t+1}^j - p_t^j)$  and pays interest on the loan at the funding rate. If the trade is kept on, the broker keeps the margin to protect against losses going forward from time  $t + 1$ . The margin can be adjusted if the risk of the collateral has changed, unless the counterparties have contractually fixed the margin for a certain period. Stock exchanges and self-regulatory organizations (e.g., NASD) also impose maintenance/continuation margins for existing stock positions. For example, the NYSE and the NASD require that investors maintain a minimum margin of 25% for long stock positions and 30% for short stock positions.

Instead of posting risk-free assets (cash), a hedge fund can also post risky assets to cover his margin. However, in this case a “haircut” is subtracted from the risky asset's market value to account for the riskiness of the collateral. The haircut is equivalent to a margin since the hedge fund could alternatively have used the risky security to raise cash and then could use this cash to cover the margins for asset  $j$ . We therefore use the terms margins and

haircuts interchangeably.

We have described how funding constraints work when margins and haircuts are set separately for each security position. It is, however, sometimes possible to “cross-margin”, i.e. to jointly finance several trades that are part of the same strategy. This leads to a lower total margin if the risks of the various positions are partially offsetting. For instance, much of the interest rate risk is eliminated in a “spread trade” with a long position in one bond and a short position in a similar bond. Hence, the margin/haircut of a jointly financed spread trade is smaller than the sum of the margins of the long and short bonds. For a strategy that is financed jointly, we can reinterpret security  $j$  as such a strategy. Prime brokers compete by, among other things, offering low margins and haircuts — a key consideration for hedge funds — which means that it is becoming increasingly easy to finance more and more strategies jointly. In the extreme, one can imagine a joint financing of a hedge fund’s total position such that the “portfolio margin” would be equal to the maximum portfolio loss with a certain confidence level. Currently, it is often not practical to jointly finance a large portfolio. This is because a large hedge fund finances its trades using several brokers; both a hedge fund and a broker can consist of several legal entities (possibly located in different jurisdictions); certain trades need separate margins paid to exchanges (e.g., futures and options) or to other counterparties of the prime broker (e.g., securities lenders); prime brokers may not have sufficiently sophisticated models to evaluate the diversification benefits (e.g., because they don’t have enough data on the historical performance of newer products such as CDOs); and because of other practical difficulties in providing joint financing. Further, if the margin requirement relies on assumed stress scenarios in which the securities are perfectly correlated (e.g., due to predatory trading as in Brunnermeier and Pedersen (2005)), then the portfolio margin constraint coincides with position-by-position margins.

## 1.2 Funding Requirements for Banks

A bank's capital consists of equity capital plus its long-term borrowings (including credit lines secured from individual or syndicates of commercial banks), reduced by assets that cannot be readily employed (e.g., goodwill, intangible assets, property, equipment, and capital needed for daily operations), and further reduced by uncollateralized loans extended by the bank to others (see e.g., Goldman Sachs 2003 Annual Report). Banks also raise money using short-term uncollateralized loans such as commercial paper and promissory notes, and, in the case of commercial banks, demand deposits. These sources of financing cannot, however, be relied on in times of funding crises since lenders may be unwilling to continue lending, and therefore this short-term funding is often not included in measures of capital.

The financing of a bank's trading activity is largely based on *collateralized* borrowing. Banks can finance long positions using collateralized borrowing from corporations, other banks, insurance companies, and the Federal Reserve Bank, and can borrow securities to shortsell from, for instance, mutual funds and pension funds. These transactions typically require margins which must be financed by the bank's capital as captured by the funding constraint (1).

The financing of a bank's proprietary trading is more complicated than that of a hedge fund, however. For instance, banks may negotiate zero margins with certain counterparties, and banks can often sell short shares held in house, that is, held in a customer's margin account (in "street name") such that the bank does not need to use capital to borrow the shares externally. Further, a bank receives margins when financing hedge funds (i.e., the margin is negative from the point of view of the bank). However, often the bank wants to pass on the trade to an exchange or another counterparty and hence has to pay a margin to the exchange. In spite of these caveats, we believe that in times of stress, banks face margin requirements and are ultimately subject to a funding constraint in the spirit of (1). For instance, Goldman Sachs (2003 Annual Report, page 62) states that it seeks to maintain net capital in excess of total margins and haircuts that it would face in periods of market stress

plus the total draws on unfunded commitments at such times. In addition, Goldman Sachs recognizes that it may not have access to short-term borrowing during a crisis, that margins and haircuts may increase during such a crisis, and that counterparties may withdraw funds at such times.

Banks must also satisfy certain regulatory requirements. Commercial banks are subject to the Basel accord, supervised by the Federal Reserve system for US banks. In short, the Basel accord of 1988 requires that a bank's "eligible capital" exceeds 8% of the "risk-weighted asset holdings," which is the sum of each asset holding multiplied by its risk weight. The risk weight is 0% for cash and government securities, 50% for mortgage-backed loans, and 100% for all other assets. The requirement posed by the 1988 Basel accord corresponds to Equation (1) with margins of 0%, 4%, and 8%, respectively. In 1996, the accord was amended, allowing banks to measure market risk using an internal model based on portfolio VaRs rather than using standardized risk weights.

U.S. broker-speculators, including banks acting as such, are subject to the Securities and Exchange Commission's (SEC)'s "net capital rule" (SEC Rule 15c3-1). This rule stipulates, among other things, that a broker must have a minimum "net capital," which is defined as equity capital plus approved subordinate liabilities minus "securities haircuts" and operational charges. The haircuts are set as security-dependent percentages of the market value. The standard rule requires that the net capital exceeds at least  $6\frac{2}{3}\%$  (15:1 leverage) of aggregate indebtedness (broker's total money liabilities) or alternatively 2% of aggregate debit items arising from customer transactions. This constraint is similar in spirit to (1).<sup>7</sup> As of August 20, 2004, SEC amended (SEC Release No. 34-49830) the net capital rule for Consolidated Supervised Entities (CSE)'s such that CSE's may, under certain circumstances, use their internal risk models to determine whether they fulfill their capital requirement.

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<sup>7</sup>Let  $L$  be the lower of  $6\frac{2}{3}\%$  of total indebtedness or 2% of debit items and  $h^j$  the haircut for security  $j$ ; then the rule requires that  $L \leq W - \sum_j h^j x^j$ , that is,  $\sum_j h^j x^j \leq W - L$ .

### 1.3 Funding Requirements for Market Makers

There are various types of market-making firms. Some are small partnerships, whereas others are parts of large investment banks. The small firms are financed in a similar way to hedge funds in that they rely primarily on collateralized financing; the funding of banks was described in Section 1.2.

Certain market makers, such as NYSE specialists, have an obligation to make a market and a binding funding constraint means that they cannot fulfill this requirement. Hence, avoiding the funding constraint is especially crucial for such market makers.

Market makers are in principle subject to the SEC's net capital rule (described in Section 1.2), but this rule has special exceptions for market makers. Hence, market makers' main regulatory requirements are those imposed by the exchange on which they operate. These constraints are often similar in spirit to (1).

## 2 Model

**Setup.** The economy has  $J$  risky assets, traded at times  $t = 0, 1, 2, 3$ . At time  $t = 3$ , each security  $j$  pays off  $v^j$ , a random variable defined on a probability space  $(\Omega, \mathcal{F}, \mathcal{P})$ . There is no aggregate risk and the risk-free interest rate is normalized to zero, so the fundamental value of each stock is its conditional expected value of the final payoff  $v_t^j = E_t[v^j]$ . Fundamental volatility has an autoregressive conditional heteroscedasticity (ARCH) structure. Specifically,  $v_t^j$  evolves according to

$$v_{t+1}^j = v_t^j + \Delta v_{t+1}^j = v_t^j + \sigma_{t+1}^j \varepsilon_{t+1}^j, \quad (2)$$

where all  $\varepsilon_t^j$  are i.i.d. across time and assets with a standard normal cumulative distribution function  $\Phi$  with zero mean and unit variance, and the volatility  $\sigma_t^j$  has dynamics

$$\sigma_{t+1}^j = \underline{\sigma}^j + \theta^j |\Delta v_t^j|, \quad (3)$$

where  $\underline{\sigma}^j, \theta^j \geq 0$ . A positive  $\theta^j$  implies that shocks to fundamentals increase future volatility.

There are three groups of market participants: “customers” and “speculators” trade assets while “financiers” finance speculators’ positions. The group of customers consists of three risk-averse agents. At time 0, customer  $k = 0, 1, 2$  has a cash holding of  $W_0^k$  bonds and zero shares, but finds out that he will experience an endowment shock of  $\mathbf{z}^k = \{z^{1,k}, \dots, z^{J,k}\}$  shares at time  $t = 3$ , where  $\mathbf{z}$  is a random variables such that the aggregate endowment shock is zero,  $\sum_{k=0}^2 z^{j,k} = 0$ .

