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Preferred-Habitat Investors and the US Term Structure of Real Rates*

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Abstract

We estimate structurally a model of the term structure of interest rates that is consistent with no arbitrage but allows for demand pressures. The term structure in our model is determined through the interaction of risk-averse arbitrageurs and preferred-habitat investors with preferences for specific maturities. The model is estimated on US real rates during the 2000s and allows for two factors: one corresponding to the short rate and one to preferred-habitat demand. We find that the puzzling drop in long rates during 2004-05 (Greenspan conundrum) is driven by the demand factor, which in turn is correlated with purchases of long-term bonds by foreign officials. For example, foreign purchases in July 2004 appear to have lowered the 10-year rate by about 100 basis points. Foreign purchases have larger effects following periods when arbitrageurs have lost money.

Abstract

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JEL Classification: F31; G10.

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1 Introduction

Prior to the financial crisis in 2007, long-term US interest rates fell to surprisingly low levels, contributing to the so-called "search for yield" environment which undermined the stability of the financial system profoundly. The decline was particularly puzzling during 2004-05, given that at that time the US authorities started to rise the policy rate (Figure 1). This phenomenon was described as a conundrum by Alan Greenspan in 2005, and since then became a popular research topic for policy makers and academics. However, to date the conundrum remains largely unexplained. Many studies failed to rationalize the phenomenon of low long-term rates on the grounds of fundamental macroeconomic variables such as inflation, inflation expectations, real activity and the rising policy rate (Rudebusch, Swanson and Wu (2006)). It is worth noting though that many studies focused on nominal rates. However, as Figure 2 shows, the fall in nominal rates of the early 2000s largely mirrored the fall in real rates. This stylized fact in turn suggests that in order to crack the conundrum in nominal rates it is key to understand the forces behind the fall in *real* rates.

Our hypothesis is that demand pressures, and foreign official purchases of US Treasury securities in particular, contributed to explain the fall in longer-term real rates. Greenspan himself in 2005 noted that "heavy purchases of longer-term Treasury securities by foreign central banks have often been cited as a factor boosting bond prices and pulling down longer-term yields." Foreign officials and foreign private investors have been among the major purchasers of US assets since the early 2000s. During this period foreign investors owned an extremely high proportion of US long-term debt. For example, in 2007 their holdings of Treasury notes due from three in ten years reached 80 percent of the whole amount outstanding.[1] In particular, foreign officials seemed to prefer Treasuries over other types of assets, and within Treasuries longer over shorter-term debt. The drivers of this foreign demand for US safe assets, and its impact on real rates, have been mostly analyzed by the macroeconomic literature on "global imbalances" (see, for example, Caballero (2006); Caballero, Farhi and Gourinchas (2008); Mendoza, Quadrini and Rios-Rull (2007); and Caballero and Krishnamurthy (2009)). There are also few empirical, mostly reduce-form, studies trying to quantify the impact of foreign officials' demand for US Treasuries on US bond prices (Warnock and Warnock, (2009), Krishnamurthy and Vissing-Jorgensen (2010); and Sierra (2010)).

The empirical and macroeconomic literature suggests that foreign institutions' purchases of long-term US Treasury securities had an impact on real bond prices. However, this literature lacks a quantitative structural analysis of bond yields. In particular, structural estimation of the foreign demand effects within a model of the term structure is missing. Such estimation can improve over reduced form analysis by delivering consistent estimates both across maturities and over time. Most

[1] In 2007, international investors owned $672 billion of the $835.4 billions of Treasuries due in three to ten years.

no-arbitrage asset pricing models, however, do not have a role for demand and supply effects, either because they are mute on the underlying economic structure or because they assume a representative agent with no preference between markets and assets. In contrast, in this paper we present a model that allows for demand and supply effects on bond prices, while imposing the discipline of no-arbitrage. We estimate the model structurally and uncover a number of new results on demand effects in the term-structure of interest rates.

To explain how the demand for US Treasuries (quantities) can affect the term-structure of interest rates (prices), our paper departs from standard no-arbitrage asset pricing. We build on, and extend, the "limited arbitrage" model of Vayanos and Vila (2009).[2] In this model the authors emphasize, similarly to Modigliani and Sutch (1966), that investor clienteles with preferences for specific maturities, the so called preferred-habitat investors, could play an important role for bond pricing. In particular, Vayanos and Vila (2009) set up a formal model of 2 types of agents: investors with a preferred habitat (maturity) and risk averse arbitragers. So, in contrast to common term structure models (Cox, Ingersoll and Ross (1985), among others), there are heterogeneous clienteles. Interest rates are determined through the interaction of preferred-habitat investors and risk-averse arbitrageurs. Precisely, these arbitrageurs incorporate expected short rates into bond prices and bring yields in line with each other by smoothing demand and supply pressure. These arbitrageurs trade the slope of the term structure by buying (selling) long-term bonds and selling (buying) short-term ones. But since arbitrageurs are risk averse, they demand higher risk premia as their relative exposure to long bonds increases. Thus, excess supply matters.

We bring the analytical model of Vayanos and Vila (2009) to the data. Our model explains the term structure of US real rates by two unobservable factors, modelled as two correlated Ornstein-Uhlenbeck (Vasicek) processes, which represent the short-term real interest rate and a demand factor. We expect the estimated demand factor to co-move with proxies of reserve accumulation, because preferred-habitat investors in Vayanos and Vila (2009), to some extent, might be thought of as the foreign official sector. For example, foreign central banks buy Treasuries because they are highly liquid assets that provides a reliable store of value. There is also anecdotal evidence that they buy Treasuries regardless of their prices relative to other assets (Krishnamurthy and Vissing-Jorgensen, 2010).[3] Thus, foreign officials are unlikely to exploit relative price differentials across the term structure, which makes them similar to preferred-habitat investors. Moreover, data on the ownership of US Treasury securities show that the foreign official sector is a major player in the Treasury market

[2] See Gromb and Vayanos (2010) for a review on the limits of arbitrage literature.

[3] Krishnamurthy and Vissing-Jorgensen (2010) argue that foreign officials' demand for US Treasuries is completely inelastic: when a foreign central bank receives a dollar capital inflow and accumulates more dollar reserves, it buys Treasuries regardless of their prices relative to other assets.

(Figure 3).[4] However, it is unlikely that foreign official investors can fully identify the preferred-habitat demand of Vayanos and Vila (2009), as other investors (e.g. demand from pension funds, life-insurance companies, and government issuance) can also be important. As a result, we model demand as an unobservable factor. However, our model still imposes structure on the data, so that we can identify demand factor and then assess its impact on the all term-structure of yields.

We estimate the model on US real rates from January 2001 to September 2009, at weekly frequency,[5] and we perform a Bayesian estimation which relies on a Markov Chain Monte Carlo (MCMC, hereafter) algorithm. The estimation output shows that the model performs well for most of the sample with pricing errors within the 10-20 basis point range, and we can explain much of the conundrum period. The performance deteriorates over the crisis period when a series of pricing anomalies have manifested in financial markets. A number of interesting results emerge from our analysis. First, the risk-free rate (or, real policy rate) has a strong impact on shorter-term rates as might be expected. By contrast, the demand factor, which might be thought as a proxy for reserve accumulation, strongly affects longer-term maturities, and it has its strongest impact on the 10-year rate. Second, we find that in general the short rate and demand move in opposite directions but during the conundrum period they are both increasing. In particular, the 2004-2005 fall in long rates is explained by the drop in the term premia, which are mainly affected by changes in preferred-habitat demand. Thus, although the expected policy rates were increasing, the long rates were pushed in the opposite direction by the negative impact of growing demand on the term premia. Third, international reserves, foreign official holdings of longer term US Treasuries and other proxies for foreign reserve demand all co-move with our estimated demand factor. We also find that the interaction between arbitrageurs and preferred-habitat investors (demand pressure) matters. Our proxies for foreign institutions' demand are more important when the arbitrageurs in the model bear losses and their activities are fund constrained to trade away demand impacts on the term structure. Finally, we run a simple experiment to quantify the impact of foreign official capital flows into long-term US Treasury securities on the term structure of real rates. For example, we find that the accumulation of U.S. government bonds in the year to July 2004 (when such purchases reached their maximum) can explain a decline of around 100 basis points in the 10-year rate, and around 50 basis points in the 1-year rate. Foreign

[4] Treasury International Capital System (TIC hereafter) data show that the US debt market has been growing constantly since 2000 over the sample, and that foreign officials and foreign private investors have been major purchasers of US assets. For example, in September 2009 non-Americans held more than 65% percent of all US government notes and bonds. Private foreign holdings are usually overstated because TIC data do not capture foreign central banks acquisitions which take place through a third country intermediary. Once we move to the empirical analysis we use the estimated data of Bertaut and Tryon (2007) which account for this and other drawbacks of the TIC data.

[5] We use inflation-indexed bonds, which are the primary source of market real interest rates. There have not been many papers on the term structure of US real rates, mainly due to the scarcity of US inflation-linked bonds. The U.S. Treasury auctioned its first tranche of Treasury inflation-protected securities (TIPS) in 1997 and then it took several years for this market to become liquid. Thus at the time of the conundrum the samples of reliable data on real bonds were too short for any sound analysis.

officials' purchases explain around 90 basis points of the decline in 5- and 20-year interest rates.

Our results are consistent with Warnock and Warnock (2009) in that the fall in the 10-year rate associated with the foreign official purchases of US Treasuries is of comparable magnitude to ours. However, only by estimating a structural no-arbitrage model we can shed new light on the fall in long rates. For example, we are able to assess the impact of demand on all maturities. More fundamentally, we can link the fall in bond risk premia to our rising demand factor. Our study also relates to Krishnamurthy and Vissing-Jorgensen (2010). They find that foreign officials' purchases reduce the supply of safe assets available to the rest of investors and hence drive up the convenience yield. They also find that if foreign officials were to sell their holdings, the effect would be to raise long-term Treasury yields by 59 basis points relative to the Baa corporate bond yield. However, as the authors concede, the demand for Treasury attributes varies by maturity, thus our preferred-habitat model can be seen as being complementary to their analysis. Moreover, while they look at the effect of foreign officials' purchases on the interest rates relative to other assets we look at their impact on the level of Treasury yields and bond risk premia. A recent paper by Sierra (2010), through a series of forecasting regressions of realized excess returns on measures of net purchases of treasuries, finds that official flows appear similar to relative supply shocks, whereas private foreign investors are more alike arbitrageurs. Thus, to some extent, his results motivate even further our hypothesis that foreign official investors can partly identify the preferred-habitat investors of Vayanos and Vila (2010). At the same time, by showing that demand affects bond risk premia, and because demand correlates with foreign officials' purchases of US Treasuries, we validate his reduce-form method.

Although in the paper we focussed on the analysis of demand effects, our model can also be used to analyze supply effects on bond prices. Such structural analysis is directly related to a number of empirical studies providing evidence of supply effects on interest rates. For example, Fleming (2002) examines the relationship between issue size and yield in the Treasury market and finds that larger issues have higher yields. Similarly, Longstaff (2004), studying the 2000-02 buyback program, when the Treasury made large scale purchases of longer-term bonds from market participants, argues that changes in the supply of Treasury securities available to investors can significantly affect the value of Treasury bonds. More recently, Greenwood and Vayanos (2010b) find that the relative supply of bonds affects bond yields through excess returns, with the effect increasing with maturity, while Hamilton and Wu (2010) observe that direct large-scale asset purchases are a feasible tool that the Fed could use to lower long-term interest.

The remaining of the paper proceeds as follows. Section 2 introduces the model. Section 3 describes the estimation methodology and presents the data. Section 4 summarizes the parameter estimates and presents ex-post estimation calculations (e.g. loadings, term premia, excess returns etc.). Section 5 deals with interpreting the demand factor. Finally, section 6 concludes.

2 Model

The theory builds on the preferred-habitat model of Vayanos and Vila (2009). In this model investors with strong preferences for specific maturities (preferred-habitat investors) trade with arbitrageurs. In the absence of arbitrageurs, each maturity would constitute a separate market, with its yield being determined by the clientele of investors for that particular maturity. Thus, arbitrageurs integrate maturity markets, rendering the term structure arbitrage-free. However, because arbitrageurs are risk averse, investor demand has an effect. Arbitrageurs thus not only bridge the disconnect between the short rate and bond yields, but also bring yields in line with each other, smoothing local demand and supply pressures. We extend the original model of Vayanos and Vila (2009) by allowing for a more general dependence structure between the short rate and the preferred-habitat demand. Section 2.1 summarizes the main elements of the model, and Section 2.2 presents its solution.

2.1 Theory

The term structure is represented by a continuum of zero-coupon bonds, with bond maturities in the interval $(0, T]$. We denote by $P_{t,\tau}$ the time-t price of the bond with maturity τ that pays \$1 at time $t + \tau$. The spot rate for that maturity, $R_{t,\tau}$, is given by

$$R_{t,\tau} \equiv -\frac{\log(P_{t,\tau})}{\tau}. \tag{1}$$

The short rate r_t, which is the limit of $R_{t,\tau}$ when τ goes to zero, follows the Ornstein-Uhlenbeck process

$$dr_t = \kappa_r(\overline{r} - r_t)dt + \sigma_r dB_{r,t}, \tag{2}$$

where $(\overline{r}, \kappa_r, \sigma_r)$ are positive constants and $B_{r,t}$ is a Brownian motion.

We assume that there are two groups of agents: preferred-habitat investors and arbitrageurs. Preferred-habitat investors form maturity clienteles, with the clientele for maturity τ only buying the bond with the same maturity. The demand for the bond with maturity τ is assumed to be a linear function of the bond's yield $R_{t,\tau}$, i.e.,

$$y_{t,\tau} = \alpha(\tau)\tau(R_{t,\tau} - \beta_{t,\tau}), \tag{3}$$

where $\alpha(\tau)$ is positive. There are overlapping generations of infinitely risk averse investors, who consume at the end of their life. The extreme level of infinite risk aversion is required to make sure that investors demand only the bond that matures at the end of their life. Thus, each generation comprise a clientele for a particular maturity. While preferred-habitat investors cannot substitute across maturities, they have an option to substitute outside the bond market. In particular, we assume that investors can save for consumption through two imperfect substituting means: investing in bonds or a private technology that yields return $\beta_{t,\tau}$.

