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JEL codes: G01, G21, G28.

Keywords: Loan insurance, adverse selection, screening, market liquidity.

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1 Introduction

Risk in credit markets is often assumed at loan origination by third parties for a fee. A typical example is insurance that protects a loan owner against borrower default and is popular in mortgage markets around the world (Blood, 2001). Various guarantees and external credit enhancements have a similar function. Governments also offer default insurance for various loan types, including student loans, small business loans, export loans, and mortgages. In 2018, the U.S. government insured and guaranteed 62% of outstanding residential mortgages (equal to 32% of GDP) via institutions such as the Federal Housing Administration (FHA) and Government Sponsored Enterprises (GSEs) such as Fannie Mae and Freddie Mac (Urban Institute, 2018). Similarly in Canada, 44% of mortgages are insured by a dominant public insurer, the Canada Housing and Mortgage Corporate (CMHC), or private insurers with explicit government backing.

The widespread use of loan default insurance and repayment guarantees in credit markets leads to several important positive and normative questions: What is the impact of loan insurance on secondary market liquidity (allocative efficiency) and on lending standards in primary markets (productive efficiency)? How do changes in loan characteristics, screening technology, or the liquidity risk of lenders affect the privately optimal amount of loan insurance? On the normative side, is this amount of loan insurance constrained efficient? If not, how should regulatory interventions be designed? Under which economic circumstances should loan insurance and guarantees (e.g., by FHA, GSEs, or CMHC) be subsidized or taxed and by how much?

To address these questions, we study loan insurance in a parsimonious model of lending and credit risk transfer. Risk-neutral lenders subject to uninsurable liquidity shocks have three options to reduce their exposure to loan default, which increases their expected payoff. First, each lender has access to a pool of borrowers and chooses whether to screen at a heterogeneous cost. Screening improves the probability of loan repayment by identifying a borrower with a low default probability. These screening choices determine lending standards.[1] The loan payoff can be thought of as a reduced-form measure of the profitability of lending options or the degree of competition in lending markets, where higher competition implies a lower loan payoff.

The second option is to wait and privately learn loan quality over time, perhaps due to relationship lending or learning by holding (Parlour and Plantin, 2008; Plantin, 2009), and sell uninsured low-quality loans to competitive deep-pocketed and uninformed outside financiers. The reason that such lemons can be sold above their fundamental value is that some lenders are hit by a liquidity shock (e.g., a superior investment opportunity or a bank run) and also want to sell to realize gains from trade. Since both realizations are private information to lenders, the secondary market for uninsured loans is subject to adverse selection. Hence, lenders without a liquidity shock can exploit private information at the expense of liquidity-shocked lenders.[2]

We explore a third option—loan insurance—and its interplay with the other options. Insurance at origination passes default risk to outside financiers (insurer) for a fee.[3] A key feature is that loan insurance occurs at origination, which makes insured loans insensitive to future private information about its quality.[4] [5] Consistent with our principal application of mortgage insurance and guarantees, whether a loan is insured is observed by financiers and the loan trades together with its insurance in secondary markets for insured loans. An implication is the segmentation of secondary market into markets for insured and for uninsured loans, such as markets for agency mortgage-backed securities (agency MBS) and for private-label MBS.[6] Since loan

[1]Work on lending standards includes Thakor (1996), Ruckes (2004), Dell'Ariccia and Marquez (2006), and Fishman et al. (2019).

[2]In our model, the source of asymmetric information in the secondary market between lenders and outside financiers is the private learning about loan quality over time, but not screening at the loan origination stage.

[3]Consistent with this timing, insurance of individual mortgages by the FHA or CMHC requires that insurance occurs at origination. Similarly, a popular business model is to specialize in origination of conforming loans, followed by the immediate sale to GSEs (Buchak et al., 2018).

[4]In the main model, there is no adverse selection in the loan insurance market. Lenders do not learned loan quality at origination, while insurers infer that a lender who chooses to insure a loan does not screen in equilibrium. In Appendix A.1, we introduce asymmetric information and adverse selection in the loan insurance market. We show that our main results are qualitatively unchanged.

[5]Another option may be the sale of loans upon origination, i.e. before the arrival of private information. We study this option in Appendix A.2 and compare it to loan insurance.

[6]FHA loans and are fully backed by the FHA and trade in a separate market enabled by secu-

insurance is often explicitly or implicitly backed by the government, we abstract from default risk of the insurer.[7]

We start with the benchmark where insurance is not available. In equilibrium, there is a screening cost threshold, so only low-cost lenders choose to screen. This cost threshold affects productive efficiency—the average quality of loans originated net of screening costs. Private learning about loan quality generates asymmetric information between lenders and financiers and thus adverse selection. The equilibrium is *liquid* when high-quality loans are sold in in a secondary market for uninsured loans upon a liquidity shock. An *illiquid* equilibrium always exists, since a low price and no trade of high-quality loans are mutually consistent. A liquid equilibrium exists for a large enough liquidity shock. Since informed lenders can profitably sell low-quality loans in the liquid equilibrium, screening is lower than in the illiquid equilibrium. Allocative efficiency refers to whether a liquid market exists (extensive margin) and to the price of uninsured loans traded (intensive margin).

When loan insurance is available, low-cost lenders screen but do not insure, while high-cost lenders do not screen but may insure. Consistent with this self-selection result, some lenders (e.g. non-banks, monoline lenders) specialize in the issuance of FHA loans or conforming loans that are sold to GSEs right after origination. Loan insurance reveals that a lender does not screen, so the competitive insurance fee reflects the higher expected default cost of non-screened loans. Since loan insurance is observable and trades with the underlying loan, secondary markets are segmented into markets for insured and uninsured loans. In the absence of insurer default risk, insured loans are risk-free and, therefore, trade in a fully liquid secondary market. Insured loans fetch a higher price than uninsured loans, which reflects adverse selection in the latter market. Consistent with this differential pricing implication, agency MBS

ritization, mainly through Ginnie Mae bonds. For Fannie Mae and Freddie Mac, the guarantee is provided when newly originated loans are sold by lenders to these GSEs, who in return for a fee provide the guarantee and securitize loans into agency MBS. Agency MBS are traded separately from private-label MBS. Insured loans in Canada are traded separately in secondary markets, typically in the form of National Housing Act Mortgage Backed Securities (NHA MBS) (Crawford et al., 2013).

[7]See Appendix A.4 for analysis of partial insurance that can be interpreted as insurer default risk in reduced form. For comprehensive analyses of counterparty risk in financial insurance contracts, see, for example, Thompson (2010) and Stephens and Thompson (2014).

maintained robust issuance and trading volumes and low spreads compared to private-label MBS even during the recent financial crisis (Vickery and Wright, 2013).

We characterize loan insurance in the liquid equilibrium. For high-cost lenders, the benefit of insurance is to sell the loan upon a liquidity shock for a higher price in the market for insured loans. The private cost of loan insurance is to lose the option to sell lemons in the market for uninsured loans without a liquidity shock. In equilibrium, both effects equalize, and high-cost lenders are indifferent about insurance. Loan insurance occurs when (i) the repayment probability is high, (ii) loan profitability is low (alternatively, competition in the lending market is high), (iii) the liquidity shock is large, and (iv) screening costs are high.

Loan insurance improves market liquidity. It creates a liquid secondary market for insured loans. Moreover, loan insurance improves the quality of *uninsured* loans traded for two reasons. First, insured lenders (all of whom have a high screening cost) have a lower average loan quality than uninsured lenders (some of whom have a low screening cost). Second, a fraction of lemons, which would have been sold by lenders without liquidity shock, is removed from the market for uninsured loans. Taken together, loan insurance eliminates part of the adverse selection and increases the price of uninsured loans up to a level consistent with high-cost lenders being indifferent about insurance (higher allocative efficiency on the intensive margin). The higher price of uninsured loans allows the liquid equilibrium to be sustained for a set of parameters for which it is inadmissible when loan insurance is not available. That is, loan insurance liquifies the market for uninsured loans (higher allocative efficiency on the extensive margin). Although the higher uninsured loan price reduces screening (lower productive efficiency), loan insurance improves welfare.

We study the comparative statics of the liquid equilibrium. The fraction of high-cost lenders who insure increases in the size of the liquidity shock and in the repayment probability, and decreases in the loan profitability and after a reduction in screening costs. A higher liquidity shock increases the benefits of insurance (higher gains from trade). A higher repayment probability reduces the screening incentives, while higher loan profitability and lower screening costs increases screening incentives. Higher (lower) screening implies a higher (lower) price for uninsured loans, making insurance less (more) attractive. The fraction of high-cost lenders who insure can be non-monotone in the probability of the liquidity shock because of two opposing effects. First, a higher probability increases the proportion of liquidity sellers and directly reduces adverse selection. Second, a higher probability reduces screening and thus indirectly increases adverse selection. If the first effect dominates, lower adverse selection implies higher price for uninsured loans, making insurance less attractive.

We turn to the normative implications of loan insurance. We characterize the constrained-efficient allocation chosen by a planner who observes screening costs, chooses loan insurance for all lenders, and can select the equilibrium by guaranteeing a minimum price of uninsured loans. The planner maximizes welfare subject to lenders choosing screening and loan sales and to outside financiers breaking even. In contrast to the competitive equilibrium, the planner internalizes the positive externality of insurance on the quality and price of uninsured loans traded. The planner chooses more loan insurance and a positive amount of insurance for a larger set of parameters in the liquid equilibrium. The planner also uses insurance to 'liquify the market', creating a liquid equilibrium where the unique unregulated equilibrium is illiquid. For some parameters, however, liquifying the market is feasible, but the implied reduction in screening incentives is so severe that the planner prefers to keep the market illiquid.

We proceed by showing that a regulator subject to a balanced budget and with no information advantage over financiers can achieve the constrained-efficient allocation. When a liquid equilibrium is constrained efficient, the regulator can promise a minimum price of uninsured loans to eliminate the illiquid equilibrium. This policy can be credibly implemented via a subsidy for outright purchases of uninsured loans, as originally envisioned by TARP programs in the United States (a subsidy for loan sales in the model).[8] Once the liquid equilibrium arises as the unique regulated equilibrium, the constrained-efficient allocation can be implemented by a subsidy on

[8]This result arose in work on the optimal intervention in illiquid markets plagued by adverse selection (e.g., Tirole 2012). Instead of tackling the adverse selection problem by removing lemons from the market, in our paper it is mitigated via market segmentation by insuring loans.

loan insurance without using the sale subsidy for uninsured loans. It induces lenders to internalize the positive externality of their individual insurance choice on the secondary market price of uninsured loans. By contrast, the sale subsidy fails to achieve constrained efficiency because it encourages the sale of lemons, while the net effect of the insurance subsidy is a reduction in the amount of lemons in the market for uninsured loans. Thus, loan sale subsidy is an important tool that allows lenders to coordinate on the liquid equilibrium when it is welfare superior, but it should not be used in this equilibrium, because loan insurance subsidy is a better tool to sustain the liquid equilibrium. When an illiquid equilibrium is constrained efficient, then all high-cost lenders fully insure loans, so there is no role for an insurance subsidy.

Finally, we discuss how our results contribute to a debate about the design of loan insurance and mortgage guarantees, for example as provided by GSEs. On the extensive margin, loan insurance subsidies should occur for higher-quality loans, e.g. borrowers with sufficiently high credit scores—consistent with the the practices of FHA and GSEs in the US and CMHC in Canada—, or in regions with lower predictable default risk (Hurst et al., 2016). Loan insurance should also occur for loans with lower payoffs or when lending markets are more competitive. In the cross-section, this occurs in countries with a less concentrated lending market, e.g. more in the United States than in Canada, while it occurs in the time series via higher recent competition from specialized online lenders. Loan insurance subsidies should arise when lenders may face larger liquidity needs or when screening costs are higher. The latter result suggests that recent technological advances and extensive data analysis of borrowers (e.g., big data or machine learning innovations) would reduce the benefits of insurance. On the intensive margin, the size of the loan insurance subsidy increases in the loan payoff. It is also non-monotonic in the size of the liquidity shock, which is determined by the margins of allocative efficiency improved via the insurance subsidy.

Literature. Our paper is related to a literature on the interaction between productive and allocative efficiency. Pennacchi (1988) and Gorton and Pennacchi (1995) show that a lender needs to retain sufficient risk exposure to borrowers to maintain monitoring incentives after loan sales. Parlour and Plantin (2008) study the interplay between liquidity in secondary loan markets plagued by adverse selection and the incentives of a relationship bank to monitor its borrower before loan sales. Vanasco (2017) studies optimal risk retention by originating lenders when screening improves productive efficiency but the induced private information reduces secondary market liquidity.[9] Daley et al. (2020) examine how credit ratings affect secondary market liquidity and screening incentives. Our contribution is to examine the implications of loan insurance and its impact on both productive and allocative efficiency. A key channel in our paper is that loan insurance at origination commits the lender to not exploiting future private information about the quality of the loan.

Perhaps closest in spirit is Parlour and Winton (2013), who study the effect of credit default swaps (CDS) as an alternative to loan sales in secondary markets. Both CDS and loan sales affect a lender's incentive to monitor its borrower but the lender retains control rights only with CDS. There are two main differences to our paper. First, we study the incentives to screen borrowers before loan sales as opposed to monitoring incentives after laying off credit risk. Second, insurance is observable and inseparable from loans in our model, which implies a segmentation of secondary markets consistent with conforming mortgages sponsored by GSEs, for example.

2 Model

There are three dates $t = 0, 1, 2$ and a single good for consumption and investment. Two groups of risk-neutral agents are protected by limited liability.[10] Outside financiers are competitive, deep-pocketed at $t = 1, 2$, and require a return normalized to one. A unit mass of lenders have one unit of funds each to make a loan at $t = 0$. Each lender has access to an individual pool of borrowers. Without screening, $s_i = 0$, lender i finds an average borrower and receives A (repayment) with probability $\mu \in (0, 1)$ or 0 (default). The loan payoff is independently and identically distributed

[9] See Kuong and Zeng (2019) for a security-design model of bank resolution in which the bank's information advantage creates an adverse selection problem in the bank funding market.

[10] Loan insurance would be desirable if agents were risk-averse but we deliberately focus on risk-neutrality throughout in order to highlight the beneficial effect of loan insurance on market liquidity.

across lenders and publicly observable at $t = 2$. Screening, $s_i = 1$, improves the repayment probability to $\psi \in (\mu, 1)$ (Figure 1). We focus on $\psi \to 1$ henceforth.[11]

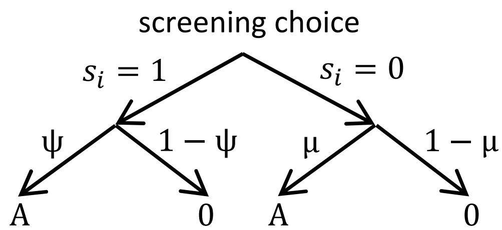

Figure 1: Screening and loan payoffs: screening improves the probability of loan repayment.

Lending opportunities are thus characterized by (i) the probability of repayment, μ, which may reflect a credit score; and (ii) and the profitability of the loan upon repayment, A. Lower values of A may reflect less profitable lending opportunities, more competitive lending market, or a higher bargaining power of borrowers.

The non-pecuniary cost of screening, η_i, is distributed across lenders according to a density function $f(\eta) > 0$ with support $[0, \overline{\eta}]$ and cumulative distribution function $F(\eta)$. The cost and choice of screening, η_i and s_i, are private information to lender i.

At $t = 1$, lenders receive two sources of private information. First, each lender learns the future loan payoff $A_i \in \{0, A\}$. This assumption is consistent with lenders forming a relationship with their borrower and the notion of learning about an asset by holding it (Plantin, 2009). This assumption also implies that screening at $t = 0$ does not increase the degree of asymmetric information at $t = 1$. Second, each lender learns an idiosyncratic liquidity shock, whereby the preference for interim consumption is $\lambda_i \in \{1, \lambda\}$ with $\lambda > 1$.[12] The liquidity shock is independently and identically distributed across lenders, independent of the loan payoff, and arises with probability $Pr\{\lambda_i = \lambda\} \equiv \nu \in (0, 1)$. Thus, the preference of lender i is

$$u_i = \lambda_i c_{i1} + c_{i2} - \eta_i s_i, \tag{1}$$

[11] We study imperfect screening, $\psi < 1$, in Appendix A.3. Our focus on the limiting case in the main text eases the exposition substantially. We show in Appendix A.3 that our results generalize to imperfect screening as long as screening sufficiently improves the repayment probability, $\psi > \underline{\psi}$.

