International Correlation Risk

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International Correlation Risk[*]

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Abstract

We provide novel evidence of priced correlation risk in the foreign exchange market. Currencies that perform badly (well) during periods of high exchange rate correlation have high (low) average returns. We also show that high (low) interest rate currencies have high (low) correlation risk exposure, providing a risk-based justification for the carry trade. To address our empirical findings, we consider a general equilibrium model that incorporates preferences characterized by external habit formation and home bias. In our model, currencies which depreciate when conditional exchange rate correlation is high command high risk premia due to their adverse exposure to global risk aversion shocks.

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This paper studies the properties of correlation risk in the foreign exchange (FX) market. First, we provide empirical evidence of priced correlation risk in currency markets. We then offer a macroeconomic justification for the pricing of correlation risk by showing that, in the context of a multi-country general equilibrium model with external habit formation, increases in conditional FX correlation are associated with increases in global conditional risk aversion.

Our first contribution is empirical. The importance of correlations in financial markets has spawned a broad literature documenting that asset return correlations are stochastic and countercyclical. Yet the existing literature has focused mainly on equities and has largely ignored the FX market. In particular, there is evidence that correlation risk is priced in the equity market, arguably due to the deterioration of investors' investment opportunities that results from a reduction in diversification benefits when asset return correlation increases. We demonstrate that correlation risk is priced in the FX market as well.

We begin by showing that the difference between risk-neutral and objective measures of FX correlation is almost always positive implying a positive association between high levels of FX correlation and high-priced, undesirable states of the world. Furthermore, the size of the difference is quantitatively significant, averaging about 15% across different exchange rate pairs against the US dollar.

After establishing the countercyclical nature of FX correlation fluctuations, we quantify FX correlation risk by constructing an empirical measure of global FX conditional correlation and using a portfolio sorting approach in line with the recent international finance literature (see, for example, Lustig and Verdelhan, 2007, Lustig, Roussanov, and Verdelhan, 2011, Burnside, 2011, and Menkhoff, Sarno, Schmeling, and Schrimpf, 2012). Intuitively, if correlation risk is priced in currency markets, then its countercyclical nature implies a negative price: currencies with low FX correlation betas should yield higher returns, whereas low correlation risk currencies (i.e., currencies that appreciate strongly when FX correlation increases and thus hedge against FX correlation risk) should yield lower returns. Our results confirm this intuition: we show that investing in (respectively, shorting) the portfolio with the highest (lowest) correlation risk exposure

generates an average annual excess return of between 5% (developed countries) and 6% (all countries) and Sharpe ratios of 0.56 and 0.76, respectively.

Furthermore, using a cross-section of currency portfolios sorted on forward premium discounts, we estimate the price of FX correlation risk to be about -1% per annum. We also contribute to the literature on the failure of the exchange rate expectations hypothesis by showing that exposure to correlation risk can explain the returns of the carry trade. Specifically, we find that high (low) interest rate currencies have negative (positive) FX correlation betas and therefore depreciate (appreciate) in bad states of the world, when global risk aversion is high. As a result, high interest rate currencies have positive risk premia, whereas low interest rate currencies have low or negative risk premia.

Given the findings of Menkhoff, Sarno, Schmeling and Schrimpf (2012) that global FX volatility risk is priced, a salient question is whether FX correlation risk is a source of systematic risk independent of FX volatility risk. To address this question, we double-sort currencies on their exposure to FX volatility and correlation risk and show that, when we condition on FX volatility risk exposure, differences in FX correlation risk exposure lead to quantitatively significant average return differentials. Therefore, FX correlation risk is not subsumed by FX volatility risk.

Our second contribution is theoretical. Motivated by our empirical findings, we examine the implications of time variation in conditional risk aversion for currency risk premia, focusing on the links between risk aversion and exchange rate moments. We consider a multi-country, multi-good general equilibrium model in which preferences are characterized by external habit formation (Menzly, Santos and Veronesi, 2004) and home bias in preferences. In the model, currency risk premia compensate investors mainly for exposure to fluctuations in global risk aversion. An agent is willing to accept lower returns for assets that have negative global risk aversion betas and, therefore, provide a hedge against increases in global risk aversion. Consequently, the price of the global risk aversion factor is negative. We also show that global risk aversion is positively associated with conditional exchange rate variance and correlation, providing a theoretical justification for priced FX volatility and correlation risk. Foreign exchange

volatility is increasing in global risk aversion because of the increasing desire of countries to insure each other and thus better align their marginal utility. Since real exchange rates represent differences in marginal utility between countries, higher international risk sharing implies smoother exchange rates. Foreign exchange correlation is rising in global risk aversion if the domestic country (in this paper, the United States) is more home-biased than foreign countries, as an increase in international risk sharing implies that it is the marginal utility of foreign countries that becomes extremely correlated.

Finally, we demonstrate that our model is able to link currency risk premia with real interest rate differentials and, therefore, explain carry trade returns. If the precautionary savings motive is sufficiently strong, then being long a high interest rate currency and short a low interest rate currency is tantamount to being highly exposed to global risk aversion. As a result, investors require high compensation in terms of expected return for engaging in the carry trade.

Related Literature: This paper builds on the literature addressing the risk–return relationship in FX markets and, especially, the carry trade. Lustig, Roussanov, and Verdelhan (2011) identify two risk factors: the average forward discount of the US dollar against foreign currencies and the return to the carry trade portfolio itself (HML factor), and show that the cross-section of interest rate sorted currency portfolio returns can be mapped to differential exposure to the HML factor. They interpret the HML factor to be a global risk factor, so carry trade excess returns compensate US investors for exposure to global risk. Our findings imply that the HML factor is a good proxy for the FX correlation factor, which, in turn, proxies for global conditional risk aversion. In that sense, our paper provides an economic justification for the HML factor.

In other recent work on the carry trade, Cenedese, Sarno, and Tsiakas (2012) find that higher (lower) average currency excess return variance (correlation) leads to larger carry trade losses (gains). Menkhoff, Sarno, Schmeling and Schrimpf (2012) show that the carry trade can be explained as compensation for global FX volatility risk. Mancini, Ranaldo, and Wrampelmeyer (2012) study the impact of FX liquidity on carry returns, and Adrian, Etula, and Shin (2010) present evidence that the funding liquidity of US

intermediaries is a good predictor of exchange rates. In their paper, funding liquidity proxies for the risk aversion of dollar-funded intermediaries.

This paper is also part of the recent literature that addresses the failure of the expectations hypothesis with respect to exchange rates. Brunnermeier, Nagel, and Pedersen (2009), Farhi, Fraiberger, Gabaix, Ranciere and Verdelhan (2009), Jurek (2009), Burnside, Eichenbaum, Kleshchelski and Rebelo (2011) and Farhi and Gabaix (2011) emphasize the importance of disaster risk for currency risk premia. Yu (2011) studies the effect of investor sentiment; Colacito and Croce (2009, 2010) and Bansal and Shaliastovich (2012) explore the implications of long-run risk in currency markets. Martin (2011) studies the failure of the uncovered interest rate parity (UIP) in a two-country economy with two goods and heterogeneity in country size. Evans (2012) shows that risk shocks are a significant driver of exchange rate dynamics. The paper closest to ours is Verdelhan (2010): he proposes a two-country, single-good model with trade frictions and time-varying risk aversion generated by external habit formation, and he illustrates the importance of procyclical real interest rates for addressing the forward premium puzzle. In contrast to Verdelhan (2010), our multi-country model features a cross-section of exchange rates and endogenizes consumption, illustrating the effects of country cross-insurance on the conditional second moments of exchange rates.

Finally, different versions of our theoretical setup have been used to address the Brandt, Cochrane, and Santa-Clara (2006) international risk-sharing puzzle (Stathopoulos, 2012a) and the portfolio home bias puzzle (Stathopoulos, 2012b); our paper extends the insights of that research by considering the effects of time variation in risk aversion for currency risk premia.

The rest of the paper is organized as follows. Section I. describes the data and details the construction of the correlation risk factor. Section II. contains our empirical results on priced correlation risk in currency markets. Section III. presents a multi–country model with external habit formation. Section IV. features the findings from the model simulation, and Section V. concludes. Proofs are deferred to the Appendix, and additional results and robustness checks are presented in an Online Appendix.

I. Data and Construction of the Risk Factor

We start by describing the data and constructing the currency correlation risk factor. We use daily option prices on the most heavily traded currency pairs to construct a measure of implied FX correlation, and we use high-frequency data on the underlying spot exchange rates to calculate the realized counterpart. Our data are from the period 1999 – 2010.

A. Data Description

High-Frequency Currency Data: The high-frequency spot exchange rate data for the Euro, Japanese Yen, British Pound, and Swiss Franc, all vis-à-vis the U.S. Dollar, are from Olsen & Associates. Given that the high liquidity in foreign exchange markets prevents triangular arbitrage opportunities in the five most heavily traded currencies, calculating the remaining cross rates using the four exchange rates is common practice. The raw data consists of all interbank bid and ask indicative quotes for the exchange rates to the nearest even second. After filtering the data for outliers, the log price at each five-minute tick is obtained by linearly interpolating from the average of the log bid and log ask quotes for the two closest ticks. Because options are traded continuously, the result is a total of 288 observations over a 24-hour period.[1]

Currency Options Data: We use daily over-the-counter (OTC) currency options data from JP Morgan for the four currency pairs EUR/USD, JPY/USD, GBP/USD, and CHF/USD plus the six cross rates; thus we have options data on ten exchange rates. Using OTC options data has several advantages over exchange-traded options data. First, the trading volume in the OTC FX options market is several times larger than the corresponding volume on exchanges such as the Chicago Mercantile Exchange, and this leads to more competitive quotes in the OTC market. Second, the conventions for writing and quoting options in the OTC markets have several features that are appealing when performing empirical studies. In particular, every day sees the issuance

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[1]We follow the empirical literature and take five minute intervals as opposed to higher frequencies, in order to mitigate the effect of spurious serial correlation stemming from microstructure noise (see Andersen and Bollerslev, 1998).

of new options series with fixed times to maturity and fixed strike prices, defined by sticky deltas; in comparison, the time to maturity of an exchange-traded option series gradually declines with the approaching expiration date and so the moneyness continually changes as the underlying exchange rate moves. As a result the OTC options data allows for better comparability over time because the series' main characteristics do not change from day to day. The options used in this study are plain-vanilla European calls and puts with five option series per exchange rate. Specifically, we consider a one-month maturity and a total of five different strikes: at-the-money (ATM), 10-delta call and 25-delta call, 10-delta-put and 25-delta put.

Spot and Forward Rates: To form our portfolios, we use daily data for spot exchange rates and one-month forward rates (versus the US dollar) obtained from Datastream. Following the previous literature (see Fama, 1984), we work with the log spot and one-month forward exchange rates, denoted $s_t^i = \ln(S_t^i)$ and $f_t^i = \ln(F_t^i)$, respectively. We use the US dollar as the home currency, so superscript i always denotes the foreign currency. Our sample consists of 20 foreign countries: Australia, Canada, Czech Republic, Denmark, Euro, Hungary, India, Japan, Kuwait, Mexico, New Zealand, Norway, Philippines, Singapore, South Africa, Sweden, Switzerland, Taiwan, Thailand, and the United Kingdom.[2] We also run a separate analysis using only developed countries: Australia, Canada, Denmark, Euro, Japan, New Zealand, Norway, Sweden, Switzerland, and the United Kingdom.