In Vayanos and Vila (2009), the intercept $\beta_{t,\tau}$ in the demand (3) takes the form

$$\beta_{t,\tau} = \overline{\beta} + \sum_{k=1}^{K} \theta_k(\tau) \widehat{\beta}_{t,k}, \tag{4}$$

where $\overline{\beta}$ is a constant, $\left\{\widehat{\beta}_{t,k}\right\}_{k=1,..,K}$ are demand risk factors and $\{\theta_k(\tau)\}_{k=1,..,K}$ are functions characterizing how each factor would impact the cross-section of maturities in the absence of arbitrageurs. For example, when $\theta_k(\tau)$ is independent of τ, a change in $\widehat{\beta}_{t,k}$ would impact all maturities equally and cause a parallel shift in the term structure. When instead $\theta_k(\tau)$ is single-peaked around a specific maturity, a change in $\widehat{\beta}_{t,k}$ would impact that maturity the most, and can be interpreted as a local demand shock.

Vayanos and Vila (2009) interpret demand factors as returns on investments outside the bond market, e.g., real estate. Demand factors could alternatively be interpreted as changes in the hedging needs of preferred-habitat investors (arising because of, e.g., changes in pension funds' liabilities or regulation), changes in the size or composition of the preferred-habitat investor pool, or changes in the supply of bonds issued by the government. Our hypothesis is that the strong accumulation of reserves by foreign officials may well have influenced this demand factor for the term structure of US real rates. In particular, foreign official demand for long-term US Treasury securities is not fully elastic, as it is mainly driven by necessity (e.g. reserve accumulation) rather than return. In addition, within long-term bonds there is no evidence that foreign institutional investors have preferences for a specific maturity. In light of these considerations, we impose that $\widehat{\beta}_{t,k} = \widehat{\beta}_t$, and $\theta(\tau) = 1$. The demand factor β_t follows the Ornstein-Uhlenbeck process

$$d\beta_t = \kappa_\beta(\overline{\beta} - \beta_t)dt + \sigma_\beta dB_{\beta,t} \tag{5}$$

where (κ_β, κ, σ_β) are positive constants and $B_{\beta,t}$ is a Brownian motion that has instantaneous correlation ρ with $B_{r,t}$. Note that since Vayanos and Vila (2009) restricted the correlation between the factor dynamics to be zero, our version represents a more general extension of their model.

Presence of arbitrageurs in the model guarantees that bonds with maturities in close proximity trade at similar prices, so that an equilibrium no-arbitrage price is established. Arbitrageurs do not have maturity preferences but trade bonds of any maturity for return considerations. For taking the risk of buying or selling bonds of different maturities (rather than for risk-free arbitrage opportunities), arbitrageurs demand a compensation in a form of risk premia. In financial markets, such fixed-income arbitrage is a portfolio strategy of an increasing number of investment professionals - in particular, hedge funds and proprietary-trading desks. We assume that arbitrageurs' investment strategy follows a mean-variance portfolio optimization, such that the arbitrageurs' optimization problem is given by

$$\max_{\{x_{t,\tau}\}_{\tau\in(0,T]}} \left[E_t(dW_t) - \frac{a}{2} Var_t(dW_t)\right], \tag{6}$$

with a denoting a risk-aversion coefficient, $x_{t,\tau}$ denoting their dollar investment in the bond with maturity τ and W_t arbitrageurs time-t wealth. We assume arbitrageurs' budget constraint to be

$$dW_t = \left(W_t - \int_0^T x_{t,\tau}\right) r_t dt + \int_0^T x_{t,\tau} \frac{dP_{t,\tau}}{P_{t,\tau}}. \tag{7}$$

2.2 Equilibrium term structure

We conjecture equilibrium spot rates that are affine in the risk factors, i.e. the short rate (r_t) and the demand factor (β_t), so that the equilibrium bond price takes the following exponential form

$$P_{t,\tau} = e^{-[A_r(\tau) r_t + A(\tau)\beta_t + C(\tau)]} \tag{8}$$

for three functions $A_r(\tau), A(\tau), C(\tau)$ that depend on maturity τ. Applying Ito's Lemma to (8) and using the dynamics (2) of r_t and (5) of β_t, we find that the instantaneous return on the bond with maturity τ is

$$\frac{dP_{t,\tau}}{P_{t,\tau}} = \mu_{t,\tau} dt - A_r(\tau)\sigma_r dB_{r,t} - A_\beta(\tau)\sigma_\beta dB_{\beta,t}, \tag{9}$$

where

$$\begin{aligned} \mu_{t,\tau} \equiv\ & A_r'(\tau) r_t + A_\beta'(\tau)\beta_t + C'(\tau) - A_r(\tau)\kappa_r(\overline{r} - r_t) - A_\beta(\tau)\kappa_\beta(\overline{\beta} - \beta_t) \\ & + \frac{1}{2} A_r(\tau)^2 \sigma_r^2 + \frac{1}{2} A_\beta(\tau)^2 \sigma_\beta^2 + \rho A_r(\tau) A_\beta(\tau)\sigma_r \sigma_\beta \end{aligned} \tag{10}$$

is the instantaneous expected return. Substituting (9) into the arbitrageurs' budget constraint (7), we can solve the arbitrageurs' optimization problem.

Lemma 1. *The arbitrageurs' first-order condition is*

$$\mu_{t,\tau} - r_t = A_r(\tau)\lambda_{r,t} + A_\beta(\tau)\lambda_{\beta,t}, \tag{11}$$

where

$$\lambda_{r,t} \equiv a\sigma_r \int_0^T x_{t,\tau} \left[\sigma_r A_r(\tau) + \rho\sigma_\beta A_\beta(\tau)\right] d\tau, \tag{12}$$

$$\lambda_{\beta,t} \equiv a\sigma_\beta \int_0^T x_{t,\tau} \left[\rho\sigma_r A_r(\tau) + \sigma_\beta A_\beta(\tau)\right] d\tau. \tag{13}$$

Equation (11) shows that returns in excess of the risk free rate are a linear function of the bond's sensitivities to the risk factors. This result is a general consequence of the no-arbitrage assumption. The economic content of our model is instead in the factor risk premia. In particular, equations (12) and (13) show that the factor risk premia relate to arbitrageurs' bond positions ($x_{t,\tau}$).

Using Lemma 1 and imposing equilibrium, i.e. $x_{t,\tau} = -y_{t,\tau}$, we find that $A_r(\tau)$ and $A_\beta(\tau)$ solve a system of two linear ODEs, with a fixed-point problem that reduces to a non-linear system of

four scalar equations. Given the functions $A_r(\tau)$ and $A_\beta(\tau)$, the function $C(\tau)$ can be determined through a linear ODE.

Proposition 1. *The functions $A_r(\tau), A_\beta(\tau)$ are given by*

$$A_r(\tau) = \frac{1-e^{-\nu_1\tau}}{\nu_1} + \gamma_r\left(\frac{1-e^{-\nu_2\tau}}{\nu_2} - \frac{1-e^{-\nu_1\tau}}{\nu_1}\right), \tag{14}$$

$$A_\beta(\tau) = \gamma_\beta\left(\frac{1-e^{-\nu_2\tau}}{\nu_2} - \frac{1-e^{-\nu_1\tau}}{\nu_1}\right), \tag{15}$$

where the $(\nu_1, \nu_2, \gamma_r, \gamma_\beta)$ scalars solve the system of (A.13)-(A.16). The function $C(\tau)$ is given by (A.20).

3 Econometric methodology

The estimation of the model is Bayesian in spirit. One of the first to implement the Bayesian estimation using MCMC with Gibbs sampling for a term structure model were Ang, Dong and Piazzesi (2007). In this paper, we also use a MCMC within a Gibbs sampling algorithm. In principle, classical statistical methods such as maximum likelihood estimation are valid because the factors are normally distributed and the yields are affine in the factors. But a Bayesian estimation has to be preferred for several reasons. First, the likelihood is highly non-linear because bond prices are complex functions of the parameters. In our model a system of four non-linear equations and two linear differential equations in the unknown parameters determine bond prices. This may complicate the numerical optimization even further than in traditional affine models. By contrast, Bayesian methods rely on simple block simulations. Second, classical methods fail to quantify parameters and factor uncertainty (Kim and Nelson (1999)). More importantly, the draws from the Gibb sampler allow us to quantify the uncertainty around post-estimation calculations (e.g. loading, term premia, reduce-form regressions etc.). Third, in a Bayesian framework we can easily specify priors and handle constraints in the parameter space (Johannes and Polson (2004)). Instead hard parameter constraints may compromise further the performance of optimization algorithms needed in maximum likelihood. All these caveats complicate the convergence of the optimization, and parameters turn out to be hardly significant in a frequentist setting. Moreover, we can easily assess the convergence of the Bayesian algorithm. And we can also incorporate maximum likelihood information into the Bayesian algorithm (Chib and Ergashev (2009)).

3.1 State Space

The model estimation combines both time series and cross-sectional properties of the observed interest rates. This framework allows us to identify the market price of interest rate risk. A natural framework to cast a panel data term-structure model is a state-space model (de Jong (2000)). In this

setting, the transition equation describes the evolution of the short rate and demand factors under the objective probability measure. And the measurement equation maps these two latent factors into the term structure of observed real rates.

To simplify notation, we group the parameters of our model as $\Theta_1 = (\rho, \sigma_r, \sigma_\beta, \kappa_r, \kappa_\beta, \overline{r}, \overline{\beta})$, $\Theta_2 = (a, \delta, \alpha)$, and $\Theta = (\Theta_1, \Theta_2)$. The transition equation describes the evolution of a $k \times 1$ vector of unobserved factors, where in our case $X_t = [r_t; \beta_t]$. We use the Euler scheme to discretize the continuous-time specification of the factors in equation (2) and (5).[6] The transition equation becomes

$$X_{t+\Delta} = G(\Theta_1) + F(\Theta_1)X_t + u_{t+\Delta}, \qquad u_t \sim N(0, P),^{7} \tag{16}$$

where $G = [k_r\overline{r}\Delta; k_\beta\overline{\beta}\Delta]$ is a $k \times 1$ matrix, whereas $F = diag([1 - k_r\Delta, 1 - k_\beta\Delta])$ and $P = \Delta[\sigma_r^2, \rho\sigma_r\sigma_\beta; \rho\sigma_r\sigma_\beta, \sigma_\beta^2]$ are $k \times k$ matrices. Note that the transition equation is a function only of the Θ_1 parameters. Finally, we denote the panel of observed yields as $Y_t = [y_{t,1}, \ldots, y_{t,\tau}]$ and the measurement equation as

$$Y_{t+\Delta} = f(\Theta, X_t, \tau) + \varepsilon_{t+\Delta}, \qquad \varepsilon_{t+\Delta} \sim N(0, Q), \text{ and } Q = \sigma_\varepsilon^2 I_n \tag{17}$$

where f relates to the formula of the bond pricing which is described in the Appendix A. This shows that the market price of risk parameters Θ_2 enter only in the observation equation. This explains why is generally difficult to estimate these parameters. Q is a $n \times n$ diagonal matrix, and we assume that the measurement errors ($\varepsilon_{t+\Delta}$) are independent and normally distributed with zero mean and common variance. This set up allows us to handle measurement errors at all maturities and to integrate out the latent factors by means of the Kalman filter.[8]

3.2 Bayesian Inference

MCMC methods facilitate the estimation of complex models. By combining the prior distribution with the likelihood function we get the posterior distribution. Our objective is to sample from the joint posterior distribution of model parameters and latent factors. In principle, the Gibbs sampler decomposes the original (intractable) estimation problem and allows us to sample iteratively from the conditional densities of independent parameter blocks. But if some of the conditional posteriors are not known in closed form, we need to sample by using Metropolis-Hastings steps within the Gibbs sampler. In our model many conditional distributions are not recognizable distributions. In

[6] Because the factor evolution is Gaussian, an alternative approach is to use the strong solution of the stochastic differential equations. But for parameters estimated at a weekly frequency the discretization bias of using an Euler scheme is negligible (Johannes and Polson, 2004).

[7] At a weekly frequency the time bewteen two observation Δ is 1/52.

[8] An alternative technique to the Kalman filter, introduced by Chen and Scott (1993), consists of evaluating the maximum likelihood by inverting the observed yields to infer the factors. But if there are more yields than factors $n > k$, respectively, only k yields can be priced without error. And the choice of which yields are observed without error is rather arbitrary.

particular, the functional form of the density is non-analytic because the observation equation is highly nonlinear in the parameters. Therefore we use a MCMC algorithm to update the parameters. The details of the estimation are left to the Appendix, here we sketch the basic algorithm.

Let define $\Theta_{-\theta}$ as all the Θ parameters but θ. The joint posterior density $\pi(\Theta, \sigma_\varepsilon^2, X^T|Y^T)$ can be decomposed into several independent full conditional densities:

$$\begin{aligned}
\text{Drift parameters} &: \pi\left(k_i|\Theta_{-k_i}, \sigma_\varepsilon^2, X^T, Y^T\right), \text{ for } i = r, \beta \\
\text{Volatility parameters} &: \pi\left(\sigma_i^2|\Theta_{-\sigma_i^2}, \sigma_\varepsilon^2, X^T, Y^T\right), \text{ for } i = r, \beta \\
\text{Correlation parameter} &: \pi\left(\rho|\Theta_{-\rho}, \sigma_\varepsilon^2, X^T, Y^T\right) \\
\text{Market price of risk parameters} &: \pi\left(\lambda|\Theta_{-\lambda}, \sigma_\varepsilon^2, X^T, Y^T\right), \text{ for } \lambda = \delta, a\alpha \\
\text{Pricing error variance} &: \pi\left(\sigma_\varepsilon^2|\Theta, X^T, Y^T\right) \\
\text{State variables} &: \pi\left(X^T|\Theta, \sigma_\varepsilon^2, Y^T\right)
\end{aligned}$$

To implement the MCMC algorithm we iteratively sample from these conditional densities. But with the exception of the parameter σ_ε^2, these densities are not known in closed form. The Metropolis step consists of drawing a candidate parameter from a proposal distribution. And we accept or reject the draw based on the information in the yields, state evolution and priors. The last step to draw the factors is standard because the Vasicek model is linear and Gaussian, and we can use the forward-filtering backward sampling by Carter and Kohn (1994). Finally, the priors are uninformative, but several parameters are subject to constraints. For example, the factors are stationary and arbitrageurs risk aversion must be positive. If these restrictions are not satisfied then the draws are simply discarded.

We do not estimate all parameters. The arbitrageurs' risk aversion (a) and the demand elasticity (α) are not separately identified (see Appendix A), so we estimate the product of the two ($a\alpha$). The parameter δ is fixed to 0.1. And the unconditional mean of the short rate process is assumed to be 2%, consistent with the standard assumption on the natural rate. This tells us where the model expects short rates to converge in the very long run, rather than where they should be today. The estimation results are robust to the choice of this parameter.

