# Trading and Voting in Distressed Firms 

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# Trading and Voting in Distressed Firms 

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#### Abstract

We investigate the effect of the ability of "non-traditional" funds to short-sell the equity of their debtors. This enables the funds to vote on the restructuring proposals of distressed firms, while at the same time they separate their voting rights from their economic exposure. The effect on firm value depends on the discrepancy between the markets for debt and equity, discrepancy in how each assesses the probability of a proposal being accepted. We show that if the assessments between the two markets are different then the presence of a non-traditional fund decreases firm value. Firm value, however, is unaffected if the assessments are the same.


Keywords: short-selling, debt restructuring, distressed securities, empty creditors

[^0]
## 1 Introduction

Companies with publicly-traded equity also often issue public bond or bond-like securities. Either outright default or the violation of a debt covenant, both of which are occurrences of financial distress, can then trigger debt restructuring procedures. These procedures, either inside or outside bankruptcy, rely on the assumption that claimholders' economic exposure and voting rights are perfectly intertwined. The increase in the multi-securities issuance activity is, however, paralleled by the increase in the number and assets under management of non-traditional investment funds, such as hedge funds and private equity funds. These non-traditional funds have the ability to enter a more diverse spectrum of strategies that are forbidden to other more regulated financiers. ${ }^{1}$ While the advantages of access to financing are well understood, we are interested in exploring a type of behavior that might affect the efficiency of debt restructuring, in particular, the ability of these funds to short the equity of their debtors, which significantly weakens the intertwinement hypothesis. ${ }^{2}$

There is a plethora of popular press articles as well as articles in the law literature (see following subsection on Related Literature) which provide anecdotal evidence on the "misbehavior" of funds going after "good" companies and forcing them into bankruptcy to make their short positions worth more. There is, however, a lack of an analytical framework that would allow us to study these phenomena from a theoretical perspective. Our paper endeavours to fill this gap by proposing a parsimonious model of debt restructuring proposals and their outcome in the presence of nontraditional funds. We derive the strategic interaction between a firm's manager, who represents the equityholders and optimally decides on the restructuring plan, and a fund; the fund can buy debt, and buy or short-sell equity. ${ }^{3}$ If it acquires debt it decides on its optimal voting decision according to its aggregate exposure to the firm's securities. ${ }^{4}$

[^1]Our main goal is to determine the conditions under which the fund's behavior may or may not be to the detriment of the value of the distressed firm or any of its claimholders, under the premise that continuing with the firm's operations is better than liquidation. We do so for different cases regarding the manager's information about the fund's positions at the time of the proposal. This highlights different aspects of how the fund's presence and trading behavior affects the restructuring procedure. Firm value (as measured by the probability of continuation after the proposal is made but before voting) is not affected in the following cases: (i) when the fund's positions are known by the manager when she makes the proposal and no trading is allowed after that, or (ii) when, even if she does not know the positions when she makes the proposal and trading is allowed, the markets for debt and equity have the same assessments over the probability of continuation, a case we term "Connected Markets." Firm value, however, decreases otherwise.

No matter the effect on firm value, the ability of the fund to short-sell the equity of the firm is to the detriment of equityholders but to the benefit of debtholders, both relative to the case where there are only pure debtholders, i.e., debtholders who do not have positions in the firm's equity. Intuitively it is the ability of the fund to trade, and in particular short equity, that drives our results. The fund trades not because it has superior information (or wants to manipulate the market) but because it can affect the voting outcome (if it acquires enough debt). A consequence of our model is that allowing for trading of distressed securities might hurt firm value in the presence of short-sellers, although it is beneficial in terms of offering liquidity to market participants. A remedy is offered by our result on "connectedness" which highlights the need for transparency and coordination between markets. In the context of our model connectedness means that marketmakers can see aggregate order-flows in other markets besides their own. ${ }^{5}$

The specifics of our model are as follows: We consider a firm with debt and equity outstanding. The firm is in financial distress and needs to restructure its debt. The main assumption is that the value of the firm if it continues with its operations is larger than if it liquidates. ${ }^{6}$ The restructuring proposal is a take-it-or-leave-it offer of a promised payment to debtholders conditional on continuation of the firm's operations; the promised payment is credible, and the continuation and liquidation values of the firm are known. Furthermore, the value of equity in the event of liquida-

[^2]tion is zero, i.e., debtholders receive all the liquidation value. The proposal is made by the firm's manager, who is not subject to agency problems, and voted on by the debtholders. To capture the frictions between claimholders in a stylized manner we assume that the firm manager's primary objective is to maximize equity value. If the voting outcome is yes the firm continues with its operations, while if it is no the firm is liquidated. ${ }^{7}$ After the proposal is made public and before voting there is a trading round. Trade (in either debt or equity) is intermediated by a competitive market-maker, à la Kyle (1985). Two types of agents trade with the market-maker: the fund, and noise traders with uninformative and liquidity-driven trades.

We begin our analysis in Section 3 by considering two benchmark cases in which trading is not allowed after the proposal is made. If there are only pure debtholders, subsection 3.1, the manager proposes a promised payment to debtholders which makes them indifferent between continuation and liquidation, in which case they vote for continuation. Similarly, when the fund's positions are known to the manager before the proposal and remain the same until voting, subsection 3.2, we show that the manager can always find a proposal that will guarantee the continuation of the firm's operations. Hence the fund's existence and its potential short equity positions do not hurt firm value. This outcome is intuitive, since in this case, we are in a bargaining game with complete information in which the party with the bargaining power (here the manager) can always find a split of the available surplus that the other party (here the fund and other debtholders) will accept. If the fund is a pivotal debtholder, i.e., it can decide unilaterally on the voting outcome, then: equityholders benefit when the fund is also long the equity of the firm, since then their incentives are aligned, while debtholders benefit when the fund is short the equity of the firm, since then the fund requires a bigger payment for debtholders to vote for continuation rather than liquidation.

In Section 4 we look into the case where the fund is a pivotal debtholder, and explore the effect of equity trading (the fund initially has no equity) after the proposal but before voting. This is motivated by the fact that Chapter 11 filings may include key debtholders but may not have timely information on the equityholders. ${ }^{8}$ We show that the manager makes a proposal that may lead to liquidation (which is inefficient in our model) with positive probability. The mechanism that

[^3]leads to this is as follows: the fund will mix/randomize in equilibrium between going long or short in equity in order to conceal its trading positions from the market-maker (this allows it to realize gains from trade). This uncertainty leads to a bargaining game of incomplete information in which there is a positive probability that a take-it-or-leave-it offer may be rejected. The ability of the fund to trade in equity, and in particular to short, reduces equityholders' expected payoff, while increases debtholders' expected payoff, both relative to the case where the fund was absent and we only had pure debtholders.

In Section 5 we allow for trading in both debt and equity after the proposal but before voting (the fund initially has no debt and no equity). We do so for two reasons: one is to investigate a fund's incentives to enter the distressed securities' market overall, and the second is to contrast the consequences of different assumptions about the connectedness between the markets for debt and equity. Recall that in the context of our model connectedness is whether market-makers can see aggregate order-flows in other markets besides their own, Connected Markets, or not, Disconnected Markets. This in turn translates to whether the assessments over the probability of continuation are the same across markets, or not. ${ }^{9}$

Our main result is that in the case of Connected Markets the existence of the fund does not affect firm value, while in the case of Disconnected Markets it reduces it, for some values of noise trading. In both cases the fund will become the pivotal debtholder in equilibrium but it will follow different mixing strategies in each case. We show that the equilibrium for Disconnected Markets is the same as the one studied in Section 3, and so it will lead to a positive probability of liquidation. For Connected Markets, however, the fund faces a more informationally complete market and so, in equilibrium, it will mix more aggressively to realize gains from trade. This will increase the manager's uncertainty when she makes the proposal, and significantly limit the payoff she can secure for equityholders, in turn forcing her to make a proposal that guarantees continuation. Regardless of the connectedness of markets, equityholders are hurt and debtholders benefit, as before. ${ }^{10}$

[^4]
### 1.1 Related Literature

Our paper relates to several strings of literature. The most relevant empirical paper is Li, Jiang, and Wang (2009), where the authors explore the role of hedge funds in distressed firms. According to statistics accumulated by Li, Jiang, and Wang (2009) hedge funds are present in $94 \%$ of the biggest Chapter 11 (debt restructuring) filings in the United States between 1996-2007. Their exposure in $79 \%(55 \%)$ of the cases is in the debt (equity) of the distressed firm, while their time of entry varies between before and after the filing. Bharath, Panchapegesan, and Werner (2007) observe in the data that post 2000 , creditors seem to benefit more during Chapter 11 renegotiations, an observation consistent with our model's predictions.

Theoretical models of debt restructuring/renegotiation appear in Gertner and Scharfstein (1991), Bolton and Scharfstein (1996), and Hart and Moore (1998). White (1994), and Chatterjee, Dhillon, and Ramirez (1996) provide institutional details about debt restructuring for companies in default. The interplay between trading and voting has been studied in different contexts. Musto and Yilmaz (2003) show how trading can affect the voting outcome in a political voting context. Maug (1999) shows how trading and voting can jointly help aggregate information about a firm. In the context of Chapter 11 and multiclass voting, Maug and Yilmaz (2002) prove the efficiency of two separate votes (one for each security class) rather than one.

How strategic traders take actions that affect firm value is also the topic of Kyle and Vila (1991), Maug (1998), and Kahn and Winton (1998). In our paper, however, we endogenize the trading decisions and voting behavior of the fund. In this sense a very related paper is Brav and Mathews (2009) where the authors study the issue of "empty voting," i.e., when a hedge fund can establish separate positions in the firm's shares and votes, and show how this can actually increase efficiency in some cases.

Goldstein and Guembel (2008), and Khanna and Mathews (2009) focus on short-selling, and show how funds can use their information to manipulate the market. The main difference from these papers is that in our model trading is not driven by information but by the ability to decide on the voting outcome. This is the difference also with Caballé and Krishnan (1994) where the authors study trading in multiple-securities in a Kyle (1985) environment. Furthermore, Caballé the ability of debtholders to short equity can reduce these deviations from priority.
and Krishnan (1994) make the assumption of connectedness so to the best of our knowledge ours is the first paper to contrast the trading behavior of a trader when market-makers observe order flows on other markets vs. when they do not.