The basic liquidity problem arises because customers may arrive sequentially, which gives rise to order imbalance. In particular, customer  $k$  only begins trading at time  $t = k$  with probability  $a$ , while all customers start trading at once at  $t = 0$  with probability  $(1 - a)$ . Before a customer arrives at the marketplace, his demand is  $\mathbf{y}_t^k = 0$ . After he arrives, he chooses his security position  $\mathbf{y}_t^k$  in order to maximize his exponential utility function  $U(W_3^k) = -\exp\{-\rho W_3^k\}$  over final wealth. Wealth  $W_t^k$ , including the value of the anticipated endowment shock of  $\mathbf{z}^k$  shares, evolves according to

$$W_{t+1}^k = W_t^k + (\mathbf{p}_{t+1} - \mathbf{p}_t)' (\mathbf{y}_t^k + \mathbf{z}^k). \quad (4)$$

The total demand shock of customers who have arrived in the market at time  $t$  is denoted by  $Z_t := \sum_{k=0}^t \mathbf{z}^k$ .

The early customers’ trading need is accommodated by speculators who provide liquidity/immediacy. Speculators are risk-neutral and maximize expected final wealth  $W_3$ . Speculators face the constraint that the total margin on their position  $\mathbf{x}_t$  cannot exceed their capital  $W_t$ :

$$\sum_j \left( x_t^{j+} m_t^{j+} + x_t^{j-} m_t^{j-} \right) \leq W_t, \quad (1)$$

where  $x_t^{j+} \geq 0$  and  $x_t^{j-} \geq 0$  are the positive and negative parts of  $x_t^j = x_t^{j+} - x_t^{j-}$ , respectively, and  $m_t^{j+} \geq 0$  and  $m_t^{j-} \geq 0$  are the dollar margin on long and short positions, respectively. The institutional features related to this key constraint are discussed in detail above in Section 1.

Speculators start out with an initial cash position of  $W_0$  and zero shares, and their wealth evolves according to

$$W_t = W_{t-1} + (\mathbf{p}_t - \mathbf{p}_{t-1})' \mathbf{x}_{t-1} + \eta_t, \quad (5)$$

where  $\eta_t$  is an independent wealth shock arising from other activities, e.g., the speculators' investment banking arm. If a speculator loses all his capital at time  $t$ ,  $W_t \leq 0$ , he can no longer invest because of the margin constraint (1), i.e. he must choose  $x_t = 0$ . We let his utility in this case be  $\varphi_t W_t$ , where  $\varphi_t \geq 0$ . Limited liability corresponds to  $\varphi_t = 0$ , and a proportional bankruptcy cost (e.g., monetary, reputational, or opportunity costs) corresponds to  $\varphi_t > 0$ . We focus on the case in which  $\varphi_2 = 1$ , that is, negative consumption equal to time-2 the dollar loss, and we discuss  $\varphi_1$  in Section 6. Our results would be qualitatively the same with other bankruptcy assumptions.<sup>8</sup>

The financier sets the margins to limit his counterparty credit risk. Specifically, the financier ensures that the margin is large enough to cover the position's  $\pi$ -value-at-risk (where  $\pi$  is a nonnegative number close to zero, e.g., 1%):

$$\pi = Pr(-\Delta p_{t+1}^j > m_t^{j+} | \mathcal{F}_t) \quad (6)$$

$$\pi = Pr(\Delta p_{t+1}^j > m_t^{j-} | \mathcal{F}_t). \quad (7)$$

Equation (6) means that the margin on a long position  $m^+$  is set such that price drops larger than the margin only happen with a small probability  $\pi$ . Similarly, (7) means that price increases larger than the margin on a short position only happen with small probability. Clearly, the margin is larger for more volatile assets. The margin depends on financiers' information set  $\mathcal{F}_t$ . We consider two important benchmarks: "informed financiers" who know the fundamental value and the liquidity shocks  $z$ ,  $\mathcal{F}_t = \sigma\{\mathbf{z}, \mathbf{v}_0, \dots, \mathbf{v}_t, \mathbf{p}_0, \dots, \mathbf{p}_t, \eta_1, \dots, \eta_t\}$ , and "uninformed financiers" who only observe prices,  $\mathcal{F}_t = \sigma\{\mathbf{p}_0, \dots, \mathbf{p}_t\}$ . This margin specifica-

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<sup>8</sup>We could allow the speculators to raise new capital as long as this takes time. Indeed, the model would be the same if the speculators could raise capital only at time 2 (and in this case we need not assume that the customers' endowment shocks  $z^j$  aggregate to zero).



tion is motivated by the real-world institutional features described in Section 1. Theoretically, Stiglitz and Weiss (1981) show how credit rationing can be due to adverse selection and moral hazard in the lending market, and Geanakoplos (2003) considers endogenous contracts in a general-equilibrium framework of imperfect commitment.

We let  $\Lambda_t^j$  be the (signed) deviation of the price from fundamental value

$$\Lambda_t^j = p_t^j - v_t^j, \quad (8)$$

and we define our measure of market illiquidity as the absolute amount of this deviation,  $|\Lambda_t^j|$ . We consider competitive equilibria of the economy:

**Definition 1** *An equilibrium is a price process  $\mathbf{p}_t$  such that (i)  $\mathbf{x}_t$  maximizes the speculators' expected final profit subject to the margin constraint (1), (ii) each  $\mathbf{y}_t^k$  maximizes  $k$ -customers expected utility after their arrival at the marketplace and is zero beforehand, (iii) margins are set according to the VaR specification (6), and (iv) markets clear,  $\mathbf{x}_t + \sum_{k=0}^2 \mathbf{y}_t^k = 0$ .*

**Outline of Equilibrium.** We derive the optimal strategies for customers and speculators using dynamic programming, starting from time 2, and working backwards. A customer's value function is denoted  $\Gamma$  and a speculator's value function is denoted  $J$ . At time 2, customer  $k$ 's problem is

$$\Gamma_2(W_2^k, p_2, v_2) = \max_{y_2^k} -E_2[e^{-\gamma W_3^k}] \quad (9)$$

$$= \max_{y_2^k} -e^{-\gamma(E_2[W_3^k] - \frac{\gamma}{2} \text{Var}_2[W_3^k])} \quad (10)$$

which has the solution

$$y_2^{j,k} = \frac{v_2^j - p_2^j}{\gamma(\sigma_3^j)^2} - z^{j,k} \quad (11)$$

Clearly, since all customers are present in the market at time 2, the unique equilibrium is  $p_2 = v_2$ . Indeed, when the prices are equal to fundamentals, the aggregate customer demand is zero,  $\sum_k y_2^{j,k} = 0$ , and the speculator also has a zero demand. We get the

customer's value function  $\Gamma_2(W_2^k, p_2 = v_2, v_2) = -e^{-\gamma W_2^k}$ , and the speculator's value function  $J_2(W_2, p_2 = v_2, v_2) = W_2$ .

The equilibrium before time 2 depends on whether the customers arrive sequentially or all at time 0. If all customers arrive at time 0, then the simple arguments above show that  $p_t = v_t$  at any time  $t = 0, 1, 2$ .

We are interested in the case with sequential arrival of the customers such that the speculators' liquidity provision is needed. At time 1, customers 0 and 1 are present in the market, but customer 2 has not arrived yet. As above, customer  $k = 0, 1$  has a demand and value function of

$$y_1^{j,k} = \frac{v_1^j - p_1^j}{\gamma(\sigma_2^j)^2} - z^{j,k} \quad (12)$$

$$\Gamma_1(W_1^k, p_1, v_1) = -\exp \left\{ -\gamma \left[ W_1^k + \sum_j \frac{(v_1^j - p_1^j)^2}{2\gamma(\sigma_2^j)^2} \right] \right\} \quad (13)$$

At time 0, customer  $k = 0$  arrives in the market and maximizes  $E_0[\Gamma_1(W_1^k, p_1, v_1)]$ .

At time  $t = 1$ , if the market is perfectly liquid so that  $p_1^j = v_1^j$  for all  $j$ , then the speculator is indifferent among all possible positions  $x_1$ . If some securities have  $p_1 \neq v_1$ , then the risk-neutral speculator invests all his capital such that his margin constraint binds. The speculator optimally trades only in securities with the highest expected profit per dollar used. The profit per dollar used is  $(v_1^j - p_1^j)/m_1^{j+}$  on a long position and  $-(v_1^j - p_1^j)/m_1^{j-}$  on a short position. A speculator's shadow cost of capital, denoted  $\phi_1$ , is 1 plus the maximum profit per dollar used as long as he is not bankrupt:

$$\phi_1 = 1 + \max_j \left( \max \left\{ \frac{v_1^j - p_1^j}{m_1^{j+}}, \frac{-(v_1^j - p_1^j)}{m_1^{j-}} \right\} \right), \quad (14)$$

where the margins for long and short positions are set by the financier as described in the next section. If the speculator is bankrupt  $W_1 < 0$  then  $\phi_1 = \varphi_1$ . Each speculator's value

function is therefore

$$J_1(W_1, p_1, v_1, p_0, v_0) = W_1 \phi_1. \quad (15)$$

At time  $t = 0$ , the speculator maximizes  $E_0[W_1 \phi_1]$  subject to his capital constraint (1).

The equilibrium prices at times 1 and 0 do not have simple expressions. However, after having derived the margin conditions, we characterize several important properties of these prices, which illuminates the connection between market liquidity,  $|\Lambda|$ , and speculators' funding situation.