[12] Our reduced-form modelling of the gains from the loan sale before maturity captures investment opportunities, consumption needs, capital constraints, bank runs, or risk management and is standard in the literature (e.g., Aghion et al. (2004), Holmstrom and Tirole (2011), Vanasco (2017)).

where c_{it} is the consumption of lender i at date t and $s_i \in \{0, 1\}$ is the screening choice. In sum, there are two motives for lenders to sell loans at $t = 1$: a higher valuation upon a liquidity shock and exploiting the information advantage about loan quality.

At $t = 0$, each lender chooses whether to insure the loan against default, $\ell_i \in \{0, 1\}$. Without loss of generality, we focus on full insurance—the transferal of all default risk.[13] If a loan is insured, its idiosyncratic default risk passes to outside financiers. The insurance contract guarantees the payoff A to the loan owner for a competitive fee k. Both the insurance payoff and the fee are charged at $t = 2$, resulting in a safe payoff $\pi = A - k$.[14] Insurance thus swaps a loan's payoff for a fixed payment $A - k$ at $t = 2$.[15] Whether a loan is insured is publicly observable at $t = 1$.

At $t = 1$, each lender can sell the loan in secondary markets to outside financiers. These potential buyers are uninformed about the screening cost and choice, liquidity shock, and loan quality, but they do observe whether a loan is insured. Consistent with our applications (see introduction), loans are sold together with their insurance. Thus, segmented markets for insured (I) and uninsured (U) loans may exist. The respective prices are p_I and p_U and sale choices are $q_i^I \in \{0, \ell_i\}$ (if the loan is insured) and $q_i^U \in \{0, 1 - \ell_i\}$ (if uninsured).[16] Figure 2 shows the timeline of events.

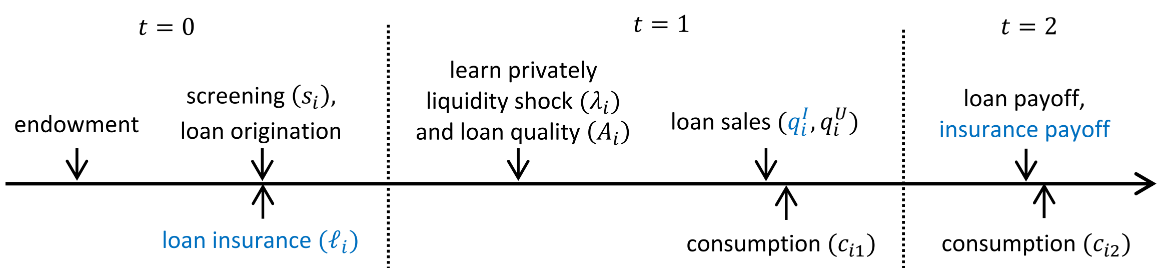

Figure 2: Timeline.

[13] We allow for partial insurance in Appendix A.4 and show that full insurance is both privately optimal and efficient.

[14] This approach parallels the non-pecuniary screening cost in that it does not affect lending volume at $t = 0$. It is feasible as contracts can be written on the observable realized the loan payoff at $t = 2$. For an extension with an insurance fee that must be paid up front (at $t = 0$), see Appendix A.5.

[15] In our model, the risk transfer via loan insurance is similar to an outright sale of loans at $t = 0$ when outside financiers cannot learn loan quality privately at $t = 1$. This case arises for the relationship lending interpretation of private learning. See Appendix A.2 for a complete analysis.

[16] We allow for partial sales in Appendix A.6 and show that our results are qualitatively unchanged.

3 Equilibrium without loan insurance

We start with the benchmark in which loan insurance is not available. This setup corresponds to the timeline without the actions highlighted in blue in Figure 2.

Definition 1. *A symmetric pure-strategy equilibrium comprises screening choices* $\{s_i\}$*, sales* $\left\{q_i^U\right\}$*, and a secondary market price of uninsured loans* p_U *such that:*

1. *At* $t = 1$*, each lender* i *optimally chooses sales for each realized liquidity shock* $\lambda_i \in \{1, \lambda\}$*, denoted by* $q_i^U(s_i, \lambda_i)$*, given the price* p_U *and screening choice* s_i*.*
2. *At* $t = 1$*, the price* p_U *is set for outside financiers to break even in expectation, given the screening choices* $\{s_i\}$ *and sales schedules* $\{q_i^U(\cdot)\}$ *of all lenders.*
3. *At* $t = 0$*, each lender* i *chooses screening* s_i *to maximize expected utility, given the price* p_U *and the sales schedule* $q_i^U(\cdot)$*:*

$$
\begin{aligned}
\max_{s_i, c_{i1}, c_{i2}} \quad & \mathbb{E}\left[\lambda_i c_{i1} + c_{i2} - \eta_i s_i\right] \qquad \textit{subject to} \\
c_{i1} \quad = \quad & q_i^U(s_i, \lambda_i)\, p_U \\
c_{i2} \quad \to \quad & \left[1 - q_i^U(s_i, \lambda_i)\right] \times \begin{cases} A & \textit{with probability} \quad s_i + \mu(1 - s_i) \\ 0 & \qquad\qquad\qquad\quad (1 - \mu)(1 - s_i). \end{cases}
\end{aligned}
$$

Due to adverse selection in the secondary market, multiple stable equilibria may exist. High-quality loans (worth A) are traded in a liquid equilibrium, but not traded in an illiquid equilibrium. We exclude the unstable equilibrium in which high-quality loans held by a fraction of liquidity-shocked lenders are traded.

Without loss of generality, lenders use a threshold strategy for their screening choice. Each lender with a screening cost below the threshold η chooses to screen:

$$
s_i = \mathbf{1}\Big\{\eta_i \le \eta\Big\}, \tag{2}
$$

where $\mathbf{1}\{\cdot\}$ is the indicator function. The screening threshold η affects productive efficiency (the average quality of loans originated net of aggregate screening costs).

Although lender types are a continuum indexed by the screening cost η_i, what matters is whether the individual screening cost is above or below the cost threshold. Hence, we refer to lenders with $\eta_i < \eta$ as low-cost lenders and to lenders with $\eta_i > \eta$ as high-cost lenders, effectively reducing the heterogeneity among lenders to two types.

Sales in the secondary market for uninsured loans. Since there is asymmetric information between lenders and outside financiers at $t = 1$, all lenders choose to sell low-quality loans (worth 0). As a result, the participation constraint of financiers implies a price $p_U \in [0, A)$. High-quality loans trade at a discount $A - p_U$, so lenders choose not to sell them without a liquidity shock. A defining feature of the equilibrium is whether lenders sell high-quality loans upon a liquidity shock:

$$\lambda p_U \geq A. \tag{3}$$

If condition (3) holds, the equilibrium is liquid, i.e., the equilibrium features a liquid secondary market for uninsured loans. In this case sales of lender i at $t = 1$ are:[17]

$$q_i^U(s_i, \lambda_i) \rightarrow \mathbf{1}\Big\{\lambda_i = \lambda\Big\} + \mathbf{1}\Big\{\lambda_i = 1\Big\}\mathbf{1}\Big\{A_i = 0\Big\}. \tag{4}$$

Allocative efficiency refers to the interim-date distribution of liquid funds across lenders and outside financiers. In our economy, allocative efficiency has two dimensions. Its extensive margin refers to whether trade of (uninsured) high-quality loans at $t = 1$ takes place, that is whether the market for uninsured loans is liquid. Asymmetric information at $t = 1$ results in a standard adverse selection problem. If severe, adverse selection implies that an equilibrium with a liquid market is not sustainable (condition 3 is violated). In this case, no gains from trade are realized.

There is also an intensive margin of allocative efficiency. When condition 3 is satisfied, all high-quality loans are sold. However, adverse selection reduces the price p_U, so the amount of liquid funds obtained by liquidity shocked-lenders is lowered. Indeed, adverse selection redistributes resources from lenders selling high-quality loans

[17] Similar to Parlour and Plantin (2008), the binary choice of loan sales and limited liability preclude signaling in this market. See Appendix A.6 for an analysis of partial loan sales.

to lenders selling low-quality loans (lemons). High-quality loans are sold exclusively by liquidity-shocked lenders (who have a high utility of interim-date consumption, λ), while low-quality loans are sold by a mix of lenders with an average utility, $\kappa \equiv \nu\lambda + 1 - \nu \in (1, \lambda)$ (as shown in panel (a) of Figure 5). Therefore, even in the liquid equilibrium, adverse selection reduces the (social) gains from trade, $\nu(\lambda - 1)p_U$.

Secondary-market price. In the illiquid equilibrium (see Lemma 2 below), the price is $p_U = 0$ due to adverse selection. In the liquid equilibrium, all lenders sell their loans upon a liquidity shock. Due to private learning about loan payoffs A_i, lenders sell all lemons at $t = 1$. These are a fraction $1 - \mu$ of loans by high-cost lenders and a (vanishing) fraction $1 - \psi \to 0$ of loans by low-cost lenders. The break-even condition of outside financiers ensures that the price equals the value of high-quality loans sold by liquidity sellers—$\nu\psi F(\eta)$ loans sold by high-cost lenders and $\nu\mu(1 - F(\eta))$ loans by low-cost lenders—divided by the total quantity of loans sold:

$$p_U \to \nu A \frac{F(\eta) + \mu(1 - F(\eta))}{\nu + (1 - \nu)(1 - \mu)(1 - F(\eta))} \equiv p_U(\eta). \tag{5}$$

Screening unambiguously supports the price, $\frac{dp_U}{d\eta} > 0$. First, more screening leads to fewer low-quality loans originated at $t = 0$, which improves the quality of loans and directly supports the price. Second, screening does not exacerbate adverse selection. Since all lenders privately learn loan quality at $t = 1$, screening at $t = 0$ does not increase the asymmetric information between lenders and financiers at $t = 1$.

Screening. The marginal lender ($\eta_i = \eta$) is indifferent between screening and not screening. Suppose a liquid equilibrium exists, so lenders sell all loans upon a liquidity shock and after learning they are of low quality. Lender who screen originate loans of higher quality on average and, therefore, sells fewer lemons (a vanishing amount for $\psi \to 1$). Thus, screening yields the expected payoff $\nu\lambda p_U + (1 - \nu)[\psi A + (1 - \psi)p_U] - \eta$. Not screening results in more frequent loan sales for informational reasons, yielding $\nu\lambda p_U + (1 - \nu)[\mu A + (1 - \mu)p_U]$. Equating both payoffs yields the cost threshold

$$\eta \to (1 - \nu)(1 - \mu)(A - p_U) \equiv \eta(p_U). \tag{6}$$

The benefit of screening arises only in the absence of a liquidity shock (with probability $1-\nu$) since liquidity-shocked lenders sell all loans irrespective of the screening choice in the liquid equilibrium. Without a liquidity shock, the screening benefit is the higher probability of identifying a high-quality loan ($\psi - \mu \to 1-\mu$), multiplied by the benefit of keeping the high-quality loan to maturity instead of selling a lemon, $A - p_U$. Because of the option to sell lemons for informational reasons, a higher secondary market price at $t=1$ reduces the benefit of screening at $t=0$, $\frac{d\eta}{dp_U} < 0$.

Lemma 1 describes the liquid equilibrium and Figure 3 shows its construction.

Lemma 1. ***Liquid equilibrium when loan insurance is unavailable.*** *If $\lambda \geq \underline{\lambda}_L$, then there exists a unique interior equilibrium in the class of liquid equilibria. It is characterized by a screening cost threshold, $\eta^* \in (0, \bar{\eta})$, which is implicitly given by*

$$\eta^* = \frac{(1-\nu)(1-\mu)^2\,[1-F(\eta^*)]}{\nu + (1-\nu)(1-\mu)\,[1-F(\eta^*)]}A; \tag{7}$$

a price of uninsured loans in the secondary market, $p_U^* \to A - \frac{\eta^*}{(1-\nu)(1-\mu)} \in \left[\frac{A}{\lambda}, A\right)$; and the lower bound on the size of the liquidity shock

$$\underline{\lambda}_L = \frac{A}{p_U^*} \to \frac{\nu + (1-\nu)\,(1-\mu)\,(1-F(\eta^*))}{\nu\,[F(\eta^*) + \mu(1-F(\eta^*))]} \in (1, \infty). \tag{8}$$

The threshold η^* increases in A and decreases in μ, ν, and after a first-order stochastic dominance (FOSD) reduction in F. The price p_U^* increases in A and after a FOSD reduction in F. Both η^* and p_U^* are independent of λ. If $\eta_\mu^* \frac{f(\eta_\mu^*)}{1-F(\eta_\mu^*)} > 1$, where $\eta_\mu^* \equiv \lim_{\mu\to 0}\eta^*$, then the price is non-monotonic in μ, decreasing first. The price can also be non-monotonic in ν. Similarly, the bound $\underline{\lambda}_L$ decreases in A and after a FOSD reduction and can be non-monotonic in ν and μ.

Proof. See Appendix B.1. ■

We offer some intuition for why the solution is interior. First, the benefit of screening is a discrete improvement in the probability of repayment, $\psi > \mu$. Since some loans are kept to maturity, lenders internalize part of the (social) benefit of

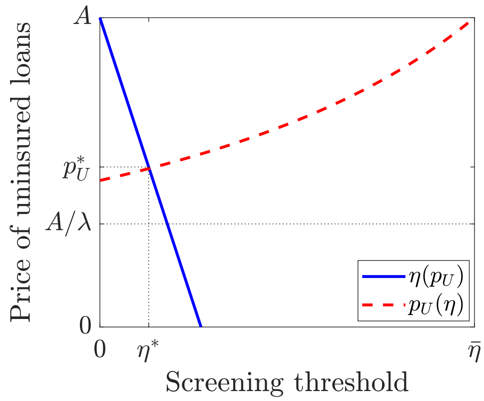

Figure 3: Existence of a unique equilibrium in the class of liquid equilibria ($\lambda \geq \underline{\lambda}_L$). The increasing red dashed curve is the competitive price in the secondary market of uninsured loans, while the decreasing blue solid curve is the screening cost threshold. The bound on the liquidity shock size ensures that high-quality loans are sold after a shock, $\lambda p_U^* \geq A$.

screening benefit and, thus, the private benefit of screening is strictly positive and exceeds the cost of some lenders. Hence, some screening occurs in equilibrium, $\eta^* > 0$. Second, suppose all lenders screen, $\eta = \bar{\eta}$. Then there is a vanishing degree of adverse selection in the secondary market and the price reflects the high average quality of loans traded, $p_U \to A$. However, this implies a vanishing benefit of keeping loans to maturity and thus the incentives to screen are vanishing when loans are traded at such a high price (see equation 6), violating the supposed screening by all lenders. Hence, some lenders do not screen in equilibrium, $\eta^* < \bar{\eta}$.

We turn to the comparative statics. The size of the liquidity shock, λ, affects the existence of liquid equilibrium. Once it exists, $\lambda > \underline{\lambda}_L$, however, the shock size has no further impact on the loan sale decisions and the quality of traded loans.

A first-order stochastic dominance reduction in the screening cost distribution, $\tilde{F} \geq F$, makes screening cheaper and increases the share of low-cost lenders. Hence, the secondary market price p_U^* increases and it is easier to support a liquid equilibrium ($\underline{\lambda}_L$ falls). Higher loan profitability A (or lower bargaining power of borrowers or lower lending market competition) increases the benefit of screening and thus the screening threshold η^*. As a result of the better pool of loans traded, the price in the secondary market p_U^* increases by more than the initial increase in loan profitability, which makes it easier to sustain the liquid equilibrium ($\underline{\lambda}_L$ falls).

A higher repayment probability μ (for example, a higher credit score) improves the average quality of non-screened loans. A higher probability of liquidity shock ν in turn implies that lenders are more likely to sell a high-quality loan in the secondary market. Therefore, both parameter changes lower the benefits of screening and, as a result, lower the screening threshold η^*. The overall effect on the price p_U^* and the bound $\underline{\lambda}_L$ can be ambiguous, however. First, lower screening tends to depresses the price. Second, higher μ lowers the probability of default conditional on not-screening and higher ν increases relative share of liquidity sellers. Both effects tend to increase the average quality of traded loans, which increases the price.

Figure 4 shows the area for which a liquid equilibrium exists. It shows the non-monotonic relationship between the bound $\underline{\lambda}_L$ and the repayment probability μ.