3.3 Data

The key source of data on market real rates is inflation-indexed bonds. These assets are designed to protect investors from inflation risk. In the US, the inflation-indexed bonds are issued by the US Treasury and their principals are adjusted to the Consumer Price Index (CPI). Since its launch in 1997, the market for Treasury Inflation-Protected Securities (TIPS) has grown considerably and now

represents the largest and the most liquid market for inflation-indexed bonds, with about $500 billion in issuance. TIPS bear fixed-coupon rates, and interest is paid every six months. The zero-coupon equivalent US real rates we use in this paper are from FED TIPS-yields estimates, where both on-the-run (newly issued) and off-the-run (previously issued) bonds are included in estimation of the TIPS yield curve.[9] Although data on US real yields are available back to 1999, we have restricted our sample period to be from January 2001 and so we exclude initial years, when TIPS yields suffer most from illiquidity issues and hence were less reliable.[10]

Our data set consists of real yields with maturities of two, five, ten and twenty years. However, the lack of short-maturity TIPS prior to 2004 implies that real market yields on two-year TIPS is available only from the first week of January 2004. The data are displayed in Figure 4. From 2001 to 2005 long-term real interest rates fell substantially. After the slight recovery in 2006-2007 the rates experienced dramatic swings during the financial crisis starting in second half of 2007. The odd behavior of real yields at the end of 2008, when real rates spiked dramatically so that at some point their levels exceeded nominal rates, coincided with the deflation episode, when consumer prices dropped for six months in a row to end-August 2009.

Table 1 displays some summary statistics for our real yield data set. It is apparent that the average yield curve is upward sloping. The term structure of yields' volatility is instead downward sloping, which suggests that longer term yields are less prone to react to temporary market and economic conditions. Autocorrelations are close to one for all maturities, which means that real rates are very persistent.

The theoretical model in Section 2 explains the real interest rates movements by two factors. However, a natural question is how many factors are needed to explain our dataset. We analyze this question empirically by principal components analysis. Table 2 shows the results from the principal components analysis on our data over the weekly sample period ranging from January 2004 to September 2009. We restrict the data set such that all the four real yields derived from index-linked bonds are available without missing observations. The largest two principal components account for more than 99% of the total variation in the four real yields. Over 94% of the variance of real rates is explained by just the first principal component of the group, which can be labelled as 'short level' because its loadings on the individual yields are positive and decreasing with maturity. The second factor is more related to the 'slope' of the curve because its loadings are negative at short maturities

[9] It is assumed that there is no particular liquidity premium in on-the-run TIPS securities. TIPS with less than 18 months to maturity are dropped from the estimation of the TIPS yield curve, because the effect of the indexation lag makes the prices of these securities erratic.

[10] Other papers tried to overcome the lack of market-derived real rates by developing synthetic real rates from nominal rates and inflation, as in Ang, Bekaert and Wei (2008). Unfortunately, they focus mostly on inflation risk premia and do not provide estimations of real term premia and hence cannot explain why (and if) real term premia might have declined.

and positive at long maturities. The contribution of other principal components is relatively small, suggesting that two factors would suffice to model all four real rates.

4 Empirical results

This section first presents parameter estimates, then deals with the estimated (latent) factors. The last sub-section looks at bond risk premia. We find that the risk-free rate has a significant influence on shorter-term rates, while long term yields are mostly affected by demand for bonds from the so-called preferred-habitat investors. The decline in long rates during 2004-05 is explained by the fall in the term premia, which is mainly affected by an increase in demand from preferred-habitat investors. In general, the short rate and demand factor tend to move in opposite directions but during the conundrum period they are both increasing. Thus, although expected policy rates were increasing, long rates were being pushed in the opposite direction by the increasing demand from preferred-habitat investors pushing down on term premia.

4.1 Parameter estimates

Table 3 displays the estimation results of the two-factor model. As well as showing the parameter estimates and standard deviations, the table shows the numerical standard errors and the absolute values of the convergence diagnostics, as in Geweke (1992). All parameters are statistically significant.

The main results are as follows. The first two parameters κ_r and κ_β represent, respectively, the speeds of mean reversion of the instantaneous rate and demand factors. The estimation reveals that the factors are persistent and return to their long run means (the $\bar{r}$ and $\bar{\beta}$ parameters) rather slowly, with half-lives of 2.23 and 1.98 years for r_t and β_t, respectively. The low speed of mean reversion for the interest rate factor seems reasonable as it reflects the high autocorrelation of the real yields observed in the data. The expected long-run real short rate in the model is assumed to be 2%, consistent with the level of the natural rate. However, it is more difficult to assess the corresponding coefficients of the demand factor. We return to the factor interpretation in the next section; at this point we only note that the shocks to the demand factor are positively correlated with the interest rate factor risks (correlation coefficient is 0.36). And the short rate process is more volatile than the demand factor process (σ_r is higher than σ_β).

In our model, bond risk premia are directly linked to the arbitrageurs' risk aversion coefficient, (whereby the higher the risk aversion a, the stronger the under-reaction of forward rates to changes in expected spot rates). Unfortunately, we cannot estimate a separately from α. The estimated product $a\alpha$ is significantly different from zero ($a\alpha = 46$). This means that the results are consistent with time-varying market prices of risk and hence with time-varying real bond premia. Moreover, a positive degree of risk aversion also implies that excessive bond demand or supply have a direct

impact on yields in our model. However, it is difficult to interpret the parameter estimate for several reasons. First of all, risk aversion is not observed directly, and we are not aware of any similar model to conduct a reliable comparison of the estimates.[11] Second, the risk aversion coefficient cannot be identified separately from α.

Finally, the estimated standard deviation of the measurement errors, σ_ϵ, is small (less than nine basis points) implying that our model provides a good fit for the yield curve. As it is also demonstrated by Figure 5, the model fits the data closely on most parts of the sample, apart from the 2008-2009 period of the financial crisis when the financial markets (including those for TIPS) were in severe distress. Moreover, during this period US CPI deflation made it more difficult to interpret the rates, because on-the-run (more sensitive to current deflation) and off-the-run bonds (less sensitive to current deflation) signalled different prices and so real rates estimated on a joint sample of these bonds were less reliable. Thus, in interpreting our results we concentrate mostly on the behavior of real rates prior to 2008.

4.2 Factors r_t and β_t

Figure 6 displays the estimated latent factors r_t and β_t. The two factors behaved differently during the conundrum period: the short rate factor was increasing, while the demand factor was falling. To understand each factor's contribution to the yield curve dynamics, we examine the estimated loadings of the two factors on the term-structure of yields. The factor loadings $A_r(\tau)$ and $A_\beta(\tau)$ on the yields, and the loadings $A'_r(\tau)$ and $A'_\beta(\tau)$ on the forwards, are illustrated in Figure 7. Because of the way we have defined the short-rate factor, its loading on the instant maturity yield is normalized to one. The contribution of the short rate factor decreases with maturity, such that its loadings for maturities longer than ten years are close to none. The risk-free factor loadings become slightly negative for very long maturities, implying that an increase in expected short rates can lead to a decrease in long-term rates. The negative loadings on risk-free rate factor are the result of arbitrageurs' need to hedge against two different risks. First, an increase in the short rate induces the arbitrageurs to engage in a reverse carry trade, i.e. they short bonds and invest in the short rate. Since the change in expected short rates in response to an increase in r_t is much more pronounced at short maturities, arbitrageurs go short more in short term bonds than long term bonds. But this shorting activity leave them exposed to the risk that the bond prices will move against them, i.e. that bond prices will increase, either because the short rate decreases (like in the one-factor model) or because investor demand increases. At the same time, to hedge the demand risk, arbitrageurs are willing to buy long term bonds, which are most sensitive to this risk. So, a decrease in long-maturity forward rates is

[11] Vayanos and Vila (2009) discuss model implications for extreme cases, when the risk aversion coefficient is zero or infinity.

the result of arbitrageurs' buying activity of long-maturity bonds (for hedging purposes) dominating their shorting activity.

The role of the demand factor is different. It has a small but positive impact on short maturity yields, but an absolutely dominant one for longer maturities. The demand factor reaches its maximum explanatory power for maturities around ten years and decreases for longer maturities. This pattern is not surprising. TIPS at the 10-year maturity are the most standard and widely traded assets whereas longer maturities are less demanded. For example, in 2005-2008, the trade volumes of 20-year and 30-year TIPS were respectively 11 and 30 times lower than trade volume for 10-year bonds (see Fleming and Krishnan (2009)). Overall, it emerges from the analysis of the factor loadings that movements in the demand factor drive long-term rates, whereas movements of the short-rate are more relevant for shorter maturity yields.

4.3 Bond risk premia

The role of the demand factor becomes more obvious once we consider the decomposition of the real rates on expected risk-free rates and bond premia. More precisely, the rate on a bond which expires $T-t$ periods from time t can be decomposed as

$$R_{t,T} = \frac{1}{T}\sum_{i=0}^{T-t} E_t(r_{t+i}) + TPR_{t,T}, \tag{18}$$

where $TPR_{t,T}$ denotes the term premium.[12]

Figure 8 shows this decomposition for the five and ten year rates. The estimated term premia average above 100 basis points in the period up to 2004. They fall below zero in 2005, where they roughly remained until the crises started. But even when term premia increased during the crisis, they have never reached the highs of the early 2000s. The fall in the term premium may well explain the fall in the real rate over the conundrum period. On the contrary, the expected short rate five and ten years ahead, which could be thought of as a proxy for the neutral real rate of interest, were instead increasing over the conundrum period. This upward trend continued until the start of the crisis. Therefore, the behavior of the expected risk-free rates was not puzzling: they indeed followed the rising policy rates, as Greenspan would have expected. The behavior of the long-term real rates did not follow the path of expected real policy rates, simply because it was dominated by the second real rate component, that is real bond risk premia. Thus, it looks like the conundrum was never there. The puzzle in the behavior of US real rates was a result of the wrong beliefs that long-term real rates reflect only the expectations of future real policy rates.

[12] To make the analysis simpler, we are ignoring the convexity term. The impact of convexity increases with maturity and factor volatility. Convexity would affect interest rates even in the case of zero market prices of risk, but is constant across time. Therefore our analysis of the yields decomposition dynamics is robust to including the convexity term.

Our model shows that the term premium on real bonds is extremely important in explaining movements in long real rates. This result is consistent with other papers (Backus and Wright (2007) and Joyce, Kaminska and Lildholdt (2008), among others). What is new is that by means of our structural model we can interpret the movements of the term premia inside the model framework. By contrast, reduced form models do not allow a structural interpretation of the term premium. As it can be also seen from a comparison of Figures 6 and 8, term premia on long real rates strongly co-move with the demand factor. Indeed, the correlation between five-year term premium and the demand factor is 0.75, the correlation increases to 0.78 for the ten-year maturity. This suggests that the movements of the term premia are largely driven by the demand factor. This result is also consistent with the evolution of excess returns which resembles the one of the demand factor (Figure 9). Figure 10 presents the contribution of each factor to excess returns. It emerges that the short rate contribution to excess returns for long-term maturities is negligible, whereas the demand has a significant strong impact. At this point to complete the analysis it remains to interpret the dynamics of the demand factor. This is the objective of the next section.

5 Interpreting the demand factor

In this section we try to interpret the estimated demand factor. The section is structured as follows. Section 5.1 relates the demand factor to foreign officials' holdings of foreign reserves. Section 5.2 sheds lights on the interaction between arbitrageurs and foreign official investors. Section 5.3 performs a simple exercise to quantify the economic impact of reserve accumulation on the term-structure of yields. Section 5.4 considers alternative proxies of demand.

5.1 Foreign officials, preferred-habitat Investors and the demand factor (β)

This sub-section deals with foreign official purchases of US Treasury securities. By doing this, it clarifies why we can identify, at least to some extent, preferred-habitat investors with foreign officials. First of all, foreign central banks, and foreign officials more generally, are major players in the US Treasury market.[13] For example, as of 2008, foreign official investors held 37% of total Treasury supply, making them the largest holder of Treasury debt. As figure (3) suggests, foreign private investors are also an important player, but raw TIC data may overstate the true figure of foreign private holdings, as the split between foreign officials and foreign investors in the TIC data is blurred (Warnock and Warnock (2009)). In particular, because TIC data on foreign officials holding of long-term US Treasuries do not account for acquisitions through a third-party intermediary ("indirect

[13] Foreign official investors mainly consist of foreign central banks and finance ministries but also government investment funds (sovereign wealth funds). However, sovereign wealth funds typically seek riskier investments and a higher rate of return, so that their share of Treasury securities is relatively small. However, sovereign wealth funds increased their purchases of US Treasuries during the crisis.

transactions"), they should be interpreted as a lower bound.[14]

More fundamentally, the characteristic of foreign officials' demand for US Treasuries seems consistent with our preferred-habitat investors demand factor (β). Foreign central banks buy Treasuries to accumulate reserves regardless of the relative price of Treasuries to other US fixed income assets (Krishnamurthy and Vissing-Jorgensen (2010)). This may also suggest that, differently from the arbitrageurs of our model, foreign central banks do not engage in carry-trade strategies on the US term-structure. Moreover, Sierra (2010) finds that foreign official flows appear similar to relative supply shocks, and thus suggests that foreign officials are similar to preferred-habitat investors. More generally, foreign central banks buy US long-term Treasury securities for "necessity" as part of their reserve accumulation strategy: changes in the capital account or exchange rate policies of these countries determine changes in the foreign central banks' holdings of Treasury securities. The optimal level of reserves depends on several other factors such as "fear of floating", the need to self-insure against sudden stops and to cover contingent liabilities (see Jeanne and Ranciere (2007), among others).[15] Financial underdevelopment may also affect the optimal size of reserves (Dominguez (2009)). So, foreign reserve accumulation may help to identify that part of the excess demand, (β), which is inelastic to changes in the yields.

A final issue regards modelling β as common risk factor across maturities. In principle, the original framework of Vayanos and Vila (2009) is flexible enough to accommodate several demand factors. Our choice of using a single demand factor is to some extent a simplifying assumption. However, it is also consistent we the nature of reserve accumulation, whereby foreign officials though have a strong preference for US Treasuries seems not to prefer any particular maturity. Thus, foreign central banks tend to diversify foreign reserve accumulation by buying longer term bonds over a range of maturities. This choice might also respond to a liquidity requirement; the size of a maturity-specific market might be too small relative to the size of foreign central bank's intervention to not trigger large price changes. In sum, it remains an empirical question to quantify how the estimated demand factor loads on the different maturities.