### 3 Margin Setting and Liquidity (Time 1)

We want to determine the financiers' margin,  $m_1$ , at time 1, both in the case of informed and uninformed financiers. The financier sets the margin such that it covers the position's value-at-risk, knowing that prices equal the fundamental values in the next period,  $p_2 = v_2$ .

If informed financiers know the fundamental values  $v_1$  (or, equivalently, know the demand shocks  $z_0, z_1$ ), they are aware of  $\Lambda_1$ . Since  $\Lambda_2 = 0$ , margins on long positions at  $t = 1$  are set according to

$$\begin{aligned} \pi &= Pr(-\Delta p_2^j > m_1^{j+} | \mathcal{F}_1) \\ &= Pr(-\Delta v_2^j + \Lambda_1^j > m_1^{j+} | \mathcal{F}_1) \\ &= 1 - \Phi\left(\frac{m_1^{j+} - \Lambda_1^j}{\sigma_2^j}\right), \end{aligned} \quad (16)$$

which implies that

$$\begin{aligned} m_1^{j+} &= \Phi^{-1}(1 - \pi) \sigma_2^j + \Lambda_1^j \\ &= \bar{\sigma}^j + \bar{\theta} |\Delta v_1^j| + \Lambda_1^j \end{aligned} \quad (17)$$

where we define

$$\bar{\sigma}^j = \underline{\sigma}^j \Phi^{-1}(1 - \pi) \quad (18)$$

$$\bar{\theta} = \theta \Phi^{-1}(1 - \pi). \quad (19)$$

The margin on a short position can be derived similarly.

**Proposition 1 (Stabilizing Margins and the Cushioning Effect)** *When the financier is informed about the fundamental value and knows that prices will equal fundamentals in the next period  $t = 2$ , then the margins on long and short positions are, respectively,*

$$m_1^{j+} = \max\{\bar{\sigma}^j + \bar{\theta}|\Delta v_1^j| + \Lambda_1^j, 0\} \quad (20)$$

$$m_1^{j-} = \max\{\bar{\sigma}^j + \bar{\theta}|\Delta v_1^j| - \Lambda_1^j, 0\} \quad (21)$$

*The more prices are below fundamentals  $\Lambda_1^j < 0$ , the lower is the margin on a long position  $m_1^{j+}$ , and the more prices are above fundamentals  $\Lambda_1^j > 0$ , the lower is the margin on a short position  $m_1^{j-}$ . Hence, in this case illiquidity reduces margins for speculators who buy low and sell high.*

The margins are reduced by illiquidity because the speculator is expected to profit when prices return to fundamentals at time 2, and this profit “cushions” the speculators from losses due to fundamental volatility. Thus, we denote the margins set by informed financiers at  $t = 1$  as *stabilizing margins*.

Stabilizing margins are an interesting benchmark, and they are hard to escape in a theoretical model. However, real-world liquidity crises are often associated with increases in margins, not decreases. To capture this, we assume that fundamentals follow an ARCH process and turn to the case of a financier who is uninformed about the current fundamental so that he must set his margin based on the observed prices  $p_0$  and  $p_1$ . This is in general a complicated problem since the financier needs to filter out the probability that a liquidity shock occurred, and the values of  $z_0$  and  $z_1$ . The expression becomes simple however, if the

financier's prior probability of a liquidity shock is small so that he finds it likely that  $p_t^j = v_t^j$ , implying a common margin  $m_1^j = m_1^{j+} = m_1^{j-}$  for long and short positions in the limit:

**Proposition 2 (Destabilizing Margins)** *When the financier is uninformed about the fundamental value, then as  $a \rightarrow 0$  the margins on long and short positions approach*

$$m_1^j = \bar{\sigma}^j + \bar{\theta}|\Delta p_1^j| = \bar{\sigma}^j + \bar{\theta}|\Delta v_1^j + \Delta\Lambda_1^j|. \quad (22)$$

*Margins are increasing in price volatility and illiquidity shocks can increase margins.*

Intuitively, since liquidity risk tends to increase price volatility, and since an uninformed financier may interpret price volatility as fundamental volatility, this increases margins.<sup>9</sup> Equation (22) corresponds closely to real-world margin setting, which is primarily based on volatility estimates from past price movements. Equation (22) shows that illiquidity increases margins when the liquidity shock  $\Delta\Lambda_1^j$  has the same sign as the fundamental shock  $\Delta v_1^j$  or is greater in magnitude, but margins are reduced if the liquidity shock counterbalances a fundamental move. We denote the phenomenon that margins can increase as illiquidity rises by *destabilizing margins*. As we will see next, the information available to the financier — i.e., whether margins are stabilizing or destabilizing — has important implications for the equilibrium.

## 4 Fragility and Liquidity Spirals (Time 1)

This section discusses liquidity spirals and fragility — the property that a small change in fundamentals can lead to a large jump in liquidity. For simplicity we illustrate this with a single security  $J = 1$ .

**Fragility.** We say that liquidity is fragile if the equilibrium price  $p_t(\eta_t, v_t)$  cannot be chosen to be continuous in the exogenous shocks, namely  $\eta_t$  and  $\Delta v_t$ . Fragility arises when the excess

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<sup>9</sup>In the analysis of time 0, we shall see that margins can also be destabilizing when price volatility signals future liquidity risk (not necessarily fundamental risk).

demand for shares  $x_t + \sum_{k=0}^1 \mathbf{y}_1$  can be non-monotonic in the price. While under “normal” circumstances, a high price leads to a low total demand (i.e., excess demand is decreasing), binding funding constraints along with destabilizing margins (margin effect) or speculators’ losses (loss effect) can lead to an increasing demand curve.

It is natural to focus on *stable* equilibria. An equilibrium is stable if a small negative (positive) price perturbation leads to excess demand (supply), which, intuitively, “pushes” the price back up (down) to its equilibrium level.

**Proposition 3 (Fragility)** *(i) With informed financiers, the market is fragile at time 1 if speculators’ position  $x_0$  is large enough.*

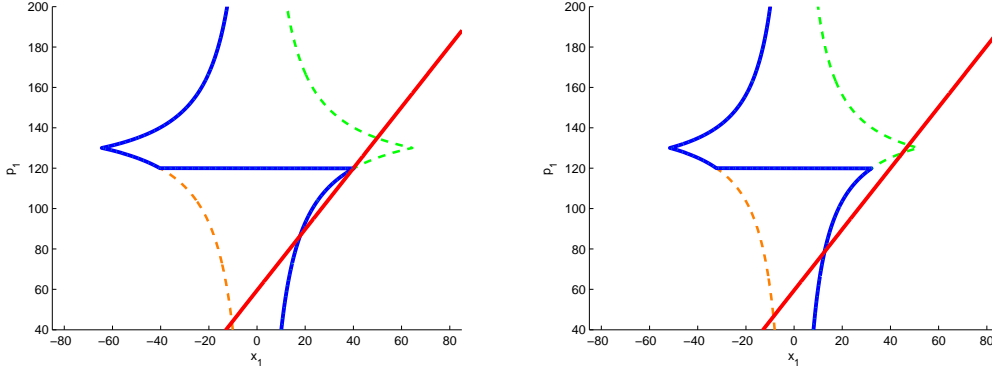
*(ii) With uninformed financiers, the market is fragile at time 1 if  $x_0$  large enough or if the ARCH parameter,  $\theta$ , is large enough and the financiers’ prior probability,  $a$ , of a liquidity shock is small enough.*

**Numerical Example.** We illustrate how fragility arises due to destabilizing margins or dealer losses by way of a numerical example. We consider the more interesting (and arguably more realistic) case in which the financiers are uninformed, and we choose parameters as follows.

The fundamental value has ARCH volatility parameters  $\underline{\sigma} = 5$  and  $\theta = 0.3$ , which implies clustering of volatility. The price in the previous time period was  $p_0 = 130$ , the aggregate demand shock of the customers who have arrived at time 1 is  $Z_1 = z_0 + z_1 = 40$ , and the customers’ risk aversion is  $\gamma = 0.025$ . The speculators’ have an initial position of  $x_0 = 0$  and a cash wealth of  $W_1 = 750$ . Finally, the financier uses a VaR with  $\pi = 1\%$  and  $a \rightarrow 0$ , leading to a margin requirement of  $m = \bar{\sigma} + \bar{\theta}|\Delta p_1| = 2.326(5 + 0.3|\Delta p_1|)$ .

We first show how speculators’ choose their optimal time-1 position  $x_1$ , and how the equilibrium price  $p_1$  depends on the exogenous shocks to their wealth  $\eta_1$  and on the security’s fundamentals  $\Delta v_1$ . Panel A of Figure 2 illustrates how the speculators’ demand  $x_1$  and the customers’ supply (i.e., the negative of the customers’ demand as per Equation (12)) depend on the price  $p_1$  when the fundamental value is  $v_1 = 120 < p_0 = 130$  and the speculators’

wealth shock is  $\eta_1 = 0$ . Customers' supply is given by the upward sloping line since, naturally, their supply is greater when the price is higher. Customers supply  $Z_1 = 40$  shares, namely the shares that they anticipate receiving at time  $t = 3$ , when the market is perfectly liquid,  $p_1 = v_1$  (i.e. illiquidity is  $|\Lambda_1| = 0$ ). For lower prices, they supply fewer shares.



