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Liquidity and Asset Returns under Asymmetric Information and Imperfect Competition

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Abstract

We analyze how asymmetric information and imperfect competition affect liquidity and asset prices. Our model has three periods: agents are identical in the first, become heterogeneous and trade in the second, and consume asset payoffs in the third. We show that asymmetric information in the second period raises ex ante expected asset returns in the first, comparing both to the case where all private signals are made public and to that where private signals are not observed. Imperfect competition can instead lower expected returns. Each imperfection can move common measures of illiquidity in opposite directions.

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1 Introduction

Financial markets deviate, to varying degrees, from the perfect-market ideal in which there are no impediments to trade. A large body of empirical work has quantified these deviations using various measures of illiquidity, and has linked illiquidity to expected asset returns. While theoretical work has provided useful guidance on the empirical findings, the guidance has been incomplete, especially concerning the relationship between illiquidity and expected returns.

Consider, for example, asymmetric information, a market friction that has been studied extensively in the literature. Seminal papers by Glosten and Milgrom (1985) and Kyle (1985) have shown that asymmetric information is positively related to illiquidity as measured by the bid-ask spread and by Kyle's lambda (price impact). In these papers and most of the subsequent literature, however, market makers are risk-neutral, competitive and can take unlimited positions. Hence, the autocovariance of asset returns, which is also a widely used measure of illiquidity, is zero. Moreover, expected asset returns are equal to the riskless rate. These papers thus offer little guidance on what the empirical relationship between illiquidity and expected returns should be under asymmetric information.

Few papers, to our knowledge, study the effect of asymmetric information on expected returns. O'Hara (2003) and Easley and O'Hara (2004) show in a multi-asset extension of Grossman and Stiglitz (1980) that prices are lower and expected returns higher when agents receive private signals than when signals are public. This comparison, however, is driven not by asymmetric information per se but by the average quality of agents' information. Indeed, while prices in their model are lower under asymmetric information than when signals are public, they are higher than under the alternative symmetric-information benchmark where no signals are observed. Garleanu and Pedersen (2004) show in a model with risk-neutral agents and unit demands that asymmetric information can raise or lower expected returns, with the effect being zero when probability distributions are symmetric—as is the case under normality, an assumption used in much of the literature. These papers thus suggest an ambiguous effect of asymmetric information on expected returns.[1]

In this paper, we study how asymmetric information affects liquidity and expected returns. Our model builds on Grossman and Stiglitz's canonical framework, and thus assumes normality. We replace the noise traders in Grossman and Stiglitz by rational hedgers. More importantly, we examine how the effects of the asymmetric-information friction are priced in an ex-ante period, in a spirit similar to Garleanu and Pedersen, and to much of the earlier literature on transaction costs

[1]See also Ellul and Pagano (2006) who show that asymmetric information in the post-IPO stage can reduce the IPO price. Their post-IPO stage involves exogenous noise traders and an insider who is precluded from bidding for the IPO. So the IPO price is influenced only by a subset of agents trading in the post-IPO stage.

(e.g., Amihud and Mendelson (1986)). Our model can incorporate a variety of market frictions in addition to asymmetric information. In particular, we also study the impact of imperfect competition, a friction closely related to asymmetric information since large traders, whose trades can move prices, are often privately informed (e.g., Kyle (1985)).[2]

We show three main results. First, asymmetric information raises expected returns, compared both to a symmetric-information benchmark where all private signals are made public and to one where private signals are not observed. Second, asymmetric information and imperfect competition raise Kyle's lambda but can bring the autocovariance of asset returns closer to zero. Thus, lambda reflects both frictions more accurately than autocovariance. Third, imperfect competition can lower expected returns.

Our results imply that the empirical relationship between illiquidity and expected returns is sensitive to the underlying imperfection and to the measure of illiquidity being used. For example, if illiquidity is measured by lambda, the relationship with expected returns is positive under asymmetric information but can turn negative under imperfect competition. Moreover, the relationship can turn negative even under asymmetric information, if illiquidity is measured by autocovariance.

Our model has three periods, $t = 0, 1, 2$. In Periods 0 and 1, risk-averse agents can trade a riskless and a risky asset that pay off in Period 2. In Period 0, agents are identical so no trade occurs. In Period 1, agents can be one of two types: liquidity demanders who will receive in Period 2 an endowment covarying with the risky asset's payoff, and liquidity suppliers who will receive no endowment. The covariance between the endowment and the risky asset's payoff is privately observed by liquidity demanders and is the source of trade. When, for example, the covariance is positive, liquidity demanders are overly exposed to the risk that the risky asset's payoff will be low, and hedge by selling that asset. Frictions concern trade in Period 1. In the case of asymmetric information, liquidity demanders can observe in Period 1 a private signal about the payoff of the risky asset. In the case of imperfect competition, liquidity demanders can collude and behave as a single monopolist in Period 1. We study the effects of each friction in isolation and of both simultaneously.

We measure illiquidity using lambda and price reversal. We define lambda as the regression coefficient of the price change between Periods 0 and 1 on liquidity demanders' signed volume in Period 1. Lambda characterizes the price impact of liquidity demanders' trades. In our model, these trades can be motivated by hedging or information, and their price impact has a transitory and a permanent component. We define price reversal as minus the autocovariance of price changes. Price

[2] A previous version of this paper (Vayanos and Wang (2010)) also considers participation costs, transaction costs, leverage constraints, and search frictions.

reversal characterizes the importance of the transitory component in price, which in our model is entirely driven by volume. Both measures are positive even in the absence of imperfections. Indeed, because agents are risk-averse, liquidity demanders' trades move the price in Period 1 (implying that lambda is positive), and the movement is away from fundamental value (implying that price reversal is positive). We examine how each imperfection impacts the two measures of illiquidity and the expected return of the risky asset. To determine the effect on expected return, we examine how the price in Period 0 is influenced by the anticipation of imperfections in Period 1.

Our first main result is that asymmetric information raises the expected return of the risky asset. We compare with two symmetric-information benchmarks: the no-information case, where information is symmetric because no agent observes the private signal available to liquidity demanders in Period 1, and the full-information case, where all agents observe that signal. We consider both benchmarks so that the effects of asymmetric information are purely driven by the dispersion in information across agents and not by any changes in the average quality of information.

The expected return of the risky asset is higher under full information than under no information. This result is related to the Hirshleifer (1971) effect, which is that public revelation of information can reduce the welfare of all agents because it hampers risk sharing. We derive the implications of the Hirshleifer effect for asset pricing, showing that the reduced risk sharing in Period 1 renders agents less willing to buy the asset in Period 0. Indeed, agents are concerned in Period 0 that the endowment they might receive in Period 1 will increase their existing risk exposure. Therefore, if they are less able to hedge in Period 1, they are less willing to take risk in Period 0 and require a higher expected return. When information is asymmetric, the quality of publicly available information (revealed through the price) is between the two symmetric-information benchmarks, so one might expect the expected return to be also in between. The expected return is higher, however, than under either benchmark. This is because risk sharing in Period 1 is further hampered by the unwillingness of the uninformed to accommodate the trades of the informed.

Our second main result is that both asymmetric information and imperfect competition increase lambda but can reduce price reversal (i.e., render the autocovariance less negative). A discrepancy between these measures of illiquidity can arise because lambda measures the price impact per unit trade, while price reversal concerns the impact of the entire trade. Market imperfections generally raise the price impact per unit trade, but because they also reduce trade size, the price impact of the entire trade can decrease.

Our third main result is that imperfect competition by liquidity demanders can lower the expected return of the risky asset. Intuitively, since non-competitive liquidity demanders can extract better terms of trade in Period 1, they are less concerned with the event where their risk

exposure increases in that period. Therefore, they are less averse to holding the asset in Period 0.

While we mainly focus on the positive analysis of imperfections, our model is also suitable for a normative analysis. We illustrate the normative analysis in the case of asymmetric information. We show that asymmetric information makes both liquidity demanders and suppliers worse off relative to either symmetric-information benchmark, i.e., no information and full information.

The perfect-market benchmark version of our model borrows from Lo, Mamaysky and Wang (2004) and Huang and Wang (2009, 2010). As in these papers, agents receive endowments correlated with the payoff of a risky asset, and the expected return compensates them for the risk that their exposure to that asset will increase. None of these papers, however, considers asymmetric information or imperfect competition.

The equilibrium in Period 1 with asymmetric information is closely related to Grossman and Stiglitz (1980). We model, however, non-informational trading through random endowments, as in the differential-information model of Diamond and Verrecchia (1981), rather than through a random asset supply. The results on how the asymmetric-information friction is reflected in ex-ante prices and expected returns (Period 0 equilibrium) are new, and so are the results on how asymmetric information affects price reversal. Subsequent work by Qiu and Wang (2010) shows that asymmetric information can raise expected returns and lower welfare in an infinite-horizon setting and under a more general information structure than ours. These results, which are numerical, indicate that the closed-form results of our three-period model are more general.

The equilibrium in Period 1 with imperfect competition is closely related to Bhattacharya and Spiegel (1991), who assume that an informed monopolist with a hedging motive trades with competitive risk-averse agents.[3] The results on how the imperfect-competition friction is reflected in ex-ante prices and expected returns (Period 0 equilibrium) are new, and so are the results on how imperfect competition affects price reversal.

The result that asymmetric information can make all agents worse off goes back to Akerlof (1970) and Glosten and Milgrom (1985), who show that asymmetric information can cause market breakdowns. In our model there are no market breakdowns and the trading mechanism is a Walrasian auction. Within a Walrasian auction model, Rahi (1996) shows that a hedger prefers to issue an asset about which he has no information than one about which he is informed. We consider instead the welfare of both informed and uninformed agents, and compare asymmetric information

[3]Strategic behavior under asymmetric information has mainly been studied in a setting introduced by Kyle (1985), where strategic informed traders trade with competitive risk-neutral market makers and noise traders. See also Glosten and Milgrom (1985), Easley and O'Hara (1987) and Admati and Pfleiderer (1988).

with both no information and full information.[4]

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 treats the perfect-market benchmark. Sections 4 and 5 add asymmetric information and imperfect competition, respectively. Sections 6 and 7 discuss empirical and welfare implications, respectively, and Section 8 concludes. All proofs are in an online Appendix.

2 Model

There are three periods, $t = 0, 1, 2$. The financial market consists of a riskless and a risky asset that pay off in terms of a consumption good in Period 2. The riskless asset is in supply of B shares and pays off one unit with certainty. The risky asset is in supply of $\bar{\theta}$ shares and pays off D units, where D has mean $\bar{D}$ and variance σ^2. Using the riskless asset as the numeraire, we denote by S_t the risky asset's price in Period t, where $S_2 = D$.

There is a measure one of agents, who derive utility from consumption in Period 2. Utility is exponential,

$$-\exp(-\alpha C_2), \tag{2.1}$$

where C_2 is consumption in Period 2, and $\alpha > 0$ is the coefficient of absolute risk aversion. Agents are identical in Period 0, and are endowed with the per capita supply of the riskless and the risky asset. They become heterogeneous in Period 1, and this generates trade. Because all agents have the same exponential utility, there is no preference heterogeneity. We instead introduce heterogeneity through agents' endowments and information.

A fraction π of agents receive an endowment $z(D - \bar{D})$ of the consumption good in Period 2, and the remaining fraction $1 - \pi$ receive no endowment. The variable z has mean zero and variance σ_z^2, and is independent of D. While the endowment is received in Period 2, agents learn whether or not they will receive it before trade in Period 1, in an interim period $t = 1/2$. Only those agents who receive the endowment observe z, and they do so in Period 1. Since the endowment is correlated with D, it generates a hedging demand. When, for example, $z > 0$, the endowment exposes agents to the risk that D will be low, and agents hedge against that risk by selling the risky asset. We assume that the endowment is perfectly correlated with D for simplicity; what matters for our analysis is that the correlation is non-zero. We denote by W_t the wealth of an agent in Period t. Wealth in Period 2 is equal to consumption, i.e., $W_2 = C_2$.

[4]See Dow and Rahi (2000) and Marin and Rahi (2000) for further results on financial innovation under asymmetric information, and Liu and Wang (2010) for a market-maker model in which asymmetric information can make the informed agents worse off.

For tractability, we assume that D and z are normal. Under normality, the endowment $z(D-\bar{D})$ can take large negative values, and this can generate an infinitely negative expected utility. To guarantee that utility is finite, we assume that the variances of D and z satisfy the condition

$$\alpha^2\sigma^2\sigma_z^2 < 1. \tag{2.2}$$

In equilibrium, agents receiving an endowment initiate trades with others to share risk. Because the agents initiating trades can be thought of as consuming market liquidity, we refer to them as liquidity demanders and denote them by the subscript d. Moreover, we refer to z as the liquidity shock. The agents who receive no endowment accommodate the trades of liquidity demanders, thus supplying liquidity. We refer to them as liquidity suppliers and denote them by the subscript s.

Because liquidity suppliers require compensation to absorb risk, the trades of liquidity demanders affect prices. Therefore, the price in Period 1 is influenced not only by the asset payoff, but also by the liquidity demanders' trades. Our measures of liquidity, defined in Section 3, are based on the price impact of these trades.

The assumptions introduced so far describe our model's perfect-market benchmark, to which we subsequently add asymmetric information and imperfect competition.[5] We maintain the perfect-market assumption in Period 0 when determining the ex-ante effect of the imperfections, i.e., how the anticipation of imperfections in Period 1 impacts the Period 0 price. Imperfections in Period 0 are, in fact, not relevant in our model because agents are identical in that period and there is no trade.

We model asymmetric information through a private signal s about the asset payoff D that some agents observe in Period 1. The signal is

$$s = D + \epsilon \tag{2.3}$$

where ϵ is normal with mean zero and variance σ_ϵ^2, and is independent of (D, z). We assume that only those agents who receive an endowment observe the signal, i.e., the set of informed agents coincides with that of liquidity demanders. Assuming that all liquidity demanders are informed is without loss of generality: even if they do not observe the signal, they can infer it perfectly from the price because they observe the liquidity shock. Asymmetric information can therefore exist only if some liquidity suppliers are uninformed. We assume that they are all uninformed for simplicity.

[5] Our perfect-market benchmark has one market imperfection built in: agents cannot write contracts in Period 0 contingent on whether they are a liquidity demander or supplier in Period 1. Thus, the market in Period 0 is incomplete in the Arrow-Debreu sense. If agents could write complete contracts in Period 0, they would not need to trade in Period 1, in which case liquidity would not matter. In our model, complete contracts are infeasible because whether an agent is a liquidity demander or supplier is private information.

We model non-competitive behavior by assuming that some agents can collude and exert market power in Period 1. We focus on the case where liquidity demanders collude and behave as a single monopolist, but we also consider more briefly monopolistic behavior by liquidity suppliers. We consider both the case where liquidity demanders have no private information on asset payoffs, and so information is symmetric, and the case where they observe the private signal (2.3), and so information is asymmetric.

3 Perfect-Market Benchmark

In this section we solve our model's perfect-market benchmark. We first compute the equilibrium, going backwards from Period 1 to Period 0. We next construct measures of market liquidity in Period 1, and study how liquidity impacts the price dynamics and the price level in Period 0.

3.1 Equilibrium

In Period 1, a liquidity demander chooses holdings θ_1^d of the risky asset to maximize the expected utility (2.1). Consumption in Period 2 is

$$C_2^d = W_1 + \theta_1^d(D - S_1) + z(D - \bar{D}),$$

i.e., wealth in Period 1, plus capital gains from the risky asset, plus the endowment. Therefore, expected utility is

$$-\mathrm{E}\exp\left\{-\alpha\left[W_1 + \theta_1^d(D - S_1) + z(D - \bar{D})\right]\right\}, \tag{3.1}$$

where the expectation is over D. Because D is normal, the expectation is equal to

$$-\exp\left\{-\alpha\left[W_1 + \theta_1^d(\bar{D} - S_1) - \tfrac{1}{2}\alpha\sigma^2(\theta_1^d + z)^2\right]\right\}. \tag{3.2}$$

A liquidity supplier chooses holdings θ_1^s of the risky asset to maximize the expected utility

$$-\exp\left\{-\alpha\left[W_1 + \theta_1^s(\bar{D} - S_1) - \tfrac{1}{2}\alpha\sigma^2(\theta_1^s)^2\right]\right\}, \tag{3.3}$$

which can be derived from (3.2) by setting $z = 0$. The solution to the optimization problems is straightforward and summarized in Proposition 3.1.

Proposition 3.1 *Agents' demand functions for the risky asset in Period 1 are*

$$\theta_1^s = \frac{\bar{D} - S_1}{\alpha\sigma^2}, \tag{3.4a}$$

$$\theta_1^d = \frac{\bar{D} - S_1}{\alpha\sigma^2} - z. \tag{3.4b}$$

Liquidity suppliers are willing to buy the risky asset as long as it trades below its expected payoff $\bar{D}$, and are willing to sell otherwise. Liquidity demanders have a similar price-elastic demand function, but are influenced by the liquidity shock z. When, for example, z is positive, liquidity demanders are willing to sell because their endowment is positively correlated with the asset.

Market clearing requires that the aggregate demand equals the asset supply $\bar{\theta}$:

$$(1-\pi)\theta_1^s + \pi\theta_1^d = \bar{\theta}. \tag{3.5}$$

Substituting (3.4a) and (3.4b) into (3.5), we find

$$S_1 = \bar{D} - \alpha\sigma^2\left(\bar{\theta} + \pi z\right). \tag{3.6}$$

The price S_1 decreases in the liquidity shock z. When, for example, z is positive, liquidity demanders are willing to sell, and the price must drop so that the risk-averse liquidity suppliers are willing to buy.