Carry Portfolios: At the end of each month t, we allocate currencies into four portfolios based on their end-of-the-month forward discount and rebalance every month. Since covered interest rate parity holds in the data at the monthly frequency, sorting on forward discounts is equivalent to sorting on interest rate differentials. Portfolio 1 contains currencies with the lowest interest rates (smallest forward discounts) while Portfolio 4 contains currencies with the highest interest rates (largest forward discounts). Monthly excess returns from holding foreign currency i are computed as $rx_{t+1}^i \approx f_t^i - s_{t+1}^i$.

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[2]These are the same countries as in Lustig, Roussanov, and Verdelhan (2011), minus the 10 Euro countries and Hong Kong, Indonesia, Malaysia, Poland, Saudi Arabia and South Korea for which we do not have a full sample of forward rates.

We follow Lustig, Roussanov, and Verdelhan (2011) and build a long–short carry factor (HML) by investing in Portfolio 4 and shorting Portfolio 1. We also build a zero-cost dollar portfolio (DOL), which is an equally weighted average of the four currency portfolios and thus consists of borrowing United States dollars and investing in global money markets outside the United States in equal weights.

[Insert Table 1 here.]

Summary statistics on the interest rate sorted currency portfolios, HML factor, and DOL factor are presented in Table 1. In line with previous findings, there is a monotonic increase in the average excess return from the lowest to the highest forward discount-sorted portfolio. The unconditional average excess return from holding an equally weighted average carry portfolio is 4% per annum. The HML portfolio is highly profitable; it has an average annual return of 9.2% and an annualized Sharpe ratio of 1.19.[3]

B. Construction of Realized and Implied Volatility and Correlation Measures

In this section, we construct empirical measures of realized and implied exchange rate correlation. For the former we use high-frequency data on the underlying spot exchange rates, and for the latter we rely on options written on those exchange rates.

B.1. Realized Variance and Correlation

Currencies are traded continuously around the world. To match the recording time of daily option prices, we record the daily spot exchange rate at 4pm GMT. Overall, we have 288 intraday currency returns over five-minute intervals (from 4pm each day to 4pm the next day):

$$r_{k,5\text{min}} = \ln{(S_k)} - \ln(S_{k-5\text{min}}).$$

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[3]We also calculate real interest rate differentials using data on forward discounts and inflation differentials. We sort currencies in portfolios based on real interest rate differentials and consider the properties of a real carry trade strategy. The results remain quantitatively the same and are reported in the Online Appendix.

We follow Andersen, Bollerslev, Diebold, and Labys (2000) and compute the realized variance by summing the squared five-minute frequency returns over the day:

$$\text{RV}_t = \sum_{k=1}^{K} r_{k,5\text{min}}^2.$$

In a similar spirit, we derive the realized covariance between exchange rates s^i and s^j:

$$\text{RCov}_t^{i,j} = \sum_{k=1}^{K} r_{k,5\text{min}}^i r_{k,5\text{min}}^j.$$

The realized correlation is then the ratio between the realized covariance and the product of the respective standard deviations:

$$\text{RC}_t^{i,j} = \text{RCov}_t^{i,j} / \sqrt{\text{RV}_t^i} \sqrt{\text{RV}_t^j}.$$

B.2. Implied Variance and Correlation

We follow Demeterfi, Derman, Kamal, and Zhou (1999) and Britten-Jones and Neuberger (2000) to obtain a model-free measure of implied volatility. They show that if the underlying asset price is continuous, then the risk-neutral expectation over a horizon $T - t$ of total return variance is defined as an integral of option prices over an infinite range of strike prices:

$$\text{E}_t^{\mathbb{Q}} \left(\int_t^T \left(\sigma_u^i \right)^2 du \right) = 2 e^{r(T-t)} \left(\int_0^{S_t^i} \frac{1}{K^2} \text{P}(K,T) \, dK + \int_{S_t^i}^{\infty} \frac{1}{K^2} \text{C}(K,T) \, dK \right), \quad (1)$$

where S_t is the underlying spot exchange rate and $\text{P}(K,T)$ and $\text{C}(K,T)$ are the respective put and call prices with maturity date T and strike K. In practice, the number of traded options for any underlying asset is finite; hence the available strike price series is a finite sequence. Calculating the model-free implied variance involves the entire cross-section of option prices: for each maturity T, all five strikes are taken into account. These are quoted in terms of the corresponding delta of the option. To convert the quoted delta-strikes into dollar strike prices, we use the option prices from the Garman and Kohlhagen

(1983) model and solve for the delta of the corresponding option. The corresponding spot rates are extracted from the high-frequency dataset, using the observation that corresponds to the exact time of the daily options quote (4pm GMT). The risk-free interest rate for each of the USD, EUR, JPY, GBP, and CHF is proxied by the London Interbank Offered Rate (LIBOR).

To approximate the integral in equation (1), we adopt a trapezoidal integration scheme over the range of strike prices covered by our dataset. Jiang and Tian (2005) report two types of implementation errors: (i) truncation errors due to the non-availability of an infinite range of strike prices; and (ii) discretization errors that arise due to the un-availability of a continuum of available options. We find that both errors are extremely small when currency options are used. For example, the size of the errors totals only half a percentage point in terms of volatility.

Model-free implied correlations are constructed from the available model-free implied volatilities.[4] We require all cross rates for three currencies, S_t^i, S_t^j, and S_t^{ij}. The absence of triangular arbitrage then implies that:[5] $S_t^{ij} = S_t^i / S_t^j$. Taking logs, we derive the following equation:

$$\ln\left(\frac{S_T^{ij}}{S_t^{ij}}\right) = \ln\left(\frac{S_T^i}{S_t^i}\right) - \ln\left(\frac{S_T^j}{S_t^j}\right).$$

Finally, taking variances yields:

$$\int_t^T \left(\sigma_u^{ij}\right)^2 du = \int_t^T \left(\sigma_u^i\right)^2 du + \int_t^T \left(\sigma_u^j\right)^2 du - 2\int_t^T \gamma_u^{i,j} du,$$

[4]Brandt and Diebold (2006) use the same approach to construct realized covariances of exchange rates from range-based volatility estimators.

[5]Recent studies report that the average violation of triangular arbitrage is about 1.5 basis points with an average duration of 1.5 seconds (Kozhan and Tham, 2012). However, we observe that most papers examining violations of triangular arbitrage use indicative quotes, which give only an approximate price at which a trade can be executed. Executable prices can differ from indicative prices by several basis points. Using executable FX quotes, Fenn, Howison, McDonald, Williams, and Johnson (2009) report that triangular arbitrage is less than 1 basis point and the duration less than 1 second. Our data also indicate that triangular arbitrage is less than 1 basis point. We therefore conclude that these violations have no effect on calculated quantities.

where $\gamma_t^{i,j}$ denotes the realized covariance of returns between currency pairs s_t^i and s_t^j. Solving for the covariance term, we obtain:

$$\int_t^T \gamma_u^{i,j} du = \frac{1}{2} \int_t^T \left(\sigma_u^i\right)^2 du + \frac{1}{2} \int_t^T \left(\sigma_u^j\right)^2 ds - \frac{1}{2} \int_t^T \left(\sigma_u^{ij}\right)^2 du.$$

Using the standard replication arguments, we find that:

$$
\begin{aligned}
\mathrm{E}_t^{\mathbb{Q}}\left(\int_t^T \gamma_u^{i,j}\, du\right) = {} & e^{r(T-t)}\bigg(\int_t^{S_t^i} \frac{1}{K^2}\mathrm{P}^i(K,T)\, dK + \int_{S_t^i}^\infty \frac{1}{K^2}\mathrm{C}^i(K,T)\, dK \\
& + \int_t^{S_t^j} \frac{1}{K^2}\mathrm{P}^j(K,T)\, dK + \int_{S_t^j}^\infty \frac{1}{K^2}\mathrm{C}^j(K,T)\, dK \\
& - \int_t^{S_t^{ij}} \frac{1}{K^2}\mathrm{P}^{ij}(K,T)\, dK - \int_{S_t^{ij}}^\infty \frac{1}{K^2}\mathrm{C}^{ij}(K,T)\, dK\bigg).
\end{aligned}
\tag{2}
$$

The model-free implied correlation can then be calculated using expression (2) and the model-free implied variance expression (1):[6]

$$\mathrm{E}_t^{\mathbb{Q}}\left(\int_t^T \rho_u^{i,j} du\right) \equiv \frac{\mathrm{E}_t^{\mathbb{Q}}\left(\int_t^T \gamma_u^{i,j} ds\right)}{\sqrt{\mathrm{E}_t^{\mathbb{Q}}\left(\int_t^T \left(\sigma_u^i\right)^2 du\right)}\sqrt{\mathrm{E}_t^{\mathbb{Q}}\left(\int_t^T \left(\sigma_u^j\right)^2 du\right)}}.
\tag{3}$$

B.3.   Global Correlation Risk

To construct a global exchange rate correlation factor, we average all available exchange rate realized correlations on any given day.[7] We prefer to use a measure of realized, as opposed to implied, exchange rate correlation in the interest of consistency with our theoretical model, to be presented in a later Section. Nonetheless, we show in the

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[6]Our expression for the risk-neutral correlation does not imply that the correlation need to be bounded between -1 and 1. One way to ensure that the absolute implied correlations never reach unity would be to impose a normalization in the spirit of the Dynamic Conditional Correlation model of Engle (2002). Because we find no implied correlations as high as 1, we do not apply this normalization.

[7]It is not clear a priori how to construct a global risk factor. In theory, we could calculate principal components and use the first principal component to represent global correlation risk. Yet the unconditional correlation between the average and the first principal component is 99%, so we prefer to use the average because it is the simplest measure. We use an equal-weighted average to represent a global risk factor though clearly one could devise more elaborate ways to construct an average. We explore a turnover-weighted average for the volatility factor in the Online Appendix and show that the two methods lead to the same results.

Online Appendix that all results are robust to using measures of implied exchange rate correlation.

We have data on four exchange rates, the EUR, GBP, JPY, and CHF against the USD, which allows us to calculate six exchange rate realized correlation measures. Hence, our measure of global FX correlation is the simple average of the six correlation measures,

$$\mathrm{RC}_t^G = \frac{1}{6} \sum_{i=1}^{4} \sum_{j>i} \mathrm{RC}_t^{ij}, \tag{4}$$

where RC_t^{ij} is the realized correlation for currency pair i and j. Likewise, we construct a global volatility risk factor by taking the average of the realized volatilities: $\mathrm{RV}_t^G = \frac{1}{4} \sum_{i=1}^{4} \mathrm{RV}_t^i$, where RV_t^i is the realized volatility for exchange rate i.