In a contemporaneous paper Bolton and Oehmke (2010) study the debt financing of a project by an entrepreneur with no personal wealth when her debtholders can trade in the credit default swaps (CDS) market. Bolton and Oehmke (2010) show that although overinsurance in the CDS markets can increase the probability of liquidation (ex post), the existence of the CDS market disciplines the entrepreneur and deters her from strategic default (ex ante). In terms of theme Bolton and Oehmke (2010) and our paper are very related, however, the focus of the analysis is different: they derive the initial debt contract and the corresponding optimal CDS position, to study the aforementioned ex post and ex ante effects. On the other hand, we analyze a parsimonious model of trading of distressed securities, and describe the equilibrium of the strategic game between the manager and the fund, to study the effects of the connectedness of markets.

Non-traditional funds' involvement in distressed firms has received significant coverage in the popular press, see Durfee (2006), Taub (2005), Sakoui (2008), and Economist Staff (2009). In particular in Economist Staff (2009), there is mention of the case of Six Flags, the amusement park, which filed for Chapter 11 but had its debtholders reject a restructuring plan that otherwise seemed beneficial. The claim is that some debtholders had hedged their economic exposure by purchasing credit derivatives. This kind of hedging either with equity short-selling or credit derivatives has been highlighted extensively in the law literature, see Skeel Jr. and Partnoy (2006), Kahan and Rock (2007), Hu and Black (2008), and Ayotte and Morrison (2009).

In Skeel Jr. and Partnoy (2006) regarding the case of Chapter 11 filing by Tower Automotive the authors comment "...But one widely rumored explanation is that, in addition to their position as financiers of Tower, the hedge funds also had shorted its stock, that is, they borrowed Tower stock and stood to profit if the value of the stock declined. Some bankers, as the Wall Street Journal later reported, believe hedge funds triggered the filing to make their short positions worth more..." [emphasis added]. Issues like this may influence impending regulation, and our paper is an attempt to derive the conditions under which short-selling by creditors may lead to inefficient liquidations.

## 2 Model

Our canonical example is a firm which has both debt and (external) equity outstanding and which has defaulted on its debt obligations. ${ }^{11}$ The firm's manager makes a restructuring proposal which is a split of the firm's value, conditional on continuation, between debt and equity holders. She chooses the proposal to maximize equityholders' expected value, an objective which is usually rationalized by postulating that the manager holds some (small) equity stake in the company and no other claims in it. In our case, the manager's assumed objective emphasizes that the outcome of the debt restructuring process depends on the conflict between claimholders. In reality the process is a sequence of offers and counter-offers by debtholders and equityholders and a final vote by debtholders. The outcome of the vote may even be challenged by a judge. Our assumption of a take-it-or-leave offer made by the manager, essentially the equityholders, to the debtholders is a simplification which, nonetheless, allows us to highlight the role of trading. Note that our restructuring model is a particular case of Hart and Moore (1998) in which equityholders have all the bargaining power. ${ }^{12}$

The expected continuation firm value, $y$, is known and certain at the time of the proposal, i.e., there are no agency problems arising from the manager's private information and/or choice of effort. The proposal is voted on only by investors who hold debt claims on the day of the vote and the voting outcome is binding. Voting is compulsory and bears no cost. In the event of rejection of the proposal the firm is liquidated and ceases its operations with value of liquidation $l$, known to all when the proposal is made. The main assumption we make throughout the paper is that $y>l$, i.e., continuation is better than liquidation from a firm value perspective. This allows us to analyze inefficient liquidation as in our motivating example about Tower Automotive. Furthermore, equityholders receive nothing in the event of liquidation, i.e., the liquidation value is not enough to cover residual debt claims; this last assumption is standard in the literature and in our case offers some expositional clarity.

Our focus is an unregulated profit-maximizing investment fund, which has the following distinguishing feature from other investors: it can hold both debt and equity claims and, in particular,

[^5]it can short the equity of its debtor. Other investors in the firm, in addition to not being able to hold both claims at the same time, are viewed as noise, i.e., liquidity-driven traders. All are risk neutral, and the risk-free interest rate is zero. In the next section we consider two benchmark cases in which there is no trading allowed between the proposal date and the voting date.

## 3 Benchmark Cases

### 3.1 Pure Debtholders

The manager's proposal is equivalent to a payment to equityholders conditional on continuation, $E$, so that the promised payment to debtholders is $D \triangleq y-E .{ }^{13}$ Hence, after the proposal has been made and before voting an equity claim on the firm is an asset that pays $E$ in the event of continuation and zero otherwise, while a debt claim is an asset that pays $D$ in the event of continuation and $l$ otherwise. The probability of each outcome depends on the incentives of the debtholders who vote. It is easy to show that it is a weakly dominant strategy for a pure debtholder, i.e., one who holds debt but no equity in the firm, to vote yes to a proposal if $D \geq l \Rightarrow E \leq y-l$, and no to a proposal if $D<l$. Let then $E_{0} \triangleq y-l$, which is strictly greater than zero since $y>l$; $E_{0}$ is the maximum promised payment to equityholders, conditional on continuation, for which pure debtholders would vote yes for continuation.

### 3.2 Fund with Exogenous Positions

We start from the basic case in which the fund's positions in debt and equity are exogenous and known, and do not change until the voting date. Let $x_{E} \in\{-1,0,1\}$, and $x_{D} \in\{0,1\}$, be the fund's positions in equity and debt, respectively. The discrete possible values of the fund's equity and debt positions allow us to focus on the direction of trade rather than quantity. The symmetry between long and short positions in equity, and the choice of one unit are mere normalizations that facilitate exposition.

Then, given the fund's possible positions, its payoff conditional on continuation, i.e., a yes vote to the proposal is

$$
x_{E} E+x_{D} D=x_{E} E+x_{D}(y-E)
$$

[^6]while its payoff conditional on liquidation, i.e., a no vote to the proposal is
$$
x_{E} 0+x_{D} l=x_{D} l .
$$

The fund is pivotal for the voting outcome if $x_{D}=1$ and not pivotal if $x_{D}=0$. So for a proposal $E$ by the manager we have concerning the fund's payoff:

- If $x_{D}=0$, the fund is not pivotal. Others vote yes for $E \leq E_{0}$ in which case the fund gets $x_{E} E$, while they vote no for $E>E_{0}$ in which case the fund gets 0 .
- If $x_{D}=1$, the fund is pivotal. It then votes yes for

$$
x_{E} E+D \geq l \Rightarrow\left(1-x_{E}\right) E \leq E_{0}
$$

and no otherwise. Hence, it votes yes: (i) for all proposals if $x_{E}=1$, (ii) for proposals such that $E \leq E_{0}$ if $x_{E}=0$, (iii) for proposals such that $E \leq E_{0} / 2$ if $x_{E}=-1$.

Hence continuation occurs in the following cases:

1. $\left\{x_{E}=-1\right.$ and $\left.x_{D}=1\right\}$, for $E \leq E_{0} / 2$,
2. $\left\{x_{E} \in\{-1,0,1\}\right.$ and $\left.x_{D}=0\right\}$ or $\left\{x_{E}=0\right.$ and $\left.x_{D}=1\right\}$, for $E \leq E_{0}$,
3. $\left\{x_{E}=1\right.$ and $\left.x_{D}=1\right\}$, for all $E$.

If the manager knows the fund's positions she proposes $E_{0} / 2$ in case $1, E_{0}$ in case 2 , and the maximum possible promised value of equity, let $\bar{E}$ (necessarily less than $y$ ) in case 3 . All these proposals achieve continuation with probability one, and hence, in this full information case, maximize the expected value of equity, which is the manager's objective. In the fund's absence, the manager, as mentioned, guarantees continuation by proposing $E_{0}$. From the above we have the following implications in this full information case. First, in all cases 1-3 expected firm value before voting is not affected by the fund's existence since continuation is achieved with probability one, as in the pure debtholders case. The existence of a fund that has shorted its debtor's equity benefits debtholders and hurts equityholders.

However, if the manager does not know the fund's positions and has some exogenous prior beliefs $p_{1}, p_{2}, 1-p_{1}-p_{2}$ over cases 1,2 , and 3 , above, respectively, then we might get liquidation with positive probability. To see this, note that in this case the manager picks $E$ to maximize

$$
\begin{gathered}
p_{1} E \mathbb{I}\left(E \leq E_{0} / 2\right)+p_{2} E \mathbb{I}\left(E \leq E_{0}\right)+\left(1-p_{1}-p_{2}\right) E= \\
{\left[p_{1} \mathbb{I}\left(E \leq E_{0} / 2\right)+p_{2} \mathbb{I}\left(E \leq E_{0}\right)+\left(1-p_{1}-p_{2}\right)\right] E,}
\end{gathered}
$$

where $\mathbb{I}(\cdot)$ is the indicator function. The term in the square brackets above is the probability of continuation. Clearly the manager chooses between expected values of equity $E_{0} / 2$, $\left(1-p_{1}\right) E_{0}$, and $\left(1-p_{1}-p_{2}\right) \bar{E}$, which correspond to proposals $E_{0} / 2, E_{0}$, and $\bar{E}$, and probabilities of continuation $1,\left(1-p_{1}\right)$, and $\left(1-p_{1}-p_{2}\right)$, respectively. Depending on the parameter values any of these three proposals may be optimal. In general, in the uncertainty case, expected firm value before voting weakly decreases, since the probability of continuation might be less than one. This uncertainty, however, might benefit equityholders in the expected sense. In what follows we endogenize these probabilities by studying the role of trading, and analyze the equilibria that arise.

## 4 Exogenous Positions in Debt, Trading in Equity

As mentioned in the introduction, reporting of positions in the debt claims of a company is more frequent than the ones in equity. Hence, in this section we are going to take the fund's position in debt as given and see what its optimal policy is in acquiring the equity of the firm in the market, after the proposal but before voting.

We assume that trading in the firm's equity is facilitated by a market-maker. The equity marketmaker has the same information as the manager, and is risk neutral. In the spirit of Kyle (1985) he receives aggregate orders (from the fund and the rest of investors), computes the probability of continuation, and sets the price of equity equal to its expected value.

As before, let $x_{E} \in\{-1,0,1\}$ and $x_{D} \in\{0,1\}$ be the fund's positions in equity and debt, respectively. It is assumed that the fund has initially no equity position and no capital constraints. If $x_{D}=0$ the manager always proposes $E_{0}$ that leads to continuation with probability one. The market-maker, who has the same information as the manager, anticipates this and prices equity at
exactly $E_{0}$. In this case the fund is indifferent between trading or not since its profit is zero in both cases.