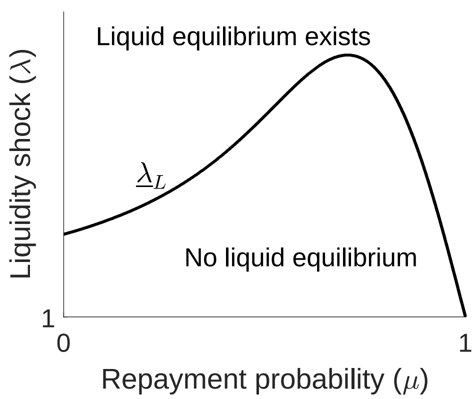

Figure 4: Existence of a liquid equilibrium (when loan insurance is unavailable).

We turn to the illiquid equilibrium in which high-quality loans are not sold after a liquidity shock. Lemma 2 characterizes this equilibrium.

Lemma 2. ***Illiquid equilibrium when loans insurance is unavailable.*** *There generically exists an illiquid equilibrium, $p_U^* = 0$. The screening threshold is $\eta^* \to (1-\mu)A$ and exceeds the screening threshold in the liquid equilibrium. If $\lambda < \underline{\lambda}_L$, the illiquid equilibrium is the unique equilibrium.*

An equilibrium with an illiquid secondary market exists whenever $\psi \neq 1$. If the price is zero, lenders do not sell high-quality loans. Only low-quality loans are traded, which is consistent with the zero price. Imperfect screening, $\psi < 1$, ensures the existence of an illiquid equilibrium, $p_U = 0$. If screening were perfect ($\psi = 1$) and sufficiently cheap, $\bar{\eta} < (1-\mu)A$, then each lender would choose to screen, $F(\eta^*) = 1$, because $\eta^* = (1-\mu)A > \bar{\eta}$, and no lemons would be sold in the secondary market. Therefore, the competitive price would be $p_U = A$, contradicting the supposed illiquid equilibrium. As a result, the illiquid equilibrium does not exist with perfect screening for parameters $\bar{\eta} < (1-\mu)A$. In the limit $\psi \to 1$, however, a positive but vanishing amount of lemons is sold in the market at price $p_U^* = 0$, ensuring existence.

The screening threshold again arises from the indifference condition of the marginal lender. Since loans are kept until maturity in the illiquid equilibrium, the expected payoff from screening is $A - \eta$, while it is μA upon screening. Since all loans are kept until maturity, lenders have higher incentives to screen.

4 Equilibrium with loan insurance

Having reviewed the benchmark, we turn to the equilibrium in which loan insurance is available. Whether a loan is insured is publicly observable at $t = 1$, so the secondary market for insured loans is separate from the market for uninsured loans.

Definition 2. *An equilibrium with loan insurance comprises the choices of screening* $\{s_i\}$ *and insurance* $\{\ell_i\}$*, the sales of insured and uninsured loans* $\left\{q_i^I, q_i^U\right\}$*, secondary market prices* p_I *and* p_U*, and an insurance fee* k *such that:*

1. *At* $t = 1$*, each lender* i *optimally chooses sales of insured and uninsured loans for each realized liquidity shock* $\lambda_i \in \{1, \lambda\}$*, denoted by* $q_i^I(s_i, \lambda_i, \ell_i)$ *and* $q_i^U(s_i, \lambda_i, \ell_i)$*, given the prices* p_I *and* p_U *and the choices of screening* s_i *and insurance* ℓ_i*.*

2. *At* $t = 1$*, prices* p_I *and* p_U *are set for outside financiers to expect to break even, given screening* $\{s_i\}$ *and insurance* $\{\ell_i\}$ *choices, the fee* k*, and sales* $\{q_i^I, q_i^U\}$*.*

3. *At* $t = 0$*, the fee* k *is set for outside financiers to expect to break even at* $t = 0$*, given screening* $\{s_i\}$ *and insurance* $\{\ell_i\}$ *choices.*

4. *At $t = 0$, each lender i chooses its screening s_i and loan insurance ℓ_i to maximize expected utility, given prices p_I and p_U, the fee k, and sale schedules q_i^I and q_i^U:*

$$\max_{s_i,\ell_i,c_{i1},c_{i2}} \mathbb{E}\left[\lambda_i c_{i1} + c_{i2} - \eta_i s_i\right] \qquad \textit{subject to}$$
$$c_{i1} = q_i^U(s_i,\lambda_i,\ell_i)\,p_U + q_i^I(s_i,\lambda_i,\ell_i)\,p_I,$$
$$c_{i2} \to \left[\ell_i - q_i^I\right](A-k) + \left[1-\ell_i - q_i^U\right] \times \begin{cases} A & w.\ p. \quad s_i + \mu(1-s_i) \\ 0 & \qquad\quad (1-\mu)(1-s_i). \end{cases}$$

Let m denote the fraction of insured loans among high-cost lenders, $\eta_i > \eta^*$.

Proposition 1. ***Loan insurance.*** *Low-cost lenders screen but never insure: $s_i^* = 1$ and $\ell_i^* = 0$ if $\eta_i \le \eta^*$. Competitive loan insurance charges the fee $k^* = (1-\mu)A$, so we have $\pi^* = \mu A = p_I^*$. In a liquid equilibrium, at most some high-cost lenders insure, $m^* \in [0,1)$. In an illiquid equilibrium, all high-cost lenders insure, $m^* = 1$.*

Proof. See Appendix B.2. ■

Insurance converts the risky loan payoff to a risk-free payoff π that is independent of the unobserved screening choice. Since screening is costly, lenders who insure loans do not screen them. As a result, only non-screened loans may be insured, and the competitive fee for them is $(1-\mu)A$, the expected cost of guaranteeing the loan. That is, insuring a loan perfectly reveals that the lender did not screen.

An insured loan is sold together with its insurance at $t = 1$. Hence, insured loans are risk-free and the secondary market for insured loans is not subject to adverse selection and is always liquid. The competitive price of insured loan equals its risk-free payoff at $t = 2$:

$$p_I^* = \pi^* = A - k^* = \mu A. \tag{9}$$

In the liquid equilibrium with insurance, $0 < m^* < 1$, high-cost lenders are indifferent about loan insurance.[18] With insurance, a high-cost lender strictly prefers selling the insured loan after the liquidity shock at $t = 1$ at price p_I^* and, without a liquidity

[18] Since market for insured loans is always liquid, we continue to refer to the equilibrium in which high-quality loans are sold in the secondary market for uninsured loans as the "liquid equilibrium".

shock, is indifferent about an insured loan sale because the price equals the loan payoff at maturity, $p_I^* = \pi^*$. Thus, the expected payoff of a high-cost lender from insuring is κp_I^*. Without insurance, a high-cost lender sells the uninsured loan after a liquidity shock in the market for uninsured loans at p_U^*. Without a shock, the loan is also sold if it is of low quality, else it is kept until maturity. Similar to the benchmark, the expected payoff from not insuring is $\nu\lambda p_U^* + (1-\nu)[\mu A + (1-\mu)p_U^*]$. Equating both payoffs yields the indifference condition for loan insurance:

$$\nu\lambda\left(p_I^* - p_U^*\right) = (1-\nu)\,(1-\mu)\,p_U^*. \tag{10}$$

This condition has an intuitive interpretation. Its left-hand side (LHS) is the benefit of insurance, which is the gain of selling the loan, after a liquidity shock, at a higher price in the insured market than in the uninsured market, $p_I^* > p_U^*$.[19] Insurance also has the (private) cost of losing the option to sell low-quality loans in the uninsured market without the liquidity shock—the right-hand side (RHS) of equation (10). Equalizing both effects pins down the uninsured loans price consistent with loan insurance, p_U^*.

There does not exist a liquid equilibrium in which all high-cost lenders insure, $m^* < 1$. If they did, $m = 1$, only high-quality loans would be sold (the quantity of lemons sold in secondary markets vanishes for $\psi \to 1$), so the price of uninsured loans would satisfy $p_U \to A$. This price, however, contradicts the equilibrium condition for high-cost lenders preferring insurance (with payoff $\kappa\mu A$) over no insurance (with payoff κA at the implied price), which is required for sustaining the supposed $m = 1$.

We turn to loan insurance in the illiquid equilibrium, where uninsured loans must be kept until maturity and gains from trade at $t = 1$ cannot be realized. Hence, the payoff of a high-cost lender without insurance is μA. The market for insured loans is always liquid, so insurance allows such lenders to exploit the gains from trade after the liquidity shock. Thus, the payoff with insurance, $\kappa\mu A$, is strictly higher, so full insurance of high-cost lenders is optimal, $m^* = 1$. This corner solution is

[19] A necessary condition for loan insurance in equilibrium is that price of insured loans exceeds the price of uninsured loans. This differential is consistent with evidence from the recent financial crisis when agency MBS maintained lower spreads than private-label MBS (Vickery and Wright, 2013).

intuitive because the choice of loan insurance is no longer associated with a trade-off for high-cost lenders. Indeed, insurance still has the benefit of a higher price in the secondary market, $p_I > p_U$. But the private cost of insurance, foregoing the option to sell uninsured lemons, does not apply because the uninsured loans market is illiquid.

We are now ready to characterize the liquid equilibrium with loan insurance.

Proposition 2. ***Liquid equilibrium when loan insurance is available.*** *There exist unique bounds $\tilde{\mu}_I$ and $\tilde{\lambda}_L$. If $\mu > \tilde{\mu}_I$ and $\lambda \geq \tilde{\lambda}_L$, there exist a liquid equilibrium in which loan insurance is used, $m^* > 0$. In this equilibrium:*

1. *Loan insurance increases the price of uninsured loans and lowers screening.*
2. *The screening threshold is $\eta^* \to \frac{(1-\nu)(1-\mu)^2\kappa A}{\nu\lambda+(1-\nu)(1-\mu)}$, the price is $p_U^* \to \frac{\nu\lambda\mu A}{\nu\lambda+(1-\nu)(1-\mu)} \in \left[\frac{A}{\lambda}, p_I^*\right)$, and the fraction of insured loans is $m^* \to 1 - \frac{\kappa F(\eta^*)}{\mu(\lambda-1)(1-\nu)(1-F(\eta^*))}$.*
3. *The screening threshold η^* increases in A, decreases in μ, ν and λ, and is independent of F. The price p_U^* increases in A, μ, ν and λ, and is independent of F. The bound on the size of the liquidity shock, $\tilde{\lambda}_L = \frac{1}{2\mu} + \sqrt{\frac{1}{4\mu^2} + \frac{(1-\mu)(1-\nu)}{\mu\nu}}$, decreases in μ and ν. The bound $\tilde{\mu}_I$ decreases in λ and increases in A and after a FOSD reduction in F. The proportion of high-cost lenders who insure m^* increases in μ and λ and decreases in A and after a FOSD reduction in F. If $\frac{F(\eta_\nu^*)(1-F(\eta_\nu^*))}{f(\eta_\nu^*)} < (1+(\lambda-1)\mu)A\lambda^{-1}$, where $\eta_\nu^* \equiv \lim_{\nu\to 0}\eta^*$, then m^* is non-monotonic in ν: it first increases and then decreases.*

Proof. See Appendix B.3 (which also defines the bound $\tilde{\mu}_I$.) ■

A key mechanism is how loan insurance affects the quality of uninsured loans traded, shown in Figure 5. Panel (a) shows loan sales in the liquid equilibrium, where loan payoffs at $t = 2$ depend on the screening choice at $t = 0$. The area shaded in blue lines depicts loans traded in the uninsured market, which depends on the liquidity shock at $t = 1$ and the loan payoffs at $t = 2$. Panel (b) shows the impact of the availability of loan insurance at $t = 0$. Loan insurance effectively segments secondary markets at $t = 1$. A fraction of high-quality and low-quality loans are removed from the uninsured market and trade in a separate market for insured loans. Since insurance is chosen only by high-cost lenders (Proposition 1), the loans insured—and hence removed from the market for uninsured loans, shaded in red crosses—are relatively more of low quality.

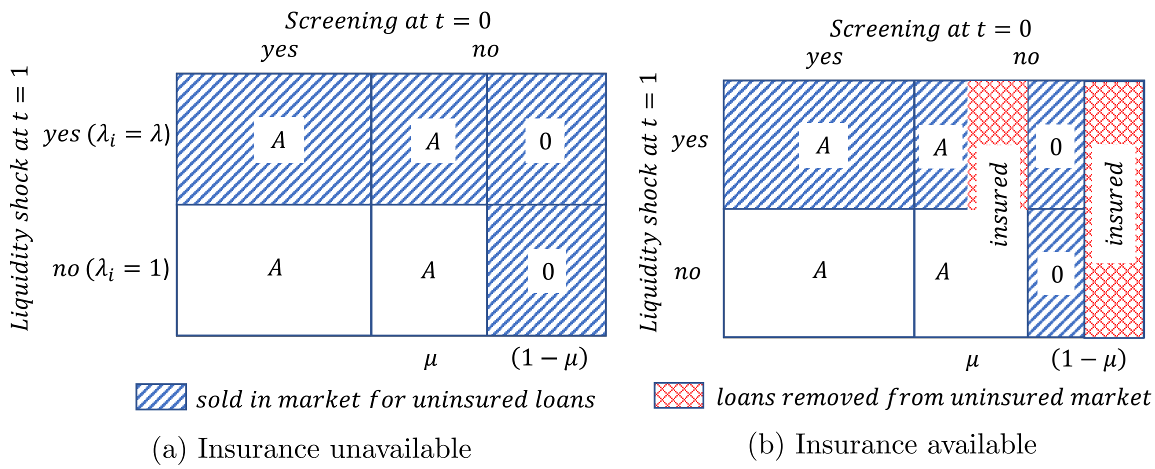

Figure 5: Loan insurance improves the quality of uninsured loans traded.

Hence, loan insurance improves the average quality of loans traded on the uninsured market. This result arises for two reasons. First, liquidity-shocked lenders who have insured have worse average quality of loans (with an expected payoff of μA) than liquidity-shocked lenders who have not insured (some of them screened their loans, with an expected payoff of $F(\eta)A + (1 - F(\eta))\mu A$). Second, a fraction of low-quality loans, which would have been traded for informational reasons, is removed from the market for uninsured loans (the quantity $(1 - \nu)(1 - \mu)(1 - F(\eta))m$). This refers to lemons owned by high-cost lenders without a liquidity shock, who have insured their loans and thus given up the option to act on private information learned at $t = 1$.

Loan insurance changes the break-even condition of outside financiers to

$$p_U \to \nu A \frac{F(\eta) + \mu (1 - F(\eta)) (1 - m)}{\nu (1 - (1 - F(\eta)) m) + (1 - \nu) (1 - \mu) (1 - F(\eta)) (1 - m)}, \tag{11}$$

where the price equals the value of uninsured loans sold by liquidity-shocked lenders divided by the amount of uninsured loans from liquidity-shocked lenders, $\nu (1 - (1 - F) m)$, and lenders with lemons and without a liquidity shock, $(1-\nu) (1 - \mu) (1 - F) (1 - m)$.

Figure 6 shows the areas for which a liquid equilibrium exists and for which loan insurance occurs in equilibrium. The liquid equilibrium exists when $\lambda \geq \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$, where $\tilde{\lambda}_L \equiv \frac{A}{p_U}$ applies when loan insurance is used, $m^* > 0$. Because of the improved quality of uninsured loans traded at $t = 1$, loan insurance improves the secondary market price p_U^*. Hence, the liquid equilibrium can be sustained for a larger range of parameters as the respective thresholds on λ decrease in p_U, resulting in $\tilde{\lambda}_L < \underline{\lambda}_L$ when insurance is used. Conditional on the existence of liquid equilibrium, loan insurance is used if $\mu > \tilde{\mu}_I$. Combining these two conditional expressions defines the parameter space where there exists a liquid equilibrium in which loan insurance is used. These bounds can also be expressed as $\tilde{\mu}_I$ and $\tilde{\mu}_L$, so loan insurance generically occurs in the liquid equilibrium if $\mu \geq \max\{\tilde{\mu}_I, \tilde{\mu}_L\}$.