**Figure 2: Speculator Demand and Customer Supply.** This figure illustrates how margins can be destabilizing when financiers are uninformed and the fundamentals have volatility clustering. Specifically, a speculator's margin constraint means that his position must be inside the four dashed lines (the “hyperbolic star”). The solid curve is the speculators' optimal demand. The upward sloping line is the customers' supply, that is, the negative of their demand. In Panel A the speculators experience a zero wealth shock,  $\eta_1 = 0$ , while in Panel B they face a negative wealth shock of  $\eta_1 = -150$ , otherwise everything is the same. In Panel A, perfect liquidity  $p_1 = v_1 = 120$  is one of two stable equilibria, while in Panel B the unique equilibrium is illiquid.

The speculators must satisfy their margin constraints  $|x_1| \leq W_1/(\bar{\sigma} + \bar{\theta}|\Delta p_1|)$ , which graphically means that their demand must be inside the “hyperbolic star” defined by the four (dashed) hyperbolas. At the price  $p_1 = p_0 = 130$ , the margin is smallest and hence the constraint is most relaxed. As  $p_1$  departs from  $p_0 = 130$ , margins increase and speculators become more constrained — the horizontal distance between two hyperbola shrinks.

The speculators' demand curve is given by the solid line in the figure: For  $p_1 = v_1 = 120$ , the security's expected return is zero and the speculator is indifferent between all his possible positions on the horizontal line. For price levels  $p_1 < v_1$  below this line, the risk neutral

speculators want to buy the asset,  $x_1 > 0$ , and their demand is constrained by the lower right side of the star. Similarly, for prices above  $v_1$ , speculators short the asset,  $x_1 < 0$ , and their demand is limited by the left side of the star. Interestingly, the speculators' demand is upward sloping for prices below  $v_1 = 120$  because a larger price drop  $\Delta p_1$  increases the financiers' estimate of fundamental volatility and consequently of margins.

There are two stable equilibria: a perfect liquidity equilibrium with price  $p_1 = v_1 = 120$  and an illiquid equilibrium with a price of about 86.5 (and an uninteresting unstable equilibrium between the two stable ones).

Panel B of Figure 2 shows the same plot as Panel A, but with a negative wealth shock to speculators of  $\eta_1 = -150$  instead of  $\eta_1 = 0$ . In this case, perfect liquidity with  $p_1 = v_1$  is no longer an equilibrium since the speculator cannot fund a large enough position. The unique equilibrium is highly illiquid because of the speculators' lower wealth and, importantly, because of endogenously higher margins.

This “disconnect” between the perfect-liquidity equilibrium and the illiquid equilibrium and the resulting fragility is illustrated more directly in Figure 3. Panel A plots the equilibrium price correspondence for different exogenous funding shocks  $\eta_1$  (with fixed  $\Delta v_1 = -5$ ) and shows that a marginal reduction in funding cannot always lead to a smooth reduction in market liquidity. Rather, there must be a level of funding such that an infinitesimal drop in funding leads to a discontinuous drop in market liquidity.

Panel B of Figure 3 plots the equilibrium price correspondence for different realizations of the fundamental shock  $\Delta v_1$  (with fixed  $\eta_1 = 0$ ) and shows the same form of discontinuity for adverse fundamental shocks to  $v_1$ . The discontinuity with respect to  $\Delta v_1$  is most easily understood in conjunction with Figure 2 Panel A. As  $\Delta v_1$  falls, the horizontal line of speculator demand shifts downward within the star, and the customer supply line moves downward. As a result, the perfect liquidity equilibrium vanishes. Panel B also reveals the interesting asymmetry that negative fundamental shocks lead to larger price movements than corresponding positive shocks (for  $Z_1 := z_0 + z_1 > 0$ ).



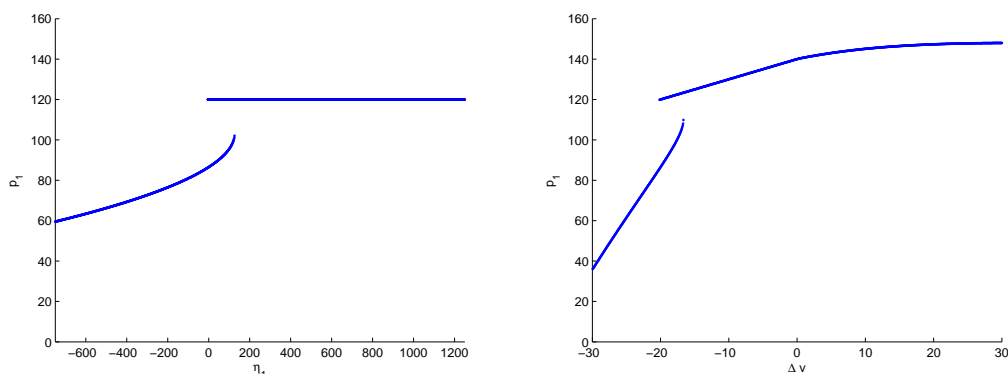


Figure 3: **Fragility due to Destabilizing Margins.** The figure shows the equilibrium price correspondence as a function of the speculators’ wealth shock  $\eta_1$  (Panel A) and of fundamental shocks  $\Delta v_1$  (Panel B). The margins are destabilizing since financiers are uninformed and fundamentals exhibit volatility clustering. The discontinuity of the equilibrium prices reflects fragility in liquidity since a small shock can lead to a disproportionately large price effect.

Fragility can also arise because of shocks to customer demand or volatility. For instance, the price correspondence  $p_1$  is discontinuous in the customers’ shock  $Z_1$  since a larger  $Z_1$  shifts customers’ supply in Figure 2 down such that the perfect liquidity equilibrium vanishes. When this happens, a marginally larger demand for liquidity by customers leads, paradoxically, to a drastic reduction of liquidity supply by the speculators.

So far we have considered speculators with zero initial positions,  $x_0 = 0$ . If  $x_0 > 0$ , then lower prices lead to losses for the speculators, and graphically this means that the constraints in the “hyperbolic star” tighten (i.e., the gap between the hyperbolas narrows) at low prices. Because of this “loss effect,” the discontinuous price drop associated with the illiquid equilibrium is even larger.

The discontinuity in prices driven by destabilizing margins and dealer losses can help to explain the sudden market liquidity dry-ups observed in many markets. For example, Russia’s default in 1998 was in itself only a trivial wealth shock relative to global arbitrage capital. Nevertheless, it had a large effect on liquidity in global financial markets, consistent with our fragility result that a small wealth shock can push the equilibrium over the edge.

**Liquidity Spirals.** This subsection shows that once the economy enters an illiquid equilibrium, market liquidity becomes highly sensitive to shocks. We identify two amplification mechanisms, a “margin spiral” due to increasing margins as speculator financing worsens, and a “loss spiral” due to escalating speculator losses. Figure 4 illustrates these “liquidity

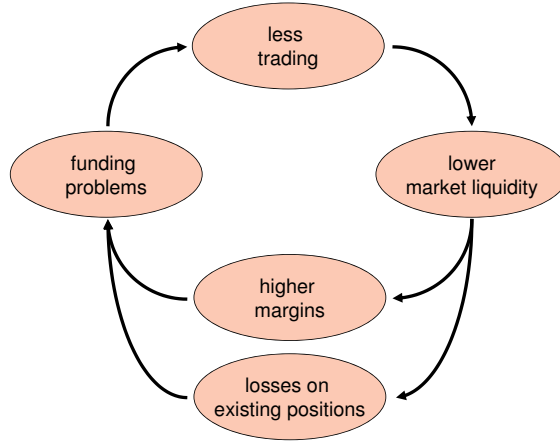


Figure 4: **Liquidity Spirals**

spirals”: A shock to speculator capital,  $\eta_1 < 0$ , forces speculators to provide less market liquidity, which increases the price impact of the customer demand pressure. With uninformed financiers and a positive ARCH effect  $\theta > 0$ , the resulting price swing increases financiers’ estimate of the fundamental volatility and, hence, increases the margin, thereby worsening speculator funding problems even further, and so on, leading to a “margin spiral.” Similarly, increased market illiquidity can lead to losses on speculators’ existing positions, worsening their funding problem and so on, leading to a “loss spiral.” Mathematically, the spirals can be expressed as follows:

**Proposition 4**

- (i) *If speculators’ capital constraint is slack then the price  $p_1$  is equal to  $v_1$  and insensitive to local changes in speculator wealth.*
- (ii) **(Liquidity Spirals)** *In a stable illiquid equilibrium with selling pressure from customers,*

$Z_1, x_1 > 0$ , the price sensitivity to speculator wealth shocks  $\eta_1$  is

$$\frac{\partial p_1}{\partial \eta_1} = \frac{1}{\frac{2}{\gamma(\sigma_2)^2}m_1^+ + \frac{\partial m_1^+}{\partial p_1}x_1 - x_0} \quad (23)$$

and with buying pressure from customers,  $Z_1, x_1 < 0$ ,

$$\frac{\partial p_1}{\partial \eta_1} = \frac{-1}{\frac{2}{\gamma(\sigma_2)^2}m_1^- + \frac{\partial m_1^-}{\partial p_1}x_1 + x_0}. \quad (24)$$

A margin spiral arises if  $\frac{\partial m_1^+}{\partial p_1} < 0$  or  $\frac{\partial m_1^-}{\partial p_1} > 0$ , which happens with positive probability if speculators are uninformed and  $a$  is small enough. A loss spiral arises if speculators' previous position is in the opposite direction as the demand pressure  $x_0 Z_1 > 0$ .