In Period 0, all agents are identical. An agent choosing holdings θ_0 of the risky asset has wealth

$$W_1 = W_0 + \theta_0(S_1 - S_0) \tag{3.7}$$

in Period 1. The agent can be a liquidity supplier in Period 1 with probability $1-\pi$, or liquidity demander with probability π. Substituting θ_1^s from (3.4a), S_1 from (3.6), and W_1 from (3.7), we can write the expected utility (3.3) of a liquidity supplier in Period 1 as

$$-\exp\left\{-\alpha\left[W_0 + \theta_0(\bar{D} - S_0) - \alpha\sigma^2\theta_0(\bar{\theta} + \pi z) + \tfrac{1}{2}\alpha\sigma^2(\bar{\theta} + \pi z)^2\right]\right\}. \tag{3.8}$$

The expected utility depends on the liquidity shock z since z affects the price S_1. We denote by U^s the expectation of (3.8) over z, and by U^d the analogous expectation for a liquidity demander. These expectations are agents' interim utilities in Period 1/2. An agent's expected utility in Period 0 is

$$U \equiv (1-\pi)U^s + \pi U^d. \tag{3.9}$$

Agents choose θ_0 to maximize U. The solution to this maximization problem coincides with the aggregate demand in Period 0, since all agents are identical in that period and are in measure one.

In equilibrium, aggregate demand has to equal the asset supply $\bar{\theta}$, and this determines the price S_0 in Period 0.

Proposition 3.2 *The price in Period 0 is*

$$S_0 = \bar{D} - \alpha\sigma^2\bar{\theta} - \frac{\pi M}{1-\pi+\pi M}\Delta_1\bar{\theta}, \tag{3.10}$$

where

$$M = \exp\left(\tfrac{1}{2}\alpha\Delta_2\bar{\theta}^2\right)\sqrt{\frac{1+\Delta_0\pi^2}{1+\Delta_0(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}}, \tag{3.11}$$

$$\Delta_0 = \alpha^2\sigma^2\sigma_z^2, \tag{3.12a}$$

$$\Delta_1 = \frac{\alpha\sigma^2\Delta_0\pi}{1+\Delta_0(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}, \tag{3.12b}$$

$$\Delta_2 = \frac{\alpha\sigma^2\Delta_0}{1+\Delta_0(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}. \tag{3.12c}$$

The first term in (3.10) is the asset's expected payoff in Period 2, the second term is a discount arising because the payoff is risky, and the third term is a discount due to illiquidity (i.e., low liquidity). In the next section we explain why illiquidity in Period 1 lowers the price in Period 0.

3.2 Illiquidity and its Effect on Price

We construct two measures of illiquidity, both based on the price impact of the liquidity demanders' trades in Period 1. The first measure, to which we refer as price impact, is the coefficient of a regression of the price change between Periods 0 and 1 on the signed volume of liquidity demanders in Period 1:

$$\lambda \equiv \frac{\mathrm{Cov}\left[S_1 - S_0, \pi(\theta_1^d - \bar{\theta})\right]}{\mathrm{Var}\left[\pi(\theta_1^d - \bar{\theta})\right]}. \tag{3.13}$$

Intuitively, when λ is large, trades have large price impact and the market is illiquid. Eq. (3.6) implies that the price change between Periods 0 and 1 is

$$S_1 - S_0 = \bar{D} - \alpha\sigma^2\left(\bar{\theta} + \pi z\right) - S_0. \tag{3.14}$$

Eqs. (3.4b) and (3.6) imply that the signed volume of liquidity demanders is

$$\pi(\theta_1^d - \bar{\theta}) = -\pi(1-\pi)z. \tag{3.15}$$

Eqs. (3.13)-(3.15) imply that

$$\lambda = \frac{\alpha\sigma^2}{1-\pi}. \tag{3.16}$$

Price impact λ is higher when agents are more risk-averse (α large), the asset is riskier (σ^2 large), or liquidity suppliers are less numerous ($1-\pi$ small).

Since the signed volume of liquidity demanders is minus that of liquidity suppliers, λ is also minus the regression coefficient of the price change between Periods 0 and 1 on suppliers' signed volume in Period 1:

$$\lambda = -\frac{\text{Cov}\left[S_1 - S_0, (1-\pi)(\theta_1^s - \bar{\theta})\right]}{\text{Var}\left[(1-\pi)(\theta_1^s - \bar{\theta})\right]}. \tag{3.17}$$

The supplier-based definition of λ can be easier to implement empirically than the equivalent demander-based definition. Indeed, an important class of liquidity suppliers in some markets are designated market makers, and information on their trades is often available.

The second measure of illiquidity is based on the autocovariance of price changes. The liquidity demanders' trades in Period 1 cause the price to deviate from fundamental value, while the two coincide in Period 2. Therefore, price changes exhibit negative autocovariance, and more so when trades have large price impact. We use minus autocovariance

$$\gamma \equiv -\text{Cov}\left(S_2 - S_1, S_1 - S_0\right), \tag{3.18}$$

as a measure of illiquidity, and refer to it as price reversal. Eqs. (3.6), (3.14), (3.18) and $S_2 = D$ imply that

$$\gamma = -\text{Cov}\left[D - \bar{D} + \alpha\sigma^2\left(\bar{\theta} + \pi z\right), \bar{D} - \alpha\sigma^2\left(\bar{\theta} + \pi z\right) - S_0\right] = \alpha^2\sigma^4\sigma_z^2\pi^2. \tag{3.19}$$

Price reversal γ is higher when agents are more risk-averse, the asset is riskier, liquidity demanders are more numerous (π large), and liquidity shocks are larger (σ_z^2 large).[6]

The measures λ and γ have been defined in models focusing on specific market imperfections, and have been widely used in empirical work ever since. Using our model, we can examine the

[6]The comparative statics of autocorrelation are similar to those of autocovariance. We use autocovariance rather than autocorrelation because normalizing by variance adds unnecessary complexity.

behavior of these measures across a variety of imperfections, and provide a broader perspective on their properties. We emphasize basic properties below, leaving a more detailed discussion of the measures and their empirical estimation to Section 6.

Kyle (1985) defines λ in a model where an informed insider trades with uninformed market makers and noise traders. The price impact measured by λ concerns the aggregate order that market makers receive, which is driven both by the insider's private information and by noise trading. Our definition of λ parallels Kyle's since the trades of our liquidity demanders can be motivated by hedging or information. In Kyle, however, market makers are risk neutral, and trades affect prices only because they can contain information. Thus, λ reflects purely the amount of information that trades convey, and is permanent because the risk-neutral market makers set the price equal to their expectation of fundamental value. In general, as in our model, λ has both a transitory and a permanent component. The transitory component, present even in our perfect-market benchmark, arises because liquidity suppliers are risk averse and require a price movement away from fundamental value to absorb a liquidity shock. The permanent component arises only when information is asymmetric, for the same reasons as in Kyle.[7]

Roll (1984) links γ to the bid-ask spread, in a model where market orders cause the price to bounce between the bid and the ask. Grossman and Miller (1988) link γ to the price impact of liquidity shocks, in a model where risk-averse liquidity suppliers must incur a cost to participate in the market. In both models, price impact is purely transitory because information is symmetric. In our model, price impact has both a transitory and a permanent component, and γ isolates the effects of the transitory component. Note that besides being a measure of imperfections, γ provides a useful characterization of price dynamics: it measures the importance of the transitory component in price arising from temporary liquidity shocks, relative to the random-walk component arising from fundamentals.

Illiquidity in Period 1 lowers the price in Period 0 through the illiquidity discount, which is the third term in (3.10). To explain why the discount arises, consider the extreme case where trade in Period 1 is not allowed. In Period 0, agents know that with probability π they will receive an endowment in Period 2. The endowment amounts to a risky position in Period 1, the size of which is uncertain because it depends on z. Uncertainty about position size is costly to risk-averse agents. Moreover, the effect is stronger when agents carry a large position from Period 0 because the cost of holding a position in Period 1 is convex in the overall size of the position. (The cost is the quadratic term in (3.2) and (3.3).) Therefore, uncertainty about z reduces agents' willingness to

[7] An alternative definition of λ, which isolates the permanent component, involves the price change between Periods 0 and 2 rather than between Periods 0 and 1. This is because the transitory deviation between price and fundamental value in Period 1 disappears in Period 2.

buy the asset in Period 0.

The intuition is similar when agents can trade in Period 1. Indeed, in the extreme case where trade is not allowed, the shadow price faced by liquidity demanders moves in response to z to the point where these agents are not willing to trade. When trade is allowed, the price movement is smaller, but non-zero. Therefore, uncertainty about z still reduces agents' willingness to buy the asset in Period 0. Moreover, the effect is weaker when trade is allowed in Period 1 than when it is not (this follows from the more general result of Proposition 4.6), and therefore corresponds to a discount driven by illiquidity. Because market imperfections hinder trade in Period 1, they tend to raise the illiquidity discount in Period 0.

The illiquidity discount is the product of two terms. The first term, $\frac{\pi M}{1-\pi+\pi M}$, can be interpreted as the risk-neutral probability of being a liquidity demander: π is the true probability, and M is the ratio of marginal utilities of wealth of demanders and suppliers, where utilities are interim in Period 1/2. The second term, $\Delta_1\bar{\theta}$, is the discount that an agent would require conditional on being a demander.

The illiquidity discount is higher when liquidity shocks are larger (σ_z^2 large) and occur with higher probability (π large). It is also higher when agents are more risk averse (α large), the asset is riskier (σ^2 large), and in larger supply ($\bar{\theta}$ large). In all cases, the risk-neutral probability of being a liquidity demander is higher, and so is the discount that an agent would require conditional on being a demander. For example, an increase in any of $(\sigma_z^2, \pi, \alpha, \sigma^2)$ increases the discount required by a demander because the liquidity shock z generates higher price volatility in Period 1 (as can be seen from (3.6)). Furthermore, in the case of $(\sigma_z^2, \alpha, \sigma^2)$, the risk-neutral probability of being a demander increases because so does the ratio M of marginal utilities of wealth of demanders and suppliers: suppliers, who benefit from the higher price volatility in Period 1, become better off relative to demanders, who are hurt by this volatility. In the case of π, both M and the physical probability of being a demander increase.[8]

Proposition 3.3 gathers the comparative statics of the illiquidity measures and the illiquidity discount with respect to the parameter σ_z^2, which measures the magnitude of liquidity shocks. We derive comparative statics with respect to the same parameter under the market imperfections that we consider, and in Section 6 draw their empirical implications. The parameter σ_z^2 has different

[8]The comparative statics of the illiquidity discount extend to its ratio relative to the discount $\alpha\sigma^2\bar{\theta}$ driven by payoff risk. Thus, while risk aversion α, payoff risk σ^2, or asset supply $\bar{\theta}$ raise the risk discount, they have an even stronger impact on the illiquidity discount. For example, an increase in α raises the risk discount because agents become more averse to payoff risk. The effect on the illiquidity discount is even stronger because not only agents become more averse to the risk of receiving a liquidity shock, but also the shock has larger price impact and hence generates more risk.

effects on the illiquidity measures and the illiquidity discount: it has no effect on λ, while it raises γ and the discount. The intuition is that λ measures the price impact per unit trade, while γ and S_0 concern the impact of the entire liquidity shock.

Proposition 3.3 *An increase in the variance σ_z^2 of liquidity shocks leaves price impact λ unchanged, raises price reversal γ, and lowers the price in Period 0.*

4 Asymmetric Information

In this section we assume that liquidity demanders observe the private signal (2.3) before trading in Period 1. We examine how asymmetric information affects the illiquidity measures and the illiquidity discount.

4.1 Equilibrium

The price in Period 1 incorporates the signal of liquidity demanders, and therefore reveals information to liquidity suppliers. To solve for equilibrium, we conjecture a price function (i.e., a relationship between the price and the signal), then determine how agents use their knowledge of the price function to learn about the signal and formulate demand functions, and finally confirm that the conjectured price function clears the market.

We conjecture a price function that is affine in the signal s and the liquidity shock z, i.e.,

$$S_1 = a + b(s - \bar{D} - cz) \tag{4.1}$$

for three constants (a, b, c). For expositional convenience, we set $\xi \equiv s - \bar{D} - cz$. We also refer to the price function as simply the price.

Agents use the price and their private information to form a posterior distribution about the asset payoff D. For a liquidity demander, the price conveys no additional information relative to observing the signal s. Given the joint normality of (D, ϵ), D remains normal conditional on $s = D + \epsilon$, with mean and variance

$$\mathrm{E}[D|s] = \bar{D} + \beta_s(s - \bar{D}), \tag{4.2a}$$

$$\sigma^2[D|s] = \beta_s \sigma_\epsilon^2, \tag{4.2b}$$

where $\beta_s \equiv \sigma^2/(\sigma^2 + \sigma_\epsilon^2)$. For a liquidity supplier, the only information is the price S_1, which is

equivalent to observing ξ. Conditional on ξ (or S_1), D is normal with mean and variance

$$\mathrm{E}[D|S_1] = \bar{D} + \beta_\xi \xi = \bar{D} + \frac{\beta_\xi}{b}(S_1 - a), \tag{4.3a}$$

$$\sigma^2[D|S_1] = \beta_\xi(\sigma_\epsilon^2 + c^2\sigma_z^2), \tag{4.3b}$$

where $\beta_\xi \equiv \sigma^2/\sigma_\xi^2$ and $\sigma_\xi^2 \equiv \sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2$. Agents' optimization problems are as in Section 3, with the conditional distributions of D replacing the unconditional one. Proposition 4.1 summarizes the solution to these problems.

Proposition 4.1 *Agents' demand functions for the risky asset in Period 1 are*

$$\theta_1^s = \frac{\mathrm{E}[D|S_1] - S_1}{\alpha\sigma^2[D|S_1]}, \tag{4.4a}$$

$$\theta_1^d = \frac{\mathrm{E}[D|s] - S_1}{\alpha\sigma^2[D|s]} - z. \tag{4.4b}$$

Substituting (4.4a) and (4.4b) into the market-clearing equation (3.5), we find

$$(1-\pi)\frac{\mathrm{E}[D|S_1] - S_1}{\alpha\sigma^2[D|S_1]} + \pi\left(\frac{\mathrm{E}[D|s] - S_1}{\alpha\sigma^2[D|s]} - z\right) = \bar{\theta}. \tag{4.5}$$

The price (4.1) clears the market if (4.5) is satisfied for all values of (s, z). Substituting S_1, $\mathrm{E}[D|s]$, and $\mathrm{E}[D|S_1]$ from (4.1), (4.2a) and (4.3a), we can write (4.5) as an affine equation in (s, z). Therefore, (4.5) is satisfied for all values of (s, z) if the coefficients of (s, z) and of the constant term are equal to zero. This yields a system of three equations in (a, b, c), solved in Proposition 4.2.

Proposition 4.2 *The price in Period 1 is given by (4.1), where*

$$a = \bar{D} - \alpha(1-b)\sigma^2\bar{\theta}, \tag{4.6a}$$

$$b = \frac{\pi\beta_s\sigma^2[D|S_1] + (1-\pi)\beta_\xi\sigma^2[D|s]}{\pi\sigma^2[D|S_1] + (1-\pi)\sigma^2[D|s]}, \tag{4.6b}$$

$$c = \alpha\sigma_\epsilon^2. \tag{4.6c}$$

To determine the price in Period 0, we follow the same steps as in Section 3. The calculations are more complicated because expected utilities in Period 1 are influenced by two random variables (s, z) rather than only z. The price in Period 0, however, takes the same general form as in the perfect-market benchmark.

Proposition 4.3 *The price in Period 0 is given by (3.10), where M is given by (3.11),*

$$\Delta_0 = \frac{(b-\beta_\xi)^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)}{\sigma^2[D|S_1]\pi^2}, \tag{4.7a}$$

$$\Delta_1 = \frac{\alpha^3 b\sigma^2(\sigma^2+\sigma_\epsilon^2)\sigma_z^2}{1+\Delta_0(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}, \tag{4.7b}$$

$$\Delta_2 = \frac{\alpha^3\sigma^4\sigma_z^2\left[1+\frac{(\beta_s-b)^2(\sigma^2+\sigma_\epsilon^2)}{\sigma^2[D|s]}\right]}{1+\Delta_0(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}. \tag{4.7c}$$

4.2 Asymmetric Information and Illiquidity

We next examine how asymmetric information impacts the illiquidity measures and the illiquidity discount. When some agents observe a private signal, this not only generates dispersion in information across agents, but also renders each agent more informed because the signal is partially revealed through the price. The improvement in each agent's information is not a distinguishing feature of asymmetric information: information can also improve if all agents observe a public signal. To focus on the dispersion in information, which is what distinguishes asymmetric information, we compare with two symmetric-information benchmarks: the no-information case, where information is symmetric because no agent observes the signal s, and the full-information case, where all agents observe s. The analysis in Section 3 concerns the no-information case, but can be extended to the full-information case (online Appendix, Proposition A.1). Price impact λ and price reversal γ under full information are given by (3.16) and (3.19), respectively, where σ^2 is replaced by $\sigma^2[D|s]$.

Proposition 4.4 *Price impact λ under asymmetric information is*

$$\lambda = \frac{\alpha\sigma^2[D|S_1]}{(1-\pi)\left(1-\frac{\beta_\xi}{b}\right)}. \tag{4.8}$$

Price impact is highest under asymmetric information and lowest under full information. Moreover, price impact under asymmetric information increases when the private signal (2.3) becomes more precise, i.e., when σ_ϵ^2 decreases.

Proposition 4.4 shows that price impact is higher under asymmetric information than under either of the two symmetric-information benchmarks. Asymmetric information thus raises price impact because information differs across agents and not because of any changes in the average quality of information.

The comparison between the asymmetric-, no- and full-information cases is driven by two effects: an uncertainty and a learning effect. Price impact increases in the uncertainty faced by liquidity suppliers, measured by their conditional variance of the asset payoff. Because of this uncertainty effect, price impact tends to be lowest under full information, since liquidity suppliers observe the signal perfectly, next lowest under asymmetric information, since the signal is partially revealed to liquidity suppliers through the price, and highest under no information.

An additional source of price impact, present only under asymmetric information, is that liquidity suppliers seek to learn the signal from the price. Because, for example, liquidity suppliers attribute selling pressure partly to a low signal, they require a larger price drop to buy. This learning effect corresponds to the term β_ξ/b in (4.8), which lowers the denominator and raises price impact λ.