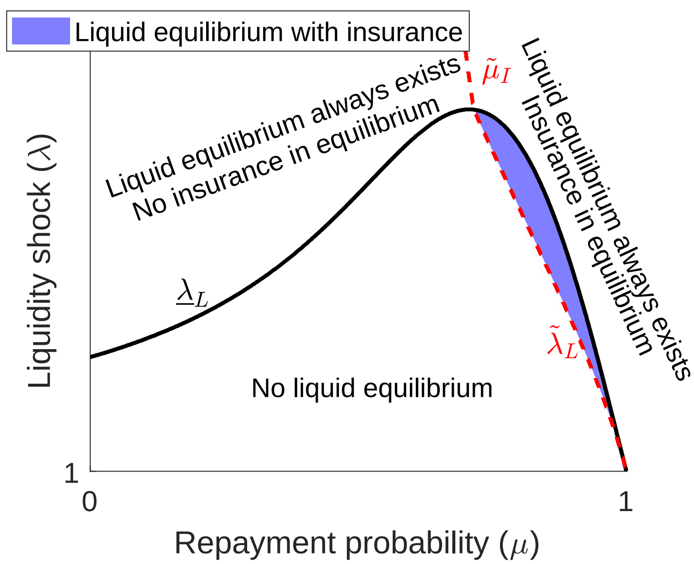

Figure 6: Existence of a liquid equilibrium (when loan insurance is available). A liquid equilibrium with loan insurance arises when $\mu > \tilde{\mu}_I$ and $\lambda \geq \tilde{\lambda}_L$. Insurance allows for the existence of a liquid equilibrium for $\tilde{\lambda}_L \leq \lambda < \underline{\lambda}_L$ (liquifying the uninsured loans market). Below $\underline{\lambda}_L$, the more restrictive condition is the sale of high-quality uninsured loans, $\lambda \geq \tilde{\lambda}_L$, while above $\underline{\lambda}_L$, the more restrictive condition is the usage of insurance, $\mu > \tilde{\mu}_I$.

Loan insurance widens the range of parameters for which the liquid equilibrium exists and increases the secondary market price of uninsured loans. Similar to benchmark, the option to shed default risk in the market for uninsured loans (i.e. sell lemons at price p_U) reduces screening incentives. First, on the intensive margin the insurance-induced higher price of uninsured loans lowers screening (equation 6). Second, on the extensive margin, insurance enables existence of the liquid equilibrium in which screening is lower than in the illiquid equilibrium without insurance.[20] Figure 7 in Section 5 shows these two effects of the availability of loan insurance.

We turn to the comparative statics of the liquid equilibrium. When loan insurance is not used in equilibrium, $m^* = 0$, then comparative statics in Lemma 1 apply. Our focus is thus on the case in which some loans are insured, $m^* > 0$. A larger liquidity shock ($\lambda \uparrow$) increases the benefits of insurance (the LHS of equation 10). More insurance improves the quality of uninsured loans traded and results in higher price of uninsured loans, which in turn lowers screening incentives (see equation 6). In sum, the size of the liquidity shock affects both the screening incentives and the secondary market price of uninsured loans, which contrasts with the benchmark model.

Higher loan profitability A directly scales both secondary market prices p_U and p_I, with no direct net effect on the incentives to insure. However, higher profitability indirectly increases screening incentives (equation 6), resulting in better average quality of uninsured loans traded. Because of this indirect effect, the competitive price of uninsured loans tends to increase more than price of insured loans (equation 11). Hence, the relative private costs of insurance—the option to sell lemons in the uninsured market (see equation 10)—rises and reduces the fraction of high-cost lenders who insure, m^*. In sum, loan profitability (which may also proxy for lower competitiveness in lending or lower borrower bargaining power) affects both screening incentives and the price of uninsured loans, as in the benchmark.

A FOSD reduction in screening costs $F(\cdot)$ (which may proxy for a more efficient screening technology, e.g. better data processing by Fintechs) directly increases the incentives to screen, which in turn tends to improve the average quality of uninsured loans traded and puts upward pressure on price of uninsured loans p_U. Hence, the relative cost of loan insurance increases and reduces the fraction of high-cost lenders who insure. The equilibrium effect is a reduction in insurance without a change in screening or the price of uninsured loans, which contrasts with the benchmark model.

[20] This result follows from Lemma 2 and the negative effect of insurance on screening incentives.

A higher probability of repayment μ and of the liquidity shock ν directly increase the benefit of insurance and reduce its cost. Moreover, these changes also lower screening incentives with negative effect on the average quality of uninsured loans traded, putting downward pressure on the price p_U, further increasing the incentives to insure. However, higher μ and ν also directly tend to increase the price of uninsured loans due to a lower fraction of lemons and a higher fraction of liquidity sellers in the uninsured loans market, respectively, which tends to increase the cost of insurance.

For a higher repayment probability μ (which may be proxied with a credit score of the borrower), the direct insurance and indirect screening effects dominate the direct price effect, resulting in more loan insurance. For the probability of liquidity shock ν, the effect on insurance is ambiguous. For a high probability of a liquidity shock the direct price effect dominates $\left(\frac{dm^*}{d\nu} < 0\right)$, while otherwise the direct insurance effect and indirect screening effect dominate under the stated sufficient condition $\left(\frac{dm^*}{d\nu} > 0\right)$. Since a higher μ or ν support the price, it is easier to sustain the liquid equilibrium (see equation 3), lowering the threshold shock size $\tilde{\lambda}_L$.

We further characterize when loan insurance is used in a liquid equilibrium. The parameters changes described above affect loan insurance, which in turn affects the price of uninsured loans p_U by changing the quality of uninsured loans traded until the insurance indifference condition (10) holds, or until insurance stops being used in equilibrium. Conditional on the existence of the liquid equilibrium, the latter condition on the extensive margin of loan insurance, $m > 0$, is affected by parameters. Since loan insurance m^* increases in μ and decreases in A and after a FOSD reduction in F, the extensive margin can be expressed as $\mu > \tilde{\mu}_I$, or, analogously, as $A < \tilde{A}$, or a sufficiently costly screening technology $F(\cdot)$. Loan insurance also increases in λ. If there is enough adverse selection in the uninsured loans market (for which $\nu \leq \frac{2\mu}{1+2\mu}$ is sufficient), the extensive margin of loan insurance can be expressed as $\lambda > \tilde{\lambda}_I$. Given the ambiguous effect of the probability of the liquidity shock ν on insurance incentives, there generally is no unique bound on ν for the usage of loan insurance.

We turn to the illiquid equilibrium in the market for uninsured loans.

Proposition 3. ***Illiquid equilibrium when loan insurance is available.*** *There generically exists an illiquid equilibrium,* $p_U^* = 0$*. If* $\lambda < \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$*, the illiquid equilibrium is unique. The screening threshold is* $\eta^* \to (1 - \kappa\mu)A$*, so loan insurance lowers screening in the illiquid equilibrium as well.*

Loan insurance generates a liquid secondary market for insured loans, allowing high-cost lenders to sell upon a liquidity shock (even in the illiquid equilibrium). Thus, insurance increases the payoff without screening, lowering screening incentives.[21]

Proof. See Appendix B.4. ■

5 Constrained efficiency and regulation

We turn to normative implications of loan insurance. We study the constrained-efficient allocation as a welfare benchmark and whether a regulator can achieve it.

5.1 Constrained efficiency

A constrained planner, P, observes the screening costs of lenders, chooses loan insurance $\{\ell_i\}$, and picks the preferred equilibrium in the secondary market for uninsured loans (liquid or illiquid) by guaranteeing a minimum price in this market. The planner maximizes utilitarian welfare subject to the individually optimal loan sales and screening choices of lenders and to outside financiers breaking even (both are conditional on $\{\ell_i\}$). We first study the planner's problem subject to a liquid and illiquid equilibrium, respectively, and then characterize the choice of equilibrium.

Liquid equilibrium in unregulated economy. Suppose the planner picks the liquid equilibrium, L, when it exists in the unregulated economy, $\lambda \geq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$. While lenders take the price of uninsured loans p_U as given, the planner internalizes

[21] Consistent with this implication, Choi and Kim (2018) document higher screening incentives due to an illiquid secondary market for uninsured loans.

the positive impact of loan insurance $\{\ell_i\}$ on it (Proposition 2). Since it is never privately optimal to screen if the loan is insured and it is efficient that lower-cost lenders screen, the choice of insurance for each lender $\{\ell_i\}$ is equivalent to choosing the proportion m of high-cost lenders who insure. Taken together, the planner solves:

$$
\begin{aligned}
W^L \quad &\equiv \quad \max_m W \equiv \max_m \overbrace{\nu(\lambda-1)\left[p_U + m\,(1-F(\eta))\,(p_I - p_U)\right]}^{\text{Gains from trade}} \\
&\qquad + \underbrace{\left[F(\eta)+\mu(1-F(\eta))\right]A}_{\text{Fundamental value}} - \underbrace{\int_0^{\eta} \tilde{\eta}\, dF(\tilde{\eta})}_{\text{Screening costs}} \qquad (12) \\
\text{s.t.} \quad &\qquad (6), \quad (11), \quad p_U\lambda \geq A, \quad \text{and} \quad p_I = \mu A.
\end{aligned}
$$

Welfare is the sum of expected payoffs of all lenders (up to a constant representing the expected payoff of outside financiers) and is derived in Appendix B.5. Welfare comprises productive efficiency and allocative efficiency. Productive efficiency is captured by the average quality of loans originated (their fundamental value) net of the aggregate screening costs of (low-cost) lenders. Allocative efficiency on the extensive margin arises by supposition of a liquid secondary market for uninsured loans. Allocative efficiency on the intensive margin refers to the magnitude of (social) gains from trade, which are proportional to the fraction of liquidity-shocked lenders, ν, the difference in marginal utilities, $\lambda - 1$, and the market value of loans sold by liquidity-shocked lenders. The market value's first part is the price of uninsured loans p_U, while its second part is proportional to the share of insured loans—a fraction m of non-screened loans, $1 - F(\eta)$—and the price differential between insured and uninsured loans. The social value of loan insurance arises from the commitment of high-cost lenders at $t = 0$ to not acting on private information about loan quality at $t = 1$ (and sell lemons). This raises the average quality of uninsured loans traded in the liquid equilibrium and thus increases the gains from trade.

On the intensive margin of allocative efficiency, the gains from trade in (12) are

$$
(\lambda-1)\Big[\underbrace{\nu(F(\eta)+\mu(1-F(\eta)))A}_{\text{Fundamental value of sold loans}} - p_U \underbrace{(1-\nu)\,(1-\mu)\,(1-F(\eta))\,(1-m)}_{\text{Lemons sold by lenders without liquidity shock}}\Big], \qquad (13)
$$

where the first term of this decomposition is the fundamental value of loans sold by liquidity-shocked lenders in both markets. The second term is the liquid funds diverted from liquidity-shocked lenders by sellers of lemons without a liquidity shock due to the private information about loan quality. It reflects the negative effect of adverse selection on the social gains from trade (given the equilibrium is liquid).

Proposition 4 states the efficient allocation if the planner chooses the liquid equilibrium when it exists in the unregulated economy. Figure 8 illustrates.

Proposition 4. ***Efficient loan insurance in the liquid equilibrium.*** *Suppose the planner chooses the liquid equilibrium for $\lambda \geq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$. There exists a constrained-efficient allocation $(m^P,\, p_U^P,\, \eta^P)$. For $\mu \leq \mu_I^P$, the unregulated level of insurance is efficient, $m^P = m^* = 0$. Otherwise, loan insurance exceeds the unregulated level at the intensive and extensive margins, $m^* < m^P < 1$, resulting in a higher price, $p_U^P > p_U^*$, and less screening, $\eta^P < \eta^*$. For $\mu > \tilde{\mu}_I$, efficient insurance exceeds the unregulated level at the intensive margin, $m^P > m^* > 0$. For $\tilde{\mu}_I \geq \mu > \mu_I^P$, efficient insurance exceeds the unregulated level at the extensive margin, $m^* = 0 < m^P < 1$.*

Proof. See Appendix B.5 (which also defines the bound μ_I^P). ■

In the liquid equilibrium with insurance used in the unregulated economy, insurance and screening are chosen privately optimally $\left(\frac{\partial W}{\partial m}\mid_{m=m^*}= 0 \text{ and } \frac{\partial W}{\partial \eta}\mid_{\eta=\eta^*}= 0\right)$.[22] But marginally more insurance increases welfare $\left(\frac{dW}{dm}\mid_{m=m^*}= \frac{\partial W}{\partial p_U}\frac{dp_U}{dm} > 0\right)$ because insurance positively affects the secondary market price of uninsured loans. When insurance increases above its unregulated level, the positive welfare effect of higher gains from trade $\left(\frac{\partial W}{\partial p_U} > 0\right)$—higher allocative efficiency—are counteracted by the negative welfare effect of less screening $\left(\frac{\partial W}{\partial \eta}\mid_{m>m^*}> 0\right)$—lower productive efficiency. That is, lenders who in the unregulated economy do not insure (and some of whom screen) are forced to insure and, therefore, do not screen. These lenders are individually worse off, while other lenders are better off due to higher gains from trade upon a liquidity shock. Full insurance, $m = 1$, is not efficient because the positive pecuniary

[22] These partial derivatives arise from a welfare expression as the sum of the expected payoffs of all lenders (before substituting for the price of uninsured loans), as stated in equation (44).

externality is exhausted $\left(\lim_{m\to 1} \frac{\partial W}{\partial p_U} = 0\right)$ and screening incentives are eliminated.

Figure 7 shows the higher efficient loan insurance on the intensive and extensive margin and how the insurance-induced increase in the average quality of uninsured loans affects the price and the incentives to screen. Higher loan insurance than in the unregulated economy, $m^P > m^*$, arises for a high probability of repayment, $\mu > \mu^P$, or, alternatively, for a low loan payoff, $A < A^P$, or for a sufficiently expensive screening technology, $F(\cdot)$. If $\nu \leq \frac{2\mu}{1+2\mu}$, the extensive margin of insurance for the planner can also be expressed as $\lambda > \lambda_I^P$. These bounds are similar to those for the usage of loan insurance in the unregulated equilibrium in Section 4.

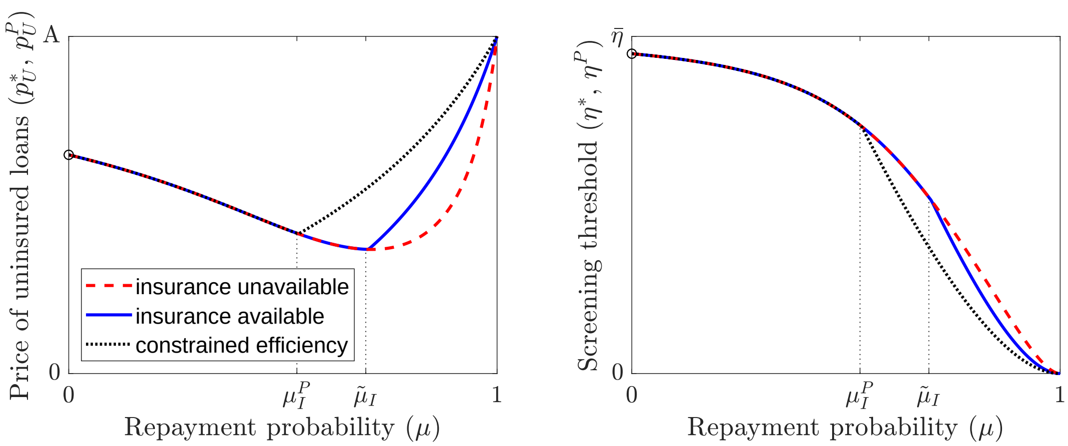

Figure 7: The constrained-efficient level of loan insurance is higher than the privately optimal level on both the intensive and extensive margin, increasing the price of uninsured loans and lowering screening (plotted for $\lambda \geq \max\{\underline{\lambda}_L, \tilde{\lambda}_L\}$).

Liquifying the market for uninsured loans. Suppose that the planner picks the liquid equilibrium when it does not exist in the unregulated economy, $\lambda < \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$. To achieve this, the planner can liquify the market by exploiting the pecuniary externality of loan insurance. By choosing a high enough m, the planner can create a liquid equilibrium in which the price satisfies $p_U \geq A/\lambda$, and thus improve the allocative efficiency on the extensive margin. The existence of a constrained-efficient allocation extends from Proposition 4 to this case.

Illiquid market. Suppose the planner picks the illiquid (or not liquid, NL) equilibrium. The price of uninsured loans is zero, so insurance has no pecuniary externality. The planner maximizes welfare subject to the individual screening choices:

$$W^{NL} = \max_{m} \; W \text{ s.t. } \eta \to (1 - \kappa\mu)A \text{ and } p_U = 0.$$

The fraction of insured high-cost lenders, m, appears only within the gains from trade, $\nu(\lambda - 1)m\,(1 - F(\eta))\,p_I$, because only the market for insured loans is liquid at $t = 1$. Hence, full insurance is constrained-efficient in the illiquid equilibrium, which is the same corner solution as in the unregulated equilibrium, $m^P = m^* = 1$.