However, if $x_{D}=1$ the fund is pivotal and what the manager proposes depends on her beliefs about the fund's trade. Given proposal $E$ the fund's problem is to choose $x_{E} \in\{-1,0,1\}$ to maximize its profit

$$
\begin{aligned}
\Pi^{1}\left(x_{E} ; E\right) & =\left\{\begin{array}{cc}
\max \{E+D, l\}-\mathbb{E}\left[p_{E} \mid x_{E}=1\right], & x_{E}=1, \\
\max \{-E+D, l\}+\mathbb{E}\left[p_{E} \mid x_{E}=-1\right], & x_{E}=-1, \\
\max \{y-E, l\}, & x_{E}=0,
\end{array}\right. \\
& =\left\{\begin{array}{cc}
\max \{y, l\}-\mathbb{E}\left[p_{E} \mid x_{E}=1\right], & x_{E}=1, \\
\max \{-2 E+y, l\}+\mathbb{E}\left[p_{E} \mid x_{E}=-1\right], & x_{E}=-1, \\
\max \{y-E, l\}, & x_{E}=0, \\
y-\mathbb{E}\left[p_{E} \mid x_{E}=1\right], & x_{E}=1, \\
\max \left\{E_{0}-E, 0\right\}+l, & x_{E}=0,
\end{array}\right.
\end{aligned}
$$

where we used the definitions of $D=y-E, E_{0}=y-l$, and the fact that $y>l$. The "max" operators above signify the voting decision of the fund, which determines the voting outcome since the fund is pivotal. Moreover, the expectations over the price of equity $p_{E}$ capture the fact that the fund does not know the aggregate demand in equity when it places its order.

Observe that when the fund goes long in equity, $x_{E}=1$, it always votes yes for any proposal $E$. Similarly, it votes yes when it goes short, $x_{E}=-1$, only when $E \leq E_{0} / 2$, and votes yes if it is a pure debtholder, $x_{E}=0$, only when $E \leq E_{0}$. Hence, for proposals $E \leq E_{0} / 2$ the fund votes yes regardless its position, that is we get continuation with probability one, and the price of equity is $p_{E}=E$. This makes the fund indifferent between trading (long or short) or not, in equity, since its profit is $y-E$ in all cases.

Now, for $E \in\left(E_{0} / 2, E_{0}\right]$ the fund chooses $x_{E}$ to maximize,

$$
\Pi_{C}^{1}\left(x_{E} ; E\right)=\left\{\begin{array}{cc}
y-\mathbb{E}\left[p_{E} \mid x_{E}=1\right], & x_{E}=1 \\
l+\mathbb{E}\left[p_{E} \mid x_{E}=-1\right], & x_{E}=-1 \\
y-E, & x_{E}=0
\end{array}\right.
$$

where the subscript $C$ in $\Pi_{C}^{1}$ signifies that for this regime of proposals pure debtholders vote for continuation. We see that the strategy $\left\{x_{E}=0\right\}$ is always weakly dominated by the strategy $\left\{x_{E}=1\right\}$ since for all $E, \mathbb{E}\left[p_{E} \mid x_{E}=1\right] \leq E$. Hence we ignore it and focus on the strategies $\left\{x_{E}=1\right\}$ and $\left\{x_{E}=-1\right\} .{ }^{14}$

Similarly for $E \in\left(E_{0}, \bar{E}\right]$ the fund chooses $x_{E}$ to maximize,

$$
\Pi_{L}^{1}\left(x_{E} ; E\right)=\left\{\begin{array}{cc}
y-\mathbb{E}\left[p_{E} \mid x_{E}=1\right], & x_{E}=1 \\
l+\mathbb{E}\left[p_{E} \mid x_{E}=-1\right], & x_{E}=-1 \\
l, & x_{E}=0
\end{array}\right.
$$

where $L$ in $\Pi_{L}^{1}$ signifies that for this regime of proposals pure debtholders vote for liquidation. We see that the strategy $\left\{x_{E}=0\right\}$ is always weakly dominated by the strategy $\left\{x_{E}=-1\right\}$ since for all $E, \mathbb{E}\left[p_{E} \mid x_{E}=1\right] \geq 0$. Again, we ignore it and focus on the strategies $\left\{x_{E}=1\right\}$ and $\left\{x_{E}=-1\right\}$. That is the fund always (weakly) prefers to trade for any proposal $E \in\left(E_{0} / 2, \bar{E}\right]$ and so we drop the subscripts $C$ and $L$ from its profit function and write,

$$
\Pi^{1}\left(x_{E} ; E\right)=\left\{\begin{array}{cc}
y-\mathbb{E}\left[p_{E} \mid x_{E}=1\right], & x_{E}=1 \\
l+\mathbb{E}\left[p_{E} \mid x_{E}=-1\right], & x_{E}=-1
\end{array}\right.
$$

To proceed with our equilibrium calculation we need to specify the derivation of the price by the market-maker. Following standard practice, let $z_{E} \in\{-1,+1\}$ be the level of noise trading, and $\nu_{E} \triangleq \mathbb{P}\left[z_{E}=1\right]$. The market-maker observes aggregate demand $y_{E} \triangleq x_{E}+z_{E}$ and sets the

[^7]equilibrium price as follows, ${ }^{15}$
\[

$$
\begin{aligned}
p_{E} & =\mathbb{E}\left[\mathbb{I}(\text { continuation }) E \mid y_{E}\right] \\
& =\mathbb{P}\left[\text { continuation } \mid y_{E}\right] E \\
& =\mathbb{P}\left[x_{E}=1 \mid y_{E}\right] E \\
& =\left\{\begin{array}{cc}
E, & y_{E}=2, \\
\frac{\left(1-\nu_{E}\right) \lambda_{E}}{\left(1-\nu_{E}\right) \lambda_{E}+\nu_{E}\left(1-\lambda_{E}\right)} E, & y_{E}=0, \\
0, & y_{E}=-2,
\end{array}\right.
\end{aligned}
$$
\]

where $\lambda_{E} \triangleq \mathbb{P}\left[x_{E}=1\right]$, i.e., $\lambda_{E}$ is the market-maker's belief that the fund will go long in equilibrium. The following proposition describes the fund's equilibrium behavior as given by $\lambda_{E}$, and the resulting probability of continuation, for $E \in[0, \bar{E}] .{ }^{16}$

Proposition 1. In the case where the fund is pivotal, $x_{D}=1$, its equity trading behavior varies as follows with the value of the proposal $E$.

- For $E \in\left[0, E_{0} / 2\right], \lambda_{E} \in(0,1)$, continuation with probability one.
- For $E \in\left(E_{0} / 2, E_{0} /\left(1+\nu_{E}\right)\right], \lambda_{E}=1$, continuation with probability one.
- For $E \in\left(E_{0} /\left(1+\nu_{E}\right), E_{0} / \nu_{E}\right), \lambda_{E}=\nu_{E} \frac{E_{0} / E-\nu_{E}}{E_{0} / E\left(2 \nu_{E}-1\right)-\left(2 \nu_{E}^{2}-1\right)}$, continuation with probability $\lambda_{E}(<1)$.
- For $E \in\left[E_{0} / \nu_{E}, \bar{E}\right], \lambda_{E}=0$, continuation with probability zero, i.e., we always have liquidation.

The manager will always pick proposals $E \in\left[E_{0} /\left(1+\nu_{E}\right), E_{0} / \nu_{E}\right]$, and her objective in that region is to maximize

$$
J(E) \triangleq \nu_{E} \frac{\frac{E_{0}}{E}-\nu_{E}}{\frac{E_{0}}{E}\left(2 \nu_{E}-1\right)-\left(2 \nu_{E}^{2}-1\right)} E .
$$

The above expression attains a maximum since we are maximizing a continuous function over a compact interval (Weierstrass Theorem). The solution is described by the following proposition.

[^8]Proposition 2. We have that

- For $\nu_{E}<\underline{\nu_{E}} \triangleq \sqrt{5} / 2-1 / 2$, the manager picks $E=E_{0} /\left(1+\nu_{E}\right)$ with $J\left(E_{0} /\left(1+\nu_{E}\right)\right)=$ $E_{0} /\left(1+\nu_{E}\right)$, and probability of continuation equal to one.
- For $\nu_{E} \geq \nu_{E}$, she picks

$$
E=E_{1} \triangleq \frac{2 \nu_{E}^{2}-\nu_{E}-\sqrt{\nu_{E}} \sqrt{-2 \nu_{E}^{2}+3 \nu_{E}-1}}{\nu_{E}\left(2 \nu_{E}^{2}-1\right)} E_{0}<E_{0},
$$

with

$$
J\left(E_{1}\right)=\left(\frac{\sqrt{2 \nu_{E}\left(\nu_{E}-1 / 2\right)}-\sqrt{1-\nu_{E}}}{2 \nu_{E}^{2}-1}\right)^{2} \nu_{E} E_{0}<E_{0},
$$

and probability of continuation

$$
\sqrt{\nu_{E}} \frac{\sqrt{2 \nu_{E}\left(\nu_{E}-1 / 2\right)}-\sqrt{1-\nu_{E}}}{\left(2 \nu_{E}^{2}-1\right) \sqrt{2\left(\nu_{E}-1 / 2\right)}}<1 .
$$

From the solution to the manager's problem we see that the existence of the fund hurts equityholders, as it reduces their expected payoff, with respect to the case where we had only pure debtholders. With respect to the same case, debtholders have a higher expected value, this follows from the fact that $E_{1}<E_{0}$. However, the fund's existence also hurts firm value since in some cases we get liquidation with positive probability. These cases occur when it is more probable to have noise trading on the long side of the market, thus allowing the fund to "hide" better its short-selling, and make a profit.

## 5 Trading in Both Debt and Equity

In this section we investigate the optimal strategy of the fund when it can trade both in debt and equity after the proposal but before voting. It is assumed that the fund has no initial positions in either debt or equity, and no capital constraints. Since we now have two markets, one for debt and one for equity we will study two cases in terms of the information each market-maker has. In the Connected Markets case an aggregate order in one market is perfectly observed from the other,
and vice versa. In the Disconnected Markets case each market-maker just observes the aggregate order in his own market and has no information on the other market's aggregate order. ${ }^{17}$

### 5.1 Connected Markets

Let $x_{E} \in\{-1,0,1\}$, and $x_{D} \in\{0,1\}$ be the trading positions of the fund in equity and debt, respectively. As before for proposals $E \in\left[0, E_{0}\right]$ pure debtholders vote yes, while for $E \in\left(E_{0}, \bar{E}\right]$ they vote no. This is relevant in the case where the fund chooses not to trade in debt, $x_{D}=0$, since then it does not vote and cannot be pivotal. So the profit the fund seeks to maximize when $E \in\left[0, E_{0}\right]$ is,

$$
\begin{array}{cc}
\max \{E+D, l\}-\mathbb{E}\left[p_{E} \mid x_{E}=1, x_{D}=1\right]-\mathbb{E}\left[p_{D} \mid x_{E}=1, x_{D}=1\right], & \Pi_{C}^{2}\left(x_{E}, x_{D} ; E\right)= \\
E-\mathbb{E}\left[p_{E} \mid x_{E}=1, x_{D}=0\right], & x_{E}=1, x_{D}=0, \\
0, & x_{E}=0, x_{D}=0, \\
\max \{-E+D, l\}+\mathbb{E}\left[p_{E} \mid x_{E}=-1, x_{D}=1\right]-\mathbb{E}\left[p_{D} \mid x_{E}=-1, x_{D}=1\right], & x_{E}=-1, x_{D}=1, \\
-E+\mathbb{E}\left[p_{E} \mid x_{E}=-1, x_{D}=0\right], & x_{E}=-1, x_{D}=0,
\end{array}
$$

Each expectation now depends on both trading positions, since both markets observe both aggregate (together with those of other investors) demands.