The learning effect works in the same direction as the uncertainty effect when comparing asymmetric to full information, but in the opposite direction when comparing asymmetric to no information. Proposition 4.4 shows that in the latter comparison the learning effect dominates. Therefore, price impact is higher under asymmetric information than under either of the two symmetric-information benchmarks.

Price reversal is not unambiguously highest under asymmetric information. Indeed, consider two extreme cases. If $\pi \approx 1$, i.e., almost all agents are liquidity demanders (informed), then the price processes under asymmetric and full information approximately coincide, and so do the price reversals. Since, in addition, liquidity suppliers face more uncertainty under no information than under full information, price reversal is highest under no information.

If instead $\pi \approx 0$, i.e., almost all agents are liquidity suppliers (uninformed), then price impact λ converges to infinity (order $1/\pi$) under asymmetric information. This is because the trading volume of liquidity demanders converges to zero, but the volume's informational content remains unchanged. Because of the high price impact, price reversal is highest under asymmetric information.

Proposition 4.5 *Price reversal γ under asymmetric information is*

$$\gamma = b(b - \beta_\xi)(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2). \tag{4.9}$$

Price reversal is lowest under full information. It is highest under asymmetric information if $\pi \approx 0$, and under no information if $\pi \approx 1$.

The analysis of the illiquidity discount involves an effect that goes in the direction opposite to the uncertainty effect. This is that information revealed about the asset payoff in Period 1

reduces uncertainty and hence the scope for risk sharing. Less risk sharing, in turn, renders agents less willing to buy the asset in Period 0 and raises the illiquidity discount. The negative effect of information on risksharing and welfare has been shown in Hirshleifer (1971). We derive the implications of the Hirshleifer effect for asset pricing: Proposition 4.6 shows that the reduced scope for risksharing in Period 1 lowers the asset price in Period 0 and raises the illiquidity discount.

Because of the Hirshleifer effect, the illiquidity discount under full information is higher than under no information—a comparison which is exactly the reverse than for the measures of illiquidity. A corollary of this result is that the illiquidity discount under no trade is higher than in the perfect-market benchmark of Section 3. Indeed, the perfect-market benchmark corresponds to the no-information case, while no trade is a special case of full information when the signal (2.3) is perfectly precise ($\sigma_\epsilon^2 = 0$).[9]

The Hirshleifer effect implies that the illiquidity discount under asymmetric information should be between that under no and under full information. The discount under asymmetric information, however, is also influenced by the learning effect, which raises price impact, reduces the scope for risk sharing and hence raises the discount. The learning effect works in the same direction as the Hirshleifer effect when comparing asymmetric to no information, but in the opposite direction when comparing asymmetric to full information. Proposition 4.6 shows that in the latter comparison the learning effect dominates. Therefore, the illiquidity discount is higher under asymmetric information than under either of the two symmetric-information benchmarks. Asymmetric information thus raises the illiquidity discount because information differs across agents and not because of any changes in the average quality of information.

Proposition 4.6 *The price in Period 0 is lowest under asymmetric information and highest under no information.*

The comparative statics with respect to the variance σ_z^2 of liquidity shocks are the same as in the perfect-market benchmark case, except for the price impact λ. Under asymmetric information, an increase in σ_z^2 lowers λ because liquidity shocks make prices less informative and attenuate learning.

Proposition 4.7 *An increase in the variance σ_z^2 of liquidity shocks lowers price impact λ, raises price reversal γ, and lowers the price in Period 0.*

[9] Recall from Section 3 that the illiquidity discount is the product of $\frac{\pi M}{1-\pi+\pi M}$, the risk-neutral probability of being a liquidity demander, times $\Delta_1\bar{\theta}$, the discount that an agent would require conditional on being a demander. No trade renders both demanders and suppliers worse off relative to the perfect-market benchmark, and hence has an ambiguous effect on the ratio M of their marginal utilities of wealth. The increase in the illiquidity discount is instead driven by the increase in the discount $\Delta_1\bar{\theta}$ required by a demander.

5 Imperfect Competition

In this section we assume that liquidity demanders can collude and exert market power in Period 1. We consider both the case where liquidity demanders have no private information on asset payoffs, and so information is symmetric, and the case where they observe the private signal (2.3), and so information is asymmetric. Since the second case nests the first by setting the variance σ_ϵ^2 of the signal noise to infinity, we treat both cases simultaneously. We examine how imperfect competition affects the illiquidity measures and the illiquidity discount.

The trading mechanism in Period 1 is that liquidity suppliers submit a demand function and liquidity demanders submit a market order, i.e., a price-inelastic demand function. Restricting liquidity demanders to trade by market order is without loss of generality: they do not need to condition their demand on price because they know all information available in Period 1.

5.1 Equilibrium

We conjecture that the price in Period 1 has the same affine form (4.1) as in the competitive case, with possibly different constants (a, b, c). Given (4.1), the demand function of liquidity suppliers is (4.4a) as in the competitive case. Substituting (4.4a) into the market-clearing equation (3.5), and using (4.3a), yields the price in Period 1 as a function of the liquidity demanders' market order θ_1^d:

$$S_1(\theta_1^d) = \frac{\bar{D} - \frac{\beta_\xi}{b}a + \frac{\alpha\sigma^2[D|S_1]}{1-\pi}(\pi\theta_1^d - \bar{\theta})}{1 - \frac{\beta_\xi}{b}}. \tag{5.1}$$

Liquidity demanders choose θ_1^d to maximize the expected utility

$$-\mathrm{E}\exp\left\{-\alpha\left[W_1 + \theta_1^d\left(D - S_1(\theta_1^d)\right) + z(D - \bar{D})\right]\right\}. \tag{5.2}$$

The difference with the competitive case is that liquidity demanders behave as a single monopolist and take into account the impact of their order θ_1^d on the price S_1. Proposition 5.1 characterizes the solution to the liquidity demanders' optimization problem.

Proposition 5.1 *The liquidity demanders' market order in Period 1 satisfies*

$$\theta_1^d = \frac{\mathrm{E}[D|s] - S_1(\theta_1^d) - \alpha\sigma^2[D|s]z + \hat{\lambda}\bar{\theta}}{\alpha\sigma^2[D|s] + \hat{\lambda}}, \tag{5.3}$$

where $\hat{\lambda} \equiv \frac{dS_1(\theta_1^d)}{d\theta_1^d} = \frac{\alpha\pi\sigma^2[D|S_1]}{(1-\pi)\left(1-\frac{\beta_\xi}{b}\right)}$.

Eq. (5.3) determines θ_1^d implicitly because it includes θ_1^d in both the left- and the right-hand side. We write θ_1^d in the form (5.3) to facilitate the comparison with the competitive case. Indeed, the competitive counterpart of (5.3) is (4.4b), and can be derived by setting $\hat{\lambda}$ to zero. The parameter $\hat{\lambda}$ measures the price impact of liquidity demanders, and is closely related to the price impact λ. Because in equilibrium $\hat{\lambda} > 0$, the denominator of (5.3) is larger than that of (4.4b), and therefore θ_1^d is less sensitive to changes in $\mathrm{E}[D|s] - S_1$ and z than in the competitive case. Intuitively, because liquidity demanders take price impact into account, they trade less aggressively in response to their signal and their liquidity shock.

Substituting (4.4a) and (5.3) into the market-clearing equation (3.5), and proceeding as in Section 4, we find a system of three equations in (a, b, c). Proposition 5.2 solves this system.

Proposition 5.2 *The price in Period 1 is given by (4.1), where*

$$b = \frac{\pi\beta_s\sigma^2[D|S_1] + (1-\pi)\beta_\xi\sigma^2[D|s]}{2\pi\sigma^2[D|S_1] + (1-\pi)\sigma^2[D|s]}, \tag{5.4}$$

and (a, c) are given by (4.6a) and (4.6c), respectively. The linear equilibrium exists if $\sigma_\epsilon^2 > \hat{\sigma}_\epsilon^2$, where $\hat{\sigma}_\epsilon^2$ is the positive solution of

$$\alpha^2\hat{\sigma}_\epsilon^4\sigma_z^2 = \sigma^2 + \hat{\sigma}_\epsilon^2. \tag{5.5}$$

The price in the competitive market in Period 0 can be determined through similar steps as in Sections 3 and 4.

Proposition 5.3 *The price in Period 0 is given by (3.10), where*

$$M = \exp\left(\tfrac{1}{2}\alpha\Delta_2\bar{\theta}^2\right)\sqrt{\frac{1+\Delta_0\pi^2}{1+\Delta_0\left(1+\frac{2\hat{\lambda}}{\alpha\sigma^2[D|s]}\right)(1-\pi)^2 - \alpha^2\sigma^2\sigma_z^2}}, \tag{5.6}$$

$$\Delta_1 = \frac{\alpha^3 b\sigma^2(\sigma^2+\sigma_\epsilon^2)\sigma_z^2}{1+\Delta_0\left(1+\frac{2\hat{\lambda}}{\alpha\sigma^2[D|s]}\right)(1-\pi)^2 - \alpha^2\sigma^2\sigma_z^2}, \tag{5.7a}$$

$$\Delta_2 = \frac{\alpha^3\sigma^4\sigma_z^2\left[1+\frac{\alpha(\beta_s-b)^2(\sigma^2+\sigma_\epsilon^2)\left(\alpha\sigma^2[D|s]+2\hat{\lambda}\right)}{\left(\alpha\sigma^2[D|s]+\hat{\lambda}\right)^2}\right]}{1+\Delta_0\left(1+\frac{2\hat{\lambda}}{\alpha\sigma^2[D|s]}\right)(1-\pi)^2 - \alpha^2\sigma^2\sigma_z^2}, \tag{5.7b}$$

and Δ_0 is given by (4.7a).

5.2 Non-Competitive Behavior and Illiquidity

We next examine how imperfect competition by liquidity demanders impacts the illiquidity measures and the illiquidity discount.

Proposition 5.4 *Price impact λ is given by (4.8). It is the same as under competitive behavior when information is symmetric, and higher when information is asymmetric.*

Although price impact is given by the same equation as under perfect competition, it is higher when competition is imperfect because the coefficient b is smaller. Intuitively, when liquidity demanders take into account their effect on price, they trade less aggressively in response to their signal and their liquidity shock. This reduces the size of both information- and liquidity-generated trades. The relative size of the two types of trades remains the same, and so does price informativeness, measured by the signal-to-noise ratio. Monopoly trades thus have the same informational content as competitive trades, but are smaller in size. As a result, the signal per trade size is higher, and so is the price impact λ of trades. Imperfect competition has no effect on price impact when information is symmetric because trades have no informational content.

An increase in information asymmetry, through a reduction in the variance σ_ϵ^2 of the signal noise, generates an illiquidity spiral. Because illiquidity increases, liquidity demanders scale back their trades. This raises the signal per trade size, further increasing illiquidity. When information asymmetry becomes severe, illiquidity becomes infinite and trade ceases, leading to a market breakdown. This occurs when $\sigma_\epsilon^2 \leq \hat{\sigma}_\epsilon^2$, i.e., for values of σ_ϵ^2 such that the equilibrium of Proposition 5.2 does not exist. Non-competitive behavior is essential for the non-existence of an equilibrium with trade because such an equilibrium always exists under competitive behavior.[10]

Proposition 5.5 *Price reversal γ is given by (4.9), and is lower than under competitive behavior.*

Although price reversal is given by the same equation as under competitive behavior, it is lower when behavior is non-competitive because the coefficient b is smaller. Intuitively, price reversal arises because the liquidity demanders' trades in Period 1 cause the price to deviate from fundamental value. Under imperfect competition, these trades are smaller and so is price reversal. Note that imperfect competition has opposite effects on the two illiquidity measures: price impact λ increases but price reversal γ decreases.

[10]There exist settings, however, where asymmetric information leads to market breakdowns even with competitive agents. See Akerlof (1970) for a setting where agents trade heterogeneous goods of different qualities, and Glosten and Milgrom (1985) for an asset-market setting.

While imperfect competition raises the price impact λ, it can lower the illiquidity discount. Indeed, since liquidity demanders scale back their trades, they render the price less responsive to their liquidity shock. Therefore, they can obtain better insurance against the shock, and become less averse to holding the asset in Period 0. This effect drives the illiquidity discount below the competitive value when information is symmetric. When information is asymmetric, the comparison can reverse. This is because the scaling back of trades generates the spiral of increasing illiquidity, and this reduces the insurance received by liquidity demanders.

Proposition 5.6 *The price in Period 0 is higher than under perfect competition when information is symmetric, but can be lower when information is asymmetric.*

The comparative statics with respect to the variance σ_z^2 of liquidity shocks are the same as under perfect competition.

Proposition 5.7 *An increase in the variance σ_z^2 of liquidity shocks leaves price impact λ unchanged under symmetric information but lowers it under asymmetric information. It raises price reversal and lowers the price in Period 0.*

The case where liquidity suppliers collude can be treated in a manner similar to the case where demanders collude, so we provide a brief sketch. Suppose that demanders are competitive but suppliers behave as a single monopolist in Period 1. Since suppliers do not know the liquidity shock z and signal s, their trading strategy is to submit a price-elastic demand function (rather than a market order). Non-competitive behavior renders this demand function less price-elastic than its competitive counterpart (4.4a). The lower elasticity manifests itself through an additive positive term in the denominator of the competitive demand (4.4a), exactly as is the case for liquidity demanders in (4.4b) and (5.3).

Because liquidity suppliers submit a less price-elastic demand function than in the competitive case, the trades of liquidity demanders have larger price impact. Hence, price impact λ and price reversal γ are larger than in the competitive case. The illiquidity discount is also larger because liquidity demanders receive worse insurance against the liquidity shock. Thus, imperfect competition by suppliers has the same effect as by demanders on λ, the opposite effect on γ, and the same or opposite effect on the illiquidity discount.

6 Empirical Implications

In this section we explore implications of our model for empirical studies of liquidity. These implications concern the relative merits of different empirical measures of illiquidity, as well as the empirical relationship between liquidity and expected returns.

6.1 Measures of Illiquidity

Within our model, we can compute two widely used empirical measures of illiquidity and examine how they behave across a variety of imperfections. The first measure is λ, defined as the regression coefficient of price changes on the liquidity demanders' signed volume, and based on the idea that trades in illiquid markets should have large price impact. The second is γ, defined as minus the autocovariance of price changes, and based on the idea that trades in illiquid markets should generate large transitory deviations between price and fundamental value. The measures λ and γ have been linked to illiquidity within models focusing on specific imperfections—λ in Kyle (1985), and γ in Roll (1984) and Grossman and Miller (1988)—and have been widely used in empirical work ever since. Measures closely related to λ are, for example, the regression-based measure of Glosten and Harris (1988) and Sadka (2006), and the ratio of average absolute returns to trading volume of Amihud (2002). Measures closely related to γ, are, for example, the bid-ask spread measure of Roll (1984), the Gibbs estimate of Hasbrouck (2006), the price reversal measure of Bao, Pan and Wang (2011), and the price reversal conditional on signed volume of Campbell, Grossman and Wang (1993).

In our analysis, λ captures not only the permanent component of price impact, driven by the information that trades convey (as in Kyle), but also the transitory component, driven by the risk aversion of liquidity suppliers. In this sense, λ overlaps with γ, which isolates the transitory component.[11] We further show that under the two imperfections considered here, λ reflects market imperfections more accurately than γ. Indeed, both asymmetric information and imperfect competition increase λ (Propositions 4.4 and 5.4) but can decrease γ (Propositions 4.5 and 5.5).[12]

Estimating γ requires information only on transaction prices. Estimating λ requires also information on the signed trades of liquidity demanders or suppliers. The signed trades of these agents can be partially identified using data on transaction prices, quantities and bid-ask quotes. Lee

[11] The overlap is larger between λ and the conditional price reversal of Campbell, Grossman and Wang (1993) because both measures condition on signed volume.

[12] A previous version of this paper (Vayanos and Wang (2010)) shows additionally that participation costs, transaction costs and leverage constraints increase both λ and γ, while search frictions can decrease both λ and γ, with λ decreasing under more stringent conditions than γ.

and Ready (1991) propose an algorithm to determine who initiates a trade, and hence to assign trades to liquidity demanders and suppliers. Their algorithm is based on the assumption that trade initiators—liquidity demanders—mostly use market orders, while those agreeing to take the other side of trades—liquidity suppliers—use limit orders. A number of papers (e.g., Sadka (2006)) employ Lee and Ready's algorithm to estimate λ for the US equity market, where data on transaction prices, quantities and bid-ask quotes are available.

The estimation of λ can be further facilitated when data on the identity of traders are available. For example, Madhavan and Smidt (1993) and Comerton-Forde, Hendershott, Jones, Moulton and Seasholes (2010) use data on the quotes and inventories of NYSE specialists to examine their behavior in supplying liquidity. The effective cost (price concession) that specialists extract from other traders provides an estimate of λ, at least for trades in which specialists take part. The transactions data on corporate bonds also identify dealer-customer and dealer-dealer trades (e.g., Edwards, Harris and Piwowar (2007)), allowing estimation of λ.[13]

6.2 Liquidity and Expected Returns

Many empirical studies seek to establish a link between liquidity and expected asset returns.[14] Their basic premise is that illiquidity is positively related to expected returns. Our analysis shows, however, that this relationship does not have to be positive. Moreover, its nature depends crucially on the underlying cause of illiquidity and on the measure of illiquidity being used. Suppose, for example, that illiquidity is caused by asymmetric information. If illiquidity is measured by λ, then its empirical relationship with expected returns will be positive since asymmetric information raises both λ and the illiquidity discount (Propositions 4.4 and 4.6). If, however, illiquidity is measured by γ, then the relationship can be negative since asymmetric information can reduce γ (Proposition 4.5). Furthermore, if the imperfection is imperfect competition, then a negative relationship can arise even if illiquidity is measured by λ. This is because imperfect competition raises λ but can lower the illiquidity discount (Propositions 5.4 and 5.6).