This proposition is intuitive. Imagine first what happens if speculators face a wealth shock of \$1, margins are constant, and speculators have no inventory  $x_0 = 0$ . In this case, the speculator must reduce his position by  $1/m_1$ . Since the slope of each of the two customer demand curves is<sup>10</sup>  $1/(\gamma(\sigma_2)^2)$ , we get a total price effect of  $1/(\frac{2}{\gamma(\sigma_2)^2}m_1)$ .

The two additional terms in the denominator imply amplification or dampening effects due to changes in the margin requirement and to profit/losses on the speculators' existing positions. To see that, recall that for any  $k > 0$  and  $l$  with  $|l| < k$ , it holds that  $\frac{1}{k-l} = \frac{1}{k} + \frac{l}{k^2} + \frac{l^2}{k^3} + \dots$ ; so with  $k = \frac{2}{\gamma(\sigma_2)^2}m_1$  and  $l = -\frac{\partial m_1^\pm}{\partial p_1}x_1 \pm x_0$ , each term in this infinite series corresponds to one loop around the circle in Figure 4. The total effect of the changing margin and speculators' positions amplifies the effect if  $l > 0$ . Intuitively, if e.g.  $Z_1 > 0$ , then customer selling pressure is pushing down the price, and  $\frac{\partial m_1^+}{\partial p_1} < 0$  means that as prices go down, margins increase, making speculators' funding tighter and thus destabilizing the system. Similarly, when customers are buying,  $\frac{\partial m_1^-}{\partial p_1} > 0$  implies that increasing prices leads to increased margins, making it harder for speculators to shortsell, thus destabilizing the system. The system is also destabilized if speculators' lose money on their previous position

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<sup>10</sup>See Equation (12).

as prices move away from fundamentals similar to e.g., Shleifer and Vishny (1997).

Interestingly, the total effect of a margin spiral together with a loss spiral is greater than the sum of their separate effects. This can be seen mathematically by using simple convexity arguments, and it can be seen intuitively from the flow diagram of Figure 4.

Note that spirals can also be “started” by shocks to liquidity demand  $Z_1$ , fundamentals  $v_1$ , or volatility. It is straightforward to compute the price sensitivity with respect to such shocks. They are just multiples of  $\frac{\partial p_1}{\partial \eta_1}$ . For instance, a fundamental shock affects the price both because of its direct effect on the final payoff and because of its effect on customers’ estimate of future volatility — and both of these effects are amplified by the liquidity spirals.

Our analysis sheds some new light on the 1987 stock market crash, complementing the standard culprit, portfolio insurance trading. In the 1987 stock market crash, numerous market makers hit (or violated) their funding constraint:

“By the end of trading on October 19, [1987] thirteen [NYSE specialist] units had no buying power”

— SEC (1988), page 4-58

Several of these firms managed to reduce their positions and continue their operations. Others did not. For instance, Tompane was so illiquid that it was taken over by Merrill Lynch Specialists and Beauchamp was taken over by Spear, Leeds & Kellogg (Beauchamp’s clearing broker). Also, market makers outside the NYSE experienced funding troubles: the Amex market makers Damm Frank and Santangelo were taken over; at least 12 OTC market makers ceased operations; and several trading firms went bankrupt.

These funding problems were due to (i) reductions in capital arising from trading losses and defaults on unsecured customer debt, (ii) an increased funding need stemming from increased inventory and, (iii) increased margins. One New York City bank, for instance, increased margins/haircuts from 20% to 25% for certain borrowers, and another bank increased margins from 25% to 30% for all specialists (SEC (1988) page 5-27 and 5-28). Other banks reduced the funding period by making intra-day margin calls, and at least two banks made

intra-day margin calls based on assumed 15% and 25% losses, thus effectively increasing the haircut by 15% and 25%. Also, some broker-dealers experienced a reduction in their line of credit and – as Figure 1 shows – margins at the futures exchanges also drastically increased. (See SEC (1988) and Wigmore (1998).)

In summary, our results on fragility and liquidity spirals imply that during “bad” times, small changes in underlying funding conditions (or liquidity demand) can lead to sharp reductions in liquidity. The 1987 crash exhibited several of the predicted features, namely capital constrained dealers, increased margins, and increased illiquidity.

## 5 Commonality and Flight to Quality (Time 1)

We now turn to the cross-sectional implications of illiquidity. Since speculators are risk-neutral, they optimally invest all their capital in securities that have the greatest expected profit  $|\Lambda^j|$  per capital use, i.e., per dollar margin  $m^j$ , as expressed in Equation (14). That equation also introduces the shadow cost of capital  $\phi_1$  as the marginal value of an extra dollar. The speculators’ shadow cost of capital  $\phi_1$  captures well the notion of funding liquidity: a high  $\phi$  means that the available funding — from capital  $W_1$  and from collateralized financing with margins  $m_1^j$  — is low relative to the needed funding, which depends on the investment opportunities deriving from demand shocks  $z^j$ .

The market liquidity of all assets depend on the speculators’ funding liquidity, especially for high-margin assets, and this has several interesting implications:

**Proposition 5** *Suppose  $\theta^j$  is close enough to zero for all  $j$ , and financiers are either informed or uninformed with probability  $a$  small enough. Then we have:*

**(i) (Commonality of Market Liquidity)** *The market illiquidities  $|\Lambda|$  of any two securities,  $k$  and  $l$ , co-move,*

$$\text{Cov}_0 \left( |\Lambda_1^k|, |\Lambda_1^l| \right) \geq 0, \tag{25}$$

*and market illiquidity co-moves with funding illiquidity as measured by speculators’ shadow*

cost of capital  $\phi_1$

$$\text{Cov}_0 \left[ |\Lambda_1^k|, \phi_1 \right] \geq 0. \quad (26)$$

**(ii) (Commonality of Fragility)** *Jumps in market liquidity occur simultaneously for all assets for which speculators are marginal investors.*

**(iii) (Quality=Liquidity)** *If asset  $l$  has lower fundamental volatility than asset  $k$ ,  $\underline{\sigma}^l < \underline{\sigma}^k$ , then  $l$  also has lower market illiquidity,*

$$|\Lambda_1^l| \leq |\Lambda_1^k| \quad (27)$$

**(iv) (Flight to Quality)** *The market liquidity differential between high- and low-fundamental-volatility securities is bigger when speculator funding is tight, that is,  $\underline{\sigma}^l < \underline{\sigma}^k$  implies that  $|\Lambda_1^k|$  increases more with a negative wealth shock to the speculator,*

$$\frac{\partial |\Lambda_1^l|}{\partial (-\eta_1)} \leq \frac{\partial |\Lambda_1^k|}{\partial (-\eta_1)}, \quad (28)$$

if  $x_1^k \neq 0$ . Further, if with large enough probability  $x^k \neq 0$ , then

$$\text{Cov}_0(|\Lambda_1^l|, \phi_1) \leq \text{Cov}_0(|\Lambda_1^k|, \phi_1). \quad (29)$$

**Numerical Example, Continued.** To illustrate these cross-sectional predictions, we extend the numerical example of Section 4 to two securities. The two securities only differ in their long-run fundamental volatility:  $\bar{\sigma}^1 = 5$  and  $\bar{\sigma}^2 = 7.5$ . The other parameters are as before, except that we double  $W_1$  to 1500 since the speculators now trade two securities, and the financiers remain uninformed.

Figure 5 depicts the assets' equilibrium prices for different values of the funding shock  $\eta_1$ . First note that as speculator funding tightens and our funding illiquidity measure  $\phi_1$  rises, the market illiquidity measure  $|\Lambda_1^j|$  rises for both assets. Hence, for random  $\eta_1$ , we see our commonality in liquidity result  $\text{Cov}_0 [|\Lambda_1^k|, |\Lambda_1^l|] > 0$ .

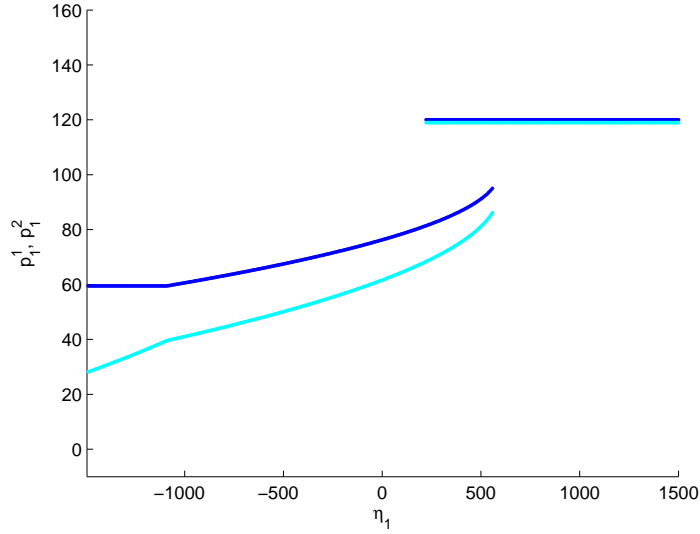


Figure 5: **Flight to Quality and Commonality in Liquidity.** The figure plots that price  $p_1^j$  of assets 1 and 2 as functions of speculators' funding shocks  $\eta_1$ . Asset 1 has lower long-run fundamental risk than asset 2,  $\bar{\sigma}^1 = 5 < 7.5 = \bar{\sigma}^2$ .