For our empirical analysis, we use innovations of the aforementioned FX correlation and volatility factors, with the innovations defined as the residuals from fitting a first-order autoregressive AR(1) process for the two global factors. We denote these innovations by $\Delta \mathrm{RC}^G$ and $\Delta \mathrm{RV}^G$, respectively.[8]

II. Empirical Analysis

In this section, we quantify the price of exchange rate correlation risk in two ways. First, we calculate the difference between the risk-neutral (implied) and objective (realized) correlation measure for each day. Second, we study the empirical relationship between the global correlation risk measure and the risk-return profile of currency portfolios. Specifically, we show that sorting currencies according to their exposure to correlation risk yields a significant spread in average returns and we assess the price of correlation risk using the two-stage methodology of Fama and MacBeth (1973). Furthermore, we

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[8]We run portfolio sorts with first differences and also with the AR(1) innovations. Our results are robust to the method used.

find that carry trade currency returns can be justified as compensation for exposure to global exchange rate correlation risk.

A. Correlation Risk Premia

Consistent with earlier literature, we define exchange rate correlation risk premia as the difference between the risk-neutral and objective measures of the FX correlation:

$$\text{CRP}_{t,T}^{i,j} \equiv \text{E}_t^{\mathbb{Q}} \left(\int_t^T \rho_u^{i,j} du \right) - \text{E}_t^{\mathbb{P}} \left(\int_t^T \rho_u^{i,j} du \right).$$

We only consider one-month premia, $T = t + 1$. Variance risk premia can be defined analogously as the difference between the risk-neutral and objective measures of FX variance.

Tables 2 and 3 provide summary statistics of the realized and implied FX volatilities and correlations respectively, as well as summary statistics for the corresponding risk premia.

[Insert Tables 2 and 3 here.]

Despite the evidence for significant variance risk premia in equity markets (e.g., Bollerslev, Tauchen, and Zhou, 2010), we find that variance risk premia in FX markets are on average small and statistically not different from zero. We also find that the variance risk premia are left-skewed, which might reflect the implicit crash risk in FX currency markets (see Brunnermeier, Nagel, and Pedersen, 2009). One possible justification for the small size of variance risk premia is the high volatility of the series themselves. The risk premia often switch sign and display large jumps — mostly negative ones in the early 2000s and a positive one during the most recent financial crisis. These observations are confirmed by Chernov, Graveline, and Zviadadze (2012), who report large jumps in the implied volatility of currency options.

In contrast, we find that implied correlation exceeds realized correlation for all currency pairs: correlation risk premia are positive and economically large. The average correlation risk premium is 14%, a figure comparable to what is observed in the equity

market, allowing us to conclude that there is evidence for compensation for correlation risk in the time-series of exchange rates.[9]

B.   Correlation Risk–Sorted Portfolios

We now turn to quantifying the price of exchange rate correlation risk in the cross-section of currency portfolios. We first sort currencies into portfolios based on their exposure to global correlation risk. Specifically, at the end of each month t, we construct four currency portfolios sorted on the pre-ranking global correlation betas of each currency. We estimate pre-ranking betas from rolling regressions of daily currency excess returns on the global correlation factor using 36-month windows that end in period $t - 1$ (as in Lustig, Roussanov, and Verdelhan, 2011 and Menkhoff, Sarno, Schmeling, and Schrimpf, 2012):

$$rx_t^i = \alpha^i + \beta^{i,\mathrm{RC}}\Delta\mathrm{RC}_t^G + \epsilon_t^i,$$

where rx_t^i is the one-month excess return of currency i, defined as $rx_t^i \equiv f_{t-1}^i - s_t^i$, and $\Delta\mathrm{RC}_t^G$ denotes innovations in the correlation risk factor. Descriptive portfolio statistics for the correlation beta sorted portfolios are reported in Table 4.

[Insert Table 4 here.]

In Panel A of Table 4, we report summary statistics for all countries; in Panel B, we report the statistics for the subsect of developed countries. Correlation-sorted portfolios yield annualized Sharpe ratios ranging between 0.21 and 1.06 (all countries) and between 0.14 and 0.67 (developed countries). Strikingly, mean portfolio returns are almost monotonically decreasing in global FX correlation betas; this is true for both set of currencies (all countries and developed countries only). Regarding the all country set, investing in currencies with high global FX correlation betas leads to significantly lower returns than investing in currencies with low correlation betas: investing in low correlation beta currencies and shorting high correlation beta currencies yields an average return of more

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[9]Driessen, Maenhout, and Vilkov (2009) estimate that the correlation risk premium on the S&P 100 is approximately 18%, with an average realized correlation of 29% and an average implied correlation of 47%.

than 6% and an annualized Sharpe ratio of 0.76. The results remain quantitatively similar when we restrict the sample to developed countries: the difference between the low correlation risk exposure currencies and the high correlation risk exposure currencies is more than 5% per annum, with an annualized Sharpe ratio of 0.56. We also report the pre-formation average forward discounts of the four currency portfolios. Interestingly, portfolio forward discounts are monotonically decreasing in correlation betas implying a connection between correlation betas and the carry trade in currencies.

C. Factor-Mimicking Portfolios

The portfolio sorting exercise yields some evidence that global correlation risk is priced in the cross-section of currency returns. To assess the price of correlation risk, we follow Ang, Hodrick, Xing, and Zhang (2006) and construct a factor-mimicking portfolio of correlation innovations by regressing innovations in the global FX correlation factor on the four interest-rate sorted currency portfolio returns:

$$\Delta \mathrm{RC}_t^G = c + b' \mathbf{rx}_t + u_t,$$

where $\mathbf{rx}_t$ is the vector of excess returns. The factor mimicking portfolio excess return, FRC_t, is the projection of global FX correlaiton innovations on the span of the four currency portfolio returns: $FRC_t \equiv \hat{b}' \mathbf{rx}_t$. We similarly construct the factor mimicking portfolio for the global FX volatility factor and denote its return by FRV_t.

In the first stage Fama and MacBeth (1973) regressions, we estimate the factor betas of the four currency portfolios. In the second stage, we run cross-sectional regressions in order to estimate the factor premia. Panel B of Table 5 reports the premia. The price of the dollar risk factor (DOL) is positive, in line with Lustig, Roussanov, and Verdelhan (2011) and equal to 0.06% per month. The price of correlation risk is -0.09% per month and statistically significant; the price of volatility risk is also negative and statistically significant. The negative price of FX correlation risk is consistent with our previous finding that currencies with higher global FX correlation betas earn lower returns on average.

To identify the currencies which provide a good hedge against global correlation risk, we consider the estimated factor betas for the different currency portfolios reported in Panel A of Table 5.

[Insert Table 5 here.]

A clear pattern emerges: low interest rate currencies have high FX correlation betas, provide a good hedge against increases in global FX correlation. On the other hand, high interest rate currencies have negative FX correlation factor betas and thus command high FX correlation risk premia. The same is true for FX variance betas: high (low) interest rate currencies have low (high) FRV betas. This finding is in line with the results in Menkhoff, Sarno, Schmeling and Schrimpf (2012): FX volatility risk appears to be relevant for explaining carry trade returns in foreign exchange markets. Lastly, the estimated loadings on the dollar factor (DOL) are statistically significant and about equal for all four currency portfolios, consistent with the results reported by Lustig, Roussanov, and Verdelhan (2011) that DOL acts as a level factor for currency returns, while HML, a portfolio long on high interest rate currencies and short on low interest rate currencies is a slope factor.

D.   What Does Correlation Tell Us beyond Volatility?

We have identified two slope factors, global FX correlation and volatility, that account for return differences across currencies with differing interest rates. Therefore, both those factors can be considered to provide economic content to the HML factor identified by Lustig, Roussanov, and Verdelhan (2011). Menkhoff, Sarno, Schmeling and Schrimpf (2012) argue in detail about the importance of the FX volatility factor, whereas we show that the FX correlation factor is priced, too. The natural question that arises is whether those two factors have independent price power for currency returns or whether one can be subsumed into the other.

Recent papers demonstrate that the compensation for volatility risk is actually compensation for correlation risk (see, e.g., Driessen, Maenhout, and Vilkov, 2009) and that

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hedging demand for correlation risk often dominates that for volatility. In order to determine whether FX correlation has pricing power beyond FX volatility, we double-sort currencies on their exposure to those two factors. Specifically, we first orthogonalize the two factors by using FX correlation innovations and the orthogonalized component of FX volatility innovations. Then, we double-sort currencies into four bins based on their FX correlation and volatility loading; we use the full country sample of 20 currencies, so each bin contains 5 currencies at any time. For each of the four portfolios formed, we report subsequent annualized returns in Table 6. We forgo performing a double sort using only developed countries since there would be too few currencies available.

[Insert Table 6 here.]

As seen in Table 6, when we hold exposure to FX volatility constant, exposure to FX correlation continues to be negatively related to returns for both low and high levels of volatility. For the low volatility beta currencies, the low minus high portfolio yields 3.5% per annum. For the the high volatility beta currencies, the return spread between high correlation beta and low correlation beta currencies is about 1.8%. Therefore, correlation risk cannot be subsumed into volatility risk. The reverse is true, as well: when we condition on the exposure to correlation risk, we find that currencies with a lower exposure to volatility risk have higher returns, consistent with the evidence in Menkhoff, Sarno, Schmeling, and Schrimpf (2012). We conclude that correlation risk is priced in the cross-section of currency returns independently of volatility risk.

E. The Link between Global Correlation Risk and Risk Aversion

We have shown that the global FX correlation measure is a priced risk factor, indicating that it may be a proxy for global systematic risk. So far, we have refrained from identifying the economic nature of that risk. However, we can argue that if conditional risk aversion is stochastic, global risk aversion would constitute such a global priced factor and, therefore, our FX correlation factor could potentially proxy for it. To evaluate the connection between our correlation risk factor and global risk aversion, we construct a

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proxy for the global surplus consumption ratio, defined as the real GDP–weighted average of all individual countries' surplus consumption ratios. Following Wachter (2006), we proxy each country's surplus consumption ratio by a weighted moving average of past consumption growth: $\sum_{k=1}^{40} \beta^k \Delta c_{t-k}$, where Δc is real per capita consumption growth and $\beta = 0.99$. We find that the unconditional correlation between our global surplus consumption proxy and the FX correlation risk factor is -0.40, indicating that global FX correlation increases in periods of low surplus consumption and, therefore, high conditional risk aversion.

Another possible empirical proxy for conditional risk aversion is consumer confidence (see Baele, Bekaert, and Inghelbrecht, 2010). To proxy for global consumer confidence, we take data from the Michigan Consumer Confidence and the European Economic Sentiment Indicator and then average these two series.[10] We find that this average has an unconditional correlation of -50% with our correlation risk factor.

Overall, these findings support a positive link between global conditional risk aversion and exchange rate correlation. In the following section, we propose a general equilibrium model that formalizes this link.

III. Model

We consider a general equilibrium model in which preferences are characterized by external habit formation and home bias. We show that, in equilibrium, there are two priced global risk factors: global consumption expenditure, with a positive price, and global conditional risk aversion, with a negative price or risk.

A. Model Setup

In our model, countries, each represented by a stand-in agent, are heterogeneous in endowments and preferences. They have access to frictionless goods markets, as well as to frictionless and dynamically complete financial markets.

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[10]The former time-series can be downloaded from the St. Louis Federal Reserve Economic Database and the latter from the website of the European Commission Economic Databases and Indicators.