Again, for $E \in\left[0, E_{0} / 2\right]$ there is always continuation since even if the fund is pivotal and it shorts equity it still prefers for the firm to continue. In that case the price of equity, $p_{E}$, and debt, $p_{D}$, are equal to $E$ and $D$ respectively. The fund is then indifferent between trading or not (in either market) since its profit is always zero.

[^9]For $E \in\left(E_{0} / 2, E\right]$ we write the above as,

$$
\Pi_{C}^{2}\left(x_{E}, x_{D} ; E\right)=\left\{\begin{array}{cl}
y-\mathbb{E}\left[p_{E} \mid x_{E}=1, x_{D}=1\right]-\mathbb{E}\left[p_{D} \mid x_{E}=1, x_{D}=1\right], & x_{E}=1, x_{D}=1, \\
E-\mathbb{E}\left[p_{E} \mid x_{E}=1, x_{D}=0\right], & x_{E}=1, x_{D}=0, \\
0, & x_{E}=0, x_{D}=0, \\
l+\mathbb{E}\left[p_{E} \mid x_{E}=-1, x_{D}=1\right]-\mathbb{E}\left[p_{D} \mid x_{E}=-1, x_{D}=1\right], & x_{E}=-1, x_{D}=1, \\
-E+\mathbb{E}\left[p_{E} \mid x_{E}=-1, x_{D}=0\right], & x_{E}=-1, x_{D}=0 .
\end{array}\right.
$$

We know that $p_{E}$ is the (perceived by the market) probability of continuation times $E$, while $p_{D}$ is the probability of continuation times $y-E$ plus the probability of liquidation times $l$. In particular the expectation over $p_{E}$ can never exceed $E$. So strategy $\left\{x_{E}=-1, x_{D}=0\right\}$ is weakly dominated by $\left\{x_{E}=0, x_{D}=0\right\}$, which is, in turn, weakly dominated by strategy $\left\{x_{E}=1, x_{D}=0\right\}$. Hence, by ignoring weakly dominated strategies the fund's profit for each case is,

$$
\Pi_{C}^{2}\left(x_{E}, x_{D} ; E\right)=\left\{\begin{array}{cc}
y-\mathbb{E}\left[p_{E} \mid x_{E}=1, x_{D}=1\right]-\mathbb{E}\left[p_{D} \mid x_{E}=1, x_{D}=1\right], & x_{E}=1, x_{D}=1 \\
E-\mathbb{E}\left[p_{E} \mid x_{E}=1, x_{D}=0\right], & x_{E}=1, x_{D}=0 \\
l+\mathbb{E}\left[p_{E} \mid x_{E}=-1, x_{D}=1\right]-\mathbb{E}\left[p_{D} \mid x_{E}=-1, x_{D}=1\right], & x_{E}=-1, x_{D}=1
\end{array}\right.
$$

Now for proposals $E \in\left(E_{0}, \bar{E}\right]$, the profit of the fund in each case is

$$
\Pi_{L}^{2}\left(x_{E}, x_{D} ; E\right)=\left\{\begin{array}{cc}
y-\mathbb{E}\left[p_{E} \mid x_{E}=1, x_{D}=1\right]-\mathbb{E}\left[p_{D} \mid x_{E}=1, x_{D}=1\right], & x_{E}=1, x_{D}=1, \\
-\mathbb{E}\left[p_{E} \mid x_{E}=1, x_{D}=0\right], & x_{E}=1, x_{D}=0, \\
0, & x_{E}=0, x_{D}=0, \\
l+\mathbb{E}\left[p_{E} \mid x_{E}=-1, x_{D}=1\right]-\mathbb{E}\left[p_{D} \mid x_{E}=-1, x_{D}=1\right], & x_{E}=-1, x_{D}=1, \\
\mathbb{E}\left[p_{E} \mid x_{E}=-1, x_{D}=0\right], & x_{E}=-1, x_{D}=0 .
\end{array}\right.
$$

Note that the change is in the terms in which the fund is not pivotal, $x_{D}=0$. Similarly as above, strategy $\left\{x_{E}=1, x_{D}=0\right\}$ is weakly dominated by $\left\{x_{E}=0, x_{D}=0\right\}$, which is, in turn, weakly dominated by strategy $\left\{x_{E}=-1, x_{D}=0\right\}$. Hence, by ignoring weakly dominated strategies the
fund's profit for each case is,

$$
\Pi_{L}^{2}\left(x_{E}, x_{D} ; E\right)=\left\{\begin{array}{cc}
y-\mathbb{E}\left[p_{E} \mid x_{E}=1, x_{D}=1\right]-\mathbb{E}\left[p_{D} \mid x_{E}=1, x_{D}=1\right], & x_{E}=1, x_{D}=1 \\
l+\mathbb{E}\left[p_{E} \mid x_{E}=-1, x_{D}=1\right]-\mathbb{E}\left[p_{D} \mid x_{E}=-1, x_{D}=1\right], & x_{E}=-1, x_{D}=1 \\
\mathbb{E}\left[p_{E} \mid x_{E}=-1, x_{D}=0\right], & x_{E}=-1, x_{D}=0
\end{array}\right.
$$

There is again noise trading in equity, of level $z_{E} \in\{-1,1\}$, with $\nu_{E}=\mathbb{P}\left[z_{E}=1\right]$, but also, now, in debt, of level $z_{D} \in\{0,1\}$, with $\nu_{D} \triangleq \mathbb{P}\left[z_{D}=1\right]$. Noise trading is independent across markets, and from the fund's orders. The market-makers observe $y_{E}=x_{E}+z_{E}$ and $y_{D} \triangleq x_{D}+z_{D}$, and set prices for equity and debt according to the following equations, respectively,

$$
\begin{aligned}
p_{E} & =\mathbb{E}\left[\mathbb{I}(\text { continuation }) E \mid y_{E}, y_{D}\right] \\
& =\mathbb{P}\left[\text { continuation } \mid y_{E}, y_{D}\right] E, \\
p_{D} & \left.=\mathbb{E}\left[\mathbb{I}(\text { continuation })(y-E) \mid y_{E}, y_{D}\right]+\mathbb{E}[\mathbb{I} \text { (liquidation) }) \mid y_{E}, y_{D}\right] \\
& =\mathbb{P}\left[\text { continuation } \mid y_{E}, y_{D}\right](y-E)+\mathbb{P}\left[\text { liquidation } \mid y_{E}, y_{D}\right] l .
\end{aligned}
$$

Let $\alpha \triangleq \mathbb{P}\left[x_{D}=1 \mid x_{E}=1\right]$, i.e., the probability the fund is pivotal given that it is long in equity, and $\beta \triangleq \mathbb{P}\left[x_{D}=1 \mid x_{E}=-1\right]$, i.e., the probability the fund is pivotal given that it is short in equity. ${ }^{18}$ We describe the equilibrium behavior of the fund as given by $\lambda_{E}, \alpha$, and $\beta$, and the resulting probability of continuation, for $E \in[0, \bar{E}]$ in the following proposition.

Proposition 3. In the case of Connected Markets for debt and equity the fund's trading behavior varies as follows with the value of the proposal E.

- For $E \in\left[0, E_{0} / 2\right]: \lambda_{E} \in(0,1), \alpha \in(0,1), \beta \in(0,1)$, continuation with probability one.
- For $E \in\left(E_{0} / 2, \bar{E}\right]: \alpha=\beta=1, \lambda_{E}=E_{0} /(2 E)$, continuation with probability $\lambda_{E}(<1)$.

The manager, now, has to solve the following problem,

$$
\max _{E \in[0, \bar{E}]} \quad E \mathbb{I}\left(E \leq E_{0} / 2\right)+\left(E_{0} / 2\right) \mathbb{I}\left(E>E_{0} / 2\right),
$$

[^10]from which we see that she is indifferent for any point $E$ between $E_{0} / 2$ and $\bar{E}$. If maximizing the probability of continuation is her secondary objective then she picks $E=E_{0} / 2$, which guarantees continuation, essentially by making the fund indifferent between trading or not. ${ }^{19}$ Hence, in this case, the presence of the fund hurts equityholders (since in its absence they would get $E_{0}$ ) and benefits debtholders as in Section 4. However, the presence of the fund does not hurt the firm as a whole, since the probability of continuation is one as it is, also, the case in the absence of the fund.

The intuition for the result is as follows: As we saw the fund becomes pivotal no matter whether it is planning to go long or short equity, although the cost is different in each case. Furthermore, the connectedness of the markets implies that the perceived probabilities of continuation between the debt and equity markets are the same. Most importantly each market has "more" information than if it were to observe only its own order flow (as it is the case in the next section). This makes it harder for the fund to "hide" and in equilibrium forces it to mix between going long or short equity over a wider range of proposals. The fund actually mixes in a way that equalizes the equityholders' payoff over that range. The best response of the manager is then to pick the proposal that at least guarantees continuation with probability one, to which the fund has no incentive to deviate.

### 5.2 Disconnected Markets

Now, we look at the case where each market-maker observes aggregate demand, only in his own market. The strategies of the fund we excluded in the Connected Markets case based on weak dominance are still excluded here since our arguments did not depend on the information of the market-makers (and the resulting conditioning of the fund in its calculations of expected prices). Also, we will have the same "indifference" equilibrium as before in the case where the proposal $E \in\left[0, E_{0} / 2\right]$.