The “commonality in fragility” cannot directly be seen from Figure 5, but it is suggestive that both assets have the same range of  $\eta_1$  with 2 equilibrium prices  $p_1^j$ . The intuition for this result is the following. Whenever funding is unconstrained, there is perfect market liquidity provision for all assets. If funding is constrained, then it cannot be the case that speculators provide perfect liquidity for one asset but not for the other, since they always would have an incentive to shift funds towards the asset with non-perfect market liquidity. Hence, market illiquidity jumps for both assets exactly at the same funding level.

Our result relating fundamental volatility to market liquidity (“Quality=Liquidity”) is reflected in  $p_1^2$  being below  $p_1^1$  for any given funding level. Hence, the high-fundamental-volatility asset 2 is always less liquid than the low-fundamental-volatility asset 1.

The graph also illustrates our result on “flight to quality.” To see this, consider the two securities' relative price sensitivity with respect to  $\eta_1$ . For large wealth shocks, market liquidity is perfect for both assets, i.e.  $p_1^1 = p_1^2 = v_1^1 = v_1^2 = 120$  so in this high range

of funding, market liquidity is insensitive to marginal changes in funding. For sufficiently small funding levels,  $\eta_1 < 222$ , market illiquidity of both assets increases as  $\eta_1$  drops since speculators must take smaller stakes in both assets. Importantly, as funding decreases,  $p_1^2$  decreases more steeply than  $p_1^1$ , that is, asset 2 is more sensitive to funding declines than asset 1. This is because speculators cut back more on the “funding-intensive” asset 2 with its high margin requirement. The speculators want to maximize their profit per dollar margin,  $|\Lambda^j|/m^j$  and therefore  $|\Lambda^2|$  must be higher than  $|\Lambda^1|$  to compensate speculators for using more capital for margin.

Both price functions exhibit a kink around  $\eta = -1086$ , because, for sufficiently low funding levels, speculators put all their capital into asset 2. This is because the customers are more eager to sell the more volatile asset 2, leading to more attractive prices for the speculators.

## 6 Liquidity Risk (Time 0)

In this section, we focus on  $t = 0$  and demonstrate that (i) funding liquidity risk matters even before margin requirements actually bind, (ii) the pricing kernel depends on future funding liquidity,  $\phi_{t+1}$ , (iii) the conditional distribution of prices  $p_1$  is skewed due to the funding constraint, and (iv) margins  $m_0$  and illiquidity  $\Lambda_0$  can be positively related due to liquidity risk even if financiers are informed.

If speculators have no dis-utility associated with negative wealth levels  $W_1 < 0$ , then they go to their limit already at time 0. While most firms legally have limited liability, we note that the capital  $W$  in our model refers to pledgable capital allocated to trading. For instance, Lehman Brothers (2001 Annual Report, page 46) states that

“the following must be funded with cash capital: Secured funding ‘haircuts,’ to reflect the estimated value of cash that would be advanced to the Company by counterparties against available inventory, Fixed assets and goodwill, [and] Operational cash ... ,”

Hence, if a speculator suffers a large loss on his pledgable capital such that  $W_t < 0$ , then he



incurs monetary costs that he has to cover with his unpledgable capital like operational cash. In addition he incurs non-monetary cost, like loss in reputation and in goodwill, that reduces his ability to exploit future profitable investment opportunities. To capture these effects, we let a speculators' utility be  $\phi_1 W_1$ , where  $\phi_1$  is given by the right-hand side of (14) both for positive and negative values of  $W_1$ . With this assumption, equilibrium prices at time  $t = 0$  are such that the speculators do not trade to their constraint at time  $t = 0$  when their wealth is large enough.<sup>11</sup> In fact, this is the weakest assumption that curbs the speculators' risk taking since it makes their objective function linear. Higher "bankruptcy costs", like e.g.  $\varphi_1 = \max\{\phi_1\}$  for  $W_1 < 0$ , would lead to more cautious trading at time 0 and qualitatively similar results.

If the speculator is not constrained at time  $t = 0$ , then the first-order condition for his position in security  $j$  is  $E_0[\phi_1(p_1^j - p_0^j)] = 0$ . (We leave the case of a constrained time-0 speculator for the appendix.) Consequently, the funding liquidity  $\phi_1$  determines the pricing kernel for the cross section of securities:

$$p_0^j = \frac{E_0[\phi_1 p_1^j]}{E_0[\phi_1]} = E_0[p_1^j] + \frac{Cov_0[\phi_1, p_1^j]}{E_0[\phi_1]}. \quad (30)$$

Equation (30) shows that the price at time 0 is the expected time-1 price — which already depends on the liquidity shortage at time-1 — further adjusted for liquidity risk in the form of a covariance term. The liquidity risk term is intuitive: The time-0 price is lower if the covariance is negative, that is, if the security has a low payoff during future funding liquidity crises when  $\phi_1$  is high.

The cost of hitting a funding constraint and the importance of funding-liquidity management are illustrated by the "LTCM crisis" after the Russian default in 1998. The hedge fund

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<sup>11</sup>Note that an adverse shock lowers speculator wealth at  $t = 1$ , but creates a profitable investment opportunity in  $t = 1$ . One might think that the latter effect provides a natural "dynamic hedge" and hence, speculators (with a relative risk aversion coefficient larger than 1) increase their  $t = 0$  hedging demand, which in turn, lowers illiquidity in  $t = 0$ . However, exactly the opposite occurs in a setting with capital constraints. Capital constraints prevent speculators from taking advantage of investment opportunities in  $t = 1$ . Hence, speculators are reluctant to trade away the illiquidity in  $t = 0$ . In this sense, our mechanism is different from one that is driven by risk-aversion.

Long Term Capital Management (LTCM) had been aware of funding liquidity risk. Indeed, they estimated that in times of severe stress, haircuts on AAA-rated commercial mortgages would increase from 2% to 10%, and similarly for other securities (HBS Case N9-200-007(A)). In response to this, LTCM had negotiated long-term financing with margins fixed for several weeks on many of their collateralized loans. Other firms with similar strategies, however, experienced increased margins. Due to an escalating liquidity spiral, LTCM could ultimately not fund its positions in spite of its numerous measures to control funding risk and was taken over by 14 banks in September 1998. Another recent example is the funding problems of the hedge fund Amaranth in September 2006, which reportedly ended with losses in excess of \$6 billion.

**Numerical Example, Continued.** To better understand funding liquidity risk, we explore the time-0 properties of our model by returning to our numerical example with one security. We consider first an uninformed financier and later turn to an informed one.

Figure 6 depicts the price  $p_0$  and expected time-1 price  $E_0[p_1]$  for different initial wealth levels,  $W_0$ , for which the speculators' funding constraint is not binding in  $t = 0$ . The figure shows that even though the speculators are unconstrained at time 0, market liquidity provision is limited with prices below the fundamental value of  $E_0[v] = 140$ . The price is below the fundamental for two reasons: First, the expected time-1 price is below the fundamental value because of the risk that speculators cannot accommodate the customer selling pressure at that time. Second, holding the security leads to losses in the states of nature when speculators are constrained and investment opportunities are good, which makes speculators require additional compensation for holding the asset.

The funding constraint not only affects the price level, it also introduces skewness in the  $p_1$ -distribution conditional on the sign of the demand pressure. For  $Z_1 > 0$ , speculators take long positions and, consequently, negative  $v_1$ -shocks lead to capital losses with resulting liquidity spirals. This amplification triggers a sharper price drops than the corresponding price increase for positive  $v_1$ -shocks. Figure 7 shows this negative skewness which is especially

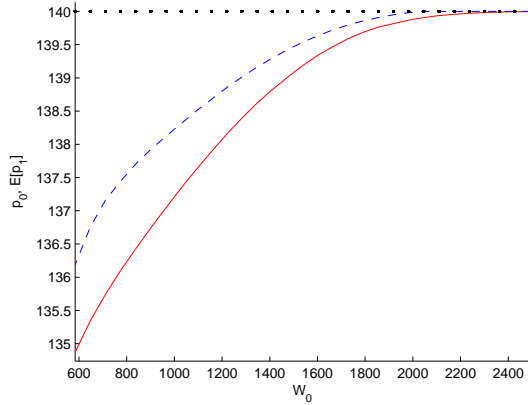


Figure 6: **Illiquidity at Time 0.** The solid line plots the price  $p_0$  at time 0 for different funding levels  $W_0$ . The dashed line depicts  $E_0[p_1]$ . The difference between  $p_0$  and the fundamental value  $E_0[v] = 140$  (dotted line) reflects illiquidity at time 0 and 1. We see that funding constraints are important already at time 0, although the constraint is not binding at this time for the depicted wealth levels.

pronounced for low wealth levels (although the effect is not monotone — zero dealer wealth implies no skewness, for instance).