A.1. Endowments

The world economy comprises $n + 1$ countries, indexed by i: the domestic country $(i = 0)$ and n foreign countries $(i = 1, ..., n)$, each of which is populated by a single representative agent. There are $n + 1$ distinct perishable goods in the world economy, which are indexed by j, and each agent is initially endowed with a claim on the entirety of the world endowment of the corresponding good. Uncertainty in the economy is represented by a filtered probability space $(\Omega, \mathcal{F}, \mathbf{F}, P)$, where $\mathbf{F} = \{\mathcal{F}_t\}$ is the filtration generated by the standard m-dimensional Brownian motion $\mathbf{B}_t$, $t \in [0, \infty)$, augmented by the null sets. The world endowment stream of good j is denoted by $\{\widetilde{X}_t^j\}$; all endowment processes are Itô processes satisfying:

$$d \log \widetilde{X}_t^j = \mu_t^{j,X} dt + \sigma_t^{j,X\prime} \mathbf{dB}_t, \quad j = 0, 1, \ldots, n$$

with $\sigma_t^{j,X} \neq \mathbf{0}$ for all j. Without loss of generality, the global numéraire is the domestic consumption basket, to be defined below. Because all goods are traded internationally without frictions, the price of each good (in units of the global numéraire) is the same in all countries; the numéraire price of good j is Q^j.

A.2. Preferences

Representative agent i has expected discounted utility:

$$E_0 \left[\int_0^\infty e^{-\rho t} \log(C_t^i - H_t^i) \, dt \right],$$

where $\rho > 0$ is the agent's subjective discount rate, C^i is her level of consumption, and H^i is the time-varying level of consumption habit. Consumption is expressed in units of a composite good, the domestic consumption basket:

$$C^i \equiv \left(\prod_{j=0}^n \left(X^{i,j} \right)^{a^{i,j}} \right),$$

where $X^{i,j}$ is the quantity of good j that agent i consumes. The preferences of agent i with respect to the $n + 1$ goods are described by the vector of preference parameters $\alpha^i = [a^{i,0}, a^{i,1}, ..., a^{1,n}]$ such that $\sum_{j=0}^{n} a^{i,j} = 1$ and $a^{i,j} > 0$ for all i and j. This specification allows for cross-country heterogeneity in consumption preferences, including consumption home bias. We collect the preference parameters in the preference matrix **A**.

The habit level of agent i is external. Instead of specifying a law of motion for the habit level H^i, we specify a law of motion for the inverse surplus consumption ratio $G^i = \frac{C^i}{C^i - H^i}$. Specifically, we assume that the inverse surplus consumption ratio solves the stochastic differential equation:

$$dG_t^i = \varphi \left(\bar{G} - G_t^i \right) dt - \delta \left(G_t^i - l \right) \left(\frac{dC_t^i}{C_t^i} - \mathrm{E}_t \left(\frac{dC_t^i}{C_t^i} \right) \right)$$

as in Menzly, Santos, and Veronesi (2004). The inverse surplus consumption ratio G^i is a stationary process, reverting to its long-run mean of $\bar{G}$ at speed φ. Furthermore, innovations in G^i are perfectly negatively correlated with innovations in the consumption growth of agent i. The parameter $\delta > 0$ scales the size of the innovation in G^i with respect to the innovation in consumption growth. The parameter $l \geq 1$ is the lower bound of the inverse surplus ratio G^i. Importantly, the sensitivity of the inverse surplus consumption ratio to consumption growth innovations is increasing in G^i, which implies large conditional variability of the inverse surplus consumption ratio in bad states of the world. This time variation in volatility is an important aspect of preferences since, as in Campbell and Cochrane (1999), it generates countercyclical time variation in the price of consumption risk. The local curvature of the utility function is:

$$-\frac{u_{CC}(C_t^i, H_t^i)}{u_C(C_t^i, H_t^i)} C_t^i = G_t^i,$$

so, in a slight abuse of terminology, we often refer to G^i as the conditional risk aversion of country i in the remainder of this paper.

A.3. Financial Markets

Financial markets are dynamically complete and frictionless, so agents are able to share risk optimally. As a result, there is a unique state-price density for cash flows expressed in units of the global numéraire, denoted by Λ. The numéraire state-price density satisfies the following law of motion:

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \eta_t' \mathbf{dB}_t,$$

where r is the global numéraire real risk-free rate and η is the market price of risk. Given financial market completeness, each agent i maximizes her utility subject to the following static budget constraint:

$$E_0 \left[\int_0^\infty \frac{\Lambda_t}{\Lambda_0} C_t^i P_t^i \, dt \right] \leq E_0 \left[\int_0^\infty \frac{\Lambda_t}{\Lambda_0} \widetilde{X}_t^i Q_t^i \, dt \right].$$

A.4. Prices and Local Numéraires

Each country i has a local numéraire, the local consumption basket C^i, the price of which in units of the global numéraire is:

$$P_t^i = \prod_{j=0}^n \left(\frac{Q_t^j}{a^{i,j}} \right)^{a^{i,j}}$$

and is defined as the minimum expenditure required to buy a unit of C^i.

Cross-sectional heterogeneity in local numéraires implies cross-sectional heterogeneity in local state-price densities. Specifically, cash flows expressed in units of the local numéraire of country i are priced by Λ^i, the local state-price density, which has the law of motion:

$$\frac{d\Lambda_t^i}{\Lambda_t^i} = -r_t^i dt - \eta_t^{i'} \mathbf{dB}_t,$$

where r^i is the real risk-free rate in units of the local numéraire and η^i is the local market price of risk. Since the global numéraire is the domestic consumption basket (the domestic local numéraire), it follows that $P_t^i = 1$ for all t and that $\Lambda = \Lambda^0$, $r = r^0$ and $\eta = \eta^0$.

A.5. Real Exchange Rates

Real exchange rates represent the relative values of local numéraires. Specifically, the real exchange rate S^i (for $i = 1, \ldots, n$) is the price of the domestic consumption basket expressed in units of the consumption basket of foreign country i:

$$S_t^i = \frac{P_t^0}{P_t^i} = \frac{1}{P_t^i},$$

(5)

so an increase of S^i denotes a real appreciation in the domestic consumption basket. Notably, real exchange rate dynamics reflect cross-country preference heterogeneity: purchasing power parity holds only if the two countries' preferences are identical, so that the two consumption baskets have the same composition and thus the same price. When preferences are heterogeneous, the real exchange rate varies across time.

Real exchange rates can be expressed as ratios of local numéraires. The absence of arbitrage in international financial markets requires real exchange rates to satisfy:

$$S_t^i = \frac{\Lambda_t}{\Lambda_t^i}.$$

(6)

As a result, the law of motion for S_t^i is:

$$\frac{dS_t^i}{S_t^i} = \left[\left(r_t^i - r_t \right) + \eta_t^{i\prime} \left(\eta_t^i - \eta_t \right) \right] dt + \left(\eta_t^i - \eta_t \right)' \mathbf{dB}_t.$$

Real exchange rate volatility arises from the differential exposure of the two local pricing kernels to endowment shocks as encoded in the vector $\eta^i - \eta$. If $\eta^i - \eta \neq \mathbf{0}$, the real exchange rate is exposed to endowment risk, so uncovered interest rate parity does not hold.

B. Equilibrium

In the Appendix, we show that the competitive equilibrium solution is equivalent to the solution of the planner's problem:

$$\max_{\{X_t^{i,j}\}} E_0 \left[\int_0^\infty e^{-\rho t} \left(\sum_{i=0}^n \varrho^i \log \left(C_t^i - H_t^i \right) \right) dt \right]$$

subject to the family of resource constraints:

$$\widetilde{X}_t^j = \sum_{i=0}^n X_t^{i,j} \text{ , for } j = 0, ..., n,$$

for all t, where ϱ^i, $i = 0, ..., n$ is the welfare weight of country i. Without loss of generality, we normalize the welfare weights so that they sum to one: $\sum_{i=0}^n \varrho^i = 1$.

B.1. Local State-Price Densities

In equilibrium, the state-price density of the local numéraire of country i is given by the discounted marginal utility of the local consumption basket, scaled by the welfare weight ϱ^i:

$$\Lambda_t^i = e^{-\rho t} \varrho^i \frac{G_t^i}{C_t^i}. \tag{7}$$

Therefore, the risk-free rate in country i is:

$$r_t^{i,C} = \rho + \mu_t^{i,C} + \varphi \left(\frac{G_t^i - \bar{G}}{G_t^i} \right) - \left(1 + \delta \left(\frac{G_t^i - l}{G_t^i} \right) \right) \sigma_t^{i,C\prime} \sigma_t^{i,C},$$

where $\mu_t^{i,C}$ is the conditional mean of the consumption growth rate of country i. An increase in G^i has two opposing effects on the equilibrium risk-free rate: while it increases current marginal utility, and thus raises the agent's desire to intertemporally smooth her marginal utility by reducing current saving, it also increases the desire for precautionary savings. The relative strength of these two effects is largely determined by the preference parameters φ and δ. On the one hand, an increase in φ, the speed of mean-reversion of conditional risk aversion, increases the importance of the intertemporal smoothing

motive by increasing the probability that future marginal utility will be lower. On the other hand, an increase in δ, the sensitivity of risk aversion changes to consumption shocks, increases the conditional variability of marginal utility and thus the agent's incentive to accumulate precautionary savings.

The local market price of risk is given by:

$$\eta_t^i = \left(1 + \delta \left(\frac{G_t^i - l}{G_t^i} \right) \right) \sigma_t^{i,C},$$

and includes two familiar components. The first component is the conditional sensitivity of the price of risk to consumption growth variability and is increasing in conditional risk aversion G_t^i. In the absence of external habit formation, the sensitivity term would be constant and equal to 1, the relative risk aversion implied by log utility. However, external habit formation induces time variation in conditional risk aversion and hence in the sensitivity of the price of risk. The second component, conditional consumption growth volatility, is largely determined by the degree of optimal international risk sharing. As in Stathopoulos (2012a), the optimal allocation of risk implies that countries with cross-sectionally low conditional risk aversion insure countries with cross-sectionally high conditional risk aversion by assuming more of the global endowment risk. As a result, countries with high conditional risk aversion relative to the rest of the world tend to have relatively high consumption growth volatility. It is important to note that we assume that all countries have identical unconditional risk aversion. This implies that the identity of the countries providing insurance to the rest of the world varies across time.

B.2. Global Risk Factors and Currency Returns

It is shown in the Appendix that the global numéraire state-price density is increasing in global conditional risk aversion G^W and decreasing in global consumption expenditure $\mathcal{C}^W$:

$$\Lambda_t = e^{-\rho t} \frac{G_t^W}{\mathcal{C}_t^W}. \tag{8}$$

Global conditional risk aversion is defined as the welfare-weighted average of all countries'
conditional risk aversions:

$$G_t^W \equiv \sum_{i=0}^{n} \varrho^i G_t^i.$$

Global consumption expenditure is the sum of all countries' consumption expenditure
and, given market clearing, equals the value of the global endowment:

$$\mathcal{C}_t^W \equiv \sum_{i=0}^{n} \left(C_t^i P_t^i \right) = \sum_{i=0}^{n} \left(\widetilde{X}_t^j Q_t^j \right).$$

We focus on the properties of the excess currency return, defined as the return of
the foreign money market account (in units of the domestic numéraire) in excess of the
domestic risk-free rate:

$$dR_t^i = \eta_t' \left(\eta_t - \eta_t^i \right) dt + \left(\eta_t - \eta_t^i \right)' \mathbf{dB}_t.$$

In terms of risk, this is a long position in the local numéraire and a short position in the
global numéraire.