Our model predicts that λ can reflect market imperfections and their impact on asset prices more accurately than γ; does this hold in the data? Suggestive evidence comes from recent studies in the corporate-bond market that compare the performance of λ and γ in explaining credit yield

[13]Besides requiring more information than γ for its estimation, λ has the drawback that it might not reflect a causal effect of volume on prices. For example, if public news cause both volume and prices, then λ can be positive even in the absence of a causal effect of volume on price changes. The causality problem does not arise in our model. Indeed, volume is generated by shocks observable only to liquidity demanders, such as the liquidity shock z and the signal s. Since these shocks can affect prices only through the liquidity demanders' trades, λ measures correctly the price impact of these trades.

[14]See, for example, the survey by Amihud, Mendelson and Pedersen (2005) for references.

spreads. Dick-Nielsen, Feldhutter and Lando (DFL 2011) examine how spreads are linked to λ, as approximated by the Amihud measure, to γ, and to more heuristic measures of illiquidity such as turnover and trading frequency.[15] They find that the positive relationship between spreads and λ is more robust than that between spreads and γ, both across different rating categories and across the pre- and post-2008-crisis sample periods (Table 3). Moreover, for the post-crisis period, the relationship between spreads and γ becomes insignificant except for AAA-rated bonds. For speculative-grade bonds, the relationship becomes even negative (with a t-statistic of -1.16). Given that speculative-grade bonds are more likely to be subject to information asymmetry, this finding, if further confirmed, would be consistent with the predictions of our model.

Rayanakorn and Wang (2011) examine how spreads are linked to λ, γ, trading frequency, bond age and maturity, and the persistence and variance of the stationary component in bond prices (presumably caused by transitory liquidity shocks). They find that λ can explain the cross-section of spreads better than γ, consistent with DFL.

One complication in measuring the relationship between illiquidity and expected returns is that cross-sectional variation might be driven by factors other than the imperfections themselves. Our analysis helps determine the effects of such variation. Suppose, for example, that assets differ mainly in the variance σ_z^2 of liquidity shocks. Under asymmetric information and imperfect competition, larger σ_z^2 lowers λ and raises expected returns (Propositions 4.7 and 5.7). Thus, if cross-sectional variation is driven by σ_z^2 and illiquidity is measured by λ, then the empirical relationship between illiquidity and expected returns will be negative. A positive relationship, however, will arise if cross-sectional variation is driven by asymmetric information.

Finally, our analysis has implications for the positive relationship between expected returns and idiosyncratic return volatility found in some empirical studies (e.g., Spiegel and Wang (2005), Ang, Hodrick, Xing and Zhang (2006)). One source of idiosyncratic volatility, especially over short horizons, is illiquidity since it affects the stationary component of prices (see, e.g., Bao, Pan and Wang (2011)). Therefore, the positive empirical relationship might be partly due to illiquidity.

7 Welfare

Our model is suitable for a normative analysis of imperfections. In this section we illustrate the normative analysis in the case of asymmetric information. We examine how asymmetric informa-

[15] Earlier studies linking credit yield spreads to a more limited set of illiquidity measures include Chen, Lesmond and Wei (2007) and Bao, Pan and Wang (2011).

tion affects the interim utilities (U^s, U^d) of liquidity suppliers and demanders in Period 1/2. As in Section 4, we compare with two symmetric-information benchmarks: no information and full information.

Since information reduces uncertainty and the scope for risk sharing, the Hirshleifer effect implies that the interim utilities (U^s, U^d) under full information are smaller than under no information. The Hirshleifer effect also implies that the interim utilities under asymmetric information should be between those under no and under full information. The interim utilities under asymmetric information, however, are also influenced by the learning effect, which raises illiquidity and reduces the scope for risk sharing. The learning effect works in the same direction as the Hirshleifer effect when comparing asymmetric to no information, but in the opposite direction when comparing asymmetric to full information. Proposition 7.1 shows that in the latter comparison the learning effect dominates. Therefore, the interim utilities are higher under asymmetric information than under either of the two symmetric-information benchmarks.

Proposition 7.1 *The interim utilities (U^s, U^d) of liquidity suppliers and demanders in Period 1/2 are lowest under asymmetric information and highest under no information.*

Proposition 7.1 carries through to the ex-ante utility in Period 0. Since the ex-ante utility is the expectation of the interim utilities, it is lowest under asymmetric information and highest under no information.

8 Conclusion

We examine how asymmetric information and imperfect competition affect liquidity and expected returns. We show three main results. First, asymmetric information raises expected returns, compared both to a symmetric-information benchmark where all private signals are made public and to one where private signals are not observed. Second, asymmetric information and imperfect competition raise Kyle's lambda but can bring the autocovariance of asset returns closer to zero. Thus, lambda reflects both frictions more accurately than autocovariance. Third, imperfect competition can lower expected returns. Our results imply that the empirical relationship between illiquidity, as measured by lambda, and expected returns is positive under asymmetric information but can turn negative under imperfect competition. Moreover, the relationship can turn negative even under asymmetric information, if illiquidity is measured by autocovariance.

Our model can incorporate additional frictions. A previous version of this paper (Vayanos and Wang (2010)) also considers participation costs, transaction costs, leverage constraints, and search

frictions. The results provide a unified treatment of many different frictions under a common set of assumptions concerning agents' preferences and trading motives. Frictions are shown to differ significantly as to their effects on illiquidity measures and expected returns, and as to the empirical implications they generate.

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Appendix

A Perfect-Market Benchmark

We start with a useful lemma.

Lemma A.1 *Let x be an $n \times 1$ normal vector with mean zero and covariance matrix Σ, A a scalar, B an $1 \times n$ vector, C an $n \times n$ symmetric matrix, I the $n \times n$ identity matrix, and $|M|$ the determinant of a matrix M. Then,*

$$\mathrm{E}_x \exp\left\{-\alpha\left[A + B'x + \tfrac{1}{2}x'Cx\right]\right\} = \exp\left\{-\alpha\left[A - \tfrac{1}{2}\alpha B'\Sigma(I + \alpha C\Sigma)^{-1}B\right]\right\} \frac{1}{\sqrt{|I + \alpha C\Sigma|}}. \quad \text{(A.1)}$$

Proof: When $C = 0$, (A.1) gives the moment-generating function of the normal distribution. We can always assume $C = 0$ by also assuming that x is a normal vector with mean 0 and covariance matrix $\Sigma(I + \alpha C\Sigma)^{-1}$. ∎

Proof of Proposition 3.1: Eqs. (3.4a) and (3.4b) follow by maximizing the term inside the exponential in (3.3) and (3.2), respectively. ∎

Proof of Proposition 3.2: We first compute the interim utilities U^s and U^d of liquidity suppliers and demanders in Period 1/2. The utility U^s is the expectation of (3.8) over z. To compute this expectation, we use Lemma A.1 and set

$$x \equiv z,$$

$$\Sigma \equiv \sigma_z^2,$$

$$A \equiv W_0 + \theta_0(\bar{D} - S_0) - \alpha\sigma^2\theta_0\bar{\theta} + \tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2,$$

$$B \equiv \alpha\sigma^2\pi(\bar{\theta} - \theta_0),$$

$$C \equiv \alpha\sigma^2\pi^2.$$

Eq. (A.1) implies that

$$\begin{aligned} U^s &= -\exp\left(-\alpha F^s\right)\frac{1}{\sqrt{1 + \alpha^2\sigma^2\sigma_z^2\pi^2}} \\ &= -\exp\left(-\alpha F^s\right)\frac{1}{\sqrt{1 + \Delta_0\pi^2}}, \end{aligned} \quad \text{(A.2)}$$

where Δ_0 is given by (3.12a) and

$$F^s = W_0 + \theta_0(\bar{D} - S_0) - \alpha\sigma^2\theta_0\bar{\theta} + \tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2 - \frac{\alpha^3\sigma^4\sigma_z^2\pi^2(\theta_0 - \bar{\theta})^2}{2\left(1 + \alpha^2\sigma^2\sigma_z^2\pi^2\right)}$$

$$= W_0 + \theta_0(\bar{D} - S_0) - \tfrac{1}{2}\alpha\sigma^2\theta_0^2 + \frac{\alpha\sigma^2(\theta_0 - \bar{\theta})^2}{2\left(1 + \alpha^2\sigma^2\sigma_z^2\pi^2\right)}. \tag{A.3}$$

To compute U^d, we derive the counterpart of (3.8) for a liquidity demander. Substituting θ_1^d from (3.4b), S_1 from (3.6), and W_1 from (3.7), we can write the expected utility (3.2) of a liquidity demander in Period 1 as

$$-\exp\left\{-\alpha\left[W_0 + \theta_0(\bar{D} - S_0) - \alpha\sigma^2(\theta_0 + z)(\bar{\theta} + \pi z) + \tfrac{1}{2}\alpha\sigma^2(\bar{\theta} + \pi z)^2\right]\right\}. \tag{A.4}$$

The utility U^d is the expectation of (A.4) over z. To compute this expectation, we use Lemma A.1 and set

$$x \equiv z,$$

$$\Sigma \equiv \sigma_z^2,$$

$$A \equiv W_0 + \theta_0(\bar{D} - S_0) - \alpha\sigma^2\theta_0\bar{\theta} + \tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2,$$

$$B \equiv -\alpha\sigma^2\left[\pi\theta_0 + (1 - \pi)\bar{\theta}\right],$$

$$C \equiv -\alpha\sigma^2(2\pi - \pi^2).$$

Eq. (A.1) implies that

$$U^d = -\exp\left(-\alpha F^d\right)\frac{1}{\sqrt{1 - \alpha^2\sigma^2\sigma_z^2(2\pi - \pi^2)}}$$

$$= -\exp\left(-\alpha F^d\right)\frac{1}{\sqrt{1 + \Delta_0(1 - \pi)^2 - \alpha^2\sigma^2\sigma_z^2}}, \tag{A.5}$$

where

$$F^d = W_0 + \theta_0(\bar{D} - S_0) - \alpha\sigma^2\theta_0\bar{\theta} + \tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2 - \frac{\alpha^3\sigma^4\sigma_z^2\left[\pi\theta_0 + (1 - \pi)\bar{\theta}\right]^2}{2\left[1 - \alpha^2\sigma^2\sigma_z^2(2\pi - \pi^2)\right]}. \tag{A.6}$$

An agent in Period 0 chooses θ_0 to maximize

$$U = (1 - \pi)U^s + \pi U^d.$$

The first-order condition is

$$(1-\pi)\exp\left(-\alpha F^s\right)\frac{dF^s}{d\theta_0}\frac{1}{\sqrt{1 + \Delta_0\pi^2}} + \pi\exp\left(-\alpha F^d\right)\frac{dF^d}{d\theta_0}\frac{1}{\sqrt{1 + \Delta_0(1 - \pi)^2 - \alpha^2\sigma^2\sigma_z^2}} = 0, \tag{A.7}$$

and characterizes a maximum since U is concave. In equilibrium, (A.7) is satisfied for $\theta_0 = \bar{\theta}$. Moreover, (A.23) and (A.27) imply that when $\theta_0 = \bar{\theta}$,

$$\frac{dF^s}{d\theta_0} = \bar{D} - S_0 - \alpha\sigma^2\bar{\theta}, \tag{A.8}$$

$$F^s = W_0 + \bar{\theta}(\bar{D} - S_0) - \tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2, \tag{A.9}$$

$$\frac{dF^d}{d\theta_0} = \frac{dF^s}{d\theta_0} - \Delta_1\bar{\theta}, \tag{A.10}$$

$$F^d = F^s - \tfrac{1}{2}\Delta_2\bar{\theta}^2, \tag{A.11}$$

where Δ_1 is given by (3.12b) and Δ_2 by (3.12c). Substituting (A.8)-(A.11) into (A.7), and solving for S_0, we find (3.10). ∎

Proof of Proposition 3.3: Eq. (3.16) implies that λ is independent of σ_z^2. Eq. (3.19) implies that γ is increasing in σ_z^2. Eqs. (3.11), (3.12a), (3.12b) and (3.12c) imply that (M, Δ_1, Δ_2) are increasing in σ_z^2. Therefore, (3.10) implies that S_0 is decreasing in σ_z^2. ∎

Proposition A.1 determines the equilibrium in the full-information case.

Proposition A.1 *In the full-information case, agents' demand functions in Period 1 are*

$$\theta_1^s = \frac{E[D|s] - S_1}{\alpha\sigma^2[D|s]}, \tag{A.12}$$

$$\theta_1^d = \frac{E[D|s] - S_1}{\alpha\sigma^2[D|s]} - z, \tag{A.13}$$

the price in Period 1 is

$$S_1 = E[D|s] - \alpha\sigma^2[D|s]\left(\bar{\theta} + \pi z\right), \tag{A.14}$$

and the price in Period 0 is given by (3.10), where M is given by (3.11) and

$$\Delta_0 = \alpha^2\sigma^2[D|s]\sigma_z^2, \tag{A.15}$$

$$\Delta_1 = \frac{\alpha^3\sigma^4\sigma_z^2\left[1 - \frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}(1-\pi)\right]}{1 + \Delta_0(1-\pi)^2 - \alpha^2\sigma^2\sigma_z^2}, \tag{A.16}$$

$$\Delta_2 = \frac{\alpha^3\sigma^4\sigma_z^2}{1 + \Delta_0(1-\pi)^2 - \alpha^2\sigma^2\sigma_z^2}. \tag{A.17}$$

Proof: In Period 1, a liquidity demander chooses holdings θ_1^d of the risky asset to maximize the expected utility

$$-\mathrm{E}\exp\left\{-\alpha\left[W_1 + \theta_1^d(D - S_1) + z(D - \bar{D})\right]\right\},$$

where the expectation is over D and conditional on s. Because of normality, the expectation is equal to

$$-\exp\left\{-\alpha\left[W_1+\theta_1^d\left(\mathrm{E}[D|s]-S_1\right)+z\left(\mathrm{E}[D|s]-\bar{D}\right)-\tfrac{1}{2}\alpha\sigma^2[D|s](\theta_1^d+z)^2\right]\right\}. \tag{A.18}$$

A liquidity supplier chooses holdings θ_1^s of the risky asset to maximize the expected utility

$$-\exp\left\{-\alpha\left[W_1+\theta_1^s\left(\mathrm{E}[D|s]-S_1\right)-\tfrac{1}{2}\alpha\sigma^2[D|s](\theta_1^s)^2\right]\right\}. \tag{A.19}$$

which can be derived from (A.18) by setting $z=0$. The solution to the optimization problems is straightforward and yields the demand functions (A.12) and (A.13). Substituting (A.12) and (A.13) into the market-clearing equation (3.5), we find that the price in Period 1 is given by (A.14).

Substituting W_1 from (3.7), θ_1^s from (A.12), S_1 from (A.14), and $\mathrm{E}[D|s]$ from (4.3a), we can write the expected utility (A.19) of a liquidity supplier in Period 1 as

$$-\exp\left\{-\alpha\left[W_0+\theta_0(\bar{D}-S_0)+\theta_0\left[\beta_s(s-\bar{D})-\alpha\sigma^2[D|s](\bar{\theta}+\pi z)\right]+\tfrac{1}{2}\alpha\sigma^2[D|s](\bar{\theta}+\pi z)^2\right]\right\}. \tag{A.20}$$

Substituting W_1 from (3.7), θ_1^d from (A.13), S_1 from (A.14), and $\mathrm{E}[D|s]$ from (4.3a), we can write the expected utility (A.18) of a liquidity demander in Period 1 as

$$-\exp\left\{-\alpha\left[W_0+\theta_0(\bar{D}-S_0)+(\theta_0+z)\left[\beta_s(s-\bar{D})-\alpha\sigma^2[D|s](\bar{\theta}+\pi z)\right]+\tfrac{1}{2}\alpha\sigma^2[D|s](\bar{\theta}+\pi z)^2\right]\right\}. \tag{A.21}$$

We next compute the expectations of (A.20) and (A.21) over (s,z), i.e., the interim utilities U^s and U^d of liquidity suppliers and demanders in Period 1/2. To compute U^s, we use Lemma A.1 and set

$$x \equiv \begin{bmatrix} s-\bar{D} \\ z \end{bmatrix}$$

$$\Sigma \equiv \begin{bmatrix} \sigma^2+\sigma_\epsilon^2 & 0 \\ 0 & \sigma_z^2 \end{bmatrix}$$

$$A \equiv W_0+\theta_0(\bar{D}-S_0)-\alpha\sigma^2[D|s]\theta_0\bar{\theta}+\tfrac{1}{2}\alpha\sigma^2[D|s]\bar{\theta}^2$$

$$B \equiv \begin{bmatrix} \beta_s\theta_0 \\ \alpha\sigma^2[D|s]\pi(\bar{\theta}-\theta_0) \end{bmatrix}$$

$$C \equiv \begin{bmatrix} 0 & 0 \\ 0 & \alpha\sigma^2[D|s]\pi^2 \end{bmatrix}.$$

Since

$$I + \alpha C\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 + \alpha^2\sigma^2[D|s]\sigma_z^2\pi^2 \end{bmatrix},$$

(A.1) implies that

$$U^s = -\exp(-\alpha F^s)\frac{1}{\sqrt{1 + \alpha^2\sigma^2[D|s]\sigma_z^2\pi^2}}, \tag{A.22}$$

where

$$F^s = W_0 + \theta_0(\bar{D} - S_0) - \alpha\sigma^2[D|s]\theta_0\bar{\theta} + \tfrac{1}{2}\alpha\sigma^2[D|s]\bar{\theta}^2 - \tfrac{1}{2}\alpha\beta_s^2(\sigma^2 + \sigma_\epsilon^2)\theta_0^2 - \frac{\alpha^3\sigma^4[D|s]\sigma_z^2\pi^2(\theta_0 - \bar{\theta})^2}{2\left[1 + \alpha^2\sigma^2[D|s]\sigma_z^2\pi^2\right]}.$$

Noting that

$$-\alpha\sigma^2[D|s]\theta_0\bar{\theta} + \tfrac{1}{2}\alpha\sigma^2[D|s]\bar{\theta}^2 - \tfrac{1}{2}\alpha\beta_s^2(\sigma^2 + \sigma_\epsilon^2)\theta_0^2 = -\tfrac{1}{2}\alpha\sigma^2\theta_0^2 + \tfrac{1}{2}\alpha\sigma^2[D|s](\theta_0 - \bar{\theta})^2,$$

we can write F^s as

$$F^s = W_0 + \theta_0(\bar{D} - S_0) - \tfrac{1}{2}\alpha\sigma^2\theta_0^2 + \frac{\alpha\sigma^2[D|s](\theta_0 - \bar{\theta})^2}{2\left[1 + \alpha^2\sigma^2[D|s]\sigma_z^2\pi^2\right]}. \tag{A.23}$$