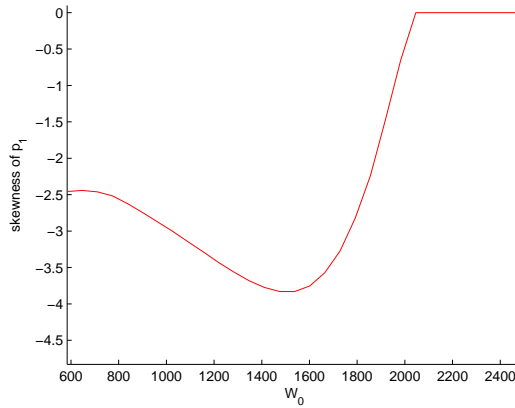


Figure 7: **Conditional Price Skewness.** The figure shows the conditional skewness of  $p_1$  for different funding levels  $W_0$ . While the funding constraint is not binding at time 0, it can become binding at time 1, leading to large price drops due to liquidity spirals. Price increases are not amplified, and this asymmetry results in skewness.

For negative realizations of  $Z_1$ , customers want to buy (not sell as above), and funding constraints induce a positive skewness in the  $p_1$ -distribution. The speculator return is negatively skewed, as above, since it is still his losses that are amplified. This is consistent with the casual evidence that hedge fund return indexes are negatively skewed. It also suggests from an ex-ante point of view, i.e., prior to the realization of  $Z_1$ , that funding constraints lead to higher kurtosis of the price distribution (fat tails).

Finally, we confirm numerically that unlike at time  $t = 1$ , margins can be positively related to illiquidity at time 0, even when financiers are fully informed. This is because of the liquidity risk between time 0 and time 1. To see this, note that if we reduce the speculators' initial wealth  $W_0$ , then the market becomes less liquid in the sense that the price is further from the fundamental value. At the same time, the equilibrium price in  $t = 1$  is more volatile and thus equilibrium margins at time 0 can actually increase.

## 7 Conclusion and New Testable Predictions

Our analysis provides a theoretical framework that delivers a unified explanation for a host of stylized empirical facts. Our analysis further suggests a novel line of empirical work that tests the model at a deeper level — namely its prediction that speculator funding is a driving force underlying these market liquidity effects.

First, it would be of interest to empirically study the determinants of margin requirements, e.g., using data from futures markets or from prime brokers. Our model suggests that both fundamental volatility and liquidity-driven volatility affects margins. Empirically, fundamental volatility can be captured using price changes over a longer time period, and the total fundamental and liquidity-based volatility is captured by short-term price changes as in the literature on variance ratios (see, e.g., Campbell, Lo, and MacKinlay (1997)). Our model predicts that, in markets where it is harder for financiers to be informed, margins depend on the total fundamental and liquidity-based volatility. In particular, in times of liquidity crisis, margins increase in such markets, and, more generally, margins should co-move with

illiquidity in the time series and in the cross section.<sup>12</sup>

Second, our model suggests that an exogenous shock to speculator capital should lead to a reduction in market liquidity. Hence, a clean test of the model would be to identify exogenous capital shocks such as an unconnected decision to close down a trading desk, a merger leading to reduced total trading capital, or a loss in one market unrelated to the fundamentals of another market, and then study the market liquidity and margin around such events.

Third, the model implies that the effect of speculator capital on market liquidity is highly non-linear: a marginal change in capital has a small effect when speculators are far from their constraints, but a large effect when speculators are close to their constraints.

Fourth, the model suggests that a cause of the commonality in liquidity is that the speculators' shadow cost of capital is a driving state variable. Hence, a measure of speculator capital tightness should help explain the empirical comovement of market liquidity. Further our result "commonality of fragility" suggests that especially sharp liquidity reductions occur simultaneously across several assets.

Fifth, the model predicts that the sensitivity of margins and market liquidity to speculator capital is larger for securities that are risky and illiquid on average. Hence, the model suggests that a shock to speculator capital would lead to a reduction in market liquidity through a spiral effect that is stronger for illiquid securities.

Sixth, speculators are predicted to have negatively skewed returns since, when they hit their constraints, they make significant losses because of the endogenous liquidity spirals, and, in contrast, their gains are not amplified when prices return to fundamentals. This leads to conditional skewness and unconditional kurtosis of security prices.

Finally, our analysis suggests that central banks can help mitigate market liquidity problems by controlling funding liquidity. If a central bank is better than the typical financiers of speculators at distinguishing liquidity shocks from fundamental shocks, then the central

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<sup>12</sup>One must be cautious with the interpretation of the empirical results related to changes in Regulation T since this regulation may not affect speculators but affects the demanders of liquidity, namely the customers.

bank can convey this information and urge financiers to relax their funding requirements — as the Federal Reserve Bank of New York did during the 1987 stock market crash. Central banks can also improve market liquidity by boosting speculator funding conditions during a liquidity crisis, or by simply stating the intention to provide extra funding during times of crisis, which would loosen margin requirements immediately as financiers’ worst-case scenarios improve.

## A Appendix: Proofs

**Proof of Propositions 1 and 2.** These results follow from the calculations in the text. ■

**Proof of Proposition 3.** We prove the proposition for  $Z_1 > 0$ , implying  $p_1 \leq v_1$  and  $x_1 \geq 0$ . The complementary case is analogous. To see how the equilibrium depends on the exogenous shocks, we first combine the equilibrium condition  $x_1 = -\sum_{k=0}^1 y_1^k$  with the speculator funding constraint to get

$$m_1^+ \left( Z_1 - \frac{2}{\gamma(\sigma_2)^2}(v_1 - p_1) \right) \leq b_0 + p_1 x_0 + \eta_1 \quad (31)$$

that is,

$$m_1^+ \left( Z_1 - \frac{2}{\gamma(\sigma_2)^2}(v_1 - p_1) \right) - p_1 x_0 - b_0 \leq \eta_1 \quad (32)$$

For  $\eta_1$  large enough, this inequality is satisfied for  $p_1 = v_1$ , that is, it is a stable equilibrium that the market is perfectly liquid. For  $\eta_1$  low enough, the inequality is violated for  $p_1 = \frac{2v_1}{\gamma(\sigma_2)^2} - Z_1$ , that is, it is an equilibrium that the speculator is in default. We are interested in intermediate values of  $\eta_1$ . If the left-hand-side of (32) is increasing in  $p_1$  then  $p_1$  is a continuously increasing function of  $\eta_1$ , implying no fragility with respect to  $\eta_1$ .

Fragility arises if the left-hand-side of (32) can be decreasing  $p_1$ . Intuitively, this expression measures speculator funding needs at the equilibrium position, and fragility arises if the

funding need is greater when prices are lower, that is, further from fundamentals. (This can be shown to be equivalent to a non-monotonic excess demand function.)

When the financier is informed, the left-hand-side of (32) is

$$(\bar{\sigma} + \bar{\theta}|\Delta v_1| + p_1 - v_1) \left( Z_1 + \frac{2}{\gamma(\sigma_2)^2}(p_1 - v_1) \right) - p_1 x_0 - b_0 \quad (33)$$

Since the first product is a product of two positive increasing functions of  $p_1$ , the entire expression decreases in  $p_1$  only if  $x_0$  is large enough.

When the financier is uninformed and  $a = 0$ , the left-hand-side of (32) is

$$(\bar{\sigma} + \bar{\theta}|\Delta p_1|) \left( Z_1 + \frac{2}{\gamma(\sigma_2)^2}(p_1 - v_1) \right) - p_1 x_0 - b_0 \quad (34)$$

When  $p_1 < p_0$ ,  $|\Delta p_1| = p_0 - p_1$  decreases in  $p_1$  and, if  $\bar{\theta}$  is large enough, this can make the entire expression decreasing. Also, the expression is decreasing if  $x_0$  is large enough.

It can be shown that the price cannot be chosen continuously in  $\eta_1$  when the left-hand-side of (32) can be decreasing. ■

**Proof of Proposition 4.** When the funding constraint binds, we use the implicit function theorem to compute the derivatives. Using that  $y_1$  is given by (12), the equilibrium condition  $x_1 = -y_1$ , the fact that the speculator funding constraint binds in an illiquid equilibrium, and that  $v_1 - p_1 > 0$  when  $Z_1 > 0$ , we have

$$m_1^+ \left( Z_1 - \frac{2}{\gamma(\sigma_2)^2}(v_1 - p_1) \right) = b_0 + p_1 x_0 + \eta_1. \quad (35)$$

We differentiate this expression to get

$$\frac{\partial m_1^+}{\partial p_1} \frac{\partial p_1}{\partial \eta_1} \left( Z_1 - \frac{2}{\gamma(\sigma_2)^2}(v_1 - p_1) \right) + m_1^+ \frac{2}{\gamma(\sigma_2)^2} \frac{\partial p_1}{\partial \eta_1} = \frac{\partial p_1}{\partial \eta_1} x_0 + 1, \quad (36)$$

which leads to Equation (23) after rearranging. The case of  $Z_1 < 0$  (i.e., Equation (24)) is

analogous.