Choice of equilibrium. Finally, we consider whether the planner prefers the liquid or the illiquid equilibrium. The liquid equilibrium is constrained efficient if it is superior to the illiquid equilibrium, $W^L \geq W^{NL}$. This ranking of equilibria occurs whenever the social gains from trade in the secondary market for uninsured loans exceed the welfare loss due to lower ex-ante screening incentives. Proposition 5 summarizes the constrained-efficient allocation and Figure 8 illustrates.

Proposition 5. ***Constrained efficiency.*** *There exists a unique* $\lambda_L^P < \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$.

1. *For* $\lambda_L^P < \lambda$, *the planner chooses the welfare-dominant liquid equilibrium:*
   a. *For* $\lambda_L^P < \lambda < \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$, *the planner liquifies the market by choosing high enough loan insurance* m^P *consistent with a price* $p_U^P \geq A/\lambda$.
   b. *For* $\min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\} \leq \lambda$, *Proposition 4 applies.*
2. *For* $\lambda \leq \lambda_L^P$, *the planner chooses the welfare-dominant illiquid equilibrium.*

Proof. See Appendix B.6 (which also defines the bound λ_L^P). ■

When both equilibria exist in the unregulated economy, the liquid equilibrium is always welfare superior. The liquid equilibrium offers an additional option for lenders to shed default risk, which lenders use whenever it is privately optimal. Similarly, lenders screen less in the liquid equilibrium (which is again privately optimally chosen). Since all externalities are pecuniary (affecting the gains from trading uninsured loans), the liquid equilibrium with positive price (higher allocative efficiency) welfare-dominates the illiquid equilibrium (see Appendix B.4 for a formal proof).

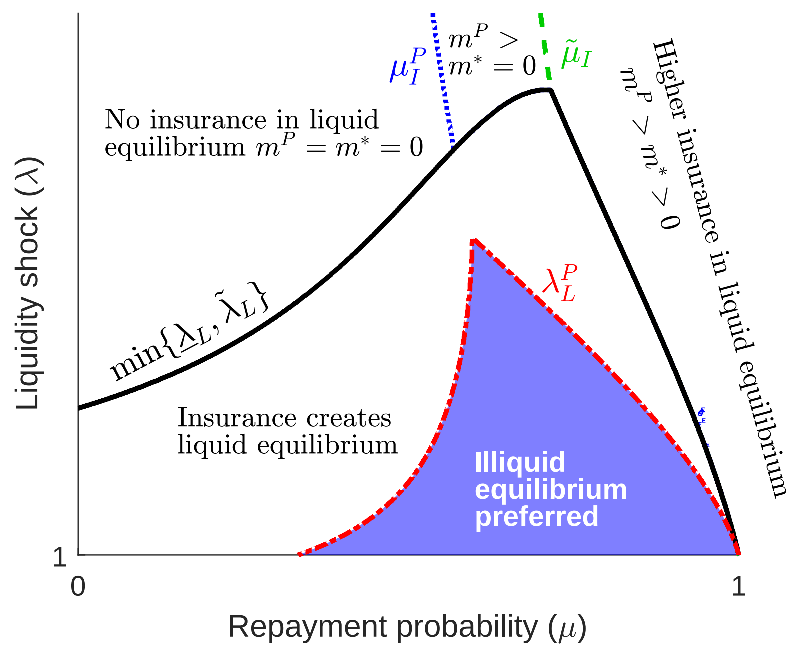

Figure 8: Constrained-efficient allocation. Proposition 4 states how insurance is used for $\lambda \geq \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$, with more insurance on the intensive and extensive margins. Proposition 5 states that the uninsured loans market is liquified for $\lambda_L^P < \lambda < \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$ as the benefit of higher allocative efficiency dominates. For $\lambda \leq \lambda_L^P$ (shaded), the planner keeps the market illiquid as liquifying would lower screening and productive efficiency too much.

On the parameter subset where the liquid equilibrium does not exist in unregulated economy, the planner achieves the liquid equilibrium by higher usage of insurance than what is privately optimal, which implies less screening than in the unregulated economy. When the cost of lower screening (lower productive efficiency) in the liquid equilibrium outweighs the benefit of liquid market for uninsured loans (higher allocative efficiency), the planner picks the illiquid equilibrium. For these parameters, the illiquid equilibrium is the unique one in the unregulated economy.

5.2 Regulation

We consider a regulator, R, with a balanced budget and no information advantage over outside financiers. Hence, a direct implementation of the constrained-efficient allocation by choosing insurance ℓ_i for each lender i is infeasible. Only high-cost lenders should insure but the screening costs of lenders $\{\eta_i\}$ are private information.

We consider two regulatory tools: (i) a subsidy $b_I \geq 0$ to lenders who insure their loan at $t = 0$ and (ii) a minimum price guarantee $p_{min} \geq 0$ in the market for uninsured loans at $t = 1$. This guarantee is implemented via a subsidy to sellers of uninsured loans

$$b_U \equiv \max\{p_{min} - p_U, 0\}, \tag{14}$$

where p_U is the competitive price. In what follows, we use p_{min} and b_U interchangeably. The regulator has commitment and announces the regulation at the beginning of $t = 0$. Given (b_I, p_{min}), lenders choose the private optimum of loan insurance.

These policies are funded by a lump-sum tax T on all lenders at $t = 1$. This tax is levied after loan sales and before consumption occurs. To ensure that lenders can always pay the tax (and to avoid unnecessary technical complications associated with limited liability), we introduce an additional non-pledgeable and perishable endowment n received when taxes are due, so these resources can be used to pay taxes or for consumption at $t = 1$.[23] Figure 9 shows the timeline. To make both policies more comparable, we assume that the loan insurance subsidy is also received at $t = 1$.

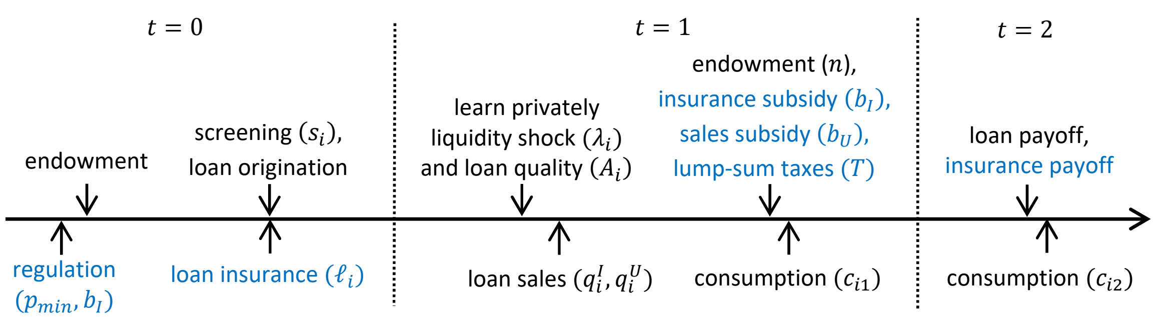

Figure 9: Timeline with loan insurance subsidy and uninsured loan sale subsidy.

Definition 3. *A competitive regulated equilibrium comprises screening $\{s_i\}$, insurance $\{\ell_i\}$, the sales of insured and uninsured loans $\{q_i^I, q_i^U\}$, an insurance subsidy b_I, a minimum price guarantee p_{min} (and the implied uninsured loan sale subsidy b_U), lump-sum taxes T, and prices p_I and p_U such that:*

[23] An endowment $n = (1-\mu)A$ covers any meaningful set of regulation. If $b_I = (1-\mu)A$, all lenders insure $m = 1$, $p_U \to A$, $\eta = 0$, and $T = (1-\mu)A$. But Proposition 4 shows that full insurance is not constrained efficient, so $b_I^R < (1-\mu)A$ and $T^R < (1-\mu)A$. Similarly for the other tool, if $p_{min} = A$, then all high-quality loans are sold irrespective of the liquidity shock and $\eta = 0$, which implies that the fundamental value of loans sold is μA and the required subsidy is $b_U = (1-\mu)A$.

1. *At $t = 1$, each lender i optimally chooses its sales of insured and uninsured loans in secondary markets for each realized shock $\lambda_i \in \{1, \lambda\}$, denoted by $q_i^I(s_i, \lambda_i, \ell_i)$ and $q_i^U(s_i, \lambda_i, \ell_i)$, given p_U, p_I, s_i, ℓ_i, b_I, and b_U.*

2. *At $t = 1$, prices p_I and p_U are set for outside financiers to break even in expectation, given $\{s_i\}$, $\{\ell_i\}$, $\{q_i^I, q_i^U\}$, b_I, and b_U.*

3. *At $t = 0$, the fee k is set for outside financiers to break even in expectation, given screening $\{s_i\}$ and insurance $\{\ell_i\}$ choices.*

4. *At $t = 0$, each lender i chooses its screening s_i and loan insurance ℓ_i to maximize expected utility, given p_I and p_U, sales q_i^I and q_i^U, b_I, b_U, and T:*

$$
\begin{aligned}
\max_{s_i,\ell_i,c_{i1},c_{i2}} \quad & \mathbb{E}\left[\lambda_i c_{i1} + c_{i2} - \eta_i s_i\right] \qquad \textit{subject to} \\
c_{i1} \quad = \quad & q_i^U(s_i,\lambda_i,\ell_i)\,(p_U + b_U) + q_i^I(s_i,\lambda_i,\ell_i)\,p_I + \ell_i\, b_I + n - T, \\
c_{i2} \quad \rightarrow \quad & \left[\ell_i - q_i^I\right](A-k) + \left[1 - \ell_i - q_i^U\right] \times \begin{cases} A & \textit{w.p.} \quad s_i + \mu(1-s_i) \\ 0 & \qquad\quad (1-\mu)(1-s_i). \end{cases}
\end{aligned}
$$

5. *At $t = 0$, the regulator chooses the insurance subsidy b_I and price guarantee p_{min} to maximize welfare subject to a balanced budget, $T = b_U \int q_i^U di + b_I \int \ell_i di$.*

The insurance subsidy b_I increases the incentives to insure and the fraction of insured loans m^R, which indirectly increases the price of uninsured loans $p_U(b_I)$ (recall Figure 5 and Proposition 2). The sale subsidy b_U directly increases the value of sold uninsured loans, $p_U + b_U$. Loan insurance is used in the regulated equilibrium (and insurance subsidies are effective), $m^R > 0$, only if high-cost lenders are indifferent:

$$\nu\lambda\,(p_I + b_I - p_U - b_U) = (1-\nu)\,(1-\mu)\,(p_U + b_U), \tag{15}$$

which generalizes the indifference condition (10) to the regulated economy with subsidies.[24] It implies that the sale subsidy lowers the incentives to insure. When (15) is not satisfied, an insurance subsidy has no effect on p_U since no lender insures, $m^R = 0$.

[24]We abstract from a strict preference for loan insurance, $m = 1$, because full insurance is never efficient (Proposition 4). We allow for $m = 0$ because no insurance is efficient for some parameters.

The screening threshold in the regulated equilibrium with subsidies is

$$\eta \to (1-\nu)(1-\mu)(A - p_U(b_I) - b_U), \tag{16}$$

which generalizes the threshold in the unregulated equilibrium in equation (6) by reflecting the negative effects of both subsidies on screening incentives.

The regulator chooses the insurance subsidy b_I and the minimum price p_{min} to maximize welfare. If the regulator implements the illiquid equilibrium, it solves:

$$\max_{b_I, p_{min}} W + \kappa(n + \underbrace{m(1-F(\eta))b_I - T}_{\text{Policy redistribution } (=0)}) \text{ s.t. } \eta \to (1-\kappa\mu)A,\ p_U = 0, \text{ and } p_{min} < \frac{A}{\lambda}.$$

If the regulator implements the liquid equilibrium, it solves:[25]

$$\max_{b_I, p_{min}} W^R \to \max_{b_I, p_{min}} \overbrace{\nu(\lambda-1)\left[p_U(b_I) + b_U + m\,(1-F(\eta))\,(p_I + b_I - p_U(b_I) - b_U)\right]}^{\text{Gains from trade}} \tag{17}$$

$$+ \underbrace{\left(F(\eta) + \mu(1-F(\eta))\right)A}_{\text{Fundamental value}} - \underbrace{\int_0^{\eta} \tilde{\eta} dF(\tilde{\eta})}_{\text{Screening costs}} + \kappa(n - T) \tag{18}$$

s.t. (16), (11), (14), $p_I = \mu A, \lambda p_U \geq A,$ and

$$m[\nu\lambda\,(p_I + b_I - p_U - b_U) - (1-\nu)\,(1-\mu)\,(p_U + b_U)] = 0, \tag{19}$$

where (19) holds with complementary slackness. A main result on regulation follows.

Proposition 6. ***Regulation achieves the constrained-efficient allocation.***

1. *If $\lambda \leq \lambda_L^P$, then the regulator implements the illiquid equilibrium, $b_U^R = 0 = p_{min} = b_I^R$. The illiquid equilibrium, $p_U^R = 0$, is unique and constrained efficient.*

2. *If $\lambda > \lambda_L^P$, the regulator guarantees a minimum price, $p_{min}^R = A/\lambda$ to eliminate the welfare-dominated illiquid equilibrium. Then, the liquid equilibrium is the unique regulated equilibrium, where $p_U^R \geq p_{min}^R$ (and therefore $b_U^R = 0$).*

[25] We focus on the interval $b_I \leq (1-\mu)A$ (respectively, $b_U \leq (1-\mu)A$) without loss of generality. Higher subsidies have no effect on welfare, as the payoff of insured loans $\mu A + b_I$ (sold loans $p_U + b_U$) would exceed the payoff from high-quality loans, so all lenders insure (sell all high quality loans irrespective of liquidity shock) and do not screen, resulting in constant welfare $W = \kappa(\mu A + n)$.

(a) If $\lambda \geq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$ *and* $\mu \leq \mu_I^P$*, there are no insurance subsidies,* $b_I^R = 0$*, as the allocation in the liquid equilibrium is constrained efficient,* $m^R = 0$*.*

(b) If either $\lambda_L^P < \lambda < \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$*, or* $\lambda \geq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$ *and* $\mu > \mu_I^P$*, the regulator implements the constrained-efficient allocation by subsidizing insurance:*

$$b_I^R = \frac{[\kappa - (1-\nu)\mu]p_U^P - \nu\lambda\mu A}{\kappa}. \tag{20}$$

For $\lambda_L^P < \lambda \leq< \lambda_B^P \leq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$*, the regulator increases allocative efficiency only on the extensive margin,* $p_U^R = A/\lambda$*. Then,* b_I^R *increases in* A*, decreases in* μ *and* λ *and is independent of screening technology* $F(\cdot)$*.*

(c) If $\lambda_B^P < \lambda < \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$ *or* $\lambda \geq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$ *and* $\mu > \mu_I^P$*, the regulator improves allocative efficiency on the intensive margin,* $p_U^R > A/\lambda$*. For a uniform screening cost,* $\eta_i \sim \mathcal{U}(0, \bar{\eta})$*, the insurance subsidy* b_I^R *increases in* A *and* λ*, is independent of* $\bar{\eta}$*, and non-monotonic in both* μ *and* ν*.*

Proof. See Appendix B.7 (which also defines the bound λ_B^P). ■

Eliminating the illiquid equilibrium when it is welfare-dominated by the liquid equilibrium can only be achieved via guaranteeing a minimum price in the secondary market for uninsured loans. Conditional on the liquid equilibrium, both an uninsured loan sale subsidy and an insurance subsidy can keep the market for uninsured loans liquid. However, the insurance subsidy is superior to the sale subsidy because of the positive pecuniary externality of loan insurance. The insurance subsidy incentivizes high-cost lenders to forgo the option of selling only lemons in the secondary market for uninsured loans and, therefore, reduces adverse selection in this market. In contrast, the sale subsidy does not take advantage of this externality and is thus more expensive. Hence, the welfare-dominated sale subsidy is not used in the liquid equilibrium, $b_U^R = 0$, and higher social gains from trade are achieved with insurance subsidies.

Under the optimal subsidy, the regulator does not increase welfare directly by redistribution between lenders, resulting in zero redistributive welfare term $\kappa(b_I m(1-F(\eta)) - T) = 0$. While the insurance subsidy redistributes from all lenders (taxpayers) to insured lenders, all of these agents have the same expected utility of consumption, κ. Hence, the welfare effects arise only from the impact of subsidies on the incentives to insure m, which affects p_U, and the incentives to screen η.