Figure 11 presents the adjusted twelve-month flows of foreign officials' purchases of long-term Treasury securities scaled by monthly GDP and the estimated demand factor. The adjusted flow

[14] Another caveat is that TIC data include both Treasury bonds and TIPS, so that disaggregated data on the foreign ownership of TIPS are not available. In addition, there is evidence that Treasury bonds are overvalued relative to TIPS due to supply factors such as Treasury debt issuance (Fleckenstein, Longstaff and Lustig (2010)). However, it seems unlikely that these facts are able to undermine the use of TIC data as proxy for our estimated preferred habitat-demand. For example, foreign demand at TIPS auctions has been remarkably strong averaging around 39 percent (Gongloff (2010)), thus, showing that not only foreign demand for Treasury bonds but also for TIPS has been remarkable. Moreover, because it is possible (to some extent) to replicate the payoffs of Treasury bonds through a combination of TIPS, STRIPS and inflation-swaps, it is reasonable to assume that demand pressure on one Treasury market transmits to the other market.

[15] Calvo and Mendoza (2000), Calvo and Reinhart (2002), Obstfeld, Schambaugh and Taylor (2008), Dominguez (2009), and Carroll and Jeanne (2009).

data are taken from Bertaut and Tryon (2007) and address many of the drawbacks of the original TIC data (see appendix C). At first glance it emerges that the two series share a similar trend (increasing demand) over the first part of the sample. Moreover, as we move on to the empirical analysis of the demand factor, we start by using two different proxies for foreign central banks' flows to safe US assets: foreign officials' purchases of long-term US Treasury bonds (both raw TIC data and adjusted, as in Bertaut and Tryon (2007)) and foreign central bank reserves in US dollar assets. The description of the variables is left to the appendix.

5.1.1 Quantitative results

Table 4 reports results from regressing the demand factor on measures of foreign reserve accumulation and capital flows, and their interaction with arbitrageurs' wealth. All regressions are at a monthly frequency.[16] Each row refers to a separate regression. Because some of the variable show persistent behaviour, the regression residuals are serially correlated and t-statistics must be adjusted accordingly. t-statistics, reported in brackets, follow Newey-West (1987), allowing up to four lags in the adjustment.

We start by testing the hypothesis that the demand factor is explained entirely by foreign officials' demand for US Treasury securities. So, we regress the demand factor on monthly foreign officials' net purchases of long-term US Treasury securities (foi_t),

$$d_t = c_0 + c_1 foi_t + v_t, \tag{19}$$

where $d_t = -\beta_t$. When d_t rises so does the total demand for bonds, ($y_{t,\tau}$), as $y_{t,\tau} = \alpha\tau(R_{t,\tau} + d_t)$. Thus, we expect c_1 to be positive and significative for our hypothesis to hold. The result of the estimation (row (1)) confirms that foreign purchases are statistically significant with the right sign, and $R^2 = 0.14$. We also run the same regression in first differences to address non-stationarity concerns (row (2)):

$$\Delta d_t = c_0 + c_1 \Delta foi_t + v_t, \tag{20}$$

Reassuringly, variables' signs and significance do not change significantly, and the R^2 remains high.

We repeat the analysis using foreign reserves data and we find that foreign reserves also co-move with our estimated demand factor. The change in official reserves ($\Delta fres_t$) may be affected by valuation effects due to changes in the US interest rate. So, we try to control for these potential valuation effects through a two-stage aprroach. We first regress the change of total official reserves minus gold ($\Delta fres_t$) on the valuation change of foreign official holdings of long-term US Treasury securities, as computed by Bertaut and Tryon (2007). Then, we use the residuals from this regression ($\widehat{\Delta fres_t}$),

[16] Although the estimated factor is available in weekly frequency, we transform it into monthly frequency to match other variables.

which identify the part of foreign reserves orthogonal to valuation changes, as an explanatory variable in the following regression

$$d_t = c_0 + c_1 \Delta \widehat{fres}_t + v_t. \tag{21}$$

The results are shown in row (3). Consistent with our expectation, $\Delta \widehat{fres}_t$ is significant and positively related to the demand factor. The result also holds in differences, although the explanatory power drops slightly.

5.2 Arbitrageurs' wealth and supply

Even if foreign officials can be thought of as preferred-habitat investors, the model is set-up such that preferred-habitat investors are not the only agent determining long rates through the demand factor. There is a complex tripartite influence - from government, different preferred-habitat investors, and arbitrageurs - which in principle makes it difficult to relate the estimated demand factor to a single variable. To shed light on the estimated demand factor, we want to construct measures which proxy for the behavior of each specific model agent and then relate these measures to the estimated factor.

Risk aversion is constant in our model. Stepping outside the model we may expect arbitrageurs' risk aversion to decrease in arbitrageurs' capital. This may imply that as arbitrageurs' wealth decreases the impact of demand (foreign official capital flows) may intensify. To receive a valid empirical support for our hypothesis that arbitrageurs' wealth is an important determinant of the demand factor, we proxy the arbitrageurs' wealth from outside sources. In particular, we use a direct measure of hedge-funds' wealth, which is Credit Suisse/Tremont Hedge Fund Index of Fixed Income Arbitrage.

Similarly, in Greenwood and Vayanos (2010a) arbitrageurs' capital decreases when trading strategies make losses. Differently from us, Greenwood and Vayanos (2010a) proxy changes of annual arbitrageurs' wealth by the product of the yield spread at the end of year $t-1$ times bond excess returns over year t. They constructed this proxy for arbitrageurs' wealth using data external to the preferred-habitat model. Differently from Greenwood and Vayanos (2010a), we have estimated the model, so that in principle we could be using the estimation's outcome to compute their measure of arbitrageurs' wealth. But excess returns, which are a key part of their measure of arbitrageurs' wealth, are model dependent. This implies that proxies for arbitrageurs' wealth, which are based on the estimation's outcome, will be related (by construction) to our demand factor. And a test which uses this measure would inherently suffer from an endogeneity problem.

Our estimated factor (β) could also be affected by supply. Although we could use the amount of Treasuries outstanding as a proxy for supply, this variable would be contaminated by demand. For example, if the Treasury times its issuance of bonds to meet favorable demand conditions, then the increase in government debt would reflect demand and not supply. To try to isolate supply from

demand, we use US Treasury's auctions data: quantities of Treasury bonds offered adjusted to the bonds accepted to represent our proxy for the excess supply. In particular, we use the difference between tendered and accepted amount of bonds. The variable and its source are explained in greater detail in the appendix. However, a detailed analysis of supply effects is beyond the scope of our study.

5.2.1 Demand, foreign official investors and arbitrageurs

Table (4) showed that proxies for foreign official purchases of US Treasuries appear to be statistically and economically significant. However, we argued that other variables could also explain the estimated demand factor, and proxies for arbitrageurs' activity would be one of the most obvious candidates. We expect that our proxies for foreign official demand are more important when the arbitrageurs become less wealthy and their risk appetite deteriorates. Therefore, we test the hypothesis that our measures of foreign demand interact with arbitrageurs' wealth (hf_t). To do this, we include arbitrageurs' wealth multiplied by our proxy for foreign demand as dependent variable, as well as arbitrageurs' wealth on its own to avoid possible omitted variables bias (Table 5). In this exercise we only present the results for the regressions in first differences to address non-stationarity concerns for some of the variables. For example, 20 becomes

$$\Delta d_t = c_0 + c_1 hf_t + c_1 \Delta foi_t + c_2 hf_t \cdot \Delta foi_t + v_t, \tag{22}$$

where we use the Credit Suisse/Tremont Hedge Fund Index of Fixed Income Arbitrage (hf_t) as a proxy for arbitrageurs' wealth. We find that c_2 has the expected negative sign and is significant when we use foreign reserves data, and marginally so when we use TIC data. Overall, the explanatory power of the regressions improves significantly compared to the univariate regressions of Table 4.[17]

5.2.2 Arbitraguers' risk aversion

In our model, when the estimated arbitrageurs' risk aversion is less than infinity arbitrageurs are able to eliminate arbitrage opportunities. The model implied interest rates are the rate that would prevail under benign market conditions, when arbitrageurs' activities are unrestricted. This seems consistent with the evidence prior to 2007 when the fitting errors were very small (see Figure 5). However, the model produces large and volatile errors during the financial crisis in 2007-09, when arbitrageurs (hedge funds) had suffered large losses and when many markets were dysfunctional. These estimated fitting errors could be seen as a measure of the degree of distress in hedge funds' markets.

[17] We also test for significance of bond supply as an explanatory variable. As proxy for the excess supply, we use the change in US Treasury auctions data, i.e. the difference between tendered and accepted amount of bonds. However, this variable is not significant and the R^2 does not improve, so we do not report the results here.

While in our model prices of risk are time varying, the arbitrageurs risk aversion is assumed to be constant. Assuming a constant arbitrage risk aversion allowed us to operate with a more tractable model. By contrast, adding time-varying risk aversion coefficient would require a third factor and may further complicate the bond pricing. The assumption of a constant risk aversion seems consistent with the evidence prior to 2007, thus the results on the conundrum period should be robust to this assumption. However, if we want the model to be able to explain the periods of crisis, then time-varying risk aversion could be a useful tool that would allow us to capture all market imperfections described above. If during the crises, arbitrageurs become infinitely risk averse, then their intolerance to risk could prohibit exploiting arbitrage opportunities, so that pricing anomalies could persist till arbitrageurs' risk appetite may go back to normal levels. In fact, setting arbitrageurs risk aversion close to infinity would correspond to periods when access to credit is restricted or when arbitrageurs are extremely intolerant to risk, e.g. after bearing large losses. We believe that introducing time-varying risk aversion would be a useful and natural extension of our model.

5.3 A simple exercise

A final exercise consists of quantifying the impact of foreign official purchases of long-term US Treasuries on the term-structure of real rates. This exercise would allow us to have a simple means of comparison of our results with the ones of Warnock and Warnock (2009) and Krishnamurthy and Vissing-Jorgensen (2010). For example, Warnock and Warnock (2009) find that if foreign governments did not accumulate US governments bonds over the twelve months ending May 2005, the 10-year Treasury yield would have been 90 basis points higher. However, it is worth noting that, differently from reduced form studies, our results go beyond quantifying the impact of capital flows on a specific maturity and tackle more structural aspects of the fall in real rates.

For this exercise, similarly to Warnock and Warnock (2009), we also use 12-month flows of foreign officials' purchases of long term US Treasuries scaled by lagged GDP ($foi_{12,t}$). But, differently from their exercise, we focus on the period around July 2004 when such purchases reached their maximum. In principle, we could assess the impact of $foi_{12,t}$ on the US real rates $R_{t,\tau}$ for each month t and maturity τ. Precisely, we first regress minus the demand factor (d_t) on a constant and on the foreign officials' flows, ($foi_{12,t}$). Then, we quantify the impact of the flows on the yield at maturity τ, $R_{t,\tau}$, such as

$$FOH\_R_{t,\tau} = -\frac{A_\beta(\tau)}{\tau} \times \widehat{c}_1 \times foi_{12,t}, \tag{23}$$

where $\widehat{c}_1$ is the loading of $foi_{12,t}$ on (d_t) similarly to equation (19). From (23) is evident that the evolution of $foi_{12,t}$ drives the time series dimension of the result, whereas the loadings, $A_\beta(\tau)$, determine the cross-section.

Similarly to Warnock and Warnock (2009), we find that the estimated effect of foreign official

acquisitions of US bonds on real rates can be significant. The accumulation of US government bonds in the year to July 2004 can explain a decline of around 100 basis points in the 10-year rate, and around 50 basis points in the 1-year rate. Foreign officials' purchases explain around 90 basis points of the decline in 5- and 20-year interest rates. If we account for the fact that the as-reported TIC data understate foreign official acquisitions of long-term securities by using the adjusted TIC data as in Bertaut and Tryon (2007), we find that foreign officials' purchases can explain a decline of around 130 basis points in the 10-year rate, and around 65 basis points in the 1-year rate. By contrast, foreign officials' purchases explain around 110 basis points of the decline in the 5- and 20-years rates.

5.4 A Closer Look at Demand

This subsection completes the analysis of the estimated latent factor (β) by focusing on several measures of foreign official investors' acquisitions of US securities and foreign reserve accumulation. Table 6 presents the results of the monthly univariate regressions of the estimated latent demand factor (d), both in levels (left panel) and in differences (right panel). Details about the definitions and sources of the data for the explanatory variables are provided in the Appendix C.

Foreign holdings of US securities grew at a striking pace between 2002 and mid-2007 mostly because Asian central banks continued accumulating dollar reserves. Moreover, the bulk of foreign official holdings of Treasury securities were concentrated in long-term bond and notes. But a significant share of foreign official portfolio of US Treasury securities were also held in agency bonds and T-bills. For example, during the period 2005 through mid-2007, purchases of agency securities accounted for about one-half of all official flows, and more than two-thirds of the net issuance of agency debt (Bertaut and Pounder, (2009)).[18] We find that foreign official holdings of agency bonds co-move with our estimate of the demand factor. Although the coefficient is still positive, once we run the regression in first differences, it becomes not significant. By contrast, our demand factor does not correlate with foreign official holding of T-bills. The coefficient is negative and not significant. Overall, the negative sign may be consistent to a "flight to liquidity" episode, when during the crisis investors have rebalanced their fixed-income portfolio toward more liquid short-term securities.[19]

Commodity exporters tend to sterilize cash inflows, because of associated destabilizing inflationary pressures, by accumulating US Treasury securities. In addition, high commodity prices may reflect a strong demand from emerging market exporters, e.g. China. But when exports are buoyant large amount of dollars flow to the exporting country creating pressures on the domestic currency.

[18] As the crisis worsened with rising concerns about Fannie and Freddie Mac around July 2008, foreign officials decided not to roll-over expiring long-term agency bonds, and later on made sizeable outright sales to support their currencies.

[19] During the crisis there has been growing demand for US Treasury bills. For example, as part of this global trend of safe haven flows into the most liquid and secure US asset, China has purchased T-bills in late 2008 through early 2009. But by mid-2009, the "flight to safety" flows into US Treasury began to decrease.

As a result foreign officials mitigate these pressures by accumulating foreign reserves.[20] So high commodity prices may be associated with larger purchases of Treasuries. Our estimated parameter has the expected positive sign, and it is significant.

Positive changes in our demand factor strongly co-move with a depreciating dollar effective exchange rate. In principle, as capital flows from a particular country into the United States, the US bilateral dollar exchange rate vis-à-vis this country should appreciate. This suggests that our demand factor should positively co-move with the dollar exchange rate. But there a few caveats. First, in our regression we use the real effective dollar exchange rate and not a number of bilateral exchange rates. Second, emerging market exporters may purchase Treasuries to resist a domestic currency appreciation or exchange rate volatility. However, this does not rule out the possibility that the dollar effective exchange rate appreciates, especially in light of a fundamental strength of the dollar versus all currencies. Precisely, even if a few countries succeed in mitigating the appreciation of the dollar versus their own currencies, the effective exchange rate may still appreciate. In support of this argument, the dollar effective exchange rate has a correlation of about -60% with the total reserves.