For $E \in\left(E_{0} / 2, E_{0}\right]$ the fund's profit for each (non-weakly dominated) trading strategy is

$$
\Pi_{C}^{3}\left(x_{E}, x_{D} ; E\right)=\left\{\begin{array}{cc}
y-\mathbb{E}\left[p_{E} \mid x_{E}=1\right]-\mathbb{E}\left[p_{D} \mid x_{D}=1\right], & x_{E}=1, x_{D}=1 \\
E-\mathbb{E}\left[p_{E} \mid x_{E}=1\right], & x_{E}=1, x_{D}=0 \\
l+\mathbb{E}\left[p_{E} \mid x_{E}=-1\right]-\mathbb{E}\left[p_{D} \mid x_{D}=1\right], & x_{E}=-1, x_{D}=1
\end{array}\right.
$$

[^11]The following proposition shows that the fund always finds it optimal to be pivotal when it is long equity, so in the notation of the previous section $\alpha=\beta=1$.

Proposition 4. Strategy $\left\{x_{E}=1, x_{D}=0\right\}$ is weakly dominated by strategy $\left\{x_{E}=1, x_{D}=1\right\}$.
This allows us to write,

$$
\Pi_{C}^{3}\left(x_{E}, x_{D} ; E\right)=\left\{\begin{array}{cc}
y-\mathbb{E}\left[p_{E} \mid x_{E}=1\right]-\mathbb{E}\left[p_{D} \mid x_{D}=1\right], \quad x_{E}=1, x_{D}=1 \\
l+\mathbb{E}\left[p_{E} \mid x_{E}=-1\right]-\mathbb{E}\left[p_{D} \mid x_{D}=1\right], & x_{E}=-1, x_{D}=1
\end{array}\right.
$$

Similarly, for $E \in\left(E_{0}, \bar{E}\right]$,

$$
\Pi_{L}^{3}\left(x_{E}, x_{D} ; E\right)=\left\{\begin{array}{cc}
y-\mathbb{E}\left[p_{E} \mid x_{E}=1\right]-\mathbb{E}\left[p_{D} \mid x_{D}=1\right], & x_{E}=1, x_{D}=1 \\
l+\mathbb{E}\left[p_{E} \mid x_{E}=-1\right]-\mathbb{E}\left[p_{D} \mid x_{D}=1\right], & x_{E}=-1, x_{D}=1 \\
+\mathbb{E}\left[p_{E} \mid x_{E}=-1\right], & x_{E}=-1, x_{D}=0
\end{array}\right.
$$

The following Proposition shows that the fund always finds it optimal to be pivotal when it is short equity, so in the notation of the previous section $\alpha=\beta=1$.

Proposition 5. Strategy $\left\{x_{E}=-1, x_{D}=0\right\}$ is weakly dominated by strategy $\left\{x_{E}=-1, x_{D}=1\right\}$.
This allows us to write,

$$
\Pi_{L}^{3}\left(x_{E}, x_{D} ; E\right)=\left\{\begin{array}{cc}
y-\mathbb{E}\left[p_{E} \mid x_{E}=1\right]-\mathbb{E}\left[p_{D} \mid x_{D}=1\right], \quad x_{E}=1, x_{D}=1 \\
l+\mathbb{E}\left[p_{E} \mid x_{E}=-1\right]-\mathbb{E}\left[p_{D} \mid x_{D}=1\right], \quad x_{E}=-1, x_{D}=1
\end{array}\right.
$$

Noise trading is as before and the market-makers set prices

$$
\begin{aligned}
p_{E} & =\mathbb{E}\left[\mathbb{I}(\text { continuation }) E \mid y_{E}\right] \\
& =\mathbb{P}\left[\text { continuation } \mid y_{E}\right] E, \\
p_{D} & \left.=\mathbb{E}\left[\mathbb{I}(\text { continuation })(y-E) \mid y_{D}\right]+\mathbb{E}[\mathbb{I} \text { (liquidation }) l \mid y_{D}\right] \\
& =\mathbb{P}\left[\text { continuation } \mid y_{D}\right](y-E)+\mathbb{P}\left[\text { liquidation } \mid y_{D}\right] l .
\end{aligned}
$$

Since the fund is pivotal for proposals $E \in\left(E_{0} / 2, \bar{E}\right]$ the pure debtholders' behavior is irrelevant and we need not distinguish between $\Pi_{C}^{3}$ and $\Pi_{L}^{3}$. Moreover, exactly because markets are discon-
nected, the price of becoming pivotal that enters the fund's calculations, $\mathbb{E}\left[p_{D} \mid x_{D}=1\right]$, is the same regardless of whether the fund decides to go long or short equity. The debt market-maker knows this and sets

$$
p_{D}=\lambda_{E}(y-E)+\left(1-\lambda_{E}\right) l,
$$

for all values of aggregate demand $y_{D}=x_{D}+z_{D},{ }^{20}$ where $\lambda_{E}=\mathbb{P}\left[x_{E}=1\right]$, and $z_{D} \in\{0,1\}$, as before. Similarly the equity market-maker sets,

$$
p_{E}=\left\{\begin{array}{cc}
E, & y_{E}=2 \\
\frac{\left(1-\nu_{E}\right) \lambda_{E}}{\left(1-\nu_{E}\right) \lambda_{E}+\nu_{E}\left(1-\lambda_{E}\right)} E, & y_{E}=0 \\
0, & y_{E}=-2
\end{array}\right.
$$

The fact that the fund becomes pivotal in equilibrium, and that the cost of doing so can be viewed as sunk (due to the disconnectedness) makes the equilibrium calculation of this section equivalent to the one of Section 4 for the case that the fund has a pivotal, i.e., $x_{D}=1$, exogenous position of debt. Thus the equilibrium also here is described by Proposition 1, and the manager's optimal decision by Proposition 2. Recalling our discussion there we see that the fund's existence hurts shareholders, as it reduces their expected payoff, and benefits debtholders. Firm value, in contrast to the Connected Markets case, decreases when it is more probable for noise trading in the equity market to be in the long side. This is because a "long-bias" in noise trading allows the fund to make a profit by being able to "hide" better its short-selling, and hence affects firm value by making liquidation more probable.

## 6 Implications and Further Issues

In the following subsections we discuss certain implications of our model, and mention how our framework can be modified to address closely related issues.

### 6.1 Transparency vs Disclosure

We showed that stringent disclosure requirements are sufficient for the firm to avoid inefficient liquidation, only when they are coupled with a trading ban for the period between the announcement

[^12]of the proposal and the voting date. Actually, if, as in Section 4, the manager knows the fund is a pivotal debtholder then trading in equity yields a positive probability of liquidation. On the contrary, when the manager does not know the fund's positions and the markets for debt and equity are connected, subsection 5.1, we get continuation with certainty. In this case it is exactly the absence of disclosure and the allowance of trading, coupled with transparency between markets, that deters inefficient liquidation. The virtue of some uncertainty is in accordance with the seminal Hirshleifer (1971), and with the notion of symmetric ignorance between traders in Dang, Gorton, and Holmström (2009). Of course in our model, without transparency between markets we also get a positive probability of liquidation, subsection 5.2.

### 6.2 Debt Capacity

We showed that it is the firm's debtholders who benefit by the fund's involvement, while equityholders get a lower expected value than if there is no fund, regardless of the effect on firm value. It would be then interesting to study whether or not the debt capacity of the firm is increased ex ante, i.e., prior to default. There are two competing forces that will determine this: the positive probability of liquidation after default in some cases, may make the manager issue less debt in order to decrease the probability of default; on the other hand, investors are more willing to lend to a firm that, if it were to default, would pass proposals that benefit debtholders.

Bolton and Oehmke (2010) find that allowing debtholders to trade in the CDS market increases the debt capacity of the firm ex ante when the firm has no equity and the manager has the option of strategic default. If we extend our model to study the ex ante problem and maintain our assumption that the manager is absent of any agency problems then strategic default is not an issue; the firm in our model, however, has equity outstanding and hence the manager in an ex ante model will choose the leverage ratio to balance the two forces we mention above (plus any other considerations, e.g., corporate tax).

### 6.3 Nash-Bargaining Solution

In our model the surplus that is split between equity and debt holders after default is $y-l$. Debtholders can guarantee themselves $l$ by voting no, in which case equityholders receive zero. We modelled the bargaining game as a take-it-or-leave-it offer from the equityholders to debtholders,
essentially giving all the bargaining power to equityholders. Then if there are only pure debtholders, equityholders get $E_{0}=y-l$ and debtholders $y$. In the cases we studied in Sections 4 and 5 the equilibrium proposal is less than $E_{0}$, and, in particular, in the Connected Markets case it is $E_{0} / 2$. Recall that $E_{0} / 2$ is the maximum value of $E$ for which the fund is indifferent between trading or not, and we always achieve the efficient outcome, i.e., continuation. It turns out that $E_{0} / 2$ is also the Nash-Bargaining solution, and hence what a social planner would want to enforce. Our study, essentially, shows how the ability of the fund to trade and short the equity of the firm moves the bargaining power towards the debtholders, and in the case of Connected Markets how it actually leads us to the Nash-Bargaining solution.

### 6.4 Credit Default Swaps

A CDS claim that is triggered by liquidation is an asset that pays zero if there is continuation and pays a positive nominal amount otherwise. So in the context of our model, a CDS claim with nominal $E$ (the proposed payment to equityholders) is equivalent to a short equity claim plus a certain amount $E$ paid regardless of the voting outcome. Of course this equivalence holds only if the CDS and the equity markets have equal probability (of continuation) assessments. So our model can also encompass the case in which there is no trade in equity but there is trade in CDS, even of this restrictive type. Hence our argument in favor of transparency and connectedness also applies between the debt and CDS markets.

An interesting point to note is that the equivalence of short equity claims and CDS in our framework also relies on the assumption that the proposed payoff is credible, and the continuation/liquidation values are known. Lack of credibility of the manager's proposal (due to agency problems) would not allow investors to infer the probability of continuation just by observing the price of equity or the price of debt. ${ }^{21}$ Then the CDS market would be the only clear indicator of the probability of continuation.

### 6.5 Penalizing Short-selling

In a straightforward extension (available upon request) we show that in all studied cases penalizing short-selling (of the equity of the distressed firm) both decreases the probability of liquidation, as

[^13]well as reduces the range of proposals for which the fund chooses to randomize between going long or short the equity of the firm. As expected, when there is a sort-selling ban, i.e., a $100 \%$ penalty, the fund is indifferent between entering the market altogether or not. For any value of the penalty less than $100 \%$, however, the probability of liquidation remains positive in the cases we studied where it was not zero. So penalizing short-selling decreases but does not eliminate the probability of liquidation, while at the same time may deprive firms from necessary financing.

### 6.6 Acquisitions

Our model about how trading and, in particular, short-selling can affect real outcomes, and how the informational structure of markets is related to these real outcomes can be applied equally well to the study of acquisitions. ${ }^{22}$ Imagine that the "manager" in our model is making an offer on behalf of an acquirer company, and that the "debtholders" in our model are the shareholders of the target company. The offer suggests a way of dividing the surplus from the merger, and if the target company rejects the offer, the "manager" would simply keep the acquirer company, which would equal liquidation in the current model. The "fund" then is a potential shareholder of the target who can long or short the equity of the acquirer. This can lead to a rejection by the shareholders of the target of a value enhancing merger proposal.