To compute U^d, we use Lemma A.1 and set

$$x \equiv \begin{bmatrix} s - \bar{D} \\ z \end{bmatrix}$$

$$\Sigma \equiv \begin{bmatrix} \sigma^2 + \sigma_\epsilon^2 & 0 \\ 0 & \sigma_z^2 \end{bmatrix}$$

$$A \equiv W_0 + \theta_0(\bar{D} - S_0) - \alpha\sigma^2[D|s]\theta_0\bar{\theta} + \tfrac{1}{2}\alpha\sigma^2[D|s]\bar{\theta}^2$$

$$B \equiv \begin{bmatrix} \beta_s\theta_0 \\ -\alpha\sigma^2[D|s]\left[\pi\theta_0 + (1-\pi)\bar{\theta}\right] \end{bmatrix}$$

$$C \equiv \begin{bmatrix} 0 & \beta_s \\ \beta_s & -\alpha\sigma^2[D|s](2\pi - \pi^2) \end{bmatrix}.$$

Using (4.2b) and the definition of β_s, we find

$$I + \alpha C\Sigma = \begin{bmatrix} 1 & \alpha\beta_s\sigma_z^2 \\ \alpha\sigma^2 & 1 - \alpha^2\sigma^2[D|s]\sigma_z^2(2\pi - \pi^2) \end{bmatrix},$$

$$|I + \alpha C\Sigma| = 1 + \alpha^2\sigma^2[D|s]\sigma_z^2(1-\pi)^2 - \alpha^2\sigma^2\sigma_z^2, \tag{A.24}$$

$$\Sigma(I + \alpha C\Sigma)^{-1} = \frac{1}{|I + \alpha C\Sigma|}\begin{bmatrix} \left[1 - \alpha^2\sigma^2[D|s]\sigma_z^2(2\pi - \pi^2)\right](\sigma^2 + \sigma_\epsilon^2) & -\alpha\sigma^2\sigma_z^2 \\ -\alpha\sigma^2\sigma_z^2 & \sigma_z^2 \end{bmatrix}. \tag{A.25}$$

Eqs. (A.1), (A.24) and (A.25) imply that

$$U^d = -\exp\left(-\alpha F^d\right)\frac{1}{\sqrt{1+\alpha^2\sigma^2[D|s]\sigma_z^2(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}}, \tag{A.26}$$

where

$$\begin{aligned}F^d =& W_0+\theta_0(\bar{D}-S_0)-\alpha\sigma^2[D|s]\theta_0\bar{\theta}+\tfrac{1}{2}\alpha\sigma^2[D|s]\bar{\theta}^2\\ &-\frac{\alpha}{2\left[1+\alpha^2\sigma^2[D|s]\sigma_z^2(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2\right]}\left\{\beta_s^2\left[1-\alpha^2\sigma^2[D|s]\sigma_z^2(2\pi-\pi^2)\right](\sigma^2+\sigma_\epsilon^2)\theta_0^2\right.\\ &\left.+2\alpha^2\beta_s\sigma^2[D|s]\sigma^2\sigma_z^2\left[\pi\theta_0+(1-\pi)\bar{\theta}\right]\theta_0+\alpha^2\sigma^4[D|s]\sigma_z^2\left[\pi\theta_0+(1-\pi)\bar{\theta}\right]^2\right\}.\end{aligned}$$

Noting that

$$-\alpha\sigma^2[D|s]\theta_0\bar{\theta}+\tfrac{1}{2}\alpha\sigma^2[D|s]\bar{\theta}^2=-\alpha\sigma^2\theta_0\bar{\theta}+\tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2+\alpha\beta_s\sigma^2\theta_0\bar{\theta}-\tfrac{1}{2}\alpha\beta_s\sigma^2\bar{\theta}^2,$$

we can write F^d as

$$\begin{aligned}F^d =& W_0+\theta_0(\bar{D}-S_0)-\alpha\sigma^2\theta_0\bar{\theta}+\tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2\\ &-\frac{\alpha\left[\beta_s\sigma^2(1-\alpha^2\sigma^2\sigma_z^2)(\theta_0-\bar{\theta})^2+\alpha^2\beta_s\sigma^4\sigma_z^2\theta_0^2+\alpha^2\sigma^2\sigma^2[D|s]\sigma_z^2\left[\pi\theta_0+(1-\pi)\bar{\theta}\right]^2\right]}{2\left[1+\alpha^2\sigma^2[D|s]\sigma_z^2(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2\right]}.\end{aligned} \tag{A.27}$$

Eqs. (A.22) and (A.26) take the form (A.2) and (A.5), with Δ_0 given by (A.15). Moreover, (A.23) and (A.27) imply that when $\theta_0=\bar{\theta}$, $(dF^s/d\theta_0, F^s, dF^d/d\theta_0, F^d)$ are given by (A.8)-(A.11), with (Δ_1,Δ_2) given by (A.16) and (A.17). Since the equations for $(U^s, U^d, dF^s/d\theta_0, F^s, dF^d/d\theta_0, F^d)$ take the same form as in Proposition 3.2, the same applies to S_0. ∎

B Asymmetric Information

Proof of Proposition 4.1: Same arguments as in the proof of Proposition A.1 imply that a liquidity demander chooses holdings θ_1^d to maximize (A.18), and a liquidity supplier chooses holdings θ_1^s to maximize

$$-\exp\left\{-\alpha\left[W_1+\theta_1^s\left(\mathrm{E}[D|S_1]-S_1\right)-\tfrac{1}{2}\alpha\sigma^2[D|S_1](\theta_1^s)^2\right]\right\}. \tag{B.1}$$

The solution to the optimization problems is straightforward and yields the demand functions (4.4a) and (4.4b). ∎

Proof of Proposition 4.2: Substituting $\mathrm{E}[D|s]$ from (4.2a) and $\mathrm{E}[D|S_1]$ from (4.3a), we can write (4.5) as

$$(1-\pi)\frac{\bar{D}+\frac{\beta_\xi}{b}(S_1-a)-S_1}{\alpha\sigma^2[D|S_1]}+\pi\left[\frac{\bar{D}+\beta_s(s-\bar{D})-S_1}{\alpha\sigma^2[D|s]}-z\right]=\bar{\theta}$$

$$\Leftrightarrow(1-\pi)\frac{\bar{D}+\frac{\beta_\xi}{b}(S_1-a)-S_1}{\alpha\sigma^2[D|S_1]}+\pi\left[\frac{\bar{D}+\frac{\beta_s}{b}(S_1-a)+\beta_s cz-S_1}{\alpha\sigma^2[D|s]}-z\right]=\bar{\theta}, \tag{B.2}$$

where the second step follows from (4.1). Eq. (B.2) can be viewed as an affine equation in the variables (S_1-a,z). Setting terms in S_1-a to zero, we find

$$(1-\pi)\frac{\frac{\beta_\xi}{b}-1}{\alpha\sigma^2[D|S_1]}+\pi\frac{\frac{\beta_s}{b}-1}{\alpha\sigma^2[D|s]}=0, \tag{B.3}$$

which yields (4.6b). Setting terms in z to zero, and using (4.2b), we find (4.6c). Setting constant terms to zero, we find

$$(1-\pi)\frac{\bar{D}-a}{\alpha\sigma^2[D|S_1]}+\pi\frac{\bar{D}-a}{\alpha\sigma^2[D|s]}=\bar{\theta}$$

$$\Leftrightarrow(1-\pi)\frac{\bar{D}-a}{\alpha\sigma^2[D|S_1]}+\pi\left[\bar{\theta}+\frac{\bar{D}-a-\alpha\sigma^2[D|s]\bar{\theta}}{\alpha\sigma^2[D|s]}\right]=\bar{\theta}. \tag{B.4}$$

Using (B.3), we can write (B.4) as

$$(1-\pi)\frac{\bar{D}-a}{\alpha\sigma^2[D|S_1]}+\pi\bar{\theta}-(1-\pi)\frac{\frac{\beta_\xi}{b}-1}{\frac{\beta_s}{b}-1}\frac{\bar{D}-a-\alpha\sigma^2[D|s]\bar{\theta}}{\alpha\sigma^2[D|S_1]}=\bar{\theta}$$

$$\Leftrightarrow\bar{D}-a=\alpha\frac{\sigma^2[D|S_1](\beta_s-b)+\sigma^2[D|s](b-\beta_\xi)}{\beta_s-\beta_\xi}\bar{\theta}. \tag{B.5}$$

Using (4.2b), (4.3b) and the definitions of (β_s,β_ξ), we can write (B.5) as (4.6a). ∎

Proof of Proposition 4.3: We first compute the expected utilities of liquidity suppliers and demanders in Period 1. Substituting W_1 from (3.7), θ_1^s from (4.4a), S_1 from (4.1), and $E(D|S_1)$ from (4.3a), we can write the expected utility (B.1) of a liquidity supplier as

$$-\exp\left\{-\alpha\left[W_0+\theta_0(a+b\xi-S_0)+\frac{\left[\bar{D}+\beta_\xi\xi-(a+b\xi)\right]^2}{2\alpha\sigma^2[D|S_1]}\right]\right\}. \tag{B.6}$$

Substituting $\mathrm{E}[D|s]-S_1$ from (4.4b), we can write the expected utility (A.18) of a liquidity demander as

$$-\exp\left\{-\alpha\left[W_1+z\left(\mathrm{E}[D|s]-\bar{D}\right)+\tfrac{1}{2}\alpha\sigma^2[D|s]\left[(\theta_1^d)^2-z^2\right]\right]\right\}$$

$$=-\exp\left\{-\alpha\left[W_1+\beta_s\xi z+\tfrac{1}{2}\alpha\sigma^2[D|s]\left[(\theta_1^d)^2+z^2\right]\right]\right\}, \tag{B.7}$$

where the second step follows from (4.2a), (4.2b), (4.6c) and the definition of ξ. Using (4.2a), (4.2b), (4.6c) and the definition of ξ, we can write (4.4b) as

$$\theta_1^d = \frac{\bar{D} + \beta_s \xi - S_1}{\alpha \sigma^2 [D|s]}. \tag{B.8}$$

Substituting W_1 from (3.7), θ_1^d from (B.8), and S_1 from (4.1), we can write (B.7) as

$$-\exp\left\{-\alpha\left[W_0 + \theta_0(a + b\xi - S_0) + \beta_s \xi z + \frac{\left[\bar{D} + \beta_s \xi - (a + b\xi)\right]^2}{2\alpha\sigma^2[D|s]} + \tfrac{1}{2}\alpha\sigma^2[D|s]z^2\right]\right\}. \tag{B.9}$$

We next compute the expectations of (B.6) and (B.9) over (s, z), i.e., the interim utilities U^s and U^d of liquidity suppliers and demanders in Period 1/2. To compute U^s, we use Lemma A.1 and set

$$\begin{aligned}
x &\equiv \xi \\
\Sigma &\equiv \sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2 \\
A &\equiv W_0 + \theta_0(a - S_0) + \frac{(\bar{D} - a)^2}{2\alpha\sigma^2[D|S_1]} \\
B &\equiv b\theta_0 - \frac{(\bar{D} - a)(b - \beta_\xi)}{\alpha\sigma^2[D|S_1]} \\
C &\equiv \frac{(b - \beta_\xi)^2}{\alpha\sigma^2[D|S_1]}.
\end{aligned}$$

Eq. (A.1) implies that

$$U^s = -\exp\left(-\alpha F^s\right) \frac{1}{\sqrt{1 + \frac{(b-\beta_\xi)^2}{\sigma^2[D|S_1]}(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)}}, \tag{B.10}$$

where

$$\begin{aligned}
F^s &= W_0 + \theta_0(a - S_0) + \frac{(\bar{D} - a)^2}{2\alpha\sigma^2[D|S_1]} - \frac{\alpha\left[b\theta_0 - \frac{(\bar{D}-a)(b-\beta_\xi)}{\alpha\sigma^2[D|S_1]}\right]^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)}{2\left[1 + \frac{(b-\beta_\xi)^2}{\sigma^2[D|S_1]}(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)\right]} \\
&= \theta_0(\bar{D} - S_0) - \frac{\alpha b^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)\theta_0^2 + 2(\bar{D} - a)\left[1 - \frac{\beta_\xi(b-\beta_\xi)}{\sigma^2[D|S_1]}(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)\right]\theta_0 - \frac{(\bar{D}-a)^2}{\alpha\sigma^2[D|S_1]}}{2\left[1 + \frac{(b-\beta_\xi)^2}{\sigma^2[D|S_1]}(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)\right]}.
\end{aligned} \tag{B.11}$$

Substituting $\bar{D} - a$ from (4.6a) into (B.11), and using (4.3b) and the definition of β_ξ, we find

$$F^s = W_0 + \theta_0(\bar{D} - S_0) - \frac{\alpha\left[b^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)\theta_0^2 + \frac{(1-b)^2\sigma^4}{\sigma^2[D|S_1]}(2\theta_0 - \bar{\theta})\bar{\theta}\right]}{2\left[1 + \frac{(b-\beta_\xi)^2}{\sigma^2[D|S_1]}(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)\right]}. \tag{B.12}$$

Eq. (4.3b) and the definition of β_ξ imply that for all b,

$$\frac{(1-b)^2\sigma^4}{\sigma^2[D|S_1]} + b^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2) = \sigma^2 + \frac{(b-\beta_\xi)^2\sigma^2}{\sigma^2[D|S_1]}(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2). \tag{B.13}$$

Using (B.13), we can write (B.12) as

$$F^s = W_0 + \theta_0(\bar{D} - S_0) - \tfrac{1}{2}\alpha\sigma^2\theta_0^2 + \frac{\alpha\frac{(1-b)^2\sigma^4}{\sigma^2[D|S_1]}(\theta_0 - \bar{\theta})^2}{2\left[1 + \frac{(b-\beta_\xi)^2}{\sigma^2[D|S_1]}(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)\right]}. \tag{B.14}$$

To compute U^d, we use Lemma A.1 and set

$$x \equiv \begin{bmatrix} \xi \\ z \end{bmatrix}$$

$$\Sigma \equiv \begin{bmatrix} \sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2 & -c\sigma_z^2 \\ -c\sigma_z^2 & \sigma_z^2 \end{bmatrix}$$

$$A \equiv W_0 + \theta_0(a - S_0) + \frac{(\bar{D} - a)^2}{2\alpha\sigma^2[D|s]}$$

$$B \equiv \begin{bmatrix} b\theta_0 + \frac{(\bar{D}-a)(\beta_s - b)}{\alpha\sigma^2[D|s]} \\ 0 \end{bmatrix}$$

$$C \equiv \begin{bmatrix} \frac{(\beta_s - b)^2}{\alpha\sigma^2[D|s]} & \beta_s \\ \beta_s & \alpha\sigma^2[D|s] \end{bmatrix}.$$

Using (4.2b), (4.6c) and the definition of β_s, we find

$$I + \alpha C\Sigma = \begin{bmatrix} 1 + \frac{(\beta_s - b)^2}{\sigma^2[D|s]}(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2) - \alpha\beta_s c\sigma_z^2 & -\frac{(\beta_s - b)^2}{\sigma^2[D|s]}c\sigma_z^2 + \alpha\beta_s\sigma_z^2 \\ \alpha\sigma^2 & 1 \end{bmatrix},$$

$$|I + \alpha C\Sigma| = 1 + \frac{(\beta_s - b)^2}{\sigma^2[D|s]}(\sigma^2 + \sigma_\epsilon^2)(1 + \alpha^2\sigma_\epsilon^2\sigma_z^2) - \alpha^2\sigma^2\sigma_z^2, \tag{B.15}$$

$$\left[\Sigma(I + \alpha C\Sigma)^{-1}\right]_{(1,1)} = \frac{(\sigma^2 + \sigma_\epsilon^2)(1 + \alpha^2\sigma_\epsilon^2\sigma_z^2)}{|I + \alpha C\Sigma|}, \tag{B.16}$$

where the subscript (1,1) refers to the term in the first row and column of a matrix. Eqs. (A.1), (B.15), and (B.16) imply that

$$U^d = -\exp\left(-\alpha F^d\right)\frac{1}{\sqrt{1+\frac{(\beta_s-b)^2}{\sigma^2[D|s]}(\sigma^2+\sigma_\epsilon^2)(1+\alpha^2\sigma_\epsilon^2\sigma_z^2)-\alpha^2\sigma^2\sigma_z^2}}, \tag{B.17}$$

where

$$F^d = W_0+\theta_0(a-S_0)+\frac{(\bar{D}-a)^2}{2\alpha\sigma^2[D|s]}-\frac{\alpha\left[b\theta_0+\frac{(\bar{D}-a)(\beta_s-b)}{\alpha\sigma^2[D|s]}\right]^2(\sigma^2+\sigma_\epsilon^2)(1+\alpha^2\sigma_\epsilon^2\sigma_z^2)}{2\left[1+\frac{(\beta_s-b)^2}{\sigma^2[D|s]}(\sigma^2+\sigma_\epsilon^2)(1+\alpha^2\sigma_\epsilon^2\sigma_z^2)-\alpha^2\sigma^2\sigma_z^2\right]}. \tag{B.18}$$

Substituting $\bar{D}-a$ from (4.6a) into (B.18), we find

$$\begin{aligned}F^d =& W_0+\theta_0(\bar{D}-S_0)-\alpha\sigma^2\theta_0\bar{\theta}+\tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2\\ &+\alpha\left\{b\sigma^2\theta_0\bar{\theta}-\tfrac{1}{2}\sigma^2\bar{\theta}^2+\frac{(1-b)^2\sigma^4}{2\sigma^2[D|s]}\bar{\theta}^2-\frac{\left[b\theta_0+\frac{(1-b)(\beta_s-b)\sigma^2}{\sigma^2[D|s]}\right]^2(\sigma^2+\sigma_\epsilon^2)(1+\alpha^2\sigma_\epsilon^2\sigma_z^2)}{2\left[1+\frac{(\beta_s-b)^2}{\sigma^2[D|s]}(\sigma^2+\sigma_\epsilon^2)(1+\alpha^2\sigma_\epsilon^2\sigma_z^2)-\alpha^2\sigma^2\sigma_z^2\right]}\right\}.\end{aligned} \tag{B.19}$$