6 Conclusions

In this study, we shed light on the fall in US long-term rates during the early 2000s using an extension of Vayanos and Vila's (2009) preferred habitat model with two unobserved factors. We estimate the model on the term structure of US real rates. We conjecture that, because foreign central banks are a major player in the US Treasury market and because they buy US long-term Treasury securities for "necessity" as part of their reserve accumulation strategy, foreign central banks can be (partly) identified as preferred-habitat investors.

Our results show that the estimated demand factor can explain the decline in long rates. If we look both at the behaviour of expected rates and risk premia then the so-called "conundrum" tends to disappear. In particular, we find that in 2004-05 the fall in bond risk premia pushed down long rates. This result is consistent with a number of previous empirical studies, but in contrast to these studies our model can help us understand what drives the drop in risk premia. In particular, we can conclude that the fall in bond risk premia does reflect the increased demand for US Treasuries over this period, in particular at longer maturities, with the maximum impact being on the 10-year rate. By contrast, the risk-free rate has a strong impact on short-term rates as might be expected. Given that foreign officials' dollar reserves positively co-move with our estimated demand factor, we

[20] In particular, when the country pegs its currency to the dollar, as large amounts flow into the country, then the central bank must buy up these dollars at the pegged exchange rate. This would likely create domestic inflationary pressures, which are sterilised by issuing interest-bearing bonds (see Renminbi adjustment will not cure trade imbalance, FT, 2010).

can conclude that these purchases did have a significant effect on US long rates. We also find that the impact of one additional dollar of foreign officials' reserves decreases in arbitrageurs' wealth. In other words, arbitrageurs require a higher compensation for trading away arbitrage opportunities, which may arise as a result of foreign central banks' purchases of US Treasuries, when their capital is particularly low. So, also the timing of reserve accumulation is important to determine its impact on equilibrium rates.

Foreign central banks' reserves reached unprecedented levels over the last decade. A natural question is whether this process will continue looking ahead. Changes in exchange rate policies may halt this ongoing process of reserve accumulation. But also the introduction of new precautionary facilities which can protect emerging market economies from sudden stops of capital may limit their desire to accumulate excessive reserves. Our results suggest that if foreign central banks were to reduce the stock of their reserves, then interest rates may rise. Besides, the crisis has emphasized a potentially new trend: foreign investors continued to absorb great flows of Treasuries in 2009, but foreign flows failed to meet the increased supply. By contrast US investors absorbed a larger amount of the issuance of Treasuries.

Our model suggests that the changing pool of investors over time is important in determining the equilibrium rates. For example, if the share of preferred-habitat investors decreases, then short-rate shocks will have an increasing impact on longer maturities. But also the composition of the pool of preferred habitat investors matters. For example, if for a constant share of preferred-habitat investors, US pension funds absorb a larger amount of the issuance of Treasuries relative to foreign central banks, then the demand factor is likely to have an increasing impact on very long maturities. This is because pension funds prefer longer maturities than foreign central banks.

One extension of this study would be to estimate the model using nominal rather than real rates. But this would complicate the estimation even further since to model the nominal term structure we would need a "third factor" to account for inflation expectations and inflation risk. On the other hand, nominal rates are available over a longer period of time and this would allow us to use a longer sample but also to carry the analysis at a lower frequency (e.g. monthly or quarterly). Moreover, we could benefit from the higher liquidity of the nominal US Treasury market (see D'Amico, Kim and Wei (2008)). This may suggest that most of the foreign central banks' trades may be concentrated in nominal bonds so that the nominal term structure would be a better angle to look at global imbalances and their impact on US long rates.

In addition, we assumed the degree of risk aversion to be constant over time, which allowed us to work with a more tractable model. But, allowing the degree of risk aversion to vary would eventually increase the performance of the model during the crisis, when arbitrageurs' trading activities were impaired. Finally, this model can be useful to look at the impact of the supply of bonds on the

term structure of yields, e.g. quantitative easing in United Kingdom. These and other extensions are promising avenues for new research.

A Appendix: Proofs

Proof of Lemma 1:

Using (9), we can write (7)

$$\begin{aligned} dW_t &= \left[W_t r_t - \int_0^T x_{t,\tau}(\mu_{t,\tau} - r_t)d\tau\right] dt \\ &\quad - \left[\int_0^T x_{t,\tau} A_r(\tau) d\tau\right] \sigma_r dB_{r,t} - \left[\int_0^T x_{t,\tau} A_\beta(\tau) d\tau\right] \sigma_\beta dB_{\beta,t}, \end{aligned} \tag{A.1}$$

and (6) as

$$\begin{aligned} &\max_{\{x_{t,\tau}\}_{\tau\in(0,T]}} \int_0^T x_{t,\tau}(\mu_{t,\tau} - r_t)d\tau - \frac{a\sigma_r}{2}\left[\int_0^T x_{t,\tau} A_r(\tau) d\tau\right]^2 - \frac{a\sigma_\beta}{2}\left[\int_0^T x_{t,\tau} A_\beta(\tau) d\tau\right]^2 \\ &-a\sigma_\beta\sigma_r\rho\left[\int_0^T x_{t,\tau} A_r(\tau) d\tau\right]\left[\int_0^T x_{t,\tau} A_\beta(\tau) d\tau\right]. \end{aligned} \tag{A.2}$$

Point-wise maximization of (A.2) yields (11).

Proof of Proposition 1:

Market clearing implies that $x_{t,\tau} = -y_{t,\tau}$. Combining with (3), (4), (8) and the definition of $R_{t,\tau}$, we find

$$x_{t,\tau} = \alpha(\tau)\left\{\beta_t \tau - [A_r(\tau) r_t + A_\beta(\tau)\beta_t + C(\tau)]\right\}. \tag{A.3}$$

Substituting $(\mu_{t,\tau}, \lambda_{r,t}, \lambda_{\beta,t}, x_{t,\tau})$ from (10) and (12)-(A.3) into (11), we find an affine equation in (r_t, β_t). Setting linear terms in (r_t, β_t) to zero yields

$$A_r'(\tau) + \kappa_r A_r(\tau) - 1 = A_r(\tau) M_{1,1} + A_\beta(\tau) M_{1,2}, \tag{A.4a}$$

$$A_\beta'(\tau) + \kappa_\beta A_r(\tau) = A_r(\tau) M_{2,1} + A_\beta(\tau) M_{2,2}, \tag{A.4b}$$

where the matrix M is given by

$$M_{1,1} \equiv -a\sigma_r \int_0^T \alpha(\tau) A_r(\tau)\left[\sigma_r A_r(\tau) + \rho\sigma_\beta A_\beta(\tau)\right] d\tau, \tag{A.5}$$

$$M_{1,2} \equiv -a\sigma_\beta \int_0^T \alpha(\tau) A_r(\tau)\left[\rho\sigma_r A_r(\tau) + \sigma_\beta A_\beta(\tau)\right] d\tau, \tag{A.6}$$

$$M_{2,1} \equiv a\sigma_r \int_0^T \alpha(\tau)\left[\tau\theta(\tau) - A_\beta(\tau)\right]\left[\sigma_r A_r(\tau) + \rho\sigma_\beta A_\beta(\tau)\right] d\tau, \tag{A.7}$$

$$M_{2,2} \equiv a\sigma_\beta \int_0^T \alpha(\tau)\left[\tau\theta(\tau) - A_\beta(\tau)\right]\left[\rho\sigma_r A_r(\tau) + \sigma_\beta A_\beta(\tau)\right] d\tau. \tag{A.8}$$

The solution to the system of (A.4a) and (A.4b) is given by equations (14) and (15) that we repeat here for convenience

$$A_r(\tau) = \frac{1-e^{-\nu_1\tau}}{\nu_1} + \gamma_r\left(\frac{1-e^{-\nu_2\tau}}{\nu_2} - \frac{1-e^{-\nu_1\tau}}{\nu_1}\right),$$
$$A_\beta(\tau) = \gamma_\beta\left(\frac{1-e^{-\nu_2\tau}}{\nu_2} - \frac{1-e^{-\nu_1\tau}}{\nu_1}\right).$$

To determine $(\nu_1, \nu_2, \gamma_r, \gamma_\beta)$, we substitute (14) and (15) into (A.4a) and (A.4b), and identify terms in $\frac{1-e^{-\nu_1\tau}}{\nu_1}$ and $\frac{1-e^{-\nu_2\tau}}{\nu_2}$. This yields

$$(1-\gamma_r)(\nu_1 - \kappa_r + M_{1,1}) - \gamma_\beta M_{1,2} = 0, \tag{A.9}$$
$$\gamma_r(\nu_2 - \kappa_r + M_{1,1}) + \gamma_\beta M_{1,2} = 0, \tag{A.10}$$

in the case of (A.4a) and

$$\gamma_\beta(\nu_1 - \kappa_\beta + M_{2,2}) - (1-\gamma_r)M_{2,1} = 0, \tag{A.11}$$
$$-\gamma_\beta(\nu_2 - \kappa_\beta + M_{2,2}) - \gamma_r M_{2,1} = 0, \tag{A.12}$$

in the case of (A.4b). Combining (A.9) and (A.10), we find the equivalent equations

$$\nu_1 + \gamma_r(\nu_2 - \nu_1) - \kappa_r + M_{1,1} = 0, \tag{A.13}$$
$$\gamma_r(1-\gamma_r)(\nu_1 - \nu_2) - \gamma_\beta M_{1,2} = 0, \tag{A.14}$$

and combining (A.11) and (A.12), we find the equivalent equations

$$\gamma_\beta(\nu_1 - \nu_2) - M_{2,1} = 0, \tag{A.15}$$
$$\kappa_\beta - \nu_2 - \gamma_r(\nu_1 - \nu_2) - M_{2,2} = 0. \tag{A.16}$$

Equations (A.13)-(A.16) are a system of four scalar non-linear equations in the unknowns $(\nu_1, \nu_2, \gamma_r, \gamma_\beta)$.

To solve the system of (A.13)-(A.16), we must assume functional forms for $\alpha(\tau), \theta(\tau)$. Many parametrizations are possible. A convenient one that we adopt from now on is $\alpha(\tau) \equiv \alpha e^{-\delta\tau}$ and $\theta(\tau) = 1$ (i.e., the demand factor affects all maturities equally in the absence of arbitrageurs). We also set $\alpha = 1$, which is without loss of generality because α matters only through the product αa.

Next, we show how to determine the function $C(\tau)$. Setting $x_{t,\tau} = -y_{t,\tau}$ in (12) and (13), and using (1), (3) and (8), we find

$$\lambda_{r,t} \equiv a\sigma_r^2 \int_0^T \alpha(\tau)\left[\beta_t\tau - A_r(\tau)r_t + A_\beta(\tau)\beta_t + C(\tau)\right]A_r(\tau)d\tau \tag{A.17}$$
$$+ a\sigma_r\rho\sigma_\beta \int_0^T \alpha(\tau)\left[\beta_t\tau - A_r(\tau)r_t + A_\beta(\tau)\beta_t + C(\tau)\right]A_\beta(\tau)d\tau,$$
$$\lambda_{\beta,t} \equiv a\sigma_\beta\rho\sigma_r \int_0^T \alpha(\tau)\left[\beta_t\tau - A_r(\tau)r_t + A_\beta(\tau)\beta_t + C(\tau)\right]A_r(\tau)d\tau \tag{A.18}$$
$$+ a\sigma_\beta^2 \int_0^T \alpha(\tau)\left[\beta_t\tau - A_r(\tau)r_t + A_\beta(\tau)\beta_t + C(\tau)\right]A_\beta(\tau)d\tau.$$

Substituting $\mu_{t,\tau}$ from (10), $\lambda_{r,t}$ from (A.17), $\lambda_{\beta,t}$ from (A.18), we find

$$
\begin{aligned}
&C'(\tau) - \kappa_r \overline{r} A_r(\tau) + \frac{1}{2}\sigma_r^2 A_r(\tau)^2 + \frac{1}{2}\sigma_\beta^2 A_\beta(\tau)^2 + \rho\sigma_r\sigma_\beta A_r(\tau)A_\beta(\tau) \\
&= a\sigma_r A_r(\tau) \int_0^T \alpha(\tau) \left[\overline{\beta}\tau - C(\tau)\right] \left[\sigma_r A_r(\tau) + \rho\sigma_\beta A_\beta(\tau)\right] d\tau \\
&+ a\sigma_\beta A_\beta(\tau) \int_0^T \alpha(\tau) \left[\overline{\beta}\tau - C(\tau)\right] \left[\rho\sigma_r A_r(\tau) + \sigma_\beta A_\beta(\tau)\right] d\tau. \qquad \text{(A.19)}
\end{aligned}
$$

The solution to (A.19) is

$$
\begin{aligned}
C(\tau) = z_r \int_0^\tau A_r(u)du + z_\beta \int_0^\tau A_\beta(u)du \qquad \text{(A.20)} \\
- \frac{\sigma_r^2}{2} \int_0^\tau A_r(u)^2 du - \frac{\sigma_\beta^2}{2} \int_0^\tau A_\beta(u)^2 du - \rho\sigma_r\sigma_\beta \int_0^\tau A_r(u)A_\beta(u)du,
\end{aligned}
$$

where

$$
z_r \equiv \kappa_r \overline{r} - a\sigma_r \int_0^T \alpha(\tau)C(\tau) \left[\sigma_r A_r(\tau) + \rho\sigma_\beta A_\beta(\tau)\right] d\tau, \qquad \text{(A.21)}
$$

$$
z_\beta \equiv \kappa_\beta \overline{\beta} - a\sigma_\beta \int_0^T \alpha(\tau)C(\tau) \left[\rho\sigma_r A_r(\tau) + \sigma_\beta A_\beta(\tau)\right] d\tau. \qquad \text{(A.22)}
$$

Substituting $C(\tau)$ from (A.20) into (A.21) and (A.22), we can derive (z_r, z_β) as the solution to a linear system of equations.

B Appendix: MCMC Algorithm

We estimate the term structure model by means of Bayesian methods. Ang, Dong and Piazzesi (2007), Feldhutter (2008) and Chib and Ergashev (2009) among others[21], have estimated multi-factor affine yield curve models using a MCMC with a Gibbs sampling algorithm. Our approach is similar in nature, but we are the first to estimate the model of Vayanos and Vila (2009), which departs from more traditional no-arbitrage affine models.