## 7 Conclusions

We presented a simplified model of a debt restructuring proposal and its outcome in the presence of a non-traditional fund. The fund has the ability to buy or short-sell equity, and also buy debt in a distressed firm. If it acquires debt then it can vote on the restructuring proposal. The manager represents the equityholders and makes the proposal anticipating the fund's possible involvement. We showed that it is the firm's debtholders who benefit by the fund's involvement, while equity holders get a lower expected value than if there is no fund, regardless of the effect on firm value.

According to our results the effect on firm value depends crucially on the connectedness between the markets for debt and equity. Our criterion for connectedness is whether there are discrepancies in the assessment of the probability of continuation (emergence) between the markets for debt and

[^14]equity; in the context of our model connectedness is whether market-makers can observe aggregate flows in markets other than their own.

In the case of Disconnected Markets we showed that the fund's existence and its ability to short equity lead to inefficient liquidation with positive probability for more probable long-equity noise trading. In contrast, in the case of Connected Markets the fund's existence does not affect firm value. Hence, from a regulatory perspective, a remedy to inefficient liquidation is advocating for more transparency and exchange of timely information between markets, rather than an outright trading ban, which can hurt market liquidity.

## Appendix

Proof of Proposition 1. For the equilibrium in the case where $E \in\left[0, E_{0} / 2\right]$ see the main text. Now for $E \in\left(E_{0} / 2, \bar{E}\right]$ we have

$$
\begin{aligned}
\mathbb{E}\left[p_{E} \mid x_{E}=1\right] & =\mathbb{E}\left[p_{E} \mid y_{E}=2\right] \mathbb{P}\left[z_{E}=1\right]+\mathbb{E}\left[p_{E} \mid y_{E}=0\right] \mathbb{P}\left[z_{E}=1\right] \\
& =E \nu_{E}+E\left(1-\nu_{E}\right) \frac{\left(1-\nu_{E}\right) \lambda_{E}}{\left(1-\nu_{E}\right) \lambda_{E}+\nu_{E}\left(1-\lambda_{E}\right)}, \\
\mathbb{E}\left[p_{E} \mid x_{E}=-1\right] & =\mathbb{E}\left[p_{E} \mid y_{E}=0\right] \mathbb{P}\left[z_{E}=1\right]+\mathbb{E}\left[p_{E} \mid y_{E}=-2\right] \mathbb{P}\left[z_{E}=-2\right] \\
& =E \nu_{E} \frac{\left(1-\nu_{E}\right) \lambda_{E}}{\left(1-\nu_{E}\right) \lambda_{E}+\nu_{E}\left(1-\lambda_{E}\right)} .
\end{aligned}
$$

Then the fund will choose an action according to the following relation

$$
\begin{aligned}
\Pi^{1}(1 ; E) & \gtrless \Pi^{1}(-1 ; E) \Longleftrightarrow \\
y-E \nu_{E}+E\left(1-\nu_{E}\right) \frac{\left(1-\nu_{E}\right) \lambda_{E}}{\left(1-\nu_{E}\right) \lambda_{E}+\nu_{E}\left(1-\lambda_{E}\right)} & \gtrless l+E \nu_{E} \frac{\left(1-\nu_{E}\right) \lambda_{E}}{\left(1-\nu_{E}\right) \lambda_{E}+\nu_{E}\left(1-\lambda_{E}\right)} \Longleftrightarrow \\
E_{0} & \gtrless E \frac{\left(1-\nu_{E}\right) \lambda_{E}\left(1+\nu_{E}\right)+\nu_{E}^{2}\left(1-\lambda_{E}\right)}{\left(1-\nu_{E}\right) \lambda_{E}+\nu_{E}\left(1-\lambda_{E}\right)} .
\end{aligned}
$$

We want to see if $\lambda_{E}=1$ is an equilibrium. For this, from above, we would need $E_{0}>E\left(1+\nu_{E}\right)$ or

$$
E<\frac{E_{0}}{1+\nu_{E}} .
$$

This can be true only in the case where $E \in\left(E_{0} / 2, E_{0}\right]$. Similarly we want to see if $\lambda_{E}=0$ is an equilibrium. For this, from above, we would need $E_{0}<E \nu_{E}$ or

$$
E>\frac{E_{0}}{\nu_{E}} .
$$

This can be true only in the case where $E \in\left(E_{0}, \bar{E}\right]$. Now, the indifference condition for a mixed equilibrium leads to

$$
\begin{aligned}
E_{0} & =E \frac{\left(1-\nu_{E}\right) \lambda_{E}\left(1+\nu_{E}\right)+\nu_{E}^{2}\left(1-\lambda_{E}\right)}{\left(1-\nu_{E}\right) \lambda_{E}+\nu_{E}\left(1-\lambda_{E}\right)} \Longleftrightarrow \\
\lambda_{E} & =\nu_{E} \frac{E_{0} / E-\nu_{E}}{E_{0} / E\left(2 \nu_{E}-1\right)-\left(2 \nu_{E}^{2}-1\right)} .
\end{aligned}
$$

To make sure this is a valid mixing probability we must have $\lambda_{E} \in(0,1)$. Note that,

- $E_{0} / E-\nu_{E}>0 \Longleftrightarrow E<E_{0} / \nu_{E}$,
- Given the above,

$$
E_{0} / E\left(2 \nu_{E}-1\right)-\left(2 \nu_{E}^{2}-1\right)>\nu_{E}\left(2 \nu_{E}-1\right)-\left(2 \nu_{E}^{2}-1\right)>1-\nu_{E}>0,
$$

- Given these

$$
E_{0} / E-\nu_{E}<E_{0} / E\left(2 \nu_{E}-1\right)-\left(2 \nu_{E}^{2}-1\right) \Longleftrightarrow E_{0} / E<1+\nu_{E} \Longleftrightarrow E>E_{0} /\left(1+\nu_{E}\right)
$$

Hence for $E \in\left[E_{0} /\left(1+\nu_{E}\right), E_{0} / \nu_{E}\right]$ the fund mixes between going long and short with probability $\lambda_{E}$ given above.

Proof of Proposition 2. We have that

- $J\left(E_{0} / \nu_{E}\right)=0<E_{0} /\left(1+\nu_{E}\right)=J\left(E_{0} /\left(1+\nu_{E}\right)\right)$, so $E_{0} / \nu_{E}$ cannot be the solution for any $\nu_{E} \in(0,1)$. So the solution is either $E_{0} /\left(1+\nu_{E}\right)$ or an internal one.
- The first order condition (FOC) has two real solutions,

$$
E_{1} \triangleq \frac{2 \nu_{E}^{2}-\nu_{E}-\sqrt{\nu_{E}} \sqrt{-2 \nu_{E}^{2}+3 \nu_{E}-1}}{\nu_{E}\left(2 \nu_{E}^{2}-1\right)} E_{0},
$$

$$
E_{2} \triangleq \frac{2 \nu_{E}^{2}-\nu_{E}+\sqrt{\nu_{E}} \sqrt{-2 \nu_{E}^{2}+3 \nu_{E}-1}}{\nu_{E}\left(2 \nu_{E}^{2}-1\right)} E_{0},
$$

for $\nu_{E}>1 / 2$ (and no real solutions otherwise).

- $E_{2}>E_{0} / \nu_{E}$ for all $\nu_{E} \in(0,1)$ and hence cannot be a solution.
- $E_{1}<E_{0} /\left(1+\nu_{E}\right)$ for $\nu_{E}<\sqrt{5} / 2-1 / 2$, and $E_{1} \in\left(E_{0} /\left(1+\nu_{E}\right), E_{0} / \nu_{E}\right)$ for $\nu_{E} \geq \sqrt{5} / 2-1 / 2$.
- $J^{\prime}\left(E_{0} /\left(1+\nu_{E}\right)\right)<0$ for $\nu_{E}<\sqrt{5} / 2-1 / 2$, and $J^{\prime}\left(E_{0} /\left(1+\nu_{E}\right)\right) \geq 0$ for $\nu_{E} \geq \sqrt{5} / 2-1 / 2$.
- $J^{\prime}\left(E_{0} / \nu_{E}\right)<0$ for all $\nu_{E} \in(0,1)$.

Hence the solution for $\nu_{E}<\sqrt{5} / 2-1 / 2$ is $E_{0} /\left(1+\nu_{E}\right)$ with $J\left(E_{0} /\left(1+\nu_{E}\right)\right)=E_{0} /\left(1+\nu_{E}\right)$, and probability of continuation one. While for $\nu_{E} \geq \sqrt{5} / 2-1 / 2$ the solution is $E_{1}$ with

$$
J\left(E_{1}\right)=\left(\frac{\sqrt{2 \nu_{E}\left(\nu_{E}-1 / 2\right)}-\sqrt{1-\nu_{E}}}{2 \nu_{E}^{2}-1}\right)^{2} \nu_{E} E_{0}
$$

and probability of continuation

$$
\sqrt{\nu_{E}} \frac{\sqrt{2 \nu_{E}\left(\nu_{E}-1 / 2\right)}-\sqrt{1-\nu_{E}}}{\left(2 \nu_{E}^{2}-1\right) \sqrt{2\left(\nu_{E}-1 / 2\right)}} .
$$

Both $J\left(E_{0} /\left(1+\nu_{E}\right)\right)$ and $J\left(E_{1}\right)$ are less than $E_{0}$.
Proof of Proposition 3. Since the markets in debt and equity are connected the perceived probabilities of continuation and liquidation between the two markets are going to be the same. This observation allows us to write the fund's payoffs when $E \in\left(E_{0} / 2, E_{0}\right]$ as, ${ }^{23}$

$$
\Pi_{C}^{L}\left(x_{E}, x_{D} ; E\right)=\left\{\begin{array}{cl}
\mathbb{P}_{m k t}\left[\text { liquidation } \mid x_{E}=1, x_{D}=1\right] E_{0}, & x_{E}=1, x_{D}=1 \\
\mathbb{P}_{m k t}\left[\text { liquidation } \mid x_{E}=1, x_{D}=0\right] E, & x_{E}=1, x_{D}=0 \\
\left(1-\mathbb{P}_{m k t}\left[\text { liquidation } \mid x_{E}=-1, x_{D}=1\right]\right)\left(2 E-E_{0}\right), & x_{E}=-1, x_{D}=1
\end{array}\right.
$$

The " $m k t$ " subscript in the probabilities above is meant to stress that these are the market perceived probabilities of continuation, which the fund tries to assess given its action and its prior on noise trading; in this case, of course, $\beta=1$.