Using (4.2b) and the definition of β_s, we can write (B.19) as

$$\begin{aligned}F^d =& W_0+\theta_0(\bar{D}-S_0)-\alpha\sigma^2\theta_0\bar{\theta}+\tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2\\ &-\frac{\alpha\left\{b^2(\sigma^2+\sigma_\epsilon^2)(1+\alpha^2\sigma_\epsilon^2\sigma_z^2)\theta_0^2+2b(\sigma^2+\sigma_\epsilon^2)\left[\alpha^2\sigma^2\sigma_z^2-b(1+\alpha^2\sigma_\epsilon^2\sigma_z^2)\right]\theta_0\bar{\theta}+X\bar{\theta}^2\right\}}{2\left[1+\frac{(\beta_s-b)^2}{\sigma^2[D|s]}(\sigma^2+\sigma_\epsilon^2)(1+\alpha^2\sigma_\epsilon^2\sigma_z^2)-\alpha^2\sigma^2\sigma_z^2\right]},\end{aligned} \tag{B.20}$$

where

$$X\equiv\left[\sigma^2-\frac{(1-b)^2\sigma^4}{\sigma^2[D|s]}\right](1-\alpha^2\sigma^2\sigma_z^2)+\frac{(\beta_s-b)^2\sigma^2}{\sigma^2[D|s]}(\sigma^2+\sigma_\epsilon^2)(1+\alpha^2\sigma_\epsilon^2\sigma_z^2).$$

Eq. (4.2b) and the definition of β_s imply that for all b,

$$\frac{(1-b)^2\sigma^4}{\sigma^2[D|s]}+b^2(\sigma^2+\sigma_\epsilon^2)=\sigma^2+\frac{(\beta_s-b)^2\sigma^2}{\sigma^2[D|s]}(\sigma^2+\sigma_\epsilon^2). \tag{B.21}$$

Using (B.21) to eliminate the term in $\sigma^2-\frac{(1-b)^2\sigma^4}{\sigma^2[D|s]}$ in the definition of X, and substituting X into (B.20), we find

$$\begin{aligned}F^d =& W_0+\theta_0(\bar{D}-S_0)-\alpha\sigma^2\theta_0\bar{\theta}+\tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2\\ &-\frac{\alpha\left\{b^2(\sigma^2+\sigma_\epsilon^2)(1+\alpha^2\sigma_\epsilon^2\sigma_z^2)(\theta_0-\bar{\theta})^2+\alpha^2(\sigma^2+\sigma_\epsilon^2)^2\sigma_z^2\left[\frac{2b\sigma^2\theta_0\bar{\theta}}{\sigma^2+\sigma_\epsilon^2}+\left[\frac{(\beta_s-b)^2\sigma^2}{\sigma^2[D|s]}-b^2\right]\bar{\theta}^2\right]\right\}}{2\left[1+\frac{(\beta_s-b)^2}{\sigma^2[D|s]}(\sigma^2+\sigma_\epsilon^2)(1+\alpha^2\sigma_\epsilon^2\sigma_z^2)-\alpha^2\sigma^2\sigma_z^2\right]}.\end{aligned} \tag{B.22}$$

Eqs. (B.10) and (B.17) take the form (A.2) and (A.5), with Δ_0 given by (4.7a). In the case of (B.10), this follows directly from (4.7a). In the case of (B.17), this is because

$$\begin{aligned}
&\frac{(\beta_s - b)^2}{\sigma^2[D|s]}(\sigma^2 + \sigma_\epsilon^2)(1 + \alpha^2\sigma_\epsilon^2\sigma_z^2) \\
=&\frac{(b - \beta_\xi)^2\sigma^2[D|s](1-\pi)^2}{\sigma^4[D|S_1]\pi^2}(\sigma^2 + \sigma_\epsilon^2)(1 + \alpha^2\sigma_\epsilon^2\sigma_z^2) \\
=&\frac{(b - \beta_\xi)^2\sigma^2[D|s](\sigma^2 + \sigma_\epsilon^2)(\sigma_\epsilon^2 + c^2\sigma_z^2)(1-\pi)^2}{\sigma^4[D|S_1]\sigma_\epsilon^2\pi^2} \\
=&\frac{(b - \beta_\xi)^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)(1-\pi)^2}{\sigma^2[D|S_1]\pi^2} = \Delta_0(1-\pi)^2,
\end{aligned} \tag{B.23}$$

where the first step follows from (B.3), the second from (4.6c), the third from (4.2b), (4.3b) and the definitions of (β_s, β_ξ), and the fourth from (4.7a). Eqs. (B.14), (B.22) and (B.23) imply that when $\theta_0 = \bar{\theta}$, $(dF^s/d\theta_0, F^s, dF^d/d\theta_0, F^d)$ are given by (A.8)-(A.11), with (Δ_1, Δ_2) given by (4.7b) and (4.7c). Since the equations for $(U^s, U^d, dF^s/d\theta_0, F^s, dF^d/d\theta_0, F^d)$ take the same form as in Proposition 3.2, the same applies to S_0. ∎

Proof of Proposition 4.4: The price change between Periods 0 and 1 is $S_1 - S_0$. The signed volume of liquidity demanders is

$$\begin{aligned}
\pi(\theta_1^d - \bar{\theta}) &= -(1-\pi)(\theta_1^s - \bar{\theta}) \\
&= -(1-\pi)\left(\frac{\mathrm{E}[D|S_1] - S_1}{\alpha\sigma^2[D|S_1]} - \bar{\theta}\right) \\
&= -(1-\pi)\left[\frac{\bar{D} + \frac{\beta_\xi}{b}(S_1 - a) - S_1}{\alpha\sigma^2[D|S_1]} - \bar{\theta}\right],
\end{aligned}$$

where the first step follows from (3.5), the second from (4.4a), and the third from (4.3a). Substituting into (3.13), and noting that variation in the numerator and denominator arises because of S_1, we find (4.8).

Illiquidity under no information is given by (3.16), under full information by

$$\lambda = \frac{\alpha\sigma^2[D|s]}{1-\pi}, \tag{B.24}$$

and under asymmetric information by (4.8). We can write (4.8) as

$$\begin{aligned}\lambda &= \frac{\alpha\left(\pi\beta_s\sigma^2[D|S_1] + (1-\pi)\beta_\xi\sigma^2[D|s]\right)}{(\beta_s-\beta_\xi)\pi(1-\pi)} \\ &= \frac{\alpha\sigma^2(\sigma_\epsilon^2 + c^2\sigma_z^2\pi)}{c^2\sigma_z^2\pi(1-\pi)},\end{aligned} \tag{B.25}$$

where the first step follows from (4.6b), and the second from (4.2b), (4.3b), and the definitions of (β_s, β_ξ). Eqs. (3.16), (B.24) and (B.25) imply that illiquidity is highest under asymmetric information and lowest under full information. Moreover, (4.6c) and (B.25) imply that illiquidity under asymmetric information increases when σ_ϵ^2 decreases. ∎

Proof of Proposition 4.5: Eqs. (3.18) and (4.1) imply that

$$\begin{aligned}\gamma &= -\text{Cov}\left[D - a - b(s - \bar{D} - cz), a + b(s - \bar{D} - cz) - S_0\right] \\ &= -\text{Cov}\left[(1-b)(D-\bar{D}) - b\epsilon + bcz, b(D-\bar{D}) + b\epsilon - bcz\right] \\ &= -b\left[\sigma^2 - b(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)\right].\end{aligned} \tag{B.26}$$

Using the definition of β_ξ, we can write (B.26) as (4.9).

Price reversal under no information is given by (3.19), under full information by

$$\gamma = \alpha^2\sigma^4[D|s]\sigma_z^2\pi^2, \tag{B.27}$$

and under asymmetric information by (4.9). Substituting b from (4.6b), $\sigma^2[D|s]$ from (4.2b), $\sigma^2[D|S_1]$ from (4.3b), and using the definitions of (β_s, β_ξ), we can write (4.9) as

$$\gamma = \frac{\sigma^4(\sigma_\epsilon^2 + c^2\sigma_z^2)(\sigma_\epsilon^2 + c^2\sigma_z^2\pi)c^2\sigma_z^2\pi}{\left[\sigma_\epsilon^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2) + \sigma^2c^2\sigma_z^2\pi\right]^2}. \tag{B.28}$$

Price reversal under full information is lower than under no information because $\sigma^2 > \sigma^2[D|s]$, and lower than under asymmetric information if

$$\begin{aligned}&\frac{\sigma^4(\sigma_\epsilon^2 + c^2\sigma_z^2)(\sigma_\epsilon^2 + c^2\sigma_z^2\pi)c^2\sigma_z^2\pi}{\left[\sigma_\epsilon^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2) + \sigma^2c^2\sigma_z^2\pi\right]^2} > \frac{\alpha^2\sigma^4\sigma_\epsilon^4\sigma_z^2\pi^2}{(\sigma^2+\sigma_\epsilon^2)^2} \\ \Leftrightarrow 1 &> \frac{\left[\sigma_\epsilon^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2) + \sigma^2c^2\sigma_z^2\pi\right]^2\pi}{(\sigma^2+\sigma_\epsilon^2)^2(\sigma_\epsilon^2 + c^2\sigma_z^2)(\sigma_\epsilon^2 + c^2\sigma_z^2\pi)},\end{aligned} \tag{B.29}$$

where the second step follows from (4.6c). Eq. (B.29) holds because the right-hand side is increasing in π and equal to one for $\pi = 1$. Since for $\pi = 1$, price reversals under asymmetric and full

information coincide, they are lower than under no information. For $\pi \approx 0$, price reversal is of order π^2 under no information and of order π under asymmetric information. ∎

Lemma B.1 compares the parameters (Δ_0, Δ_2) under symmetric and asymmetric information. For expositional convenience, we use the following superscripts for $\{\Delta_j\}_{j=0,1,2}$ and M: ni under no information, fi under full information, and ai under asymmetric information.

Lemma B.1 $\Delta_0^{ni} > \Delta_0^{fi} > \Delta_0^{ai}$ *and* $\Delta_2^{ni} < \Delta_2^{fi} < \Delta_2^{ai}$.

Proof: Substituting b from (4.6b), $\sigma^2[D|s]$ from (4.2b), $\sigma^2[D|S_1]$ from (4.3b), and using the definitions of (β_s, β_ξ), we can write (4.7a) as

$$\Delta_0^{ai} = \frac{\sigma^2 c^4 \sigma_z^4 (\sigma_\epsilon^2 + c^2\sigma_z^2)}{\left[\sigma_\epsilon^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2) + \sigma^2 c^2 \sigma_z^2 \pi\right]^2}. \tag{B.30}$$

Eqs. (3.12a) and (A.15) imply that $\Delta_0^{ni} > \Delta_0^{fi}$. Eqs. (A.15) and (B.30) imply that $\Delta_0^{fi} > \Delta_0^{ai}$ if

$$\begin{aligned} &\alpha^2\sigma^2[D|s]\sigma_z^2 > \frac{\sigma^2 c^4 \sigma_z^4 (\sigma_\epsilon^2 + c^2\sigma_z^2)}{\left[\sigma_\epsilon^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2) + \sigma^2 c^2 \sigma_z^2 \pi\right]^2} \\ \Leftrightarrow &1 > \frac{\sigma_\epsilon^2 c^2 \sigma_z^2 (\sigma^2 + \sigma_\epsilon^2)(\sigma_\epsilon^2 + c^2\sigma_z^2)}{\left[\sigma_\epsilon^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2) + \sigma^2 c^2 \sigma_z^2 \pi\right]^2}, \end{aligned} \tag{B.31}$$

where the second step follows from (4.2b) and (4.6c). Eq. (B.31) holds for all $\pi \in [0,1]$ if it holds for $\pi = 0$, i.e.,

$$\begin{aligned} &\sigma_\epsilon^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)^2 > c^2\sigma_z^2(\sigma^2 + \sigma_\epsilon^2)(\sigma_\epsilon^2 + c^2\sigma_z^2) \\ \Leftrightarrow &\sigma_\epsilon^2\sigma^4 + 2\sigma_\epsilon^2\sigma^2(\sigma_\epsilon^2 + c^2\sigma_z^2) + \sigma_\epsilon^2(\sigma_\epsilon^2 + c^2\sigma_z^2)^2 - c^2\sigma_z^2(\sigma^2 + \sigma_\epsilon^2)(\sigma_\epsilon^2 + c^2\sigma_z^2) > 0 \\ \Leftrightarrow &\sigma_\epsilon^2\sigma^4 + (\sigma_\epsilon^2 + c^2\sigma_z^2)\left[2\sigma_\epsilon^2\sigma^2 + \sigma_\epsilon^4(1 - \alpha^2\sigma^2\sigma_z^2)\right] > 0, \end{aligned} \tag{B.32}$$

where the last step follows from (4.6c). Eq. (B.32) holds because of (2.2).

Eq. (3.12c) implies that

$$\left[1 + \Delta_0^{ni}(1-\pi)^2 - \alpha^2\sigma^2\sigma_z^2\right]\Delta_2^{ni} = \alpha^3\sigma^4\sigma_z^2. \tag{B.33}$$

Eq. (A.17) implies that

$$\left[1 + \Delta_0^{fi}(1-\pi)^2 - \alpha^2\sigma^2\sigma_z^2\right]\Delta_2^{fi} = \alpha^3\sigma^4\sigma_z^2. \tag{B.34}$$

Eq. (4.7c) implies that

$$\left[1 + \Delta_0^{ai}(1-\pi)^2 - \alpha^2\sigma^2\sigma_z^2\right]\Delta_2^{ai} = \alpha^3\sigma^4\sigma_z^2\left[1 + \frac{(\beta_s - b)^2}{\sigma^2[D|s]}(\sigma^2 + \sigma_\epsilon^2)\right]. \tag{B.35}$$

Since $\Delta_0^{ni} > \Delta_0^{fi}$, (B.33) and (B.34) imply that $\Delta_2^{ni} < \Delta_2^{fi}$. Since $\Delta_0^{fi} > \Delta_0^{ai}$, (B.34) and (B.35) imply that $\Delta_2^{fi} < \Delta_2^{ai}$. ∎

Proof of Proposition 4.6: To show the ranking for S_0, we must show the reverse ranking for the illiquidity discount in (3.10), i.e.,

$$\frac{\pi M^{ni}}{1-\pi+\pi M^{ni}}\Delta_1^{ni} < \frac{\pi M^{fi}}{1-\pi+\pi M^{fi}}\Delta_1^{fi} < \frac{\pi M^{ai}}{1-\pi+\pi M^{ai}}\Delta_1^{ai}. \tag{B.36}$$

Since $\Delta_2^{ni} < \Delta_2^{fi} < \Delta_2^{ai}$, (B.36) holds if it does so when $\{\Delta_2^j\}_{j=ni,fi,ai}$ are replaced by zero. Using (3.11), we can write the latter condition as

$$\left(\frac{1-\pi}{\pi\hat{M}^{ni}}+1\right)\frac{1}{\Delta_1^{ni}} > \left(\frac{1-\pi}{\pi\hat{M}^{fi}}+1\right)\frac{1}{\Delta_1^{fi}} > \left(\frac{1-\pi}{\pi\hat{M}^{ai}}+1\right)\frac{1}{\Delta_1^{ai}}. \tag{B.37}$$

where

$$\hat{M}^j \equiv \sqrt{\frac{1+\Delta_0^j\pi^2}{1+\Delta_0^j(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}},$$

for $j = ni, fi, ai$. Eq. (3.12b) implies that

$$\left[1+\Delta_0^{ni}(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2\right]\Delta_1^{ni} = \alpha^3\sigma^4\sigma_z^2\pi. \tag{B.38}$$

Eq. (A.16) implies that

$$\left[1+\Delta_0^{fi}(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2\right]\Delta_1^{fi} = \alpha^3\sigma^4\sigma_z^2\left[1-\frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}(1-\pi)\right]. \tag{B.39}$$

Eq. (4.7b) implies that

$$\begin{aligned}\left[1+\Delta_0^{ai}(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2\right]\Delta_1^{ai} &= \alpha^3 b\sigma^2(\sigma^2+\sigma_\epsilon^2)\sigma_z^2 \\ &= \frac{\alpha^3\sigma^4\sigma_z^2(\sigma^2+\sigma_\epsilon^2)(\sigma_\epsilon^2+c^2\sigma_z^2\pi)}{\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi},\end{aligned} \tag{B.40}$$

where the second step follows from (4.2b), (4.3b), (4.6b) and the definitions of (β_s, β_ξ). Eqs. (B.38)-(B.40) and $\Delta_0^{ni} > \Delta_0^{fi} > \Delta_0^{ai}$ imply that a sufficient condition for (B.37) is

$$\left(\frac{1-\pi}{\pi\hat{M}^{ni}}+1\right)\frac{1}{\pi} > \left(\frac{1-\pi}{\pi\hat{M}^{fi}}+1\right)\frac{1}{1-\frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}(1-\pi)} > \left(\frac{1-\pi}{\pi\hat{M}^{ai}}+1\right)\frac{\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi}{(\sigma^2+\sigma_\epsilon^2)(\sigma_\epsilon^2+c^2\sigma_z^2\pi)}. \tag{B.41}$$

We can write the first inequality in (B.41) as

$$\begin{aligned}
&\frac{1-\frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}(1-\pi)}{\pi}\left(\frac{1-\pi}{\pi\hat{M}^{ni}}+1\right)>\frac{1-\pi}{\pi\hat{M}^{fi}}+1\\
\Leftrightarrow&\left(1+\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\frac{1-\pi}{\pi}\right)\left(\frac{1-\pi}{\pi\hat{M}^{ni}}+1\right)>\frac{1-\pi}{\pi\hat{M}^{fi}}+1.
\end{aligned}\tag{B.42}$$

A sufficient condition for (B.42) is

$$\begin{aligned}
&\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}+\frac{1}{\hat{M}^{ni}}>\frac{1}{\hat{M}^{fi}}\\
\Leftrightarrow&\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}>\sqrt{\frac{1+\Delta_0^{fi}(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}{1+\Delta_0^{fi}\pi^2}}-\sqrt{\frac{1+\Delta_0^{ni}(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}{1+\Delta_0^{ni}\pi^2}}\\
\Leftrightarrow&\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}>\frac{\frac{1+\Delta_0^{fi}(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}{1+\Delta_0^{fi}\pi^2}-\frac{1+\Delta_0^{ni}(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}{1+\Delta_0^{ni}\pi^2}}{\sqrt{\frac{1+\Delta_0^{fi}(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}{1+\Delta_0^{fi}\pi^2}}+\sqrt{\frac{1+\Delta_0^{ni}(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}{1+\Delta_0^{ni}\pi^2}}}\\
\Leftrightarrow&\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}>\frac{(\Delta_0^{ni}-\Delta_0^{fi})\left[(1-\alpha^2\sigma^2\sigma_z^2)\pi^2-(1-\pi)^2\right]}{\left[\sqrt{\frac{1+\Delta_0^{fi}(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}{1+\Delta_0^{fi}\pi^2}}+\sqrt{\frac{1+\Delta_0^{ni}(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}{1+\Delta_0^{ni}\pi^2}}\right]\left(1+\Delta_0^{fi}\pi^2\right)\left(1+\Delta_0^{ni}\pi^2\right)}.
\end{aligned}\tag{B.43}$$

Eqs. (3.12a), (A.15) and the non-negativity of $(\Delta_0^{ni},\Delta_0^{fi})$ imply that a sufficient condition for (B.43) is

$$\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}>\frac{\alpha^2\sigma^2\sigma_z^2\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}(1-\alpha^2\sigma^2\sigma_z^2)\pi^2}{2\sqrt{1-\alpha^2\sigma^2\sigma_z^2}}.\tag{B.44}$$

Eq. (B.44) holds because of (2.2).