Finally, spiral effects happen if one of the last two terms in the denominator of the right-hand side of Equations (23)-(24) is negative. (The total value of the denominator is positive by definition of a stable equilibrium.) When the speculator is informed,  $\frac{\partial m_1^+}{\partial p_1} = 1$  and  $\frac{\partial m_1^-}{\partial p_1} = -1$  using Proposition 1. Hence, in this case margins are stabilizing.

If the speculators are uninformed and  $a$  approaches 0, then using Proposition 2 we have that  $\frac{\partial m_1^+}{\partial p_1} = \frac{\partial m_1^+}{\partial \Lambda_1}$  approaches  $-\bar{\theta} < 0$  for  $v_1 - v_0 + \Lambda_1 - \Lambda_0 < 0$  and  $\frac{\partial m_1^-}{\partial p_1} = \frac{\partial m_1^-}{\partial \Lambda_1}$  approaches  $\bar{\theta} > 0$  for  $v_1 - v_0 + \Lambda_1 - \Lambda_0 > 0$ . This means that there is a margin spiral with positive probability. The case of a loss spiral is immediately seen to depend on the sign on  $x_0$ . ■

**Proof of Proposition 5.** We first consider the equation that characterizes a constrained equilibrium. When there is selling pressure from customers,  $Z_1^j > 0$ , it holds that

$$|\Lambda_1^j| = -\Lambda_1^j = v_1^j - p_1^j = \min\left\{\phi_1 m_1^{j+}, \frac{\gamma(\sigma_2^j)^2}{2} Z_1^j\right\}, \quad (37)$$

and if customers are buying,  $Z_1^j < 0$ , we have

$$|\Lambda_1^j| = \Lambda_1^j = p_1^j - v_1^j = \min\left\{\phi_1 m_1^{j-}, \frac{\gamma(\sigma_2^j)^2}{2} (-Z_1^j)\right\}. \quad (38)$$

Using the equilibrium condition  $x_1^j = -\sum_k y_1^{j,k}$ , Equation (12) for  $y_1^{j,k}$ , the speculators' funding condition becomes

$$\sum_{Z_1^j > \frac{2\phi_1 m_1^{j+}}{\gamma(\sigma_2^j)^2}} m_1^{j+} \left( Z_1^j - \frac{2\phi_1 m_1^{j+}}{\gamma(\sigma_2^j)^2} \right) + \sum_{-Z_1^j > \frac{2\phi_1 m_1^{j-}}{\gamma(\sigma_2^j)^2}} m_1^{j-} \left( -Z_1^j - \frac{2\phi_1 m_1^{j-}}{\gamma(\sigma_2^j)^2} \right) = \sum_j x_0^j p_1^j + b_0 + \eta_1 \quad (39)$$

where the margins are evaluated at the prices solving (37)–(38). When  $\phi_1$  approaches infinity, the left-hand-side of (39) becomes zero, and when  $\phi_1$  approaches zero, the left-hand-side approaches the capital needed to make the market perfectly liquid. As in the case of one security, there can be multiple equilibria and fragility (Proposition 3). On a stable equilibrium



branch,  $\phi_1$  increases as  $\eta_1$  decreases.

Of course, the equilibrium shadow cost of capital  $\phi_1$  is random since  $\eta_1, \Delta v_1^1, \dots, \Delta v_1^j$  are random. To see the commonality in liquidity, we note that  $|\Lambda^j|$  is increasing in  $\phi_1$  for each  $j = k, l$ . To see this, consider first the case  $Z_1^j > 0$ . When the financiers are uninformed,  $a = 0$ , and  $\theta^j = 0$ , then  $m_1^{j+} = \bar{\sigma}^k$ , and, therefore, Equation (37) shows directly that  $|\Lambda_1^j|$  increases in  $\phi_1$  (since the minimum of increasing functions is increasing). When financiers are informed and  $\theta^j = 0$  then  $m_1^{j+} = \bar{\sigma}^k + \Lambda_1^j$ , and, therefore, Equation (37) can be solved to be  $|\Lambda_1^j| = \min\{\frac{\phi_1}{1+\phi_1}\bar{\sigma}^j, \frac{\gamma(\sigma_2^k)^2}{2}Z_1^k\}$ , which increases in  $\phi_1$ . Similarly, Equation (38) shows that  $|\Lambda^j|$  is increasing in  $\phi_1$  when  $Z_1^j < 0$ .

Now, since  $|\Lambda^j|$  is increasing in  $\phi_1$  and does not depend on other state variables under these conditions,  $\text{Cov}(|\Lambda^k(\phi)|, |\Lambda^l(\phi)|) \geq 0$  because any two functions which are both increasing in the same random variable are positively correlated.

To see part (ii) of the proposition, note that, for all  $j$ ,  $|\Lambda^j|$  is a continuous function of  $\phi_1$ , which is locally insensitive to  $\phi_1$  if and only if the speculator is not marginal on security  $j$  (i.e., if the second term in Equation (37) or (38) attains the minimum). Hence,  $|\Lambda^j|$  jumps if and only if  $\phi_1$  jumps.

To see part (iii), we write illiquidity using Equations (37)–(38) as

$$|\Lambda_1^j| = \min\left\{\phi_1 m_1^{j, \text{sign}(Z_1^j)}, \frac{\gamma(\sigma_2^j)^2}{2}|Z_1^j|\right\}. \quad (40)$$

Hence, using the expression for the margin, if the financier is uninformed and  $\theta^j = a = 0$ , then

$$|\Lambda_1^j| = \min\left\{\phi_1 \bar{\sigma}_1^j, \frac{\gamma(\sigma_2^j)^2}{2}|Z_1^j|\right\} \quad (41)$$

and, if the financier is informed and  $\theta^j = 0$ , then

$$|\Lambda_1^j| = \min\left\{\frac{\phi_1}{1+\phi_1}\bar{\sigma}_1^j, \frac{\gamma(\sigma_2^j)^2}{2}|Z_1^j|\right\}. \quad (42)$$

In the case of an uninformed financier as in (41), we see that, if  $x_1^k \neq 0$ ,

$$|\Lambda_1^k| = \phi_1 \bar{\sigma}_1^k \geq \phi_1 \bar{\sigma}_1^l \geq |\Lambda_1^l| \quad (43)$$

and, if  $|Z_1^k| \geq |Z_1^l|$ ,

$$|\Lambda_1^k| = \min\left\{\phi_1 \bar{\sigma}_1^k, \frac{\gamma(\sigma_2^i)^2}{2}|Z_1^k|\right\} \geq \min\left\{\phi_1 \bar{\sigma}_1^l, \frac{\gamma(\sigma_2^i)^2}{2}|Z_1^l|\right\} = |\Lambda_1^l|. \quad (44)$$

With an informed financier, it is seen that  $|\Lambda_1^k| \geq |\Lambda_1^l|$  using the same arguments.

For part (iv) of the proposition, we use that

$$\frac{\partial |\Lambda_1^j|}{\partial(-\eta_1)} = \frac{\partial |\Lambda_1^j|}{\partial \phi_1} \frac{\partial \phi_1}{\partial(-\eta_1)} \quad (45)$$

Further,  $\frac{\partial \phi_1}{\partial(-\eta_1)} \geq 0$  and, from Equations (41)–(42), we see that  $\frac{\partial |\Lambda_1^k|}{\partial \phi_1} \geq \frac{\partial |\Lambda_1^l|}{\partial \phi_1}$ . The result that  $\text{Cov}(\Lambda^k, \phi) \geq \text{Cov}(\Lambda^l, \phi)$  now follows from Lemma 1 below. ■

**Lemma 1** *Let  $X$  be a random variable and  $g_i$ ,  $i = 1, 2$ , be weakly increasing functions  $X$ , where  $g_1$  has a larger derivative than  $g_2$ , that is,  $g_1'(x) \geq g_2'(x)$  for all  $x$ . Then,*

$$\text{Cov}[X, g_1(X)] \geq \text{Cov}[X, g_2(X)] \quad (46)$$

**Proof.** For  $i = 1, 2$  we have

$$\text{Cov}[X, g_i(X)] = E[(X - E[X])g_i(X)] \quad (47)$$

$$= E\left[(X - E[X])\left(\int_{E[X]}^X g_i'(y) dy\right)\right]. \quad (48)$$

The latter expression is a product of two terms that always have the same sign. Hence, this is higher if  $g_i'$  is larger. ■

**Liquidity Risk (Time 0).** Section 6 focuses on the case of speculators who are uncon-

strained at  $t = 0$ . When a speculator's problem is linear and he is constrained at time 0, then he invests only in securities with the highest expected profit per capital use, where profit is calculated using the pricing kernel  $\phi_1^i/E_0[\phi_1^i]$ . In this case, his time-0 shadow cost of capital is

$$\phi_0^i = E_0[\phi_1^i] \left( 1 + \max_j \left\{ \frac{E_0[\frac{\phi_1^i}{E_0[\phi_1^i]}p_1^j] - p_0^j}{m_0^+}, -\frac{E_0[\frac{\phi_1^i}{E_0[\phi_1^i]}p_1^j] - p_0^j}{m_0^-} \right\} \right) \quad (49)$$

■

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