Figure 10 shows the superiority of insurance subsidies. It compares welfare in the liquid equilibrium when both policies have the same target price p_U^T. This price arises either directly with subsidized sales of uninsured loans, $p_U^T = p_U + b_U$, or indirectly with subsidized loan insurance, $p_U^T = p_U(b_I)$. Subsidizing insurance is better and achieves constrained efficiency, $p_U^R = p_U(b_I^R) = p_U^P > p_U^*$. Sale subsidies do not achieve constrained efficiency and can be welfare-inferior as they eliminate insurance (when used in the unregulated equilibrium, $\lambda \geq \tilde{\lambda}_L$ and $\mu > \tilde{\mu}_I$). This is the case in Figure 10, so the regulator chooses not to subsidize uninsured loan sales in liquid equilibrium, $p_U(b_U^R) = p_U^*$, even when an insurance subsidy is not available.[26]

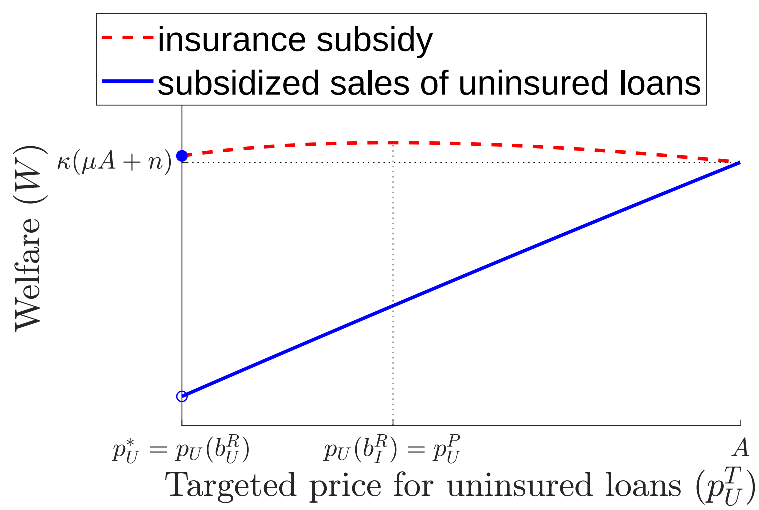

Figure 10: Welfare comparison across policies. Insurance subsidies achieve the constrained-efficient allocation and welfare-dominate uninsured loan sale subsidies. Sale subsidies eliminate insurance used in the unregulated equilibrium, resulting in a discrete drop in welfare.

The size of the optimal insurance subsidy depends on which margin of allocative efficiency the regulator improves. For relatively low social gains from trade, $\lambda_L^P < \lambda \leq \lambda_B^P \leq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$, the regulator liquifies the market for uninsured loans, but further

[26]The discontinuity between no intervention and an effective sales subsidy for uninsured loans, at $p_U^T = p_U^*$, arises because this policy eliminates insurance and its positive pecuniary externality that is compensated by costly sales subsidies. At $p_U^T = A$, all lenders receive a subsidy under both policy options and, therefore, the overall welfare levels are equalized: $W \mid_{p_U^T = A} = \kappa(\mu A + n)$.

improvement of allocative efficiency on intensive margin is not optimal due to negative effects on productive efficiency. Hence, insurance subsidies target the minimum price that sustains a liquid equilibrium, $p_U = A/\lambda$, and the subsidy decreases in λ (see Figure 11). For higher social gains from trade, it is optimal to improve allocative efficiency at both margins ($\lambda_B^P < \lambda < \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$), or only at the intensive margin ($\lambda \geq \min\left\{\underline{\lambda}_L, \tilde{\lambda}_L\right\}$ and $\mu > \mu_I^P$). In these cases, the insurance subsidy increases in the size of the liquidity shock λ, as shown below.

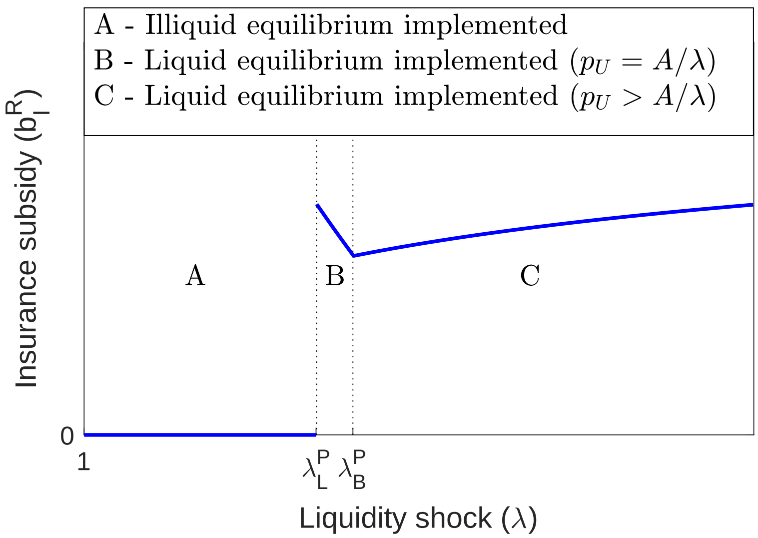

Figure 11: Optimal insurance subsidy b_I^R as a function of the size of the liquidity shock λ (plotted for $\mu > \mu_I^P$). The illiquid equilibrium is implemented for $\lambda \leq \lambda_L^P$, so no subsidies are used. For $\lambda > \lambda_L^P$, the liquid equilibrium is implemented. For $\lambda_L^P < \lambda \leq \lambda_B^P$, the regulator implements the minimum price consistent with a liquid market, $p_U^R = A/\lambda$ (improving allocative efficiency on the extensive margin only). For $\lambda > \lambda_B^P$, in contrast, the regulator also improves allocative efficiency on intensive margin, $p_U^R > A/\lambda$.

6 Interpretation and implications of results

Our normative results contribute to a debate on the government provision of repayment guarantees for various loan types. In the US after the recent financial crisis, this debate has been especially intense regarding the government intervention in the mortgage market. Some in the literature emphasize the negative implications of subsidized government-backed mortgage default insurance, for instance on bank risk-taking and its distributional implications in quantitative macro models (Jeske et al., 2013; Elenev et al., 2016). Our contribution to this debate is to uncover a new channel through which loan insurance increases market liquidity and welfare. We show that lenders with worse lending technology self-select into insurance, which makes their loans insensitive to future information about loan quality. Since these lenders give up the option to selectively sell lemons in the market for uninsured loans, the average quality of loans traded in this market increases, raising the social gains from trade.

Proposition 4 states conditions under which the privately optimal level of loan insurance in the liquid equilibrium is inefficiently low, which rationalizes insurance subsidies. First, these subsidies arise for higher-quality loans with a low default risk (high μ), such as loans to borrowers with high credit scores or loans in regions with lower predictable default risk. Indeed, conditioning insurance on sufficiently high credit scores, as our analysis suggests, is consistent with the practices of FHA and GSEs in the US or CMHC in Canada. However, government support for loan insurance does not vary over regions within countries despite large regional variation in predictable default risk (Hurst et al., 2016), which is inefficient according to our model. Second, subsidizing insurance is efficient when loans are less profitable, borrowers have a lot of bargaining power, or lending markets are more competitive (low A). This suggests that the benefits of loan insurance are higher in countries with a less concentrated lending market and lower profit margin of lenders (e.g., in the United States as opposed to Canada). Similarly, higher recent competition from Fintech (e.g., specialized online lenders) suggests that insurance has become more beneficial. Third, loan insurance subsidies arise when lenders may face larger liquidity needs (high λ). This would apply in countries with high systemic vulnerabilities in the financial sector and when lenders are highly levered or have large liquidity and maturity mismatches on their balance sheets. Finally, more insurance is desirable when screening costs are higher (a shift in F). Recent technological advances and extensive data analysis of borrowers would reduce the benefits of insurance.

We turn to the size of the optimal insurance subsidy, stated in Proposition 6 point 2c when allocative efficiency is improved on the intensive margin. First, we find that the insurance subsidy is proportional to loan profitability, A. This result suggests that the insurance subsidy should be higher in less competitive markets. Similarly, higher competition brought by Fintech lenders suggests that a smaller subsidy is necessary. Second, the insurance subsidy increases in the size of the liquidity shock, λ, because the benefits of loan insurance increase in social gains from trade. Hence, the insurance subsidy should be higher in countries with or during times of high systemic risk and when lenders are vulnerable due to high leverage or mismatches in their balance sheet. When it is efficient to improve the allocative efficiency only on the extensive margin and thus target the minimum price consistent with liquid market for uninsured loans, however, the insurance subsidy is still proportional to profitability A but decreases in the size of the liquidity shock, λ. Therefore, the size of the optimal insurance subsidy is non-monotonic in social gains from trade globally.

Our findings also have implications for preventing illiquid markets. It is useful for the regulator to have the option to intervene directly in the market for uninsured loans to eliminate the illiquid equilibrium whenever it is welfare-inferior. This intervention is similar to the Troubled Asset Purchase Program (TARP) as originally intended, and has already been studied by Tirole (2012), Philippon and Skreta (2012), and Chiu and Koeppl (2016), for example. We contribute by showing that while this tool is useful for eliminating dominated illiquid equilibria, loan insurance subsidies are a better tool to sustain the liquid equilibrium. That is, a direct intervention in the market for uninsured loans is useful as a credible option that allows lenders to coordinate on the welfare-superior equilibrium but is actually not used in equilibrium.

7 Conclusion

We have studied insurance against loan default in a parsimonious model of lending and credit risk transfer with costly screening of borrowers in primary markets and adverse selection in secondary markets. A key result is that loan insurance reduces the adverse selection in the market for uninsured loans. This raises the social gains from trade, reduces screening incentives, and increases welfare. Loan insurance is inefficiently low in the unregulated equilibrium because its positive effect on the price of uninsured loans is not internalized by lenders. Optimal regulation achieves the constrained-efficient allocation with two tools: (i) a subsidy for sale of uninsured loans eliminates the illiquid equilibrium when it is welfare-dominated by the liquid equilibrium; and (ii) an insurance subsidy that induces lenders to internalize the beneficial impact of loan insurance on secondary market liquidity and sustains the liquid equilibrium. These findings contribute to a debate about the role of mortgage insurance and the reform of government-sponsored enterprises by identifying a mechanism that suggests a positive externality on market liquidity.

We wish to discuss potential directions for further work. First, we have assumed that each lender has access to a separate pool of borrowers. If lenders share a common pool instead, then screening has a thinning effect and a lender's choice of screening reduces the quality of the residual pool, a negative externality. Since lenders who screen never insure in equilibrium, we expect loan insurance to mitigate this negative externality of thinning and the social incentives to subsidize loan insurance to be even higher. Second, we have assumed a competitive insurance fee. If a premium is required in the insurance market instead, then we expect a lower premium to increase the incentives to insure, which increases the uninsured loans price and reduces screening. Third, we have normalized the rate of return required by outside financiers to zero. If a general required return is allowed instead, the impact of a low required return (perhaps due a savings glut or stimulative monetary policy) on lending standards and the social benefits of insurance could be studied. In this case, we expect that lending standards to be lower and insurance benefits to be higher.

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A Generalizations and extensions

Unless stated otherwise, we focus on $\psi \to 1$ throughout these extensions.

A.1 Adverse selection in insurance market

To examine the possibility of adverse selection in the loan insurance market at $t = 0$, we modify the screening technology in two ways. First, screening improves the probability of loan repayment from μ to $\psi < 1$. Second, lenders privately learn loan quality upon screening at $t = 0$ (that is, earlier than in the main model). Hence, this information advantage affects the insurance market at $t = 0$, where up to a fraction $1 - \psi$ of screened loans, which are lemons, may be insured. We show that the presence of adverse selection in loan insurance strengthens the case for loan insurance subsidies compared to the main model.

Proposition 7. ***Asymmetric information at*** $t = 0$***.*** *In the modified model, there is adverse selection in both the loan insurance market at* $t = 0$ *and in the secondary market for uninsured loans at* $t = 1$*, resulting in (additional) multiplicity of equilibria:*

1. *There are equilibria with an illiquid insurance market, i.e.* $k = A$ *and* $p_I = 0$*. Thus, Lemma 1 and 2 apply.*
2. *For* $\lambda \geq \widehat{\lambda}_L > \tilde{\lambda}_L$ *and* $A < \widehat{A} < \tilde{A}$*, there exists an equilibrium with both a liquid insurance market and a liquid market for uninsured loans (whereby some lenders who do not screen insure and some lenders sell high-quality loans after a liquidity shock).*

   Among the multiple equilibria, this equilibrium has the highest price of uninsured loans, the lowest screening threshold, and the highest welfare. Since the positive effects of loan insurance are not fully internalized, insurance by high-cost lenders above the level in the unregulated economy increases welfare, motivating insurance subsidies.

Proof. See Appendix B.8, where $\widehat{\lambda}_L$ and $\widehat{A}_I$ are defined. ■

Due to private learning at $t = 0$, low-cost lenders can selectively insure lemons, so screening creates adverse selection in the loan insurance market and the related multiplicity of equilibria. The additional defining feature of the equilibrium is whether high-cost lenders insure and thus make the insurance market liquid, i.e. the competitive fee and the secondary market price of insured loans reflect that not only lemons are insured, $k < A$ and $p_I > 0$.

The insurance markets can always be illiquid since $k = A$ and $p_I = 0$ and the option of low-cost lenders to selectively insure lemons are mutually consistent. Section 3 characterizes the two possible equilibria with illiquid insurance in this case.

The equilibrium with a liquid insurance market is characterized by a higher price for uninsured loans, reduced screening incentives, and higher welfare.[27] This ranking echoes the

[27]For a knife-edge parameter constellation, there exists an equilibrium in which the insurance market is liquid but the market for uninsured loans in illiquid. We abstract from this equilibrium based on its instability and the fact that it is non-generic.

effect of the introduction of loan insurance on the liquid equilibrium in the main text (Section 4). Compared to the liquid equilibrium with insurance in Section 4, private learning by low-cost lenders at $t = 0$ reduces the benefits of insurance (because adverse selection implies $p_I < \mu A$), and increases the cost of insurance (because lemons by low-cost lenders are no longer sold in the uninsured market, putting upward pressure on p_U). Moreover, there is a strategic complementarity in the choice to insure because the benefit of insurance increases in the proportion of insuring high-cost lenders m ($dp_I/dm > 0$), resulting in potential multiplicity of equilibria in the class of equilibrium with a liquid insurance market.

In an equilibrium with a liquid insurance market, insurance again improves the average quality of uninsured loans traded, and thus improves their price in secondary market but now for three reasons. Two reasons are the same as in the main model: insured lenders have on average lower quality than uninsured lenders and high-cost lenders who insure give up the option to act on future private information. The third channel is new and arises from insurance removing all lemons owned by low-cost lenders from the uninsured market. This result is because informed low-cost lenders sell lemons in the secondary market with the highest price and, in equilibrium, this is the market for insured loans, $p_I > p_U$.

Due to the positive externality of loan insurance for the quality of uninsured loans traded, the amount of insurance by high-cost lenders in the equilibrium with liquid insurance is again excessively low. Therefore, there is again scope for a constrained planner or regulator to improve allocative efficiency and welfare. Moreover, the adverse selection in the loan insurance market creates a new and additional incentive for the planer to liquify the loan insurance market and improve allocative efficiency on the extensive margin, with a positive effect on welfare.

A.2 Early loan sale

We study the option for lenders to sell loans to outside financiers at $t = 0$ upon origination. To allow lenders to consume at $t = 1$ (when they have a high expected marginal utility of consumption), we introduce storage of the loan sale proceeds (but not of the endowment) until $t = 1$. We study the implications of early loan sales in two cases of private learning about the loan payoff A_i at $t = 1$: (a) relationship banking, whereby only lenders can learn loan quality but outside financiers receive no private information; and (b) learning-by-holding (Plantin, 2009), whereby outside financiers learn A_i at $t = 1$ upon having owned the loan since $t = 0$.

Proposition 8. ***Early loan sales.*** *The implications of loan sales to outside financiers at $t = 0$ depend on whether financiers privately learn loan quality at $t = 1$:*

1. ***Relationship lending.*** *Early loan sales are equivalent to loan insurance and all of our positive and normative implications carry over.*
2. ***Learning-by-holding.*** *Early sales exacerbate adverse selection in the secondary market for uninsured loans. Because of this negative pecuniary externality, early loan sales are excessively high:*

(a) If $\lambda_L^{PS} < \lambda < \underline{\lambda}_L^S$ or $\lambda > \bar{\lambda}_L^S$, the planner liquifies the market for uninsured loans by reducing early loan sales (extensive margin of allocative efficiency).

(b) If $\underline{\lambda}_L^S \leq \lambda \leq \bar{\lambda}_L^S$ and $A > \bar{A}^S$, the planner reduces early loan sales in the liquid equilibrium (intensive margin of allocative efficiency).