We group the parameters of our model as $\Theta_1 = (\rho, \sigma_r, \sigma_\beta, \kappa_r, \kappa_\beta, \overline{r}, \overline{\beta})$, $\Theta_2 = (a, \delta, \alpha)$, and $\Theta = (\Theta_1, \Theta_2)$. The yields are observed with error, and σ_ε^2 is the measurement error variance. Moreover, X^T and Y^T denote the latent factors and the observed panel of yields, respectively, for the all sample period.

B.1 Likelihood Functions

The density of the factors is

$$
\pi(X^T|\Theta_1) \propto \prod |P|^{-1/2} \exp(-1/2 u_t' P^{-1} u_t) \qquad \text{(B.1)}
$$

[21] Lamoureux and Witte (2002), Mikkelsen (2002), Bester (2003) and Bauer (2009) all implement MCMC algorithms to estimate term structure models. And, more in general, Johannes and Polson (2004) review MCMC methods for financial econometrics.

where $u_{t+\Delta} = X_{t+\Delta} - G(\Theta_1) + F(\Theta_1)X_t$ denote the transition equation errors. Note that the errors of the latent factors are correlated, so P is a full matrix. Conditional on a realization of the parameters and latent factors the likelihood function of the data is

$$L(Y^T|\Theta, \sigma_\varepsilon^2, X^T) \propto \prod |Q|^{-1/2} \exp(-1/2\varepsilon_t' Q^{-1} \varepsilon_t) \tag{B.2}$$

where the measurement errors $\varepsilon_{t+\Delta} = Y_{t+\Delta} - f(\Theta, X_{t+\Delta}, \tau)$. Finally, the joint posterior distribution of the model parameters and the latent factors is given by

$$\pi(\Theta, \sigma_\varepsilon^2, X^T|Y^T) \propto L(Y^T|\Theta, \sigma_\varepsilon^2, X^T)\pi(X^T|\Theta_1)\pi(\Theta), \tag{B.3}$$

i.e. the product of the likelihood of the observation, the density of the factors and the priors of the parameters $\pi(\Theta)$. Next, we present the block-wise Metropolis-Hastings (MH) algorithm within Gibbs sampler that allows us to draw from the full posterior, $\pi(\Theta, \sigma_\varepsilon^2, X^T|Y^T)$. In principle, we approximate the target density by repeatedly simulating from the conditional distributions of each block in turn. If the conditional distributions were known, this algorithm then consists of a series of Gibbs sampler steps. But in our case most of these conditional distributions are not recognizable, so we replace Gibbs sampler steps with MH steps

B.2 Drawing Latent Factors

The term structure model is linear and has a Gaussian state-space representation. The measurement and transition equations are linear in the unobserved factors, X^T. And both equations have Gaussian distributed errors. So we use the Carter and Kohn (1994) simulation smoother to obtain a draw from the joint posterior density of the factors, which is

$$\pi(X^T|\Theta, \sigma_\varepsilon^2, X^T) \propto \pi(X_T|\Theta, \sigma_\varepsilon^2, X^T) \prod_{t=1}^{T-1} \pi(X_t|X_{t+1}, \Theta, \sigma_\varepsilon^2, X^T). \tag{B.4}$$

In short, a run of the Kalman filter yields $\pi(X_T|\Theta, \sigma_\varepsilon^2, X^T)$, and the predicted and smoothed means and variances of the states. By contrast, the simulation smoother provides the updated estimates of the conditional means and variances that fully determine the remaining densities of equation (B.4) (see also Kim and Nelson, 1999).

B.3 Drawing Drift Parameters

The discretized dynamics of the factors follow a VAR process, and VAR parameters have conjugate normal posterior distribution given the factors, X^T. But in our model the drift parameters also enter the pricing of the yields, so their conditional posteriors are unknown. We draw the drift parameters of the latent factors using a Random-Walk Metropolis (RWM) algorithm (see Johannes and Polson, 2004).

Let denote with $\theta^{(g)}$ the $(g)^{th}-$draw of the parameter. At the $(g+1)-$iteration we draw a candidate parameter $\theta^{(c)}$ from the proposal normal density

$$\theta^{(c)} = \theta^{(g)} + v_\theta \epsilon \tag{B.5}$$

where $\epsilon \sim N(0,1)$ and v_θ is the scaling factor used to tune the acceptance probability around 10-50%. Let define $\Theta_{-\theta}$ as all the Θ parameters but θ, we accept the candidate draw with probability

$$a = min\left\{\frac{L(Y^T|\Theta_{-\theta}, \theta^{(c)}, \sigma_\varepsilon^2, X^T)\pi(X^T|\Theta_{-\theta,1}, \theta^{(c)})\pi(\Theta_{-\theta,1}, \theta^{(c)})}{L(Y^T|\Theta_{-\theta}, \theta^{(g)}, \sigma_\varepsilon^2, X^T)\pi(X^T|\Theta_{-\theta,1}, \theta^{(g)})\pi(\Theta_{-\theta,1}, \theta^{(g)})}, 1\right\} \tag{B.6}$$

Note that because the proposal density is symmetric it has no impact on the acceptance probability. We perform this RWM step for each of the individual drift parameters $(\overline{\beta}, \kappa_\beta, \overline{r}, \kappa_r)$.

At first, as an alternative of using a RWM, we implemented an independence Metropolis algorithm where the proposal density has the form of the latent factor posterior. The acceptance probability would simplify to the ratio of the likelihoods of the data evaluated at the candidate relative to the old draw. But this algorithm performed poorly because the Markov Chain was easily trapped. This was the case because the likelihood is highly sensitive to the drift parameters and the candidate draws were too distant from the old values. Johannes and Polson (2002) suggest of trying alternative specifications, and And, Dong and Piazzesi (2007) faced a similar issue.

B.4 Drawing the Factors Covariance Matrix (P)

We now focus on drawing the variance-covariance matrix, P, of the transition equation. The posterior of P takes the form of

$$\pi(P|Y^T) \propto L(Y^T|\Theta, \sigma_\varepsilon^2, X^T)\pi(X^T|\Theta_1)\pi(P) \tag{B.7}$$

where $\pi(P)$ is the prior distribution. If we specify an inverse Wishart prior we can easily draw from the inverse Wishart proposal distribution

$$q(P) = \pi(X^T|\Theta_1)\pi(P). \tag{B.8}$$

And the acceptance probability simplifies to

$$a = min\left\{\frac{L(Y^T|\Theta_{-\theta}, \theta^{(c)}, \sigma_\varepsilon^2, X^T)}{L(Y^T|\Theta_{-\theta}, \theta^{(g)}, \sigma_\varepsilon^2, X^T)}, 1\right\}. \tag{B.9}$$

But we perform the accept/reject step for each individual candidate draw, so for $\theta^{(c)}$equal to ρ, σ_r and σ_β, because we would otherwise accept too few draws. For example, we accept/reject $\theta^{(c)} = \sigma_r^{(c)}$ conditional on $\sigma_\beta^{(g)}$, $\rho^{(g)}$ and the rest of parameters. Then, we accept/reject $\theta^{(c)} = \sigma_\beta^{(c)}$ conditional on $\sigma_r^{(g+1)}$, $\rho^{(g)}$ and so forth.

B.5 Drawing Arbitrageurs Risk Aversion x Excess Demand Elasticity ($a\alpha$)

Arbitrageurs' risk aversion, a, and excess demand elasticity, α, are not separately identified, so we estimate $a\alpha$. The estimation of $a\alpha$ is similar in spirit to market price of risk parameters in traditional no-arbitrage models. These parameters are notably difficult to estimate because they only enter the measurement equation (bond pricing). We again use a RWM algorithm, but the acceptance probability simplifies to

$$a = min\left\{\frac{L(Y^T|\Theta_{-a\alpha}, a\alpha^{(c)}, \sigma_\varepsilon^2, X^T)}{L(Y^T|\Theta_{-a\alpha}, a\alpha^{(g)}, \sigma_\varepsilon^2, X^T)}, 1\right\}. \tag{B.10}$$

because $a\alpha$ do not enter the transition equations.

B.6 Drawing Measurement Error Variance

We simply use a Gibbs sampler to draw the variance of the measurement errors. Conditional on the the other parameters, Θ, the factors and the observed yields, we get the measurement errors, ε_t. And because we assume a common variance for all the maturities, we implicitly pool the n vectors of residuals into a single series. So the inverse Gamma distribution becomes the natural prior for the variance, σ_ε^2.

B.7 Priors

We set the priors such that they are proper but only little informative. The priors on the transition equation covariance matrix is inverse Wishart, and the one on the measurement error variance is inverse Gamma. The rest of the parameters have normal or, in a few cases, truncated normal distributions. For example, we impose arbitrageurs risk aversion to be positive, and also the mean reversion parameters to be positive, so that the factors are stationary. We discard the draws that do not fall within the desired region, and we keep drawing a proposal parameter until it respects the constraint. But to avoid that the chain gets stuck we specify a maximum number of draws, otherwise we retain the old draw (also see Mikkelsen (2002)). Note that after few iterations the draws lie away from the boundaries.

B.8 Implementations Details and Convergence Check

We perform 70,000 replications, of which the first 30,000 are "burned" to insure convergence of the chain to the ergodic distribution. We save 1 every 20 draws of the last 40,000 replications of the Markov chain to limit the autocorrelation of the draws.

The RWM algorithm converges for an acceptance level of accepted draws around 20-40% (Johannes and Polson (2004)). If the variance is too high we will reject nearly every draw, and the opposite is true for a variance that is too low. In order to reach reasonable acceptance ratios we follow the method of Feldhutter (2007). The variance is tuned over the first half of the burn-in period and we check the acceptance ratio every 100 draws. If we accepted more than 50 draws over the last 100, we double the standard deviation. If, instead, we accepted less than 10 draws we half the standard deviation.

In order to check the convergence of the Markov chain we carried on several exercises. We implemented a preliminary Maximum Likelihood (ML) estimation of the model. Chib and Ergashev (2009) show that a ML estimation of the model may efficiently help the Bayesian algorithm, in particular by tuning the priors and proposal densities. We simply use the ML estimates to initialize the parameters and, above all, the unobserved factors. But we have also estimated the model from many initial values, and the results do not change.

Moreover, the posterior distributions of the parameters are unimodal. We also use two convergence diagnostics: the numerical standard error (NSE), and the convergence diagnostic (CD) of Geweke (1992).[22] The NSE is a widely used measure of the approximation error. A good estimate of NSE has to compensate for the correlation in the draws (Koop, 2003). The second diagnostic, CD, relies on the idea that an estimate of the parameter based on the first half of the draws must be essentially the same to an estimate based on the last half. If this was not the case, then either the number of replications is too small, or the effect of the starting value has not vanished .[23] Table

[22] To compute the NSE and CD we use the codes of James P. LeSage.

[23] Following Koop (2003), the middle set of 50 percent of the draws is dropped to have the first and second set of draws to be independent.

(3) presents the posterior results, and the convergence diagnostics. And the convergence diagnostics support convergence of the chain.[24]

C Appendix: Data

Daily TIPS series could be found at http://www.federalreserve.gov/econresdata/researchdata.htm. This appendix provides further information about the data used to analyze the estimated "demand" factor.

C.1 International Foreign Reserves

International foreign reserve data in the study are obtained from International Financial Statistics database compiled by the International Monetary Fund. These data are at a monthly frequency. Total Official Reserves minus Gold (TR) refer to the total amount of reserves held in foreign assets denominated in all currencies.

C.2 Foreign Officials Holdings of US Assets

The Treasury International Capital Reporting System (TIC) provides monthly data on Foreign Official Holdings of US long-term Treasury securities. But the TIC data do not account for acquisitions through a third-country intermediary ("indirect transactions"). So the split between foreign officials and foreign investors in the TIC data is blurred (Warnock and Warnock (2009)). For example, the as-reported TIC securities transactions data understate foreign official acquisitions of long-term US Treasuries, whereas UK private holdings are often overstated. By contrast, infrequent benchmark survey of positions provide a more truthful and accurate portrait of foreign official holdings of TIC securities. So there is often a discrepancy between the measured value of Treasury securities held by foreign official investors as identified in the annual survey, and what results from summing official transactions as-reported in TIC since the last survey. This discrepancy remains even after making several needed adjustments, as taking into account price changes (see Bertaut and Tryon (2007)).

By knowing this discrepancy, one can infer that official purchases must have been larger than as reported in the TIC S. Warnock and Warnock (2009) firstly introduced a formula to distribute this error and estimate monthly positions between surveys. But Bertaut and Tryon (2007) have improved even further the estimation technique. For our study we use both the raw data and their estimates of monthly purchases of long-term Treasury securities by foreign official investors. Moreover, the discrepancy is even more severe for Agency bonds. So, we use their estimates of foreign official holdings of agency securities in Table 6.

Bertaut and Tryon (2007) monthly estimates of TIC data are available as a link from their Discussion Paper: *http://www.federalreserve.gov/pubs/ifdp/2007/910/default.htm*. They update these data files whenever there is an new survey release.

Finally, foreign official holdings of US Treasury bills are available monthly from the TIC website in the "history" file that goes along with the monthly TIC press release. Note that because the short-term Treasuries are reported as positions by custodians, they do not have the same problem of not capturing the "indirect transactions".

[24] CD is distributed as standard normal, thus values of CD less than 1.96, in absolute value, support the convergence of the Markov chain Monte Carlo.

C.3 Other variables

The commodity variable (All Commodity Index) is obtained from the International Financial Statistics database. The real effective dollar exchange rate is obtained from Datastream. Macroeconomic Advisers provides a Monthly GDP Index, and in particular we use the monthly nominal US GDP. As proxy for Arbitrageurs' wealth we use the Hedge Fund Index of Fixed Income Arbitrage which is obtained from Credit Suisse/Tremont. Finally, to construct our series of supply the data are obtained from the US Treasury website (http://www.fms.treas.gov/bulletin/index.html). We use the auction data on the amount tendered versus accepted as provided in the table PDO-3-Offerings of Marketable Securities Other than Regular Weekly Treasury Bills.