The currency risk premium is determined by the exposure of the currency return to
the two global risk factors:

$$\mathrm{E}_t \left(dR_t^i \right) = -\mathrm{E}_t \left(dR_t^i \frac{d\Lambda_t}{\Lambda_t} \right) = \lambda_t^C \beta_t^{i,C} + \lambda_t^G \beta_t^{i,G}.$$

The price of the global consumption expenditure factor is positive, since bad states of
the world are associated with low global consumption expenditure, while the price of the
global risk aversion factor is negative, as high global risk aversion states are undesirable:

$$\lambda_t^C \equiv \mathrm{Var}_t \left(\frac{d\mathcal{C}_t^W}{\mathcal{C}_t^W} \right), \qquad \lambda_t^G \equiv -\mathrm{Var}_t \left(\frac{dG_t^W}{G_t^W} \right).$$

IV. Calibration and Simulation Results

In this section, we first calibrate our model and confirm that it is able to match several
salient empirical moments. Then, we show that conditional exchange rate variance and

correlation are positively associated with conditional global risk aversion. Finally, we establish that the main driver of cross-sectional variation in currency risk premia is differential exposure to the global risk aversion factor and show that, given a sufficiently strong precautionary savings motive, differences in exposure can generate a carry trade effect.

A. Calibration

We consider a global economy of 21 countries, the United States (domestic country) and $n = 20$ foreign countries. In order to simulate our economy, we adopt a simple parametric specification for the endowment drift and diffusion processes and for the preference matrix $\mathbf{A}$.

We consider a symmetric calibration for endowment growth. In particular, we assume that endowment growth in all countries has the same constant conditional mean μ^X and constant conditional standard deviation σ^X. Furthermore, the conditional correlation coefficient for any two countries' endowment growth rates is assumed to be constant and equal to θ^X.

We allow for heterogeneity in preferences between the domestic and the foreign countries. Specifically, the preference matrix $\mathbf{A}$ is parameterized by the US preference parameter a and the foreign preference parameter a^*, so that:

$$
\mathbf{A} = \begin{bmatrix}
a & (1-a)/n & \cdots & (1-a)/n \\
(1-a^*)/n & a^* & \cdots & (1-a^*)/n \\
\vdots & \vdots & \ddots & \vdots \\
(1-a^*)/n & (1-a^*)/n & \cdots & a^*
\end{bmatrix}.
$$

To calibrate the model parameters, we use data on the United States and all foreign countries from our empirical discussion except for Singapore and Kuwait.[11] For each country, we collect quarterly data on nominal GDP, the GDP deflator, exports and imports of goods and services, private consumption expenditure and the consumer price

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[11]Kuwait is excluded for lack of quarterly data, while Singapore is excluded because its openness ratio exceeds 1 (which implies a negative preference parameter a^*).

index. To construct per capita measures, we also collect annual midyear population data and assume that population growth is constant within each four-quarter period corresponding to midyear endpoints. Our sample period is 1999:Q1 through 2010:Q4.

The annualized calibration parameters are reported in Table 7. To calibrate the endowment parameters, we proxy endowment growth by real per capita GDP growth. Given our symmetric endowment growth parameterization, we set the common endowment growth mean μ^X (volatility σ^X) equal to the cross-sectional average of the time-series average (respectively, standard deviation) of the real per capita GDP growth rates. We also set the common pairwise endowment correlation coefficient θ^X equal to the cross-sectional average pairwise correlation coefficient of real per capita GDP growth rates.

[Insert Table 7 here.]

The preference parameters a and a^* are calibrated using the openness ratio, defined as $\frac{1}{2} \times$ (Imports + Exports)/GDP; in the model, the steady-state openness ratio of the domestic (foreign) country equals $1 - a\,(1 - a^\star)$. The US preference home bias parameter, 0.87, corresponds to the time-series mean of the US openness ratio, equal to 0.13 over the sample period. Similarly, the foreign preference home bias calibrated parameter, 0.60, corresponds to the cross-sectional average of all foreign countries' mean openness ratio, equal to 0.40. We calibrate the five habit parameters, (ρ, φ, δ, $\bar{G}$, and l) following the spirit of the Menzly, Santos, and Veronesi (2004) calibration. In particular, they calibrate the parameters in order to match five US real interest rate and US real equity return moments. In the same vein, we adopt their calibrated values for the subjective rate of time preference ρ, the steady-state conditional risk aversion $\bar{G}$, and the lower bound of conditional risk aversion l. However, given the importance of real interest rates for our model, we calibrate the two key habit parameters, the parameter of conditional risk aversion mean-reversion (φ) and the parameter of conditional risk aversion sensitivity to consumption growth shocks (δ), to match two key interest rate moments in our sample. Specifically, we adopt a simulated method of moments approach and target the cross-

sectional average of the time-series mean and standard deviation of the real interest rates.[12]

Table 8 presents the simulated moments and compares them to the empirical ones.[13] For each moment, we refer to the cross-sectional average across countires for both simulated and empirical data. The simulated consumption growth mean and standard deviation are in line with the empirical counterparts. Importantly, the model can match the volatility of exchange rates. On the other hand, the model overshoots consumption growth correlation and, as a result, real exchange rate and real interest rate correlation across countries. This is due to the moderate degree of preference home bias in our calibration: although countries have heterogeneous preferences, the heterogeneity is not strong enough to significantly dampen their desire to share risk. As a result, countries therefore use the financial markets to align their marginal utility growth almost perfectly.

[Insert Table 8 here.]

B. The Cross-Section of Currency Risk Premia

To study the cross-section of currency risk premia, we set the conditional risk aversion of all countries equal to its common steady-state value and consider the effects of a negative innovation of one standard deviation in the first foreign country (country 1). The impulse response functions are presented in Figure 1. Although the negative shock affects only the endowment of country 1 (Panel A), all countries experience a negative consumption shock due to international risk sharing. However, the risk sharing is not full, due to the preference home bias: equilibrium consumption is home biased, so the negative consumption shock is higher (in absolute value) in country 1. Therefore, while conditional risk aversion rises in all countries, the increase is relatively higher in country 1 (Panel B).

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[12]We draw 1,000 simulation paths, each consisting of 384 monthly observations (32 years) and discard the first 240 observations to reduce the impact of initial conditions. Therefore, the calibration uses 1,000 paths of 144 monthly observations each, matching the 12-year size of our sample. The 95% confidence interval is $[0.037, 0.043]$ for φ and $[201.92, 294.76]$ for δ.

[13]The moments are calculated using 1,000 paths of 144 monthly observations each; for each path, 240 initial observations were discarded.

The price of risk is countercyclical: the price of both systematic risk factors, global consumption expenditure and global risk aversion, increases in absolute value (Panel C). Since the price of the global risk aversion factor is significantly higher in magnitude than the price of global consumption expenditure, it follows that risk premia are largely determined by exposure to global risk aversion. As a result, the cross-section of currency risk premia essentially mirrors the cross-section of global risk aversion betas.

[Insert Figure 1 here.]

The adverse endowment shock in country 1 lowers the relative riskiness of its currency: the consumption expenditure beta of currency 1 increases less than that of the other foreign currencies (Panel D), while its global risk aversion beta increases more (Panel E), implying that currency 1 becomes a relatively better hedge against both priced risk factors. As a result, currency 1 commands a lower risk premium (Panel F).

To understand the positive relationship between a country's conditional risk aversion and the global risk aversion beta of its currency, it is useful to consider the determinants of a currency's global risk aversion beta. If the growth rate of conditional risk aversion is highly correlated across countries, a condition satisfied in our calibration, exchange rate loading $\beta^{i,G}$ is given by:

$$\beta_t^{i,G} \simeq \frac{\sigma_t \left(\frac{dG_t^i}{G_t^i} \right)}{\sigma_t \left(\frac{dG_t^W}{G_t^W} \right)} - 1.$$

As mentioned when discussing our habit specification, the conditional volatility of conditional risk aversion changes is increasing in the level of conditional risk aversion. As a result, the cross-section of $\beta^{i,G}$ largely depends on the cross-section of G^i. Intuitively, the conditionally more risk averse countries experience larger fluctuations in risk aversion. As a result, global risk aversion shocks tend to arise primarily from risk aversion fluctuations in the more risk averse countries. It follows that the more risk averse a country is, the more it contributes to global risk aversion volatility and, therefore, the more its risk aversion shocks are correlated with global risk aversion innovations. Therefore, loadings $\beta^{i,G}$ are increasing in G^i: the currencies of countries with high conditional risk

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aversion provide a hedge against increases in global risk aversion, while the currencies of countries with low relative conditional risk aversion are very exposed to increases of global risk aversion (negative $\beta^{i,G}$) and, thus, command high risk premia.

C.   Global Risk Aversion and Exchange Rate Second Moments

In our calibration, conditional real exchange rate volatility and correlation are increasing in global risk aversion. Figure 2 illustrates the dependence of several key variables on global risk aversion, assuming that conditional risk aversion is identical across countries ($G_t^i = G_t^W$ for all i); the horizontal axis measures the value of G_t^i, ranging from 20 to 50, while the vertical axis measures different variables of interest. Eliminating cross-sectional heterogeneity in conditional risk aversion allows us to abstract from cross-country insurance effects and focus exclusively on the effects of parallel shifts in conditional risk aversion.

Given the existence of complete financial markets, countries are able to achieve the optimal level of international risk sharing. However, preference home bias implies that optimal risk sharing is not identical to perfect consumption pooling: there is a tension between the desire to share risk, which tends to induce identical behavior across countires, and preference home bias, which induces heterogeneity in consumption allocations.

[Insert Figure 2 here.]

As global risk aversion increases, the desire to share risk becomes stronger. Panels A and C of Figure 2 present the conditional variance of consumption growth rates and SDFs; we report the variance for the domestic country and for any of the foreign countries, since they are all identical. An increase in global risk aversion generates two opposing effects on conditional SDF volatility η^i: On the one hand, the increased desire of countries to share risk decreases the variance of the consumption growth rates (Panel A), reducing η^i. However, the increase in risk aversion increases the sensitivity component of the SDF and this rise overwhelms the decrease in consumption risk, which increases SDF variance (Panel C). Panels B and D present the conditional correlation

of consumption growth rates and SDFs across countries. In each panel, we report the correlation between the domestic country and any of the foreign countries, as well as between any two foreign countries. We see that as global risk aversion increases, the stronger desire of countries to share risk increases the correlation of consumption growth (Panel B) and marginal utility growth (Panel D).