To implement these allocations, a regulator taxes the early sales of loans.

Proof. See Appendix B.9 (which also defines λ_L^{PS}, $\underline{\lambda}_L^S$, $\bar{\lambda}_L^S$, and $\bar{A}^S$). ■

If only lenders can learn loan quality at $t = 1$ (e.g., due to relationship lending), then early loan sales reduce the adverse selection problem in the secondary loan market at $t = 1$ and makes the liquid equilibrium more likely—exactly as loan insurance does. This positive externality implies the same normative results for early loan sales as for loan insurance in the main text due a formal equivalence of the two instruments.

If the holder of the loan can learn at $t = 1$ (e.g., learning by holding), however, then outside financiers selectively sell lemons in the market at $t = 1$. Since outside financiers are not subject to the liquidity shock, they never sell high-quality loans—unlike the lenders who sell high-quality loans upon liquidity shock (in the liquid equilibrium). As a result, early loan sales increase adverse selection in the market at $t = 1$—a negative pecuniary externality—and make the liquid equilibrium less likely. In contrast to loan insurance, the option of early loan sales is not a Pareto improvement in the unregulated equilibrium. Moreover, the normative implication of early loan sales in this case are the opposite of loan insurance. The planner wants to reduce the volume of early loan sales and a regulator wishes to tax it accordingly.

A.3 Imperfect screening

Consider imperfect screening, whereby the success probability satisfies $\mu < \psi < 1$ (but no private learning at $t = 0$ in Appendix A.1). As a result, some low-cost lenders, who choose to screen at $t = 0$, also sell low-quality loans in the secondary market for uninsured loans at $t = 1$.

Proposition 9. ***Imperfect screening.*** *If $\psi > \underline{\psi}$, better screening ($d\psi > 0$) lowers loan insurance on both the intensive and extensive margins: fewer high-cost lenders insure, $\frac{dm^*}{d\psi} < 0$, and the parameter range for loan insurance shrinks, $\frac{d\tilde{A}}{d\psi} < 0$.*

Proof. See Appendix B.3 (which also contains the definition of $\underline{\psi}$). ■

A better screening technology implies higher screening benefits and more low-cost lenders in equilibrium. The incentives to insure, equation (10), are not directly affected because only high-costs (non-screening) lenders insure. The effect of better screening on insurance comes entirely via the price of uninsured loans. Better screening puts an upward pressure on p_U^*, because more high-quality loans are sold. The higher price lowers the relative benefits of insurance and increases the cost of insurance (loosing the option to sell lemons at p_U). Hence, better screening technology lowers insurance. In equilibrium, insurance drops enough to fully compensate for better screening and, as a result, the price remains unchanged and satisfies the insurance indifference condition (10).

A.4 Partial insurance

Suppose insurance contracts allow lenders to choose the fraction ω of default costs covered by the insurance. Such insurance contracts are equivalent to guaranteeing the non-default payment A with a deductible $(1-\omega)A$, where the owner of the loan pays the insurance fee at the time of maturity ($t=2$). As proven in Appendix B.10, only high-cost lenders insure, so the competitive insurance fee is actuarially fair and reflects the average cost of insurance, $k=\omega(1-\mu)A$. We have the following result.

Proposition 10. *Full insurance, $\omega^*=1$, is privately and socially optimal.*

Proof. See Appendix B.10. ■

With partial insurance, $\omega<1$, the value of an insured loan of low quality is $\omega A-k=\omega\mu A$, which is below the value of an insured loan of high quality, $A-k=A\left(1-(1-\mu)\omega\right)$ (in contrasts with the full-insurance case). There is adverse selection in the market for partially insured loans since lenders without a liquidity shock sell only low-quality loans. Adverse selection redistributes wealth from lenders with liquidity shock (who always sell) to lenders without liquidity shock (who sell only lemons). Since lenders have a higher utility of consumption in states with liquidity shock, they choose full coverage, $\omega^*=1$, to avoid the costs of adverse selection. As for social optimality, a higher insurance coverage has a positive externality on the price of uninsured loans, so a planner also chooses full coverage.

An alternative interpretation of partial insurance is insurer default. We have assumed so far that the insurer has deep pockets, perhaps because of (implicit) government backing. In contrast, suppose the insurer defaults on its liabilities after the fee is paid at $t=2$ with exogenous probability $1-\omega$. The expected value of an insured loan is $\omega A-k$ upon loan default ($-k$ when insurer defaults and $A-k$ otherwise) and $A-k$ upon loan repayment (irrespective of insurer default). The insurance fee is $k=\omega(1-\mu)A$. Since the expected payoffs are equal to those for partial insurance, the problem with insurer default is identical. Proposition 10 implies that welfare decreases in insurer default risk.

A.5 Upfront insurance fee

In this extension, we suppose that the insurance fee k has to be paid at $t=0$. Hence, a lender who insures can fund only a fraction $1-k$ of the loan. Despite this negative effect on lending volume, we show that our qualitative results remain unchanged.

Proposition 11. ***Upfront fee.*** *Suppose the insurance fee is paid at $t=0$.*

1. *For $A < \tilde{A}'$ and $\lambda \geq \tilde{\lambda}'_L$, some loans are insured, $m^{*\prime} = 1 - \frac{\kappa F(\eta^{*\prime})(1-\delta)}{(1-F(\eta^{*\prime}))\left[\mu(\lambda-1)(1-\nu)-\kappa\delta\frac{\nu+(1-\nu)(1-\mu)}{\nu}\right]} \in (0,1)$, the screening threshold is $\eta^{*\prime} \equiv \frac{(1-\nu)(1-\mu)^2\kappa A}{\nu\lambda+(1-\nu)(1-\mu)}(1+\delta)$, and the price of uninsured loans is $p^{*\prime}_U \equiv \frac{\nu\lambda\mu A-\kappa(1-\mu)A\delta}{\nu\lambda+(1-\nu)(1-\mu)}$, where $\delta \equiv \frac{\mu A-1}{1+A(1-\mu)}$. Loan insurance increases the price $p^{*\prime}_U$, reduces screening, and increases welfare.*

2. *The constrained efficient level of loan insurance, $m^{P\prime} \in [m^{*\prime}, 1)$, exceeds the unregulated level at both the intensive and the extensive margins.*

3. *If $\mu A > 1$, then insurance is less beneficial under upfront fee payment, $m^{*\prime} \leq m^*$ and $\tilde{A}' < \tilde{A}$ and $m^{P\prime} \leq m^P$, which implies $p^{*\prime}_U \leq p^*_U$, $\eta^{*\prime} \geq \eta^*$, $\underline{\lambda}'_I > \tilde{\lambda}_L$.*

Proof. See Appendix B.11 (which also defines the bounds $\tilde{A}'$ and $\tilde{\lambda}'_L$). ■

If the return from non-screened loans exceeds the intertemporal rate of substitution of financiers, $\mu A > 1$, then the net individual benefit of insurance is reduced by the lower lending volume. Compared to payment at $t = 2$, less insurance occurs at both the intensive margin, $m^{*\prime} \leq m^*$, and the extensive margin, $\tilde{A}' < \tilde{A}$. There is a weaker positive effect on the price of uninsured loans, which is lower than under final-date payment. The lower price, in turn, implies a higher screening threshold and a higher required threshold for the existence of liquid equilibria, $\underline{\lambda}'_I > \tilde{\lambda}_L$. But insurance continues to have a positive pecuniary externality in the uninsured loans market, so our normative results go through qualitatively.

A.6 Partial loan sales

We allow for partial sales of uninsured loans, $q^U_i \in [0, 1-\ell_i]$, and study whether retaining default risk on the uninsured loan, $1-\ell_i-q^U_i$, can signal loan quality. The quantity of uninsured loans not sold is a continuous choice that can be used by financiers to update their beliefs about loan quality and may result in a continuum of perfect Bayesian equilibria. We show that our results are qualitatively unchanged.

Proposition 12. ***Partial loan sales.***

1. *For $\bar{\eta} < (1-\mu)A$, risk retention induces the existence of perfect Bayesian equilibria with full screening, $\eta^*(q^{U*}) \geq \bar{\eta}$, sustained by out-of-equilibrium beliefs interpreting $q^U \neq q^{U*}$ as a signal of low quality, where $q^{U*} \in \left(0, 1-\frac{\bar{\eta}}{(1-\mu)A}\right]$. All originated loans are of high quality, and adverse selection in the market for uninsured loans disappears.*

2. *For $\bar{\eta} \geq (1-\mu)A$, all perfect Bayesian equilibria are pooling characterized by $q^{U*} \in (\bar{q}^U, 1]$ and by partial screening, $\eta^*(q^{U*}) < \bar{\eta}$, and are sustained by out-of-equilibrium beliefs interpreting $q^U \neq q^{U*}$ as a signal of low quality. That means that the quality of uninsured loans remains private information, adverse selection in the secondary market remains, and our results are qualitatively unchanged:*

   a. *When $A < \tilde{A}(q^U)$, some loans are insured in the liquid equilibrium, $m^* > 0$.*

b. The constrained-efficient level of loan insurance in the liquid equilibrium is higher at both the intensive margin, $m^P > m^* > 0$ when $A < \tilde{A}(q^U)$, and the extensive margin, $m^P > m^* = 0$ when $A^P(q^U) > A \geq \tilde{A}(q^U)$.

Proof. See Appendix B.12 (which formally defines the equilibrium and $\bar{q}^U$). ■

Since lenders have limited liability, any loan sale q^U would be mimicked by sellers of low-quality loans (similar to Parlour and Plantin 2008). Thus, the quality of uninsured loans is public information only in the corner solution in which everyone screens, $\eta^* \geq \bar{\eta}$, which arises for $\bar{\eta} < (1-\mu)A$. In this case, the upper bound on screening costs is low enough so that sufficient default risk retention incentivizes all lenders to screen, so all loans are of high-quality. When $\bar{\eta} \geq (1-\mu)A$, however, some lenders do not screen and the screening choice of sellers of uninsured loans and the quality of uninsured loans remain private information (pooling equilibrium). This results in adverse selection in the market for uninsured loans, so our results from the main text extend to partial loan sales. Loan insurance reduces such adverse selection and the competitive level of loan insurance is inefficiently low.

B Proofs

B.1 Proof of Lemma 1

We solve the case of $\mu < \psi < 1$ and then take the limit $\psi \to 1$. In the liquid equilibrium, the competitive price reflects that low-cost lenders sell some, but fewer, lemons than high-cost lenders:

$$p_U = \nu A \frac{\psi F(\eta) + \mu(1 - F(\eta))}{\nu + (1-\nu)\left[(1-\psi)\, F(\eta) + (1-\mu)\,(1 - F(\eta))\right]} \equiv p_U(\eta), \tag{21}$$

which yields equation (5) in the limit of $\psi \to 1$. The screening threshold is obtained by equalizing the payoff when screening, $\nu\lambda p_U + (1-\nu)(\psi A + (1-\psi)p_U) - \eta$, and when not screening, $\nu\lambda p_U + (1-\nu)(\mu A + (1-\mu)p_U)$:

$$\eta = (1-\nu)(\psi-\mu)(A - p_U) \equiv \eta(p_U), \tag{22}$$

where equation (6) arises for the limit again. The equilibrium screening threshold, η^*, is obtained by substituting equation (21) in equation (22). It is implicitly given by

$$\eta^* = \frac{(1-\nu)(\psi-\mu)\left[1 - \mu - F(\eta^*)(\psi-\mu)\right]}{\nu + (1-\nu)\left[1 - \mu - F(\eta^*)(\psi-\mu)\right]} A, \tag{23}$$

which simplifies to equation (7) in the limit.

Within the class of liquid equilibria, does a unique equilibrium exist? Regarding uniqueness, the left-hand side (LHS) of the above equation increases in η and its right-hand side (RHS) decreases in it, so at most one intersection exists. Regarding existence, we evaluate both sides at the bounds of the screening cost, using $F(0) = 0 < 1 = F(\bar{\eta})$. Note that $LHS(0) < RHS(0)$ and $LHS(\bar{\eta}) > RHS(\bar{\eta})$ if $\bar{\eta} > \frac{(1-\nu)(\psi-\mu)(1-\psi)}{\nu+(1-\nu)(1-\psi)}A$. For $\psi \to 1$, the above condition always holds. For imperfect screening, we assume that the screening cost is heterogeneous enough such that this condition holds. Hence, there exists a unique interior screening threshold $\eta^* \in (0, \bar{\eta})$. The secondary market price (for uninsured loans) follows:

$$p_U^* \equiv p_U(\eta^*) = \nu A \frac{\psi F(\eta^*) + \mu(1 - F(\eta^*))}{\nu + (1-\nu)\left[(1-\psi)F(\eta^*) + (1-\mu)(1 - F(\eta^*))\right]}, \tag{24}$$

where η^* is given in equation (23). The limit expression for the price is stated in Lemma 1.

To verify the supposed liquid equilibrium (in which high-quality loans are sold in the secondary market), we combine conditions (24) and (3). Thus, the condition for the liquid equilibrium is $\lambda \geq \underline{\lambda}_L \equiv \frac{\nu + (1-\nu)[(1-\psi)F(\eta^*) + (1-\mu)(1-F(\eta^*))]}{\nu(\psi F(\eta^*) + \mu(1-F(\eta^*)))}$, whose RHS is independent of λ.

Next, we derive comparative statics for p_U^*, η^*, and the bound $\underline{\lambda}_L$. For the effect on the screening threshold, we use equation (23) to define

$$H \equiv \eta - \frac{(1-\nu)(\psi - \mu)\left[1 - \mu - F(\eta)(\psi - \mu)\right] A}{\nu + (1-\nu)\left[1 - \mu - F(\eta)(\psi - \mu)\right]} \equiv \eta - \frac{N}{D}, \tag{25}$$

with $H(\eta^*) = 0$ and N and D being the numerator and denominator, respectively. To use the implicit function theorem, we obtain the following partial derivatives of H:

$$\begin{aligned}
H_\eta &= 1 + D^{-2}(1-\nu)(\psi-\mu)^2 \nu A f > 0, \qquad H_\nu = D^{-2}(\psi - \mu)\left[1 - \mu - F(\psi - \mu)\right] A > 0, \\
H_\mu &= D^{-2}(1-\nu)A\Big\{ \left[(1-\psi)F + (1+\psi-2\mu)(1-F)\right]\nu + \left[(1-\psi)F + (1-\mu)(1-F)\right]^2 (1-\nu) \Big\} > 0, \\
H_\lambda &= 0, \qquad H_A = -D^{-1}(1-\nu)(\psi-\mu)\left[1 - \mu - F(\eta)(\psi - \mu)\right] < 0.
\end{aligned} \tag{26}$$

These partial derivatives imply the following comparative statics:

$$\frac{d\eta^*}{d\nu} = -\frac{H_\nu}{H_\eta} < 0, \quad \frac{d\eta^*}{d\mu} = -\frac{H_\mu}{H_\eta} < 0, \quad \frac{d\eta^*}{d\lambda} = -\frac{H_\lambda}{H_\eta} = 0, \quad \frac{d\eta^*}{dA} = -\frac{H_A}{H_\eta} > 0. \tag{27}$$

For the effect on the price, we use equation (24) and obtain these partial derivatives:

$$\frac{\partial p_U^*}{\partial \lambda} = 0, \quad \frac{\partial p_U^*}{\partial A} = \frac{p_U^*}{A} > 0, \quad \frac{dp_U^*}{d\eta^*} = \frac{(\psi - \mu) A \nu f}{D^2} > 0, \quad \frac{\partial p_U^*}{\partial \mu} = \frac{\nu(1-F)A}{D^2} > 0, \tag{28}$$

$$\frac{\partial p_U^*}{\partial \nu} = D^{-2}\left[\mu + (\psi - \mu) F(\eta^*)\right]\left[(1-\psi)F(\eta^*) + (1-\mu)(1 - F(\eta^*))\right] A > 0. \tag{29}$$

The total derivatives for A and λ yield unambiguous results, while the total derivatives for ν and μ may yield ambiguous results:

$$\frac{dp_U^*}{dA} = \frac{\partial p_U^*}{\partial A} + \frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{dA} > 0, \qquad \frac{dp_U^*}{d\lambda} = \frac{\partial p_U^*}{\partial \lambda} + \frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{d\lambda} = 0, \tag{30}$$

$$\frac{dp_U^*}{d\nu} = \underbrace{\frac{\partial p_U^*}{\partial \nu}}_{>0} + \underbrace{\frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{d\nu}}_{<0} \lesseqgtr 0, \qquad \frac{dp_U^*}{d\mu} = \underbrace{\frac{\partial p_U^*}{\partial \mu}}_{>0} + \underbrace{\frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{d\mu}}_{<0} \lesseqgtr 0.$$

Higher ν and μ increase the price directly but decrease it indirectly via a negative effect on the screening threshold. A set of sufficient conditions for the non-monotonicity of p_U^* in μ is $\frac{dp_U^*}{d\mu}\big|_{\mu \to 1} > 0$ and $\frac{dp_U^*}{d\mu}\big|_{\mu \to 1} < 0$. Substituting into (30) from conditions (28) and (27), we evaluate derivatives for the two limits:

$$\frac{dp_U^*}{d\mu}\mid_{\mu\to 1} \quad = \quad \frac{A}{\nu}>0, \frac{dp_U^*}{d\mu}\mid_{\mu\to 0}= \frac{\nu A(1-F(\eta^*))}{D^2}\left\{1-\frac{A(1-\nu)f(\eta^*)\left[2\nu+(1-\nu)(1-f(\eta^*))\right]}{D^2+(1-\nu)\nu A f(\eta^*)}\right\}.$$

The second derivative is negative for $\psi\to 1$ if $\eta_\mu^*\frac{f(\eta_\mu^*)}{1-F(\eta_\mu^*)}>1$, where $\eta_\mu^*=\eta^*\mid_{\mu\to 0}$.