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Table 1
Summary statistics of the real rates data

This table presents means, standard deviations and minimum and maximum values of the 2-year (y_t^2), 5-year (y_t^5), 10-year (y_t^{10}) and 20-year (y_t^{20}) real interest rates used in the estimation over the sample Jan. 2001 - Sep. 2009 . However, since 2-year yields are available only from January 2004, the statistics for y_t^2 are estimated on the sample Jan. 2004 - Sep. 2009.

	y_t^2	y_t^5	y_t^{10}	y_t^{20}
Mean	1.36	1.87	2.31	2.51
Std Deviation	1.18	0.77	0.59	0.50
Autocorrelation	0.98	0.98	0.98	0.98
Maximum	5.16	3.68	3.64	3.61
Minimum	-0.79	0.12	1.14	1.73

Table 2
Principal component analysis of the real rates data

This table presents principle component (PC) statistics for the set of data on 2-year (y_t^2), 5-year (y_t^5), 10-year (y_t^{10}) and 20-year (y_t^{20}) real interest rates used for the model estimation. Due to the lack of 2-year yields before January 2004, the PC analysis is done on the sample Jan. 2004 - Sep. 2009.

		Yield loadings			
PC	Proportion of total variance explained (%)	y_t^2	y_t^5	y_t^{10}	y_t^{20}
1	94.69	0.84	0.47	0.25	0.09
2	4.42	-0.40	0.30	0.57	0.65
3	0.79	0.34	-0.65	-0.17	0.66
4	0.10	-0.10	0.52	-0.76	0.38

Table 3
Parameter Estimates

This table presents the posterior mean, the standard deviation (st.dev.), the numerical standard error (NSE), and the absolute value of the convergence diagnostic (CD), as in Geweke (1992), for the model parameters. These estimates result from the Bayesian estimation, described in section 3, based on weekly US real rates from Jan-2001 through Sep-2009. for the 2-, 5-,10- and 20-yr maturities.

	mean	st. dev	NSE	CD
κ_r	0.31	0.02	0.0014	0.10
κ_β	0.35	0.05	0.0036	0.61
$\overline{r}$ (%)	2	-	-	-
$\overline{\beta}$ (%)	2.55	0.06	0.0014	-0.76
σ_r (%)	2.18	0.09	0.0019	0.09
σ_β (%)	0.85	0.04	0.0010	0.27
ρ (%)	36.03	4.50	0.1005	-1.07
$a\alpha$	46.34	3.48	0.0778	-0.09
δ	0.1	-	-	-
σ_ε (bps)	8.88	0.41	0.0091	-0.84

Table 4
Regression Analysis of the Demand Factor.

This table reports the OLS regressions of our estimated Demand Factor ($d_t = -\beta_t$) on proxies of foreign official demand based on the all sample Jan.2001 - Sep.2009. All variables are monthly; the monthly d_t is obtained as monthly average of our weekly estimates. foi_t is foreign official monthly net purchases of long-term US Treasury securities. $\widehat{fres_t}$ is our estimate of total official reserves minus gold controlling for valuation effects. Precisely, $\Delta\widehat{fres_t}$ is the residual of the regression of $\Delta fres_t$ on the valuation change of foreign official holdings of long-term US Treasury securities, as computed by Bertaut and Tryon (2007). See the Appendix C for a detailed description of the data. t-statistics, reported in brackets, follow Newey-West (1987), allowing up to four lags in the adjustment.

	$constant$ (t-stat)	foi_t (t-stat)	Δfoi_t (t-stat)	$\Delta\widehat{fres_t}$ (t-stat)	$\Delta^2\widehat{fres_t}$ (t-stat)	adjR^2(%)
d_t	-0.03 (-16.63)	0.30 (3.69)	-	-	-	14
Δd_t	0.00 (1.05)	-	0.07 (2.96)	-	-	10
d_t	-0.02 (-17.69)	-	-	0.14 (2.04)	-	6
Δd_t	0.00 (0.88)	-	-	-	0.04 (1.67)	4

Table 5
Interaction effects of foreign demand and arbitraguers wealth

This table reports the OLS regressions of our estimated Demand Factor ($d_t = -\beta_t$), in first difference (Δd_t), on proxies of foreign official demand based on the all sample Jan.2001 - Sep.2009. All variables are monthly; the monthly d_t is obtained as monthly average of our weekly estimates. foi_t is foreign official monthly net purchases of long-term US Treasury securities. $\widehat{fres}_t$ is our estimate of total official reserves minus gold controlling for valuation effects. Precisely, $\Delta\widehat{fres}_t$ is the residual of the regression of $\Delta fres_t$ on the valuation change of foreign official holdings of long-term US Treasury securities, as computed by Bertaut and Tryon (2007). The arbitrageurs' wealth is proxied by returns on the Credit Suisse/Tremont Hedge Fund Index of Fixed Income Arbitrage ($rhfw_t$). See the Appendix C for a detailed description of the data. t-statistics, reported in brackets, follow Newey-West (1987), allowing up to four lags in the adjustment.

	con. (t-stat)	hf_t (t-stat)	Δfoi_t (t-stat)	$\Delta foi_t \times hf_t$ (t-stat)	$\Delta^2\widehat{fres}$ (t-stat)	$\Delta^2\widehat{fres} \times hf_t$ (t-stat)	adjR^2
Δd_t	0.00	0.03	0.07	-0.95	-	-	19
	(-0.02)	(2.50)	(3.41)	(-1.75)			
Δd_t	0.00	0.02	-	-	0.02	-0.99	14
	(1.10)	(0.96)			(2.07)	(-3.48)	
Δd_t	0.00	0.03	0.07	-0.99	0.02	-	19
	(0.73)	(2.48)	(3.42)	(-1.62)	(1.10)		
Δd_t	0.00	0.02	0.06	-	0.01	-1.02	24
	(1.24)	(1.54)	(3.01)		(1.46)	(-5.76)	

Table 6
Regression analysis of the demand factor on demand proxies

This table reports the univariate OLS regressions of our estimated Demand Factor (d_t) on proxies of foreign official demand based on the all sample Jan.2001 - Sep.2009. All variables are monthly, in levels (left panel) and in differences (d), returns (r) or percentage change (pc) in the right panel. All the explanatory variables are scaled by monthly nominal US GDP. First, we use Total Foreign Official Holdings in US agency bonds (FOH_AG$_t$) and US Treasury Bills (FOH_TB$_t$). Then there is the return of the IFS commodity index (IFS_COM$_t$) and the real effective dollar exchange rate (EF_FX_t$). See the Appendix C for a detailed description of the data. t-statistics, reported in brackets, follow Newey-West (1987), allowing up to four lags in the adjustment.

	Levels $d_t = \alpha_0 + \alpha_1 D_t + \varepsilon_t$			*Differences* $\Delta d_t = \alpha_0 + \alpha_1 \Delta D_t + \varepsilon_t$		
D_t	α_0	α_1	Adj-R^2(%)	α_0	α_1	Adj-R^2(%)
FOH_AG$_t$	-0.03	0.25	7.60	0.00	0.02	0
	(-17.7)	(-2.10)		(0.87)	(0.57)	
FOH_TB$_t$	-0.02	-0.07	0	0.00	-0.06	3.61
	(15.79)	(-0.79)		(0.98)	(-1.09)	
IFS_COM$_t$	-0.02	0.03	4.47	0.00	0.01	5.21
	(-22.09)	(2.51)		(0.82)	(2.12)	
EF_FX_t$	0.02	-0.00	74.54	0.00	-0.00	15.42
	(5.14)	(-10.59)		(0.30)	(-2.51)	

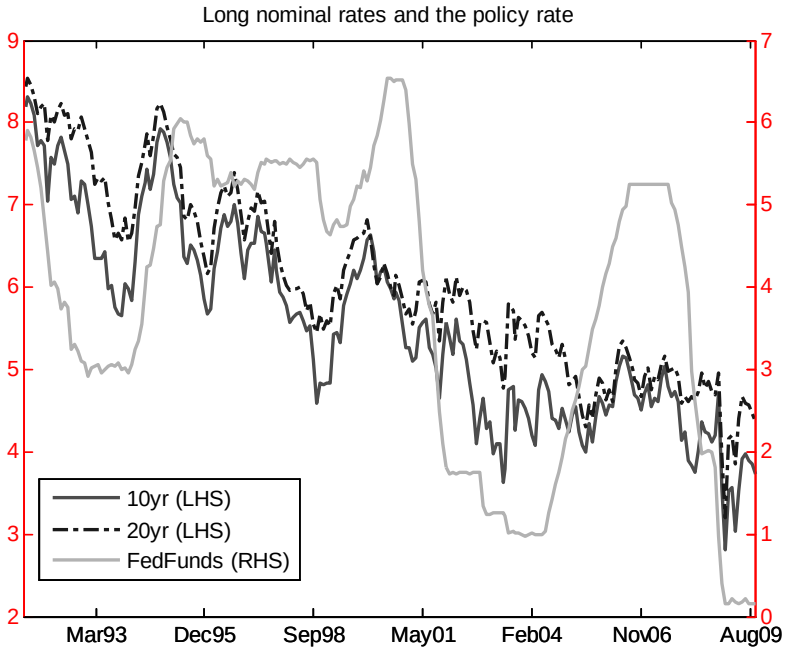


Figure 1: US nominal long-term rates and policy rate (%). Weekly data on nominal US interest rates for the 10- and 20-years maturities and the effective Federal Funds Rate.

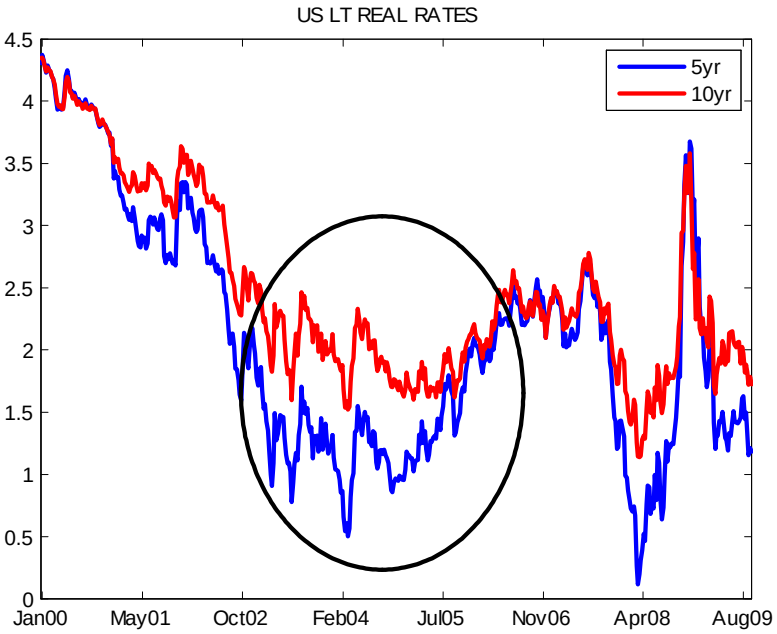
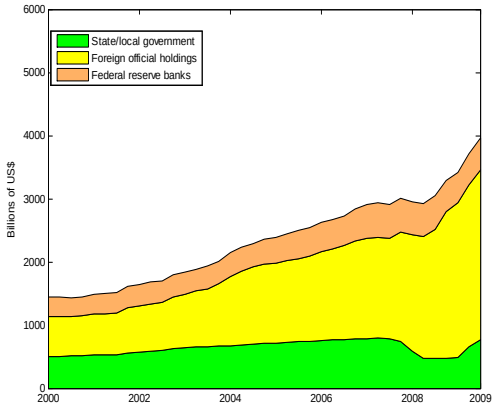


Figure 2: US real rates (%). Weekly data on real US interest rates for the 5- and 10-years maturities. The data are obtained from the Federal Reserve and are TIPS-yieds estimates.

Government



Private holders

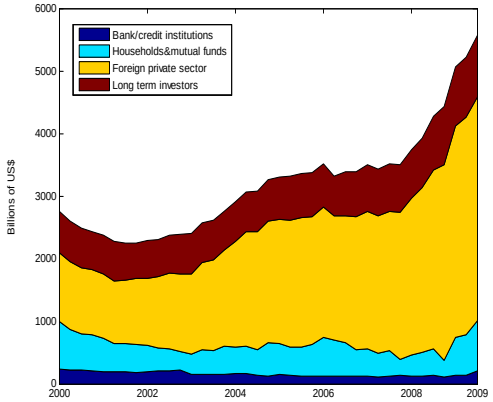


Figure 3: Estimated ownership of US Treasury securities, 2000–2009. Federal securities presented in the figure comprise savings bonds, bills, notes, and bonds that the Treasury issues. The debt ownership data is taken from Datastream and Federal Reserve estimates. Long term investors comprise pension funds and insurance companies. Foreign private investor category includes "intermediaries", buying US securities for foreign officials, so, as stated in the Appendix, the data understate foreign official holdings of US Treasuries.

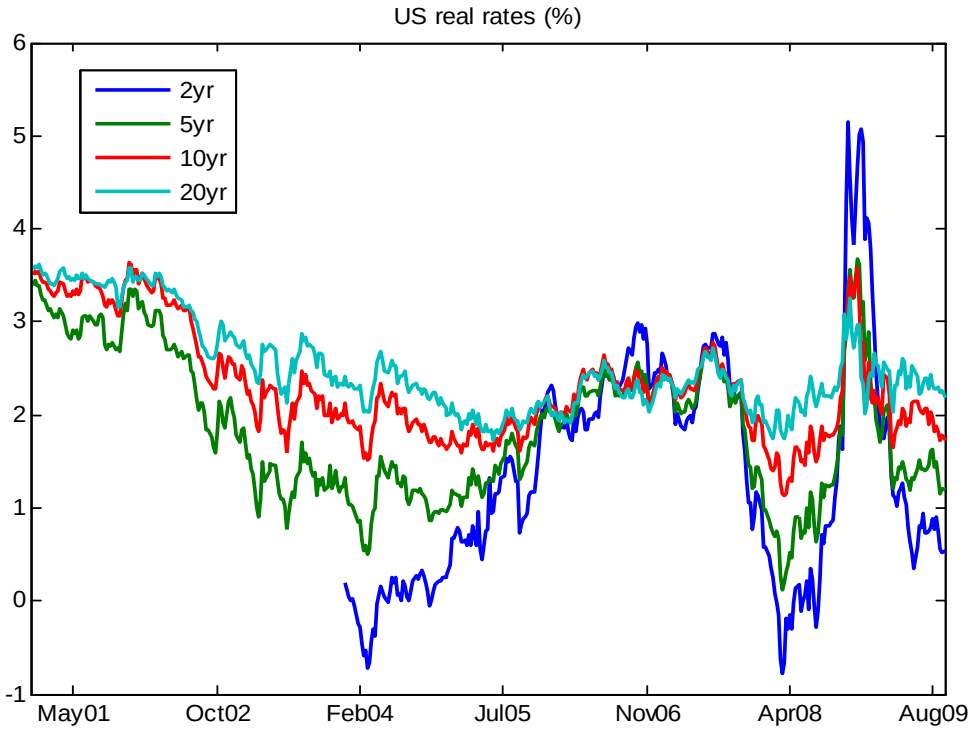


Figure 4: US Real Spot Yields (%). Weekly data on real US interest rates for the 2-, 5-, 10- and 20-years maturities. The data are obtained from the Federal Reserve and are TIPS-yieds estimates. See section (3) for a detailed description of the data.