Panels E, F, G, and H focus on the conditional second moments of real exchange rates. The conditional real exchange rate volatility of currency i against the domestic currency can be decomposed into the amount of aggregate risk, defined as:

$$\text{RP}_t^{i,0} \equiv \text{Var}_t\left(\frac{d\Lambda_t^i}{\Lambda_t^i}\right) + \text{Var}_t\left(\frac{d\Lambda_t^0}{\Lambda_t^0}\right) = \eta_t^{i\prime}\eta_t^i + \eta_t^{0\prime}\eta_t^0,$$

and the proportion of aggregate risk that is internationally shared, as given by the Brandt, Cochrane and Santa-Clara (2006) risk sharing index:

$$\text{RS}_t^{i,0} = 1 - \frac{\left(\sigma_t^i\right)^2}{\text{Var}_t\left(\frac{d\Lambda_t^i}{\Lambda_t^i}\right) + \text{Var}_t\left(\frac{d\Lambda_t^0}{\Lambda_t^0}\right)} = 1 - \frac{\left(\eta_t^i - \eta_t^0\right)'\left(\eta_t^i - \eta_t^0\right)}{\eta_t^{i\prime}\eta_t^i + \eta_t^{i\prime}\eta_t^0}$$

The index ranges between 0 when there is no risk sharing between the two countries, and 1, when risk sharing between the two countries is perfect. Exchange rate variance represents the amount of unshared risk, so it equals the product of the proportion of aggregate risk not shared and the amount of aggregate risk:

$$\left(\sigma_t^i\right)^2 = (1 - \text{RS}_t^{i,0})\text{RP}_t^{i,0}.$$

When global risk aversion increases, the decrease in consumption risk is not enough to offset the increase in global risk aversion, so $\text{RP}_t^{i,0}$ increases (Panel G). On the other hand, risk sharing increases (Panel H). The risk pricing component dominates, increasing exchange rate volatility. The intuition is similar for exchange rate covariances: they are increasing in global risk aversion (Panel E). In sum, when global risk aversion increases, the increase in the market price of risk is large enough to more than offset the reduction in the disparity in marginal utility growth across countries. As a result, exchange rate variances and covariances are increasing in global risk aversion regardless of the degree

of preference home bias. Therefore, our model provides a global equilibrium justification for the global exchange rate volatility factor proposed by Menkhoff, Sarno, Schmeling and Schrimpf (2012).

In our calibration, conditional real exchange rate correlation is also increasing in global risk aversion (Panel F). This result is solely due to the assumption that the domestic country is more home biased than the foreign countries. Intuitively, the domestic country has a relatively lower desire to engage in risk sharing with the rest of the world: the correlation of marginal utility growth is higher between any two foreign countries than between the domestic country and any foreign country for all levels of global risk aversion (Panel D). As global risk aversion increases, the increased desire for international risk sharing aligns the marginal utility growth of the foreign countries. Since the real exchange rate measures the disparity between the marginal utility of a given foreign country with that of the domestic country, the aforementioned alignment of marginal utility growth across foreign countries increases real exchange rate correlation. Exactly the reverse would happen if the domestic country was characterized by lower home bias than the foreign countries: in such a case, exchange rate correlation would be decreasing in global risk aversion. Lastly, if home bias is identical across all countries, exchange rate correlation does not change with global risk aversion. Empirically, the United States is more home biased than the average foreign country, so the model prediction for the conditional correlation of exchange rates against the USD is as given in Panel F. Therefore, for exchange rates against the USD, both average exchange rate variance and correlation can serve as proxies for global risk aversion.

D.   Sorting on Conditional Global Risk Aversion Betas

As discussed previously, risk premia compensate investors for exposure to two priced global risk factors: the global consumption expenditure factor and the global risk aversion factor. In our calibration, the price of the latter is significantly higher than the price of the former. This result is typical in models that reply on the variability of the surplus consumption ratio in order to generate substantial volatility in the SDF. As a

result, the cross-section of currency excess returns largely mirrors the cross-section of global risk aversion betas.

[Insert Table 9 here.]

Table 9 illustrates that point. We sort 20 foreign currencies into four portfolios according to their conditional global risk aversion beta, with Portfolio 1 containing currencies in the lowest $\beta^{i,G}$ quartile and Portfolio 4 containing the currencies in the highest $\beta^{i,G}$ quartile. As expected, there is a monotonic negative relationship between global risk aversion betas and average currency portfolio returns: Portfolio 1 which has the lowest risk aversion beta and, thus, the highest adverse exposure to the global risk aversion factor, outperforms Portfolio 4 by about 8% in annual terms.

E.   Sorting on Forward Discounts

Finally, we explore the ability of our model to address the forward premium puzzle. Table 1 reports the summary statistics on portfolios sorted on interest rate differentials (forward discounts): Portfolio 1 contains currencies in the bottom forward discount quantile (low interest rate currencies), while Portfolio 4 comprises the high interest rate currencies. Since the cross-section of currency risk premiums is largely determined by the cross-section of global risk aversion betas, Table 10 implies that high (low) interest rate currencies have low (high) global risk aversion betas, i.e. that they depreciate (appreciate) in bad states of the world, when global risk aversion is high.

[Insert Table 10 here.]

This result follows from the strong precautionary savings motive implied by our calibrated parameters. Countries with relatively high (low) conditional risk aversion tend to have relatively low (high) real interest rates.[14] As a result, the high interest

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[14]To generate a carry trade effect, Verdelhan (2010) also assumes that the precautionary savings motive dominates the intertemporal smoothing motive, inducing procyclical real interest rates. Ang, Bekaert, and Wei (2008) and Ang and Ulrich (2012) provide both theoretical and empirical support for procyclical real interest rates in the US.

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rate currencies are the currencies of low conditional risk aversion countries. As shown in Figure 1, low conditional risk aversion is associated with a low exchange rate global risk aversion beta. Therefore, the low interest rate currencies (Portfolio 1) provide a good hedge against increases in global risk aversion and thus have negative average returns, whereas the high interest rate currencies (Portfolio 4) are exposed to considerable global risk aversion risk and, therefore, command high risk premia.

Here we explore the ability of our model to address the forward premium puzzle, as illustrated in Table Table 10 reports the summary statistics for portfolios sorted on interest rate differentials (forward discounts): Portfolio 1 contains currencies ranked in the bottom forward discount quartile (low interest rate currencies), while Portfolio 4 contains the high interest rate currencies. Since the cross-section of currency risk premia is mainly determined by the cross-section of global risk aversion betas, Table 10 implies that high (low) interest rate currencies have low (high) global risk aversion betas – in other words, that such currencies depreciate (appreciate) in bad states of the world, when global risk aversion is high.

V.   Conclusion

We show that FX correlation risk is priced in the foreign exchange market. First, we establish that the difference between the risk-neutral and the objective measure of FX correlation is almost always positive and averages 15% in a cross-section of exchange rates. Then, we construct an FX correlation risk factor and demonstrate that its price is negative and economically significant (about -1% per year). Sorting currencies into portfolios on the basis of their exposure to our FX correlation factor, we find that a strategy of investing in low FX correlation beta currencies and shorting high FX correlation beta currencies yields attractive returns and Sharpe ratios. We also show that exposure to correlation risk can explain FX carry trade returns: high interest rate currencies are highly exposed to FX correlation risk, whereas low interest rate currencies provide a hedge against adverse FX correlation innovations.

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Motivated by our empirical findings, we propose a general equilibrium model that features time variation in conditional risk aversion and establishes a positive relationship between conditional risk aversion and conditional FX correlation. Therefore, hedging against increases in FX correlation is tantamount to hedging against increases in global risk aversion. Moreover, when the precautionary savings motive is sufficiently strong, we show that high interest rate currencies command high risk premia because of their high exposure to the global risk aversion factor. This finding provides a risk-based explanation for the empirically observed violations of uncovered interest rate parity that are exploited by the FX carry trade.

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Appendix A Proofs

Equilibrium Prices and Quantities

Under the assumption of market completeness, there is a unique global numéraire state-price density, Λ, that satisfies the following stochastic differential equation:

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \eta_t' \mathbf{dB}_t,$$

where r is the global numéraire risk-free rate and η is the market price of risk process.

Using Λ, the intertemporal budget constraint of agent i can be written in static form as:

$$\mathrm{E}_0 \left[\int_0^\infty \frac{\Lambda_t}{\Lambda_0} C_t^i P_t^i \, dt \right] \leq \mathrm{E}_0 \left[\int_0^\infty \frac{\Lambda_t}{\Lambda_0} \widetilde{X}_t^i Q_t^i \, dt \right]$$

or

$$\mathrm{E}_0 \left[\int_0^\infty \frac{\Lambda_t}{\Lambda_0} \left(\sum_{j=0}^{n+1} X_t^{i,j} Q_t^j \right) \, dt \right] \leq \mathrm{E}_0 \left[\int_0^\infty \frac{\Lambda_t}{\Lambda_0} \widetilde{X}_t^i Q_t^i \, dt \right].$$

After replacing each agent's intertemporal dynamic budget constraint with her static budget constraint, we can solve for the competitive equilibrium. The first order conditions (FOCs) of agent i are:

$$e^{-\rho t} a^i \frac{G_t^i}{X_t^{i,j}} = \frac{1}{\varrho^i} \Lambda_t Q_t^j, \text{ for all } j,$$

where $\frac{1}{\varrho^i}$ is the Lagrange multiplier associated with the budget constraint of agent i holding with equality. Combining the FOCs with the market clearing conditions:

$$\sum_{k=0}^n X_t^{i,j} = \tilde{X}_t^j, \text{ for all } j,$$

we derive the equilibrium consumption allocation:

$$X_t^{i,j} = \frac{a^{i,j} \varrho^i G_t^i}{\sum_{k=0}^n a^{k,j} \varrho^k G_t^k} \widetilde{X}_t^j.$$

To calculate the Lagrange multipliers $\frac{1}{\varrho^i}$, we substitute equilibrium quantities and prices in the static budget constraint of agent i (holding with equality). After some algebra, we obtain:

$$\varrho^i \left(\varphi \bar{G} + \rho G_0^i \right) = \sum_{k=0}^n a^{k,i} \varrho^k \left(\varphi \bar{G} + \rho G_0^k \right).$$

This system of equations has solutions of the form

$$\frac{\varrho^i}{\varrho^0} = b^i \frac{\varphi \bar{G} + \rho G_0^0}{\varphi \bar{G} + \rho G_0^i},$$

where the vector $\mathbf{b} = [b^1, b^2, ..., b^n]'$ is the unique solution of

$$\mathbf{b} = \begin{bmatrix} a^{0,1} & a^{1,1} & \cdots & a^{n,1} \\ a^{0,2} & a^{1,2} & \cdots & a^{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ a^{0,n} & a^{1,n} & \cdots & a^{n,n} \end{bmatrix} \begin{bmatrix} 1 \\ \mathbf{b} \end{bmatrix}.$$

The budget constraint determines only the ratios $\frac{\varrho^i}{\varrho^0}$. To pin down the values for the Lagrange multipliers, we impose the normalization $\sum_{i=0}^n \varrho^i = 1$.

It can easily be shown that, when law of motion for each agent's inverse surplus consumption ratio is taken to be exogenous, the planner's problem solution is equivalent to the competitive equilibrium solution if each country's welfare weight is set equal to ϱ^i.