[^15]We have

$$
\begin{array}{r}
\mathbb{P}_{m k t}\left[\text { liquidation } \mid x_{E}=e, x_{D}=d\right] \quad= \\
\mathbb{P}_{m k t}\left[\text { liquidation } \mid x_{E}=e, x_{D}=d, z_{E}=1, z_{D}=1\right] \mathbb{P}\left[z_{E}=1, z_{D}=1\right] \quad+ \\
\mathbb{P}_{m k t}\left[\text { liquidation } \mid x_{E}=e, x_{D}=d, z_{E}=1, z_{D}=0\right] \mathbb{P}\left[z_{E}=1, z_{D}=0\right] \quad+ \\
\mathbb{P}_{m k t}\left[\text { liquidation } \mid x_{E}=e, x_{D}=d, z_{E}=-1, z_{D}=1\right] \mathbb{P}\left[z_{E}=-1, z_{D}=1\right] \quad+ \\
\mathbb{P}_{m k t}\left[\text { liquidation } \mid x_{E}=e, x_{D}=d, z_{E}=-1, z_{D}=0\right] \mathbb{P}\left[z_{E}=-1, z_{D}=0\right] \quad= \\
\mathbb{P}\left[x_{E}=-1, x_{D}=1 \mid y_{E}=e+1, y_{D}=d+1\right] \nu_{E} \nu_{D} \quad+ \\
\mathbb{P}\left[x_{E}=-1, x_{D}=1 \mid y_{E}=e+1, y_{D}=d\right] \nu_{E}\left(1-\nu_{D}\right) \quad+ \\
\mathbb{P}\left[x_{E}=-1, x_{D}=1 \mid y_{E}=e-1, y_{D}=d+1\right]\left(1-\nu_{E}\right) \nu_{D} \quad+ \\
\mathbb{P}\left[x_{E}=-1, x_{D}=1 \mid y_{E}=e-1, y_{D}=d\right]\left(1-\nu_{E}\right)\left(1-\nu_{D}\right),
\end{array}
$$

where $e \in\{-1,1\}, d \in\{0,1\}$. Note that

$$
\mathbb{P}_{m k t}[\text { liquidation }]=1-\mathbb{P}_{m k t}[\text { continuation }]=\mathbb{P}\left[x_{E}=-1, x_{D}=1\right],
$$

so,

$$
\begin{array}{r}
\mathbb{P}\left[x_{E}=-1, x_{D}=1 \mid y_{E}=2, y_{D}=2\right]=0, \\
\mathbb{P}\left[x_{E}=-1, x_{D}=1 \mid y_{E}=2, y_{D}=1\right]=0, \\
\mathbb{P}\left[x_{E}=-1, x_{D}=1 \mid y_{E}=0, y_{D}=2\right]=K, \\
\mathbb{P}\left[x_{E}=-1, x_{D}=1 \mid y_{E}=0, y_{D}=1\right]=\Lambda, \\
\mathbb{P}\left[x_{E}=-1, x_{D}=1 \mid y_{E}=2, y_{D}=0\right]=0, \\
\mathbb{P}\left[x_{E}=-1, x_{D}=1 \mid y_{E}=0, y_{D}=0\right]=0, \\
\mathbb{P}\left[x_{E}=-1, x_{D}=1 \mid y_{E}=-2, y_{D}=2\right]=1, \\
\mathbb{P}\left[x_{E}=-1, x_{D}=1 \mid y_{E}=-2, y_{D}=1\right]=1,
\end{array}
$$

where

$$
\begin{aligned}
K & \triangleq \frac{\nu_{E}\left(1-\lambda_{E}\right)}{\nu_{E}\left(1-\lambda_{E}\right)+\left(1-\nu_{E}\right) \alpha \lambda_{E}}, \\
\Lambda & \triangleq \frac{\nu_{E}\left(1-\nu_{D}\right)\left(1-\lambda_{E}\right)}{\left(1-\nu_{E}\right)\left(1-\nu_{D}\right) \alpha \lambda_{E}+\nu_{E}\left(1-\nu_{D}\right)\left(1-\lambda_{E}\right)+\left(1-\nu_{E}\right) \nu_{D}(1-\alpha) \lambda_{E}},
\end{aligned}
$$

so that

$$
\begin{aligned}
& \mathbb{P}_{m k t}\left[\text { liquidation } \mid x_{E}=1, x_{D}=1\right]=K\left(1-\nu_{E}\right) \nu_{D}+\Lambda\left(1-\nu_{E}\right)\left(1-\nu_{D}\right), \\
& \mathbb{P}_{m k t}\left[\text { liquidation } \mid x_{E}=1, x_{D}=0\right]=\Lambda\left(1-\nu_{E}\right) \nu_{D} .
\end{aligned}
$$

We consider the following possibilities for an equilibrium,

- If $\alpha=1$, i.e., the fund prefers to become pivotal when it is long in equity, then

$$
K=\Lambda=\frac{\nu_{E}\left(1-\lambda_{E}\right)}{\nu_{E}\left(1-\lambda_{E}\right)+\left(1-\nu_{E}\right) \lambda_{E}},
$$

so that

$$
\Pi_{C}^{2}(1,1 ; E)=K\left(1-\nu_{E}\right) E_{0} \quad>K\left(1-\nu_{E}\right) v_{D} E=\Pi_{C}^{2}(1,0 ; E),
$$

and this is an equilibrium.

- If $\alpha=0$, i.e., the fund prefers not to become pivotal when it is long in equity, then

$$
\begin{aligned}
K & =1 \\
\Lambda & =\frac{\nu_{E}\left(1-\nu_{D}\right)\left(1-\lambda_{E}\right)}{\nu_{E}\left(1-\nu_{D}\right)\left(1-\lambda_{E}\right)+\left(1-\nu_{E}\right) \nu_{D} \lambda_{E}}<1
\end{aligned}
$$

so that,

$$
\begin{array}{r}
\Pi_{C}^{2}(1,1 ; E)=\left(1-\nu_{E}\right) \nu_{D}+\Lambda\left(1-\nu_{E}\right)\left(1-\nu_{D}\right)> \\
\Lambda\left(1-\nu_{E}\right) \nu_{D}=\Pi_{C}^{2}(1,0 ; E)
\end{array}
$$

so this cannot be an equilibrium.

Hence in equilibrium, the fund always becomes pivotal either it is long or short in equity, i.e., $\alpha=\beta=1$, and $K=\Lambda$. Given this we also have that,

$$
\begin{aligned}
\mathbb{P}_{m k t}\left[\text { liquidation } \mid x_{E}=-1, x_{D}=1\right] & =K \nu_{E} \nu_{D}+\Lambda \nu_{E}\left(1-\nu_{D}\right)+\left(1-\nu_{E}\right) \nu_{D}+\left(1-\nu_{E}\right)\left(1-\nu_{D}\right) \\
& =K \nu_{E}+\left(1-\nu_{E}\right),
\end{aligned}
$$

so that,

$$
\Pi_{C}^{2}(-1,1 ; E)=(1-K) \nu_{E}\left(2 E-E_{0}\right)
$$

We consider the following possibilities for an equilibrium,

- If $\lambda_{E}=1$, i.e., the fund prefers to go long in equity, then $K=0$ and

$$
\Pi_{C}^{2}(1,1 ; E)=0<\nu_{E}\left(2 E-E_{0}\right)=\Pi_{C}^{2}(-1,1 ; E)
$$

so this cannot be an equilibrium.

- If $\lambda_{E}=0$, i.e., the fund prefers to go short in equity, then $K=1$ and

$$
\Pi_{C}^{2}(-1,1 ; E)=0<\left(1-\nu_{E}\right) E_{0}=\Pi_{C}^{2}(1,1 ; E)
$$

so this cannot be an equilibrium.

Any mixed equilibria should be such that

$$
\begin{aligned}
\Pi_{C}^{2}(-1,1 ; E) & =\Pi_{C}^{2}(1,1 ; E) \Longleftrightarrow \\
(1-K) \nu_{E}\left(2 E-E_{0}\right) & =K\left(1-\nu_{E}\right) E_{0} \Longleftrightarrow \\
\lambda_{E} & =\frac{E_{0}}{2 E},
\end{aligned}
$$

which is a well defined probability since $E>E_{0} / 2$.

Similarly, for $E \in\left(E_{0}, \bar{E}\right]$, we have

$$
\Pi_{L}^{2}\left(x_{E}, x_{D} ; E\right)=\left\{\begin{array}{cc}
\left(1-\mathbb{P}_{m k t}\left[\text { continuation } \mid x_{E}=1, x_{D}=1\right]\right) E_{0}, & x_{E}=1, x_{D}=1 \\
\left.\mathbb{P}_{m k t}\left[\text { continuation } \mid x_{E}=-1, x_{D}=1\right]\right)\left(2 E-E_{0}\right), & x_{E}=-1, x_{D}=1 \\
\left.\mathbb{P}_{m k t}\left[\text { continuation } \mid x_{E}=-1, x_{D}=0\right]\right) E, & x_{E}=-1, x_{D}=0
\end{array}\right.
$$

In this regime, of course, $\alpha=1$, i.e., the fund always becomes pivotal when it is long in equity. Note that

$$
\mathbb{P}_{m k t}[\text { continuation }]=1-\mathbb{P}_{m k t}[\text { liquidation }]=\mathbb{P}\left[x_{E}=1, x_{D}=1\right],
$$

so similar calculations as before yield

$$
\begin{array}{r}
\mathbb{P}\left[x_{E}=1, x_{D}=1 \mid y_{E}=2, y_{D}=2\right]=1, \\
\mathbb{P}\left[x_{E}=1, x_{D}=1 \mid y_{E}=2, y_{D}=1\right]=1, \\
\mathbb{P}\left[x_{E}=1, x_{D}=1 \mid y_{E}=0, y_{D}=2\right]=K^{\prime}, \\
\mathbb{P}\left[x_{E}=1, x_{D}=1 \mid y_{E}=0, y_{D}=1\right]=\Lambda^{\prime}, \\
\mathbb{P}\left[x_{E}=1, x_{D}=1 \mid y_{E}=-2, y_{D}=2\right]=0, \\
\mathbb{P}\left[x_{E}=1, x_{D}=1 \mid y_{E}=-2, y_{D}=1\right]=0, \\
\mathbb{P}\left[x_{E}=1, x_{D}=1 \mid y_{E}=0, y_{D}=0\right]=0, \\
\mathbb{P}\left[x_{E}=1, x_{D}=1 \mid y_{E}=-2, y_{D}=0\right]=0,
\end{array}
$$

where

$$
\begin{aligned}
K^{\prime} & \triangleq \frac{\left(1-\nu_{E}\right) \lambda_{E}}{\nu_{E} \beta\left(1-\lambda_{E}\right)+\left(1-\nu_{E}\right) \lambda_{E}}, \\
\Lambda^{\prime} & \triangleq \frac{\left(1-\nu_{E}\right)\left(1-\nu_{D}\right) \lambda_{E}}{\left(1-\nu_{E}\right)\left(1-\nu_{D}\right) \lambda_{E}+\nu_{E}\left(1-\nu_{D}\right) \beta\left(1-\lambda_{E}\right)+\nu_{E} \nu_{D}(1-\beta)\left(1-\lambda_{E}\right)},
\end{aligned}
$$

so that

$$
\begin{aligned}
\mathbb{P}_{m k t}\left[\text { continuation } \mid x_{E}=-1, x_{D}=1\right] & =\nu_{E} \nu_{D} K^{\prime}+\left(1-\nu_{D}\right) \nu_{E} \Lambda^{\prime}, \\
\mathbb{P}_{m k t}\left[\text { liquidation } \mid x_{E}=-1, x_{D}=0\right] & =\nu_{E} \nu_{D} \Lambda^{\prime} .
\end{aligned}
$$