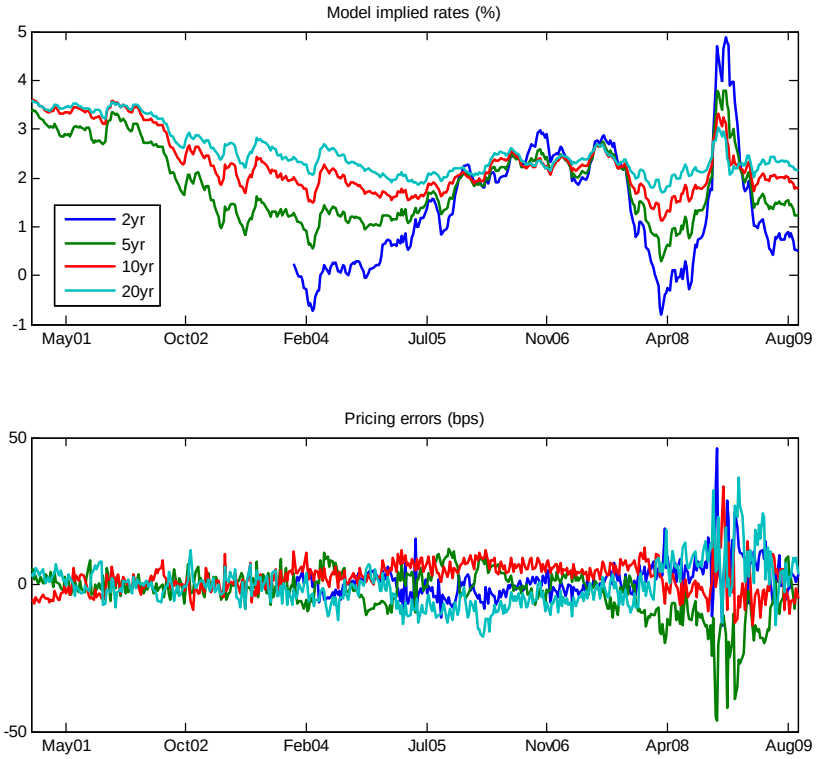


Figure 5: Model implied rates (%) and pricing errors (bps). The model implied rates are the computed using parameter estimates and the factors smoothed estimates obtained from the Carter and Kohn backward simulation. The interest rates pricing error, model implied minus market real rates, are for the 2-, 5-, 10-, and 20-years maturities over the sample from Jan.2001 through Sep.2009.

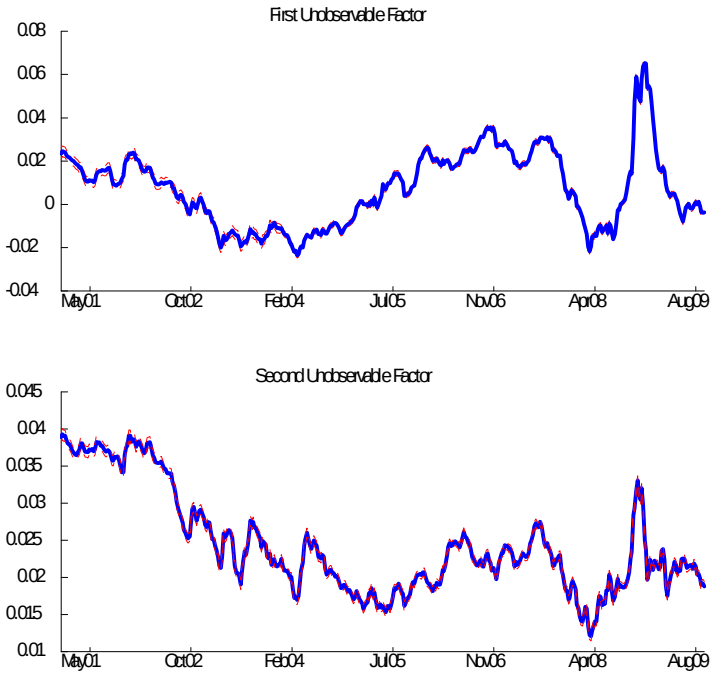


Figure 6: Estimated factors. Smoothed factors paths with one-standard deviation confidence intervals. Top plot refers to the short-term real interest rate (first unobserved factor, r_t), whereas bottom plot refers to the "demand" factor (second unobserved factor, β_t).

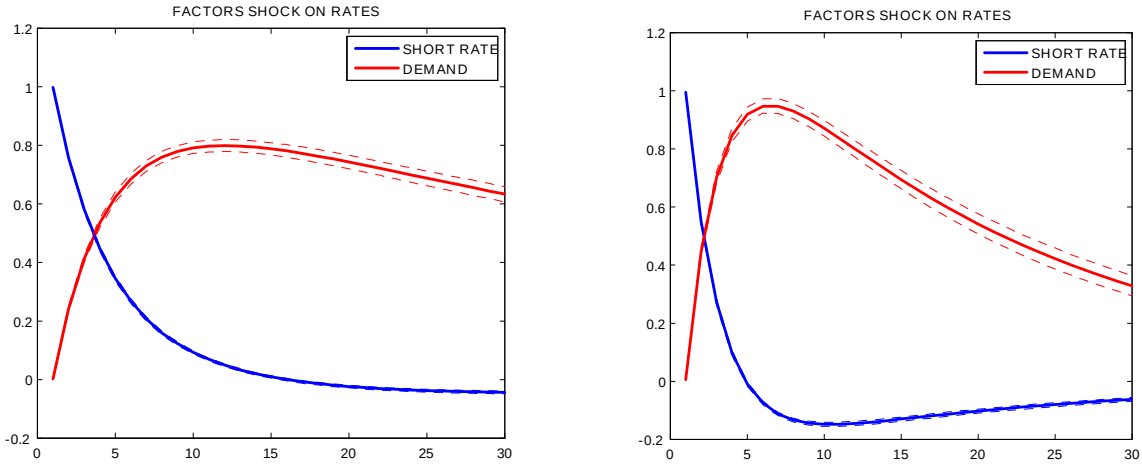


Figure 7: Factor Loadings. Left plot shows the effect a of rise in short-term real rate (blue) and the effect of a decrease in demand (red) on the term structure of spot rates. Right plot shows the effect of a rise in short-term real rate (blue) and the effect of a decrease in demand (red) on the term structure of instantaneous forward rates.

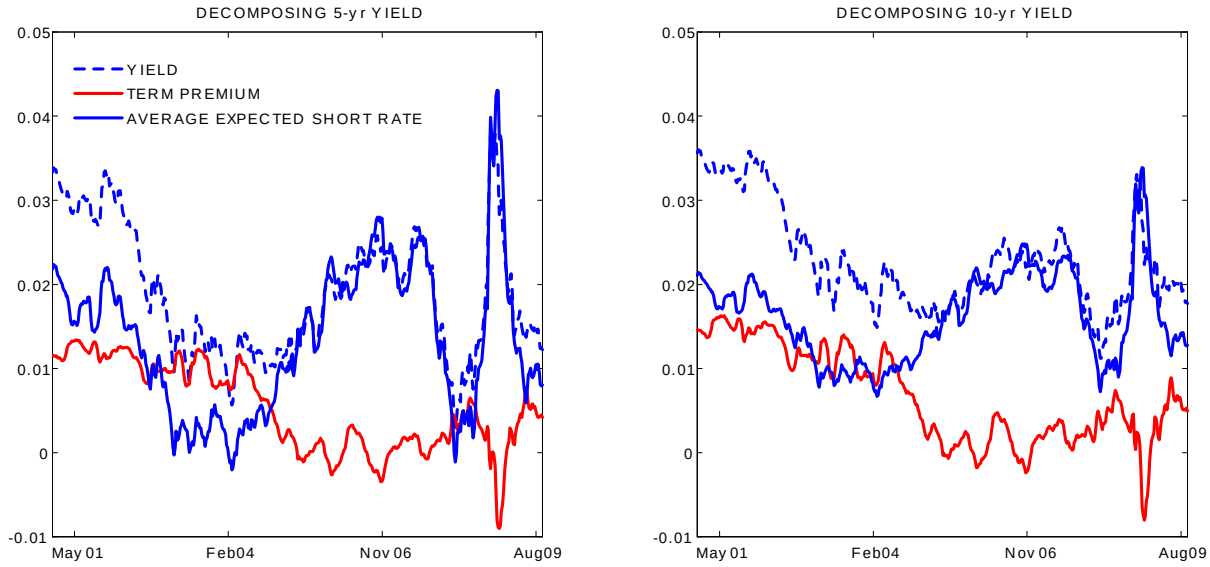


Figure 8. Real interest rate decomposition. A decomposition of the 5- and 10-years model implied rates into the term premium and the average expected short rate over a 5- and 10-year horizon.

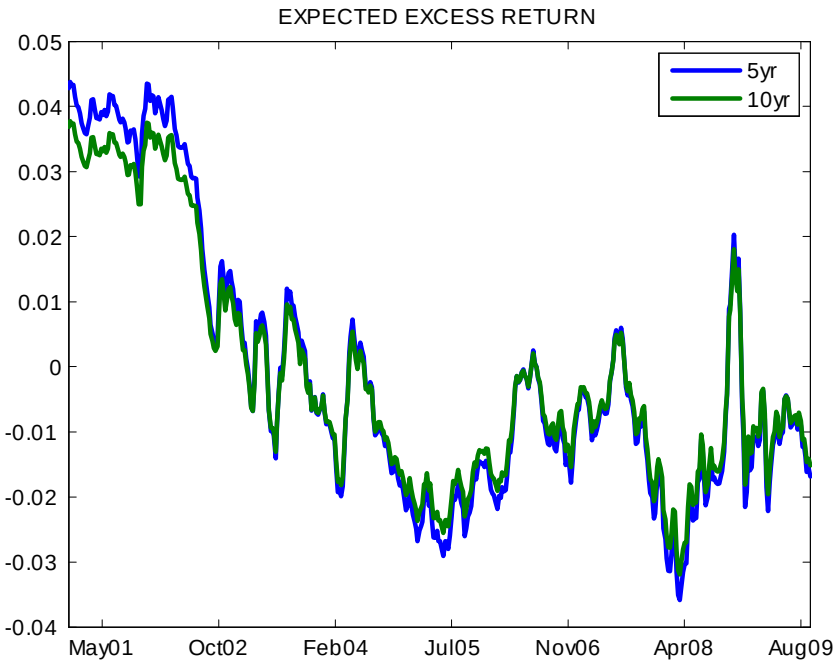


Figure 9. Expected excess returns. Expected excess returns for 5- and 10-years are computed as $\mu_{t,\tau} - r_t = A_r(\tau)\lambda_{r,t} + A_\beta(\tau)\lambda_{\beta,t}$, for τ equal to 5 and 10, respectively. The expressions for $\lambda_{r,t}$ and $\lambda_{\beta,t}$ are provided in the appendix.

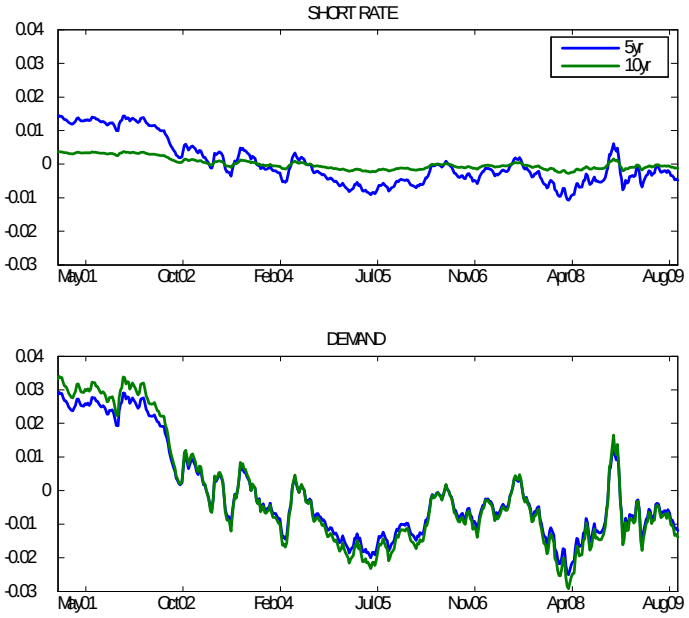


Figure 10: Excess returns decomposition. Here we presents the short rate, $A_r(\tau)\lambda_{r,t}$ (top panel), and the demand, $A_\beta(\tau)\lambda_{\beta,t}$ (bottom panel), contributions to the expected excess returns for the for 5- and 10-years maturities. The expressions for $\lambda_{r,t}$ and $\lambda_{\beta,t}$ are provided in the appendix.

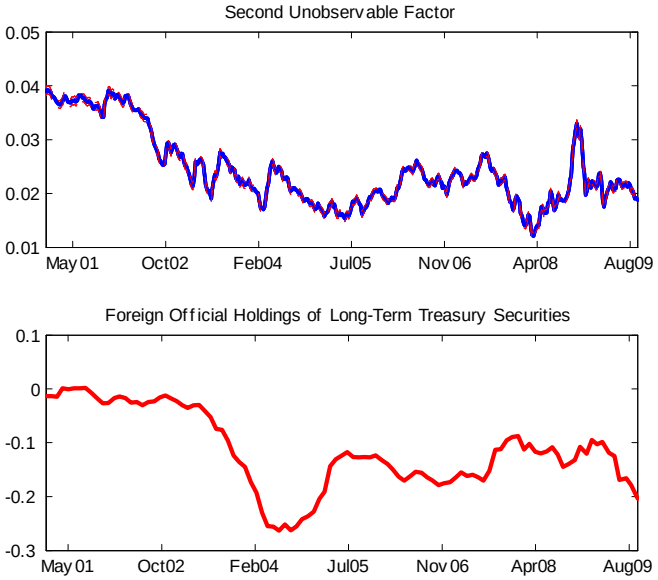


Figure 11. Interpretion of the unobserved demand factor. This plot presents the unobserved second factor (demand factor) and a measure of foreign official holdings of US long-term Treasury bonds (see appendix for a detailed description of TIC data and the particular series we use). Note that capital flows are reported with the opposite sign to be consistent with β and facilitate the interpretation. We use 12-month flow scaled by lagged GDP.