We can write the second inequality in (B.41) as

$$\begin{aligned}
&\frac{(\sigma^2+\sigma_\epsilon^2)(\sigma_\epsilon^2+c^2\sigma_z^2\pi)}{[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi]\left[1-\frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}(1-\pi)\right]}\left(\frac{1-\pi}{\pi\hat{M}^{fi}}+1\right)>\frac{1-\pi}{\pi\hat{M}^{ai}}+1\\
\Leftrightarrow&\left\{1+\frac{\sigma_\epsilon^2\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2)-\sigma^2c^2\sigma_z^2(1-\pi)\right](1-\pi)}{[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\pi\sigma^2c^2\sigma_z^2](\sigma^2+\pi\sigma_\epsilon^2)}\right\}\left(\frac{1-\pi}{\pi\hat{M}^{fi}}+1\right)>\frac{1-\pi}{\pi\hat{M}^{ai}}+1.
\end{aligned}\tag{B.45}$$

A sufficient condition for (B.45) is

$$\frac{\sigma_\epsilon^2\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2)-\sigma^2c^2\sigma_z^2(1-\pi)\right]\pi}{\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi\right](\sigma^2+\sigma_\epsilon^2\pi)}+\frac{1}{\hat{M}^{fi}}>\frac{1}{\hat{M}^{ai}}$$

$$\Leftrightarrow\frac{\sigma_\epsilon^2\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2)-\sigma^2c^2\sigma_z^2(1-\pi)\right]\pi}{\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi\right](\sigma^2+\sigma_\epsilon^2\pi)}$$

$$>\frac{(\Delta_0^{fi}-\Delta_0^{ai})\left[(1-\alpha^2\sigma^2\sigma_z^2)\pi^2-(1-\pi)^2\right]}{\left[\sqrt{\frac{1+\Delta_0^{ai}(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}{1+\Delta_0^{ai}\pi^2}}+\sqrt{\frac{1+\Delta_0^{fi}(1-\pi)^2-\alpha^2\sigma^2\sigma_z^2}{1+\Delta_0^{fi}\pi^2}}\right]\left(1+\Delta_0^{ai}\pi^2\right)\left(1+\Delta_0^{fi}\pi^2\right)}, \tag{B.46}$$

where the intermediate steps are as for (B.43). Eqs. (4.6c), (4.7a), (A.15) and the non-negativity of $(\Delta_0^{fi},\Delta_0^{ai})$ imply that a sufficient condition for (B.46) is

$$\frac{\sigma_\epsilon^2\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2)-\sigma^2c^2\sigma_z^2(1-\pi)\right]\pi}{\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi\right](\sigma^2+\sigma_\epsilon^2\pi)}$$

$$>\frac{\alpha^2\sigma^2\sigma_z^2(1-\alpha^2\sigma^2\sigma_z^2)\pi^2}{2\sqrt{1-\alpha^2\sigma^2\sigma_z^2}}\left[\frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}-\frac{\sigma_\epsilon^4c^2\sigma_z^2(\sigma_\epsilon^2+c^2\sigma_z^2)}{\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi\right]^2}\right]. \tag{B.47}$$

A sufficient condition for (B.47) is

$$\frac{2\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2)-\sigma^2c^2\sigma_z^2(1-\pi)\right]}{\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi\right](\sigma^2+\sigma_\epsilon^2)}>\left[\frac{1}{\sigma^2+\sigma_\epsilon^2}-\frac{\sigma_\epsilon^2c^2\sigma_z^2(\sigma_\epsilon^2+c^2\sigma_z^2)}{\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi\right]^2}\right]\pi, \tag{B.48}$$

which is derived from (B.47) by using (2.2) and replacing the term $\sigma^2+\sigma_\epsilon^2\pi$ in the denominator of the left-hand side by $\sigma^2+\sigma_\epsilon^2$. Multiplying by the smallest common denominator, we can write (B.48) as

$$\begin{aligned}&\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2)\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi\right](2-\pi)\\&>\sigma^2c^2\sigma_z^2\left\{2\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi\right](1-\pi)\right.\\&\left.+(\sigma_\epsilon^2+\sigma^2\pi)(\sigma_\epsilon^2+c^2\sigma_z^2\pi)\pi-\sigma_\epsilon^2(\sigma_\epsilon^2+c^2\sigma_z^2)\pi(1-\pi)\right\}.\end{aligned} \tag{B.49}$$

A sufficient condition for (B.49) is

$$\begin{aligned}&\sigma_\epsilon^4\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi\right](2-\pi)\\&>\sigma^2c^2\sigma_z^2\left\{2\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi\right](1-\pi)+(\sigma_\epsilon^2+\sigma^2\pi)(\sigma_\epsilon^2+c^2\sigma_z^2\pi)\pi\right\}.\end{aligned} \tag{B.50}$$

Eqs. (2.2) and (4.6c) imply that a sufficient condition for (B.50) is

$$\begin{aligned}&\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi\right](2-\pi)\\&>\left\{2\left[\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2)+\sigma^2c^2\sigma_z^2\pi\right](1-\pi)+(\sigma_\epsilon^2+\sigma^2\pi)(\sigma_\epsilon^2+c^2\sigma_z^2\pi)\pi\right\},\end{aligned}$$

which obviously holds. ∎

Proof of Proposition 4.7: Eq. (B.25) implies that λ is decreasing in σ_z^2. Eq. (B.28) implies that γ is increasing in σ_z^2. Eq. (B.30) implies that Δ_0 is increasing in σ_z^2, and

$$1 + \Delta_0(1-\pi)^2 - \alpha^2\sigma^2\sigma_z^2$$

is decreasing in σ_z^2. Since the left-hand side of (B.40) is increasing in σ_z^2, so is Δ_1. Eqs. (B.23) and (B.30) imply that

$$\frac{(\beta_s - b)^2}{\sigma^2[D|s]}(\sigma^2 + \sigma_\epsilon^2) = \frac{\sigma^2\sigma_\epsilon^2 c^4 \sigma_z^4 (1-\pi)^2}{[\sigma_\epsilon^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2) + \sigma^2 c^2 \sigma_z^2 \pi]^2}. \tag{B.51}$$

Since the left-hand side of (B.51) is increasing in σ_z^2, so are the left-hand side of (B.35), Δ_2 and M. Therefore, (3.10) implies that S_0 is decreasing in σ_z^2. ∎

C Non-Competitive Behavior

Proof of Proposition 5.1: Substituting W_1 from (3.7), and using normality, we can write (5.2) as

$$-\mathrm{E}\exp\left\{-\alpha\left[W_0 + \theta_0\left(S_1(\theta_1^d) - S_0\right) + \theta_1^d\left(\mathrm{E}[D|s] - S_1(\theta_1^d)\right) + z\left(\mathrm{E}[D|s] - \bar{D}\right) - \tfrac{1}{2}\alpha\sigma^2[D|s](\theta_1^d + z)^2\right]\right\}. \tag{C.1}$$

Since in equilibrium $\theta_0 = \bar{\theta}$, the first-order condition with respect to θ_1^d is

$$\mathrm{E}[D|s] - S_1(\theta_1^d) - \hat{\lambda}(\theta_1^d - \bar{\theta}) - \alpha\sigma^2[D|s](\theta_1^d + z) = 0. \tag{C.2}$$

Eq. (5.3) follows by rearranging (C.2). ∎

Proof of Proposition 5.2: The proof is similar to that of Proposition 4.2. Eq. (B.2) is replaced by

$$(1-\pi)\frac{\bar{D} + \frac{\beta_\xi}{b}(S_1 - a) - S_1}{\alpha\sigma^2[D|S_1]} + \pi\frac{\bar{D} + \frac{\beta_s}{b}(S_1 - a) + \beta_s c z - S_1 - \alpha\sigma^2[D|s]z + \hat{\lambda}\bar{\theta}}{\alpha\sigma^2[D|s] + \hat{\lambda}} = \bar{\theta}. \tag{C.3}$$

Eq. (C.3) can be viewed as an affine equation in the variables $(S_1 - a, z)$. Setting terms in $S_1 - a$ to zero, we find

$$(1-\pi)\frac{\frac{\beta_\xi}{b} - 1}{\alpha\sigma^2[D|S_1]} + \pi\frac{\frac{\beta_s}{b} - 1}{\alpha\sigma^2[D|s] + \hat{\lambda}} = 0. \tag{C.4}$$

Setting terms in z to zero, and using (4.2b), we find (4.6c). Setting constant terms to zero, we find

$$\begin{aligned}
&(1-\pi)\frac{\bar{D}-a}{\alpha\sigma^2[D|S_1]}+\pi\frac{\bar{D}-a+\hat{\lambda}\bar{\theta}}{\alpha\sigma^2[D|s]+\hat{\lambda}}=\bar{\theta}\\
\Leftrightarrow&(1-\pi)\frac{\bar{D}-a}{\alpha\sigma^2[D|S_1]}+\pi\left[\bar{\theta}+\frac{\bar{D}-a-\alpha\sigma^2[D|s]\bar{\theta}}{\alpha\sigma^2[D|s]+\hat{\lambda}}\right]=\bar{\theta}. \qquad \text{(C.5)}
\end{aligned}$$

Using (C.4) and the definition of $\hat{\lambda}$, we find (5.4). Using (C.4) and (C.5), and following the same argument as in the proof of Proposition 4.2, we find (4.6a).

A linear equilibrium exists if the liquidity demanders' second-order condition is met. Eq. (C.1) implies that the second-order condition is

$$\begin{aligned}
&\alpha\sigma^2[D|s]+2\hat{\lambda}>0\\
\Leftrightarrow&\sigma^2[D|s]+\frac{2\left[\pi\beta_s\sigma^2[D|S_1]+(1-\pi)\beta_\xi\sigma^2[D|s]\right]}{(\beta_s-2\beta_\xi)(1-\pi)}>0\\
\Leftrightarrow&\frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}+\frac{2\left(\sigma_\epsilon^2+\pi c^2\sigma_z^2\right)}{\left(c^2\sigma_z^2-\sigma^2-\sigma_\epsilon^2\right)(1-\pi)}>0, \qquad \text{(C.6)}
\end{aligned}$$

where the second step follows from (5.4) and the definition of $\hat{\lambda}$, and the third from (4.2b), (4.3b), and the definitions of (β_s,β_ξ). Eq. (C.6) is satisfied if and only if $c^2\sigma_z^2-\sigma^2-\sigma_\epsilon^2>0$, which from (4.6c) is equivalent to $\sigma_\epsilon^2>\hat{\sigma}_\epsilon^2$. ∎

Proof of Proposition 5.3: The proof is similar to that of Proposition 4.3. The expected utility of a liquidity supplier in Period 1 is (B.6), and the expectation over (s,z) is (B.10) for F_s given by (B.14). Substituting $\mathrm{E}[D|s]-S_1(\theta_1^d)$ from (C.2), we can write the expected utility (C.1) of a liquidity demander as

$$-\exp\left\{-\alpha\left[W_0+\theta_0\left(S_1-S_0\right)+\beta_s\xi z+\tfrac{1}{2}\alpha\sigma^2[D|s]\left[(\theta_1^d)^2+z^2\right]+\hat{\lambda}\theta_1^d(\theta_1^d-\bar{\theta})\right]\right\}. \qquad \text{(C.7)}$$

(Eq. (C.2) holds for $\theta_0=\bar{\theta}$ even when one agent chooses $\theta_0\neq\bar{\theta}$. This is because agents behave competitively in Period 0, and therefore a non-equilibrium choice $\theta_0\neq\bar{\theta}$ by one agent does not imply non-equilibrium choices by other agents.) Using (4.2a), (4.2b), (4.6c) and the definition of ξ, we can write (5.3) as

$$\theta_1^d=\frac{\bar{D}+\beta_s\xi-S_1+\hat{\lambda}\bar{\theta}}{\alpha\sigma^2[D|s]+\hat{\lambda}}. \qquad \text{(C.8)}$$

Substituting θ_1^d from (C.8), and S_1 from (4.1), we can write (C.7) as

$$-\exp\left\{-\alpha\left[W_0+\theta_0(a+b\xi-S_0)+\beta_s\xi z+\frac{\alpha\sigma^2[D|s]\left[\bar{D}+\beta_s\xi-(a+b\xi)+\hat{\lambda}\bar{\theta}\right]^2}{2\left(\alpha\sigma^2[D|s]+\hat{\lambda}\right)^2}\right.\right.$$

$$\left.\left.+\tfrac{1}{2}\alpha\sigma^2[D|s]z^2+\frac{\hat{\lambda}\left[\bar{D}+\beta_s\xi-(a+b\xi)+\hat{\lambda}\bar{\theta}\right]\left[\bar{D}+\beta_s\xi-(a+b\xi)-\alpha\sigma^2[D|s]\bar{\theta}\right]}{\left(\alpha\sigma^2[D|s]+\hat{\lambda}\right)^2}\right]\right\}. \quad \text{(C.9)}$$

To compute the expectation of (C.9) over (s,z), we use Lemma A.1 and set

$$x\equiv\begin{bmatrix}\xi\\ z\end{bmatrix}$$

$$\Sigma\equiv\begin{bmatrix}\sigma^2+\sigma_\epsilon^2+c^2\sigma_z^2 & -c\sigma_z^2\\ -c\sigma_z^2 & \sigma_z^2\end{bmatrix}$$

$$A\equiv W_0+\theta_0(a-S_0)+\frac{\left(\bar{D}-a+\hat{\lambda}\bar{\theta}\right)\left[(\bar{D}-a)\left(\alpha\sigma^2[D|s]+2\hat{\lambda}\right)-\alpha\sigma^2[D|s]\hat{\lambda}\bar{\theta}\right]}{2\left(\alpha\sigma^2[D|s]+\hat{\lambda}\right)^2}$$

$$B\equiv\begin{bmatrix}b\theta_0+\frac{\left[(\bar{D}-a)\left(\alpha\sigma^2[D|s]+2\hat{\lambda}\right)+\hat{\lambda}^2\bar{\theta}\right](\beta_s-b)}{\left(\alpha\sigma^2[D|s]+\hat{\lambda}\right)^2}\\ 0\end{bmatrix}$$

$$C\equiv\begin{bmatrix}\frac{(\beta_s-b)^2\left(\alpha\sigma^2[D|s]+2\hat{\lambda}\right)}{\left(\alpha\sigma^2[D|s]+\hat{\lambda}\right)^2} & \beta_s\\ \beta_s & \alpha\sigma^2[D|s]\end{bmatrix}.$$

Proceeding as in the proof of Proposition 4.3, we find

$$U^d=-\exp\left(-\alpha F^d\right)\frac{1}{\sqrt{1+\frac{\alpha(\beta_s-b)^2\left(\alpha\sigma^2[D|s]+2\hat{\lambda}\right)}{\left(\alpha\sigma^2[D|s]+\hat{\lambda}\right)^2}(\sigma^2+\sigma_\epsilon^2)(1+\alpha^2\sigma_\epsilon^2\sigma_z^2)-\alpha^2\sigma^2\sigma_z^2}}, \quad \text{(C.10)}$$

where

$$F^d=W_0+\theta_0(a-S_0)+\frac{\left(\bar{D}-a+\hat{\lambda}\bar{\theta}\right)\left[(\bar{D}-a)\left(\alpha\sigma^2[D|s]+2\hat{\lambda}\right)-\alpha\sigma^2[D|s]\hat{\lambda}\bar{\theta}\right]}{2\left(\alpha\sigma^2[D|s]+\hat{\lambda}\right)^2}$$

$$-\frac{\alpha\left[b\theta_0+\frac{\left[(\bar{D}-a)\left(\alpha\sigma^2[D|s]+2\hat{\lambda}\right)+\hat{\lambda}^2\bar{\theta}\right](\beta_s-b)}{\left(\alpha\sigma^2[D|s]+\hat{\lambda}\right)^2}\right]^2(\sigma^2+\sigma_\epsilon^2)(1+\alpha^2\sigma_\epsilon^2\sigma_z^2)}{2\left[1+\frac{\alpha(\beta_s-b)^2\left(\alpha\sigma^2[D|s]+2\hat{\lambda}\right)}{\left(\alpha\sigma^2[D|s]+\hat{\lambda}\right)^2}(\sigma^2+\sigma_\epsilon^2)(1+\alpha^2\sigma_\epsilon^2\sigma_z^2)-\alpha^2\sigma^2\sigma_z^2\right]}. \quad \text{(C.11)}$$