We consider the following possibilities for an equilibrium,

- If $\beta=1$, i.e., the fund prefers to become pivotal when it is short in equity, then

$$
K^{\prime}=\Lambda^{\prime}=\frac{\left(1-\nu_{E}\right) \lambda_{E}}{\nu_{E} \beta\left(1-\lambda_{E}\right)+\left(1-\nu_{E}\right) \lambda_{E}},
$$

so that

$$
\begin{array}{r}
\Pi_{L}^{2}(-1,1 ; E)=\nu_{E} K^{\prime}\left(2 E-E_{0}\right) \\
\nu_{E} K^{\prime} E=\nu_{E} \Lambda^{\prime} E>\nu_{E} \nu_{D} \Lambda^{\prime} E=\Pi_{L}^{2}(-1,0 ; E),
\end{array}
$$

and this is an equilibrium.

- If $\beta=0$, i.e., the fund prefers not to become pivotal when it is short in equity, then

$$
\begin{aligned}
K^{\prime} & =1, \\
\Lambda^{\prime} & =\frac{\left(1-\nu_{E}\right)\left(1-\nu_{D}\right) \lambda_{E}}{\left(1-\nu_{E}\right)\left(1-\nu_{D}\right) \lambda_{E}+\nu_{E} \nu_{D}\left(1-\lambda_{E}\right)}<1,
\end{aligned}
$$

so that,

$$
\begin{array}{r}
\Pi_{L}^{2}(-1,0 ; E)=\nu_{E} \nu_{D} \Lambda^{\prime} E< \\
\nu_{E}\left[\nu_{D}+\left(1-\nu_{D}\right) \Lambda^{\prime}\right]\left(2 E-E_{0}\right)=\Pi_{L}^{2}(-1,1 ; E),
\end{array}
$$

so this cannot be an equilibrium.

Hence in equilibrium, the fund always becomes pivotal either it is long or short in equity, i.e., $\alpha=\beta=1$, and $K^{\prime}=\Lambda^{\prime}$. Given this we also have that,

$$
\mathbb{P}_{m k t}\left[\text { liquidation } \mid x_{E}=1, x_{D}=1\right]=\left(1-K^{\prime}\right)\left(1-\nu_{E}\right),
$$

so that,

$$
\Pi_{L}^{2}(1,1 ; E)=\left(1-K^{\prime}\right)\left(1-\nu_{E}\right) E .
$$

We consider the following possibilities for an equilibrium,

- If $\lambda_{E}=1$, i.e., the fund prefers to go long in equity, then $K^{\prime}=1$ and

$$
\Pi_{L}^{2}(1,1 ; E)=0<\nu_{E}\left(2 E-E_{0}\right)=\Pi_{L}^{2}(-1,1 ; E)
$$

so this cannot be an equilibrium.

- If $\lambda_{E}=0$, i.e., the fund prefers to go short in equity, then $K^{\prime}=0$ and

$$
\Pi_{L}^{2}(-1,1 ; E)=0<\left(1-\nu_{E}\right) E_{0}=\Pi_{L}^{2}(1,1 ; E)
$$

so this cannot be an equilibrium.

Any mixed equilibria should be such that

$$
\begin{aligned}
\Pi_{L}^{2}(-1,1 ; E) & =\Pi_{L}^{2}(1,1 ; E) \Longleftrightarrow \\
K^{\prime} \nu_{E}\left(2 E-E_{0}\right) & =\left(1-K^{\prime}\right)\left(1-\nu_{E}\right) E_{0} \Longleftrightarrow \\
\lambda_{E} & =\frac{E_{0}}{2 E}
\end{aligned}
$$

which is a well defined probability since $E>E_{0} / 2$.
Hence also in this regime the unique equilibrium is $\alpha=\beta=1$ and $\lambda_{E}=E_{0} /(2 E)$.
Proof of Proposition 4. We need to show that

$$
y-E \geq \mathbb{E}\left[p_{D} \mid x_{D}=1\right],
$$

for $E \in\left(E_{0} / 2, E_{0}\right]$. Using the notation of the proof of Proposition 3,

$$
\mathbb{E}\left[p_{D} \mid x_{D}=1\right]=\left(1-\mathbb{P}_{m k t}[\text { liquidation }]\right)(y-E)+\mathbb{P}_{m k t}[\text { liquidation }] l .
$$

Since $E \leq E_{0}$ we have $y-E \geq l$, and $y-E$ is certainly greater or equal to the convex combination of itself and something smaller.

Proof of Proposition 5. We need to show that

$$
l \geq \mathbb{E}\left[p_{D} \mid x_{D}=1\right],
$$

for $E \in\left(E_{0}, \bar{E}\right]$. Using the notation of the proof of Proposition 3,

$$
\mathbb{E}\left[p_{D} \mid x_{D}=1\right]=\left(1-\mathbb{P}_{m k t}[\text { liquidation }]\right)(y-E)+\mathbb{P}_{m k t}[\text { liquidation }] l .
$$

Since $E>E_{0}$ we have $l>y-E$, and $l$ is certainly greater or equal to the convex combination of itself and something smaller.

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[^1]:    ${ }^{1}$ For example, banks in the United States are allowed to have only long positions in the equity of their clients, and only in case of financial distress. The main rationale for allowing banks to be long in both debt and equity for the same company is to have their incentives aligned to those of their debtors.
    ${ }^{2}$ Trading of distressed securities is not prohibited in the United States, though some distressed securities might be delisted from major exchanges like the New York Stock Exchange. Trading continues, however, in secondary markets, and that makes the issue of position reporting and transparency even more relevant.
    ${ }^{3}$ As it will be clear from our analysis the fund has nothing to gain by short-selling the debt of the firm, and so it is without loss of generality to ignore this possibility.
    ${ }^{4}$ In distressed firms it is the debtholders who express their preferences via voting to debt restructuring proposals. Using the United States bankruptcy law as an example, debt restructuring proposals happen typically under Chapter 11 which if rejected may lead to liquidation under Chapter 7.

[^2]:    ${ }^{5}$ Our setup can also be applied to model other interesting issues, for an example see Section 6.6.
    ${ }^{6}$ Essentially we posit that it is beneficial for the firm if the proposal is approved rather than not, abstracting from what happens after a proposal is rejected, and terming the outcome in that case as liquidation.

[^3]:    ${ }^{7}$ Admittedly this is a reduced form model of an actual debt restructuring procedure in which the proposal is the result of negotiations between equity and debt holders, and the voting outcome may be challenged by a judge. Our main goal here is, however, to highlight the role of trading in the procedure.
    ${ }^{8}$ In general, equity holdings' filings are less regular than their debt counterparts.

[^4]:    ${ }^{9}$ There are several papers, e.g., Hotchkiss and Ronen (2002), that study discrepancies between the debt market and the equity market in how quickly each reacts to firm specific news, e.g., earnings announcements. In our model, however, any discrepancy between the debt and the equity market is due to the observation of different order flows, and not to the reaction speed to firm specific news. Even for rather advanced capital markets, it is reasonable to assume that signals pertaining to order flows across markets are integrated to different degrees.
    ${ }^{10}$ The bankruptcy literature has studied "deviations from priority," e.g., Eberhart and Weiss (1998), in which equityholders come away with some value despite debtholders not being paid in full. Our model then suggests that

[^5]:    ${ }^{11}$ We are not going to distinguish between senior and junior debt, or between preferred and common shares. Debtholders are senior claimants while equityholders are residual claimants.
    ${ }^{12}$ Varying the bargaining power in favor of debtholders would make liquidation less probable in our model but would not alter our qualitative results; in this sense our analysis can be viewed as a worst-case-scenario.

[^6]:    ${ }^{13}$ We use the symbol " $\triangleq$ " to denote definition.

[^7]:    ${ }^{14}$ The non-inclusion of weakly dominated strategies is consistent with agents' caution and not just agents' rationality, as mentioned in Mas-Colell, Whinston, and Green (1995, Section 8.F). This in turn implies that the equilibria we analyze are trembling hand-perfect.

[^8]:    ${ }^{15}$ Note that we do not consider the case $z_{E}=0$ because it would not enhance the fund's ability to camouflage as $x_{E}$ cannot be zero (the action $\left\{x_{E}=0\right\}$ is weakly dominated).
    ${ }^{16}$ All proofs are in the Appendix.

[^9]:    ${ }^{17}$ These two cases are extremes of a continuum of scenarios in which each market-maker receives a noisy signal of the aggregate order in the other market. For the qualitative results we seek in this paper these two cases are sufficient.

[^10]:    ${ }^{18}$ These probabilities depend on the region in which the possible values of the proposal lie but for notational brevity we will omit this dependence.

[^11]:    ${ }^{19}$ This is a reasonable assumption to make since, although the manager is indifferent, she would otherwise be deliberately hurting firm value by proposing $E>E_{0} / 2$.

[^12]:    ${ }^{20}$ Even for the off-equilibrium aggregate order $\left\{y_{D}=0\right\}$.

[^13]:    ${ }^{21}$ This would correspond to not being able to factor out $E$ in the expectation calculation of $p_{E}$.

[^14]:    ${ }^{22}$ We thank an anonymous referee for proposing this application of our model.

[^15]:    ${ }^{23}$ For $E \leq E_{0} / 2$ see discussion in the main text.