Equilibrium Consumption Processes

Since equilibrium consumption $C = [C^0, \cdot, C^n]'$ is a function of the vector of conditional risk aversion $G = [G^0, \cdot, G^n]'$, we must solve for the fixed point that satisfies both the equilibrium consumption allocations and the law of motion for G. By definition of the consumption baskets, we have:

$$C^i \equiv \left(\prod_{j=0}^n \left(X^{i,j} \right)^{a^{i,j}} \right), \text{ for all } i.$$

Now applying Itô's lemma and equating the diffusion terms, we have, after some algebra that

$$\sigma_t^C = \left(\Psi_t^{-1} \mathbf{A} \right) \sigma^X,$$

where σ_t^C is the $(n+1) \times m$ consumption volatility matrix

$$\sigma_t^C = \begin{bmatrix} \sigma_t^{0,C\prime} \\ \vdots \\ \sigma_t^{n,C\prime} \end{bmatrix}$$

σ^X is the $(n+1) \times m$ endowment volatility matrix

$$\sigma^X = \begin{bmatrix} \sigma^{0,X\prime} \\ \vdots \\ \sigma^{n,X\prime} \end{bmatrix}$$

and Ψ is the $(n+1) \times (n+1)$ matrix defined as

$$\Psi_t = [\psi_{i,j}] = \psi_t^{i-1,j-1},$$

where

$$\psi_t^{i,i} \equiv 1 + \left(1 - \sum_{j=0}^n \frac{a^{i,j} a^{i,j} \varrho^i G_t^i}{\sum_{k=0}^n a^{k,j} \varrho^k G_t^k} \right) \delta \left(\frac{G_t^i - l}{G_t^i} \right)$$

and

$$\psi_t^{i,i'} \equiv - \left(\sum_{j=0}^n \frac{a^{i,j} a^{i',j} \varrho^{i'} G_t^{i'}}{\sum_{k=0}^n a^{k,j} \varrho^k G_t^k} \right) \delta \left(\frac{G_t^{i'} - l}{G_t^{i'}} \right), \ i \neq i'.$$

Note that (5), (6) and (7) imply that:

$$\Lambda_t = e^{-\rho t} \varrho^i \frac{G_t^i}{C_t^i} \frac{1}{P_t^i}.$$

Rearranging as:

$$\Lambda_t \left(C_t^i P_t^i \right) = e^{-\rho t} \left(\varrho^i G_t^i \right)$$

and summing over all countries, we obtain:

$$\Lambda_t \sum_{i=0}^{n} \left(C_t^i P_t^i \right) = e^{-\rho t} \sum_{i=0}^{n} \left(\varrho^i G_t^i \right).$$

Expression (8) now follows from the definition of global risk aversion and global consumption expenditure.

Appendix B Tables

Table 1
Summary Statistics for Carry Trade Portfolios

This table reports summary statistics for currency portfolios sorted on time $(t-1)$ forward discounts. Portfolio 1 (Pf1) contains 25% of all the currencies with the lowest forward discounts whereas Portfolio 4 (Pf4) contains currencies with the highest forward discounts. All returns are excess returns in USD. DOL denotes the average return of the four currency portfolios, HML denotes a long-short portfolio that is short in Pf1 and long in Pf4. Data are sampled monthly and run from January 1999 through December 2010.

	Pf1	Pf2	Pf3	Pf4	DOL	HML
Mean	-0.004	0.027	0.048	0.088	0.040	0.092
StDev	0.068	0.081	0.080	0.097	0.073	0.078
Skewness	0.108	-0.208	-0.120	-1.445	-0.435	-1.070
Kurtosis	2.613	3.632	4.808	9.394	4.683	6.773
Sharpe Ratio	-0.065	0.334	0.596	0.908	0.540	1.189

Table 2
Summary Statistics for Volatilities

This table reports summary statistics for implied and realized volatilities (i.e. the square root of variance, Panels A and B) and the variance risk premium, which is defined as the difference between the implied and realized variance (Panel C). Implied variances are calculated from daily option prices on the underlying exchange rates. Realized variances are calculated from five minute tick data on the underlying spot exchange rates. All numbers are annualized. Data run from January 1999 through December 2010.

	EUR	JPY	GBP	CHF
PANEL A: IMPLIED VOLATILITY				
Mean	0.110	0.117	0.097	0.112
Max	0.294	0.321	0.298	0.246
Min	0.049	0.065	0.050	0.057
StDev	0.035	0.036	0.036	0.027
Skewness	1.943	2.052	2.808	1.659
Kurtosis	9.602	10.215	13.087	9.127
AC(1)	0.85	0.76	0.87	0.81
PANEL B: REALIZED VOLATILITY				
Mean	0.110	0.116	0.097	0.115
Max	0.319	0.349	0.347	0.251
Min	0.049	0.042	0.050	0.054
StDev	0.039	0.044	0.038	0.032
Skewness	2.140	2.009	3.163	1.088
Kurtosis	10.986	8.987	17.525	5.138
AC(1)	0.49	0.39	0.69	0.51
PANEL C: VARIANCE RISK PREMIUM				
Mean	0.000	-0.001	-0.001	-0.002
Max	0.030	0.037	0.024	0.024
Min	-0.077	-0.045	-0.057	-0.056
StDev	0.009	0.009	0.007	0.008
Skewness	-3.923	-0.912	-3.732	-2.606
Kurtosis	36.740	9.248	29.635	16.905
AC(1)	0.19	0.16	0.08	0.30

Table 3
Summary Statistics for Correlation

This table reports summary statistics for implied and realized correlations (Panels A and B) and the correlation risk premium, which is the difference between the implied and realized correlation (Panel C). Implied correlations are calculated from daily option prices on the underlying exchange rates. Realized correlations are calculated from five-minute tick data on the underlying spot exchange rates. Data are annualized and run from January 1999 through December 2010.

	EURJPY	EURGBP	EURCHF	JPYGBP	JPYCHF	GBPCHF
PANEL A: IMPLIED CORRELATION						
Mean	0.396	0.690	0.895	0.303	0.472	0.643
Max	0.689	0.863	0.986	0.661	0.811	0.877
Min	-0.241	0.370	0.541	-0.406	-0.016	0.326
StDev	0.191	0.102	0.086	0.216	0.175	0.140
Skewness	-0.667	-0.632	-2.593	-0.562	-0.683	-0.468
Kurtosis	2.957	2.978	10.435	3.218	3.051	2.344
AC(1)	0.74	0.78	0.81	0.81	0.70	0.84
PANEL B: REALIZED CORRELATION						
Mean	0.294	0.552	0.728	0.194	0.308	0.473
Max	0.640	0.780	0.928	0.580	0.716	0.761
Min	-0.375	0.165	0.300	-0.355	-0.249	0.110
StDev	0.228	0.127	0.126	0.201	0.196	0.135
Skewness	-0.584	-0.684	-0.680	-0.342	-0.260	-0.113
Kurtosis	2.931	2.824	3.013	2.771	2.394	2.296
AC(1)	0.82	0.88	0.79	0.82	0.85	0.84
PANEL C: CORRELATION RISK PREMIUM						
Mean	0.103	0.139	0.168	0.109	0.164	0.169
Max	0.620	0.443	0.537	0.519	0.523	0.491
Min	-0.221	-0.165	-0.049	-0.345	-0.078	-0.160
StDev	0.136	0.117	0.127	0.141	0.125	0.136
Skewness	0.681	0.258	0.809	-0.364	0.421	-0.137
Kurtosis	4.212	3.079	3.029	4.118	2.829	2.574
AC(1)	0.25	0.72	0.77	0.27	0.36	0.68

Table 4
Portfolios Sorted on Betas with Correlation Risk

This table reports summary statistics for currency portfolios sorted on correlation risk betas, i.e. currencies are sorted according to their betas in a rolling time-series regression of individual currencies' daily excess returns on daily innovations in the correlation risk factor. Correlation risk is defined as the residual from an AR(1) process of realized correlation. Portfolio 1 (Pf1) contains currencies with the lowest betas whereas Portfolio 4 (Pf4) contains currencies with the highest betas. LMH is long Portfolio 1 and short Portfolio 4. Numbers are annualized. We also report pre-formation betas, Pre β and pre-formation forward discounts for each portfolio (in % per year). Pre-formation discounts are calculated at the end of each month prior to portfolio formation. Data run from January 1999 through December 2010.

	Pf1	Pf2	Pf3	Pf4	LMH
PANEL A: ALL COUNTRIES					
Mean	0.084	0.054	0.020	0.023	0.061
StDev	0.080	0.088	0.095	0.100	0.081
Skewness	0.348	-0.287	-0.466	-0.346	0.423
Kurtosis	3.601	6.767	5.223	3.422	3.446
Sharpe Ratio	1.057	0.609	0.208	0.231	0.757
Pre β	-100.315	-11.005	48.272	141.281	
Pre $f - s$	0.34	0.29	0.26	0.14	
PANEL B: DEVELOPED COUNTRIES					
Mean	0.065	0.039	0.026	0.014	0.051
StDev	0.097	0.099	0.108	0.099	0.091
Skewness	-0.030	-0.200	-0.174	-0.476	0.812
Kurtosis	4.003	3.982	3.986	4.280	6.664
SR	0.671	0.388	0.237	0.141	0.557
Pre β	-89.464	-3.663	41.651	114.288	
Pre $f - s$	0.06	0.05	0.04	0.03	

Table 5
Estimating the Price of Correlation Risk

Test assets are the four carry trade portfolios based on either all countries or developed countries only. FRC (FRV) is the mimicking factor for global correlation (volatility) innovations, DOL the average carry trade portfolio as in Lustig, Roussanov, and Verdelhan (2011). In Panel A, we report factor betas. Panel B reports the Fama and MacBeth (1973) factor prices on the carry return portfolios. Newey and West (1987) standard errors are reported in parentheses. Data run from January 1999 through December 2010.

PANEL A: FACTOR BETAS

Pf	α	ALL COUNTRIES DOL	FRC	FRV	R^2	α	DEVELOPED COUNTRIES DOL	FRC	FRV	R^2
1	-0.01	0.95	0.61	0.57	0.88	0.00	0.74	0.05	0.42	0.60
	(0.00)	(0.02)	(0.02)	(0.02)		(0.00)	(0.08)	(0.01)	(0.03)	
2	-0.01	1.11	-0.11	-0.30	0.90	0.00	0.99	0.00	0.05	0.83
	(0.00)	(0.03)	(0.03)	(0.01)		(0.00)	(0.06)	(0.00)	(0.06)	
3	0.00	1.04	-0.16	-0.45	0.60	0.00	1.06	-0.02	-0.36	0.83
	(0.00)	(0.04)	(0.04)	(0.03)		(0.00)	(0.07)	(0.00)	(0.05)	
4	0.01	0.95	-0.42	-1.10	0.98	0.00	1.20	-0.05	-0.94	0.84
	(0.00)	(0.02)	(0.02)	(0.20)		(0.00)	(0.05)	(0.01)	(0.06)	

PANEL B: FACTOR PRICES

DOL	ALL COUNTRIES FRC	FRV	R^2	DOL	DEVELOPED COUNTRIES FRC	FRV	R^2
0.06	-0.09	-0.06	0.90	0.05	-0.08	-0.06	0.95
(0.01)	(0.01)	(0.01)		(0.01)	(0.01)	(0.01)	

Table 6
Double Sorting on Volatility and Correlation

This table reports the results of double sorts of currencies. We independently sort currencies into halves based on their exposure to volatility and correlation and then form portfolios on the intersection. For each of the four portfolios formed, we report the average return (annualized). Data is monthly and runs from January 1999 through December 2010.

	Low Corr	High Corr	LMH
Low Vol	0.082	0.047	0.035
High Vol	0.064	0.048	0.016
LMH	0.018	0.001	

Table 7

Parameter and Benchmark Values of State Variables

The table reports calibrated endowment and preference parameter values. All values are annualized, where applicable.