Substituting $\bar{D} - a$ from (4.6a) into (C.11), we find

$$\begin{aligned} F^d =& W_0 + \theta_0(\bar{D} - S_0) - \alpha\sigma^2\theta_0\bar{\theta} + \tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2 \\ &+ \alpha \left\{ b\sigma^2\theta_0\bar{\theta} - \tfrac{1}{2}\sigma^2\bar{\theta}^2 + \frac{\left[\alpha(1-b)\sigma^2 + \hat{\lambda}\right]\left[(1-b)\sigma^2\left(\alpha\sigma^2[D|s] + 2\hat{\lambda}\right) - \sigma^2[D|s]\hat{\lambda}\right]}{2\left(\alpha\sigma^2[D|s] + \hat{\lambda}\right)^2}\bar{\theta}^2 \right. \\ &\left. - \frac{\left[b\theta_0 + \frac{\left[\alpha(1-b)\sigma^2\left(\alpha\sigma^2[D|s]+2\hat{\lambda}\right)+\hat{\lambda}^2\right](\beta_s - b)\bar{\theta}}{\left(\alpha\sigma^2[D|s]+\hat{\lambda}\right)^2}\right]^2 (\sigma^2 + \sigma_\epsilon^2)(1 + \alpha^2\sigma_\epsilon^2\sigma_z^2)}{2\left[1 + \frac{\alpha(\beta_s - b)^2\left(\alpha\sigma^2[D|s]+2\hat{\lambda}\right)}{\left(\alpha\sigma^2[D|s]+\hat{\lambda}\right)^2}(\sigma^2 + \sigma_\epsilon^2)(1 + \alpha^2\sigma_\epsilon^2\sigma_z^2) - \alpha^2\sigma^2\sigma_z^2\right]} \right\}. \end{aligned} \tag{C.12}$$

Using (4.2b) and the definition of β_s, we can write (C.12) as

$$\begin{aligned} F^d =& W_0 + \theta_0(\bar{D} - S_0) - \alpha\sigma^2\theta_0\bar{\theta} + \tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2 \\ &- \frac{\alpha\left\{b^2(\sigma^2 + \sigma_\epsilon^2)(1 + \alpha^2\sigma_\epsilon^2\sigma_z^2)\theta_0^2 + 2b(\sigma^2 + \sigma_\epsilon^2)\left[\alpha^2\sigma^2\sigma_z^2 - b(1 + \alpha^2\sigma_\epsilon^2\sigma_z^2)\right]\theta_0\bar{\theta} + X\bar{\theta}^2\right\}}{2\left[1 + \frac{\alpha(\beta_s - b)^2\left(\alpha\sigma^2[D|s]+2\hat{\lambda}\right)}{\left(\alpha\sigma^2[D|s]+\hat{\lambda}\right)^2}(\sigma^2 + \sigma_\epsilon^2)(1 + \alpha^2\sigma_\epsilon^2\sigma_z^2) - \alpha^2\sigma^2\sigma_z^2\right]}, \end{aligned} \tag{C.13}$$

where

$$X \equiv \left[\sigma^2 - \frac{(1-b)^2\sigma^4}{\sigma^2[D|s]}\right](1 - \alpha^2\sigma^2\sigma_z^2) + \frac{(\beta_s - b)^2\sigma^2}{\sigma^2[D|s]}(\sigma^2 + \sigma_\epsilon^2)(1 + \alpha^2\sigma_\epsilon^2\sigma_z^2) - \frac{\alpha^2(\beta_s - b)^2\hat{\lambda}^2\sigma^4(\sigma^2 + \sigma_\epsilon^2)\sigma_z^2}{\left(\alpha\sigma^2[D|s] + \hat{\lambda}\right)^2\sigma^2[D|s]}.$$

Using (B.21) to eliminate the term in $\sigma^2 - \frac{(1-b)^2\sigma^4}{\sigma^2[D|s]}$ in the definition of X, and substituting X into (B.20), we find

$$\begin{aligned} F^d =& W_0 + \theta_0(\bar{D} - S_0) - \alpha\sigma^2\theta_0\bar{\theta} + \tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2 \\ &- \frac{\alpha\left\{b^2(\sigma^2 + \sigma_\epsilon^2)(1 + \alpha^2\sigma_\epsilon^2\sigma_z^2)(\theta_0 - \bar{\theta})^2 + \alpha^2(\sigma^2 + \sigma_\epsilon^2)^2\sigma_z^2\left[\frac{2b\sigma^2\theta_0\bar{\theta}}{\sigma^2 + \sigma_\epsilon^2} + \hat{X}\bar{\theta}^2\right]\right\}}{2\left[1 + \frac{\alpha(\beta_s - b)^2\left(\alpha\sigma^2[D|s]+2\hat{\lambda}\right)}{\left(\alpha\sigma^2[D|s]+\hat{\lambda}\right)^2}(\sigma^2 + \sigma_\epsilon^2)(1 + \alpha^2\sigma_\epsilon^2\sigma_z^2) - \alpha^2\sigma^2\sigma_z^2\right]}, \end{aligned} \tag{C.14}$$

where

$$\hat{X} \equiv \frac{(\beta_s - b)^2\sigma^2}{\sigma^2[D|s]}\left[1 - \frac{\hat{\lambda}^2\sigma^2}{\left(\alpha\sigma^2[D|s] + \hat{\lambda}\right)^2(\sigma^2 + \sigma_\epsilon^2)}\right] - b^2.$$

We next note that

$$\begin{aligned}
&\frac{\alpha(\beta_s - b)^2 \left(\alpha\sigma^2[D|s] + 2\hat{\lambda}\right)}{\left(\alpha\sigma^2[D|s] + \hat{\lambda}\right)^2}(\sigma^2 + \sigma_\epsilon^2)(1 + \alpha^2\sigma_\epsilon^2\sigma_z^2) \\
=&\frac{(b - \beta_\xi)^2 \left(\alpha\sigma^2[D|s] + 2\hat{\lambda}\right)(1-\pi)^2}{\alpha\sigma^4[D|S_1]\pi^2}(\sigma^2 + \sigma_\epsilon^2)(1 + \alpha^2\sigma_\epsilon^2\sigma_z^2) \\
=&\frac{(b - \beta_\xi)^2 \left(\alpha\sigma^2[D|s] + 2\hat{\lambda}\right)(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)(1-\pi)^2}{\alpha\sigma^2[D|s]\sigma^2[D|S_1]\pi^2} \\
=&\Delta_0 \left(1 + \frac{\hat{\lambda}}{\alpha\sigma^2[D|s]}\right)(1-\pi)^2, \qquad \text{(C.15)}
\end{aligned}$$

where the first step follows from (C.4), the second from (4.2b), (4.3b), (4.6c) and the definitions of (β_s, β_ξ), and the third from (4.7a). Therefore, (C.10) takes the form (A.5), with Δ_0 replaced by $\Delta_0 \left(1 + \frac{2\hat{\lambda}}{\alpha\sigma^2[D|s]}\right)$. Eqs. (B.14), (C.14) and (C.15) imply that when $\theta_0 = \bar{\theta}$, $(dF^s/d\theta_0, F^s, dF^d/d\theta_0, F^d)$ are given by (A.8)-(A.11), with (Δ_1, Δ_2) given by (5.7a) and (5.7b). Since the equations for $(U^s, U^d, dF^s/d\theta_0, F^s, dF^d/d\theta_0, F^d)$ take the same form as in Proposition 3.2, the same applies to S_0. ∎

Proof of Proposition 5.4: The proof that λ is given by (4.8) is the same as for Proposition 4.4. When information is asymmetric ($\sigma_\epsilon^2 < \infty$), (4.6b) and (5.4) imply that b is smaller under imperfect competition. Therefore, (4.8) implies that λ is higher. To determine λ when information is symmetric, we consider the limit $\sigma_\epsilon^2 \to \infty$. Eqs. (4.2b), (4.3b), (4.6c), and the definitions of (β_s, β_ξ) imply that $\frac{\beta_\xi}{\beta_s} \to 0$ and $\frac{b}{\beta_s} \to \frac{\pi}{1+\pi}$. Eq. (4.8) then implies that $\lambda \to \frac{\alpha\sigma^2}{1-\pi}$, which coincides with the competitive counterpart (3.16). ∎

Proof of Proposition 5.5: The proof that γ is given by (4.8) is the same as for Proposition 4.5. When information is asymmetric ($\sigma_\epsilon^2 < \infty$), b is smaller under imperfect competition, and (4.9) implies that γ is lower. To determine γ when information is symmetric, we consider the limit $\sigma_\epsilon^2 \to \infty$. Since $\frac{\beta_\xi}{\beta_s} \to 0$ and $\frac{b}{\beta_s} \to \frac{\pi}{1+\pi}$, (4.9) implies that $\gamma \to \frac{\alpha^2\sigma^4\sigma_z^2\pi^2}{(1+\pi)^2}$, which is lower than the competitive counterpart (3.19). ∎

Proof of Proposition 5.6: To determine S_0 when information is symmetric, we consider the limit

$\sigma_\epsilon^2 \to \infty$. Since $\frac{\beta_\xi}{\beta_s} \to 0$ and $\frac{b}{\beta_s} \to \frac{\pi}{1+\pi}$, (4.7a), (5.7a) and (5.7b) imply that

$$\Delta_0 \to \frac{\alpha^2\sigma^2\sigma_z^2}{(1+\pi)^2} < \Delta_0^c$$

$$\Delta_0 \left(1 + \frac{2\hat{\lambda}}{\alpha\sigma^2[D|s]}\right) \to \frac{\alpha^2\sigma^2\sigma_z^2}{1-\pi^2} > \Delta_0^c$$

$$\Delta_1 \to \frac{\frac{\alpha^3\sigma^4\sigma_z^2\pi}{1+\pi}}{1 + \frac{\alpha^2\sigma^2\sigma_z^2(1-\pi)}{1+\pi} - \alpha^2\sigma^2\sigma_z^2} < \Delta_1^c$$

$$\Delta_2 \to \frac{\alpha^3\sigma^4\sigma_z^2}{1 + \frac{\alpha^2\sigma^2\sigma_z^2(1-\pi)}{1+\pi} - \alpha^2\sigma^2\sigma_z^2} < \Delta_2^c,$$

where $\{\Delta_j^c\}_{j=0,1,2}$ denote the competitive counterparts of $\{\Delta_j\}_{j=0,1,2}$, given by (3.12a)-(3.12c). The above inequalities, together with (3.10), (3.11), and (5.6) imply that S_0 is higher under imperfect competition.

To show that S_0 can be lower under imperfect competition, we consider the limit $\sigma_\epsilon^2 \to \hat{\sigma}_\epsilon^2$ (and $\sigma_\epsilon^2 > \hat{\sigma}_\epsilon^2$ so that the linear equilibrium exists). Eqs. (4.2b), (4.3b), (4.6c), and the definitions of (β_s, β_ξ) imply that $\frac{\beta_\xi}{\beta_s} \to \frac{1}{2}$ and $\frac{b}{\beta_s} \to \frac{1}{2}$. Substituting into (4.7a), (5.7a) and (5.7b), and using the definition of $\hat{\lambda}$, we find

$$\Delta_0 \to 0$$

$$\Delta_0 \left(1 + \frac{2\hat{\lambda}}{\alpha\sigma^2[D|s]}\right) \to 0$$

$$\Delta_1 \to \frac{\alpha^3\sigma^4\sigma_z^2}{2\left(1 - \alpha^2\sigma^2\sigma_z^2\right)}$$

$$\Delta_2 \to \frac{\alpha^3\sigma^4\sigma_z^2}{1 - \alpha^2\sigma^2\sigma_z^2}.$$

The competitive counterparts of $\{\Delta_j\}_{j=0,1,2}$ are given by (4.7a)-(4.7c). Since $\Delta_0^c > 0$, the following inequalities hold when $\alpha^2\sigma^2\sigma_z^2 \approx 1$: $\Delta_j > \Delta_j^c$ for $j = 1, 2$, and $M > M^c$, where M^c denotes the competitive counterpart of M. These inequalities, together with (3.10), imply that S_0 is lower under imperfect competition. ∎

Proof of Proposition 5.7: Using (4.2b), (4.3b), (4.6c), (5.4) and the definitions of (β_s, β_ξ), we can write (4.8) as

$$\lambda = \frac{\alpha\sigma^2\left(\sigma_\epsilon^2 + c^2\sigma_z^2\pi\right)}{\left(c^2\sigma_z^2 - \sigma^2 - \sigma_\epsilon^2\right)\pi(1-\pi)}, \tag{C.16}$$

(4.9) as

$$\gamma = \frac{\sigma^4(\sigma_\epsilon^2 + c^2\sigma_z^2)(\sigma_\epsilon^2 + c^2\sigma_z^2\pi)(c^2\sigma_z^2 - \sigma^2 - \sigma_\epsilon^2)\pi}{\left[2(\sigma^2 + \sigma_\epsilon^2)(\sigma_\epsilon^2 + c^2\sigma_z^2)\pi + \sigma_\epsilon^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)(1-\pi)\right]^2}, \tag{C.17}$$

(4.7a) as

$$\Delta_0 = \frac{\sigma^2(\sigma_\epsilon^2 + c^2\sigma_z^2)(c^2\sigma_z^2 - \sigma^2 - \sigma_\epsilon^2)^2}{\left[2(\sigma^2 + \sigma_\epsilon^2)(\sigma_\epsilon^2 + c^2\sigma_z^2)\pi + \sigma_\epsilon^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)(1-\pi)\right]^2}, \tag{C.18}$$

and the numerator in (5.7a) as

$$\alpha^3 b\sigma^2(\sigma^2 + \sigma_\epsilon^2)\sigma_z^2 = \frac{\alpha^3\sigma^4(\sigma^2 + \sigma_\epsilon^2)(\sigma_\epsilon^2 + c^2\sigma_z^2\pi)\sigma_z^2}{2(\sigma^2 + \sigma_\epsilon^2)(\sigma_\epsilon^2 + c^2\sigma_z^2)\pi + \sigma_\epsilon^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)(1-\pi)}. \tag{C.19}$$

Using (C.18) and the definition of $\hat{\lambda}$, we find

$$\Delta_0\left(1 + \frac{2\hat{\lambda}}{\alpha\sigma^2[D|s]}\right) = \frac{\sigma^2(\sigma_\epsilon^2 + c^2\sigma_z^2)(c^2\sigma_z^2 - \sigma^2 - \sigma_\epsilon^2)}{\sigma_\epsilon^2\left[2(\sigma^2 + \sigma_\epsilon^2)(\sigma_\epsilon^2 + c^2\sigma_z^2)\pi + \sigma_\epsilon^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)(1-\pi)\right](1-\pi)}. \tag{C.20}$$

Using (C.15) and (C.20), we find

$$\frac{\alpha(\beta_s - b)^2\left(\alpha\sigma^2[D|s] + 2\hat{\lambda}\right)}{\left(\alpha\sigma^2[D|s] + \hat{\lambda}\right)^2}(\sigma^2 + \sigma_\epsilon^2) = \frac{\sigma^2(c^2\sigma_z^2 - \sigma^2 - \sigma_\epsilon^2)(1-\pi)}{2(\sigma^2 + \sigma_\epsilon^2)(\sigma_\epsilon^2 + c^2\sigma_z^2)\pi + \sigma_\epsilon^2(\sigma^2 + \sigma_\epsilon^2 + c^2\sigma_z^2)(1-\pi)}. \tag{C.21}$$

Eq. (C.16) implies that λ is decreasing in σ_z^2. Eq. (C.17) implies that γ is increasing in σ_z^2. Eq. (C.18) implies that Δ_0 is increasing in σ_z^2. Eq. (C.20) implies that

$$1 + \Delta_0\left(1 + \frac{2\hat{\lambda}}{\alpha\sigma^2[D|s]}\right)(1-\pi)^2 - \alpha^2\sigma^2\sigma_z^2$$

is decreasing in σ_z^2. Eq. (C.19) implies that the numerator in (5.7a) is increasing in σ_z^2, and so is Δ_1. Eq. (C.21) implies that the numerator in (5.7b) is increasing in σ_z^2, and so are Δ_2 and M. Therefore, (3.10) implies that S_0 is decreasing in σ_z^2. ∎

D Welfare

Proof of Proposition 7.1: Eqs. (A.2), (A.5), (A.9) and (A.11) imply that the interim utility of liquidity suppliers under no information is

$$U^s = -\exp\left\{-\alpha\left[\bar{\theta}(\bar{D}-S_0) - \tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2\right]\right\}\frac{1}{\sqrt{1+\Delta_0^{ni}\pi^2}}, \tag{D.1}$$

and that of liquidity demanders is

$$U^d = -\exp\left\{-\alpha\left[\bar{\theta}(\bar{D}-S_0) - \tfrac{1}{2}\alpha\sigma^2\bar{\theta}^2 - \tfrac{1}{2}\Delta_2^{ni}\bar{\theta}^2\right]\right\}\frac{1}{\sqrt{1+\Delta_0^{ni}(1-\pi)^2 - \alpha^2\sigma^2\sigma_z^2}}. \tag{D.2}$$

Proposition A.1 implies that the interim utilities (U^s, U^d) under full information are given by (D.1) and (D.2), respectively, provided that $(\Delta_0^{ni}, \Delta_2^{ni})$ are replaced by $(\Delta_0^{fi}, \Delta_2^{fi})$. Proposition 4.3 similarly implies that the interim utilities (U^s, U^d) under asymmetric information are given by (D.1) and (D.2), respectively, provided that $(\Delta_0^{ni}, \Delta_2^{ni})$ are replaced by $(\Delta_0^{ai}, \Delta_2^{ai})$. The proposition then follows from Lemma B.1. ∎