PARAMETERS FOR ENDOWMENT		
Endowment expected growth rate	μ^X	0.0216
Endowment volatility parameter	σ^X	0.0727
Endowment correlation parameter	θ^X	0.32
PREFERENCE PARAMETERS		
Domestic home bias parameter	a	0.87
Foreign home bias parameter	a^*	0.60
Subjective rate of time preference	ρ	0.04
Speed of G mean reversion	φ	0.04
G sensitivity to consumption growth shocks	δ	248.34
Lower bound of G	l	20
Steady-state value of G	$\bar{G}$	34

Table 8
Simulated Moments

The first three columns of this table report the cross-sectional average, first quartile and the third quartile values of the simulated moments of interest based on 1,000 simulations of 144 months for 21 countries (20 foreign and one domestic) using the calibration parameters reported in Table 7. The last column reports the corresponding values in the sample data. All moments are annualized.

Moment	Model	25%	75%	Data
ENDOWMENT GROWTH RATE				
Mean	2.23%	1.36%	3.06%	2.16%
StDev	7.26%	7.15%	7.36%	7.27%
Correlation	0.32	0.30	0.34	0.32
CONSUMPTION GROWTH RATE				
Mean	2.34%	1.47%	3.18%	2.02%
StDev	5.79%	5.56%	5.95%	6.26%
Correlation	0.54	0.52	0.58	0.17
REAL EXCHANGE RATE CHANGES				
StDev	10.74%	9.13%	11.82%	10.00%
Correlation	0.83	0.79	0.87	0.43
REAL INTEREST RATES				
Mean	1.58%	1.09%	2.27%	1.62%
StDev	1.36%	0.83%	1.82%	1.69%
Correlation	1.00	1.00	1.00	0.16

Table 9

Summary Statistics for Sort of Simulated Conditional Risk Aversion

This table reports summary statistics for portfolios sorted on global risk aversion betas; the reported values are based on 1,000 simulations of 144 months for 21 countries (20 foreign and one domestic) using the calibration parameters reported in Table 7. Portfolio 1 (Pf1) contains the 25% of all the currencies with the lowest global risk aversion beta whereas Portfolio 4 (Pf4) contains the currencies with the highest global risk aversion beta. HML denotes a long-short portfolio that is long in Pf1 and short in Pf4. All numbers are annualized.
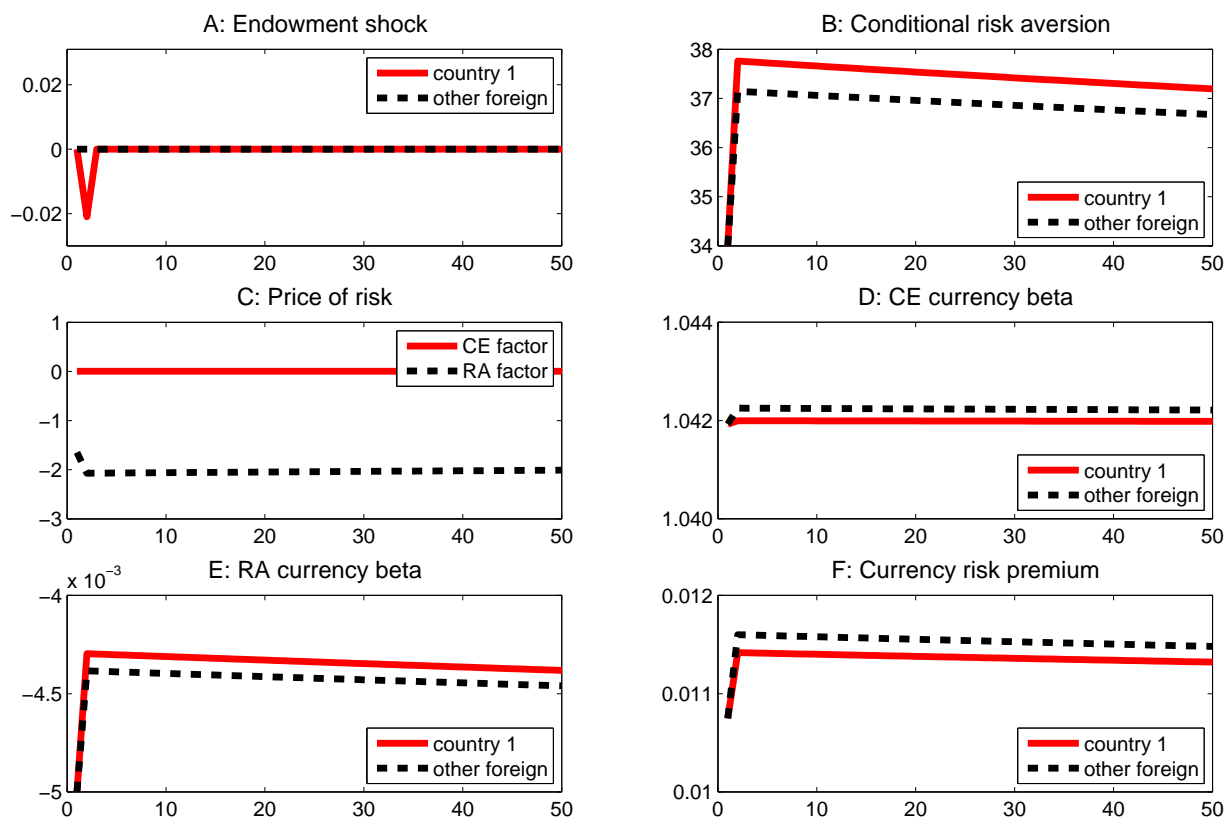
	Pf1	Pf2	Pf3	Pf4	HML
Mean	0.048	0.021	-0.001	-0.029	0.078
StDev	0.048	0.047	0.047	0.048	0.016
Skewness	0.005	0.052	0.083	0.119	0.427
Kurtosis	2.225	2.277	2.314	2.288	3.269
Sharpe Ratio	1.001	0.435	-0.026	-0.603	4.843

Table 10
Summary Statistics for Simulated Carry Trade Portfolios

This table reports summary statistics for portfolios sorted on time $(t-1)$ forward discounts, based on 1,000 simulations of 144 months for 21 countries (20 foreign and one domestic) using the calibration parameters reported in Table 7. Portfolio 1 (Pf1) contains the 25% of all the currencies with the lowest forward discounts whereas Portfolio 4 (Pf4) contains the currencies with the highest forward discounts. All returns are excess returns in USD. DOL denotes the average return of the four currency portfolios, HML denotes a long-short portfolio that is long in Pf1 and short in Pf4.

	Pf1	Pf2	Pf3	Pf4	DOL	HML
Mean	-0.019	0.001	0.016	0.034	0.008	0.052
StDev	0.049	0.045	0.046	0.049	0.045	0.021
Skewness	0.149	0.082	0.024	-0.039	0.037	-0.193
Kurtosis	2.218	2.394	2.326	2.255	2.208	2.171
Sharpe Ratio	-0.148	0.005	0.134	0.266	0.067	0.949

A: Endowment shock
B: Conditional risk aversion
C: Price of risk
D: CE currency beta
E: RA currency beta
F: Currency risk premium

country 1
other foreign
CE factor
RA factor

Figure 1. Impulse Response Functions

The figure plots a one-standard deviation shock to the endowment of country 1 and the ensuing impulse response functions for consumption and risk aversion for country 1 and the other foreign countries (Panels A and B). Panel C plots the impulse response functions for the prices of consumption and risk aversion risk. The impulse response functions for the consumption expenditure and the risk aversion betas are plotted in Panels D and E. Panel F plots the impulse response functions for currency risk premia.
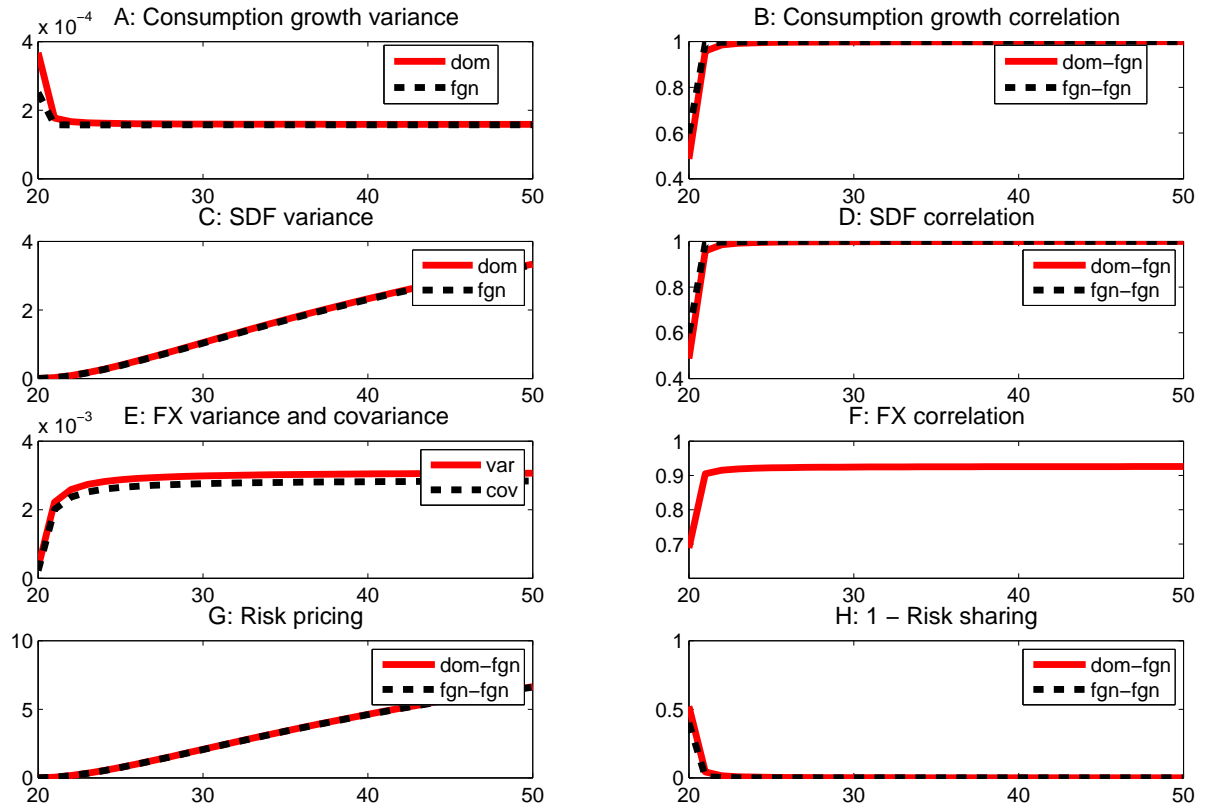
Figure 2. Conditional Global Risk Aversion on Conditional Moments

This figure gives plots of conditional second moments of consumption growth (Panels A and B), SDF (Panels C and D) and the real exchange rate (Panels E and F). Panels G and H plot the risk pricing (RP) and risk sharing (1-RS) components of real exchange rate conditional variance as defined in the main text. The horizontal axis measures global conditional risk aversion assuming that conditional risk aversion is equal in all countries; the vertical axis measures the moment of interest. The global economy comprises 21 countries, the United States (domestic country) and 20 foreign countries. The calibrated values for the parameters of the model are given in Table 7.