Next, we turn to a FOSD reduction in the screening cost distribution, $\widetilde{F}\geq F$ (lower screening costs become more likely). Since $\frac{dp_U}{dF(\eta)}=\frac{(\psi-\mu)\nu A}{D^2}>0$, the price increases, $\widetilde{p_U^*}>p_U^*$. Thus, the screening threshold decreases, $\widetilde{\eta^*}<\eta^*$. For the comparative statics of the bound $\underline{\lambda}_L$, we can use some of the derivatives in (27) and (30) and $\underline{\lambda}_L=\frac{A}{p_U^*}$ to get:

$$\frac{d\underline{\lambda}_L}{dA}=\underbrace{-\frac{(\psi-\mu)f}{\nu(\psi F(\eta^*)+(1-\mu)(1-F(\eta^*))^2}}_{<0}\underbrace{\frac{d\eta^*}{dA}}_{>0}<0,\ \frac{d\underline{\lambda}_L}{d\nu}=\underbrace{-\frac{A}{p_U^2}}_{<0}\underbrace{\frac{dp_U^*}{d\nu}}_{\lesseqgtr 0}\gtreqless 0,\qquad \frac{d\underline{\lambda}_L}{d\mu}=\underbrace{-\frac{A}{p_U^2}}_{<0}\underbrace{\frac{dp_U^*}{d\mu}}_{\lesseqgtr 0}\gtreqless 0.$$

For the price p_U^*, the threshold $\underline{\lambda}_L$ is monotonic in A but can be non-monotonic in μ and ν. Moreover, $\underline{\lambda}_L$ decreases after a FOSD reduction in the screening cost distribution, $\widetilde{F}\geq F$, because $\frac{\underline{\lambda}_L}{dF(\eta)}=-\frac{A}{p_U^2}\frac{dp_U^*}{dF(\eta)}<0$ (the second term is positive). Hence, $\widetilde{\underline{\lambda}_L}<\underline{\lambda}_L$.

B.2 Proof of Proposition 1

Since insurance transforms the loan payoff at $t=2$ from risky to risk-free, $\pi=A-k$, outside financiers break even for a price equal to this payoff, $p_I=\pi$. Next, the payoff from an insured loan is independent of the screening choice because outside financiers cannot observe the screening choice. A lender i who insures has a payoff $\nu\lambda p_I+(1-\nu)\pi=\kappa p_I$ when not screening and a payoff $\kappa p_I-\eta_i$ when screening. Thus, lenders who insure generically prefer not to screen. As a result, low-cost (screening) lenders never insure loans.

Recall that only low-cost lenders screen and the market for insured loans is not subject to adverse selection (when insuring at $t=0$, lenders do not yet know loan quality). Thus, outside financiers break even when the insurance costs reflect the costs of guaranteeing the payoff A and the probability of loan repayment μ, i.e. $k=A-\mu A=(1-\mu)A$. This implies that the payoff and market price of insured loans are $\pi=A-k=\mu A=p_I^*$.

We prove by contradiction that some non-screened loans are uninsured, $m^*<1$. If $m=1$, no high-cost lenders sell lemons in the uninsured loans market (all loans of high-cost lenders are insured and sold in a separate market) and, for $\psi\to 1$, the quantity of lemons sold by low-cost lenders vanishes. Hence, only high-quality loans are sold, $p_U\to A$. However, $m=1$ requires high-cost lenders to prefer insurance, $p_I^*\kappa\geq\nu\lambda p_U^*+(1-\nu)\left(\mu A+(1-\mu)\,p_U^*\right)$ instead of equation (10), which simplifies to $\mu\geq 1$—contradiction.

In an illiquid equilibrium, high-cost lenders have a higher payoff when insuring, $\kappa\mu A$, than when not insuring, μA, since they must keep uninsured loans until maturity. This result arises because insured loans can be traded in a liquid secondary market (even when the market for uninsured loans is illiquid). Thus, we have $m^*=1$ in any illiquid equilibrium.

B.3 Proof of Proposition 2

The secondary market price of insured loans, $p_I^* = \mu A$, is derived in Proposition 1. Next, we derive the price in the uninsured loan market and screening threshold. First, the indifference condition for loan insurance (10) pins down the price of uninsured loans at $t = 1$:

$$p_U^* = \frac{\nu\lambda\mu A}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{31}$$

Substituting p_U^* from equation (31) into equation (22), we obtain the screening threshold

$$\eta^* = \frac{(1-\nu)(1-\mu)(\psi-\mu)\kappa A}{\nu\lambda + (1-\nu)(1-\mu)}, \tag{32}$$

where the limit for $\psi \to 1$ is stated in the proposition. To ensure a liquid equilibrium (in which high-quality loans are sold upon a liquidity shock), the price in equation (31) must satisfy condition (3), so a liquid equilibrium exists when insurance is used if $\mu\nu\lambda^2 - \nu\lambda - (1-\mu)(1-\nu) \geq 0$. Since only the larger root is positive, this condition reduces to

$$\lambda \geq \tilde{\lambda}_L \equiv \frac{1}{2\mu} + \sqrt{\frac{1}{4\mu^2} + \frac{(1-\mu)(1-\nu)}{\mu\nu}}. \tag{33}$$

An equivalent expression in terms of μ is $\mu \geq \tilde{\mu}_L \equiv \frac{\kappa}{\kappa + \nu\lambda(\lambda-1)}$. When insurance is available, the liquid equilibrium exists if $\lambda \geq \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$. Using (33), the comparative statics of the bound on the size of the liquidity shock, $\tilde{\lambda}_L$, are (where $\chi \equiv \left(\frac{1}{4\mu^2} + \frac{(1-\mu)(1-\nu)}{\mu\nu}\right)^{-\frac{1}{2}} > 0$):

$$\frac{d\tilde{\lambda}_L}{d\mu} = -\frac{1}{2\mu^2} - \frac{1}{2}\chi\left(\frac{1}{2\mu^3} + \frac{1-\nu}{\nu\mu^2}\right) < 0, \qquad \frac{d\tilde{\lambda}_L}{d\mu} = -\frac{1}{2}\chi\frac{1-\mu}{\mu\nu^2} < 0.$$

To pin down the fraction of loans insured by high-cost lenders, m^*, the price of uninsured loans also satisfies the break-even condition of outside financiers:

$$p_U = \nu A \frac{\psi F(\eta) + \mu(1-F(\eta))(1-m)}{\nu\big[F + (1-F)(1-m)\big] + (1-\nu)\big[(1-\psi)F + (1-\mu)(1-F)(1-m)\big]}. \tag{34}$$

Combining (34) with (31) yields for $\frac{p_U}{A}$:

$$\nu\frac{\psi F(\eta) + \mu(1-F(\eta))(1-m)}{\nu\big[F + (1-F)(1-m)\big] + (1-\nu)\big[(1-\psi)F + (1-\mu)(1-F)(1-m)\big]} = \frac{\nu\lambda\mu}{\nu\lambda + (1-\nu)(1-\mu)}, \tag{35}$$

which can be rearranged to obtain the fraction of loans insured by high-cost lenders:

$$m^* = 1 - \frac{(\kappa(1-\mu)\psi - (1-\psi)\lambda\mu)\, F(\eta^*)}{\mu(\lambda-1)(1-\nu)(1-\mu)(1-F(\eta^*))}. \tag{36}$$

Since the LHS of (35) increases in m, insurance is used when $\frac{p_U}{A}\mid_{m=0} < \frac{\nu\lambda\mu}{\nu\lambda+(1-\nu)(1-\mu)}$, which can be expressed as (using 35):

$$\frac{p_U}{A}\mid_{m=0}=\nu\frac{\psi F(\eta(\mu,\lambda))+\mu\left(1-F(\eta(\mu,\lambda))\right)}{\nu+(1-\nu)\left[(1-\psi)F(\eta(\mu,\lambda))+(1-\mu)\left(1-F(\eta(\mu,\lambda))\right)\right]}<\frac{\nu\lambda\mu}{\nu\lambda+(1-\nu)\left(1-\mu\right)}. \tag{37}$$

The LHS of (37) increases in A and after a first-order stochastic dominance shift in $F(\cdot)$ (cheaper screening), and decreases in λ. The right-hand side (RHS) is independent of A and $F(\cdot)$ and increases in λ. Hence, the condition for loan insurance to occur can be expressed as $A<\tilde{A}$, $\lambda>\tilde{\lambda}_I$, or high enough screening costs $F(\cdot)$. The parameter thresholds $\{\tilde{A},\tilde{\mu}_I,\tilde{\lambda}_I\}$ are defined by $\frac{p_U}{A}\mid_{m=0}=\frac{\nu\lambda\mu}{\nu\lambda+(1-\nu)(1-\mu)}$ and the threshold $\tilde{A}$ can be expressed in closed form:

$$\tilde{A}\equiv\frac{\nu\lambda+(1-\nu)(1-\mu)}{(1-\nu)(1-\mu)(\psi-\mu)\kappa}F^{-1}\left(\frac{\mu(\lambda-1)(1-\nu)(1-\mu)}{\kappa(1-\mu)\psi-(1-\psi)\lambda\mu+\mu(\lambda-1)(1-\nu)(1-\mu)}\right). \tag{38}$$

The threshold $\tilde{\lambda}_I$ is defined implicitly but uniquely by

$$\frac{p_U}{A}\mid_{m=0}=\nu\frac{\psi F(\eta(\tilde{\lambda}_I))+\mu\left(1-F(\eta(\tilde{\lambda}_I))\right)}{\nu+(1-\nu)\left[(1-\psi)F(\eta(\tilde{\lambda}_I))+(1-\mu)\left(1-F(\eta(\tilde{\lambda}_I))\right)\right]}=\frac{\nu\tilde{\lambda}_I\mu}{\nu\tilde{\lambda}_I+(1-\nu)\left(1-\mu\right)}, \tag{39}$$

if, in the limit of $\lambda\to\infty$, the RHS of (39) exceeds its LHS. A sufficient condition for this is $\nu\leq\frac{2\mu}{1+2\mu}$. Next, the threshold $\tilde{\mu}_I$ is implicitly but uniquely defined by rearranging (36) and substituting $m^*=0$:

$$\tilde{\mu}_I\equiv\frac{\kappa\psi-(1-\psi)\lambda\frac{\tilde{\mu}_I}{1-\tilde{\mu}_I}}{(\lambda-1)(1-\nu)}\frac{F\left(\eta^*(\tilde{\mu}_I)\right)}{1-F\left(\eta^*(\tilde{\mu}_I)\right)}\in(0,1). \tag{40}$$

The bound $\tilde{\mu}_I$ is unique since the LHS of (40) increases in μ and the RHS decreases in μ. It is also interior because the following limits do not satisfy equation (40): first, $\lim_{\mu\to0}LHS=0$ while $\lim_{\mu\to0}RHS>0$ (implying that for $\mu\to0$ insurance costs strictly outweigh benefits); second, $\lim_{\mu\to1}LHS=1$ while $\lim_{\mu\to1}RHS\leq0$ (implying that for $\mu\to1$ insurance benefits strictly outweigh costs). Finally, existence follows from continuity in μ.

Let G be the difference between the RHS and the LHS of (37). Then, $G=0$ defines the boundary of the extensive margin of insurance. The results derived above can be expressed as $\frac{dG}{dA}<0$, $\frac{dG}{d\lambda}>0$, $\frac{dG}{d\mu}>0$ and G decreases after a FOSD reduction in F. Hence, $\frac{d\tilde{\mu}_I}{dA}=-\frac{dG}{dA}/\frac{dG}{d\mu}>0$, $\frac{d\tilde{\mu}_I}{d\lambda}=-\frac{dG}{d\lambda}/\frac{dG}{d\mu}<0$, and $\tilde{\mu}_I$ increases after a FOSD reduction in F.

Comparative statics: screening threshold and uninsured loan price The comparative statics of the liquid equilibrium without insurance, $A\geq\tilde{A}$ and $\lambda\geq\underline{\lambda}_L$, are in Appendix B.1. Thus, the focus here is on the liquid equilibrium with insurance.

The comparative statics below hold for good enough screening, $\psi>\underline{\psi}$. For inefficient screening technology, $\psi\leq\underline{\psi}$, however, insurance is strictly preferred by high-cost lenders, $m^*=1$:

$$\kappa p_I>\nu\lambda p_U+(1-\nu)(\mu A+(1-\mu)p_U). \tag{41}$$

The threshold $\underline{\psi}$ is obtained by substituting $p_I=\mu A$ and $p_U(m=1)=\frac{\nu\psi A}{\nu+(1-\nu)(1-\psi)}$ into

(41):

$$\psi > \frac{\lambda\mu}{(1-\nu)(1-\mu)+\lambda(\mu+\nu(1-\mu))} \equiv \underline{\psi} \in (\mu, 1). \tag{42}$$

Using (34) for the effect on the price, its total derivative w.r.t. loan insurance is:

$$\frac{dp_U^*}{dm^*} = \frac{\partial p_U^*}{\partial m^*} + \frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{dp_U^*}\frac{dp_U^*}{dm^*} = \frac{\frac{\partial p_U^*}{\partial m^*}}{1-\frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{dp_U^*}} > 0, \tag{43}$$

since $\frac{\partial p_U^*}{\partial m^*} = \nu A F(\eta^*)(1-F(\eta^*))(1-\mu)\Big[\nu\left(1-\left(1-F\left(\eta^*\right)\right)m\right) + (1-\nu)\big[(1-\psi)F\left(\eta^*\right) + (1-\mu)\left(1-F\left(\eta^*\right)\right)\left(1-m^*\right)\big]\Big]^{-2} > 0$, $\frac{dp_U^*}{d\eta^*} > 0$, and $\frac{d\eta^*}{dp_U^*} = -(1-\nu)(\psi-\mu) < 0$. Since the price increases in loan insurance, the screening threshold falls, $\frac{d\eta^*}{dm^*} = \frac{d\eta^*}{dp_U^*}\frac{dp_U^*}{dm^*} < 0$. Since the threshold $\tilde{\lambda}_L$ decreases in the price p_U^*, it decreases in m^*: $\frac{d\tilde{\lambda}_L}{dm^*} = \frac{d\tilde{\lambda}_L}{dp_U^*}\frac{dp_U^*}{dm^*} < 0$. As a result, when insurance is used, $m^* > 0$, the threshold for the existence of a liquid equilibrium is lower compared to the case when insurance is unavailable, $\tilde{\lambda}_L < \underline{\lambda}_L$.

For the screening threshold, we use equation (32) and $D' \equiv \nu\lambda + (1-\nu)(1-\mu)$:

$$\begin{aligned}
\frac{d\eta^*}{dA} &= \frac{(1-\nu)(1-\mu)(\psi-\mu)\kappa}{D'} > 0, \quad \frac{d\eta^*}{d\lambda} = -\frac{\nu(1-\nu)^2\mu(1-\mu)(\psi-\mu)A}{D'^2} < 0,\\
\frac{d\eta^*}{d\mu} &= -\frac{(1-\nu)\kappa A\left((1+\psi-2\mu)\nu\lambda+(1-\nu)(1-\mu)^2\right)}{D'^2} < 0,\\
\frac{d\eta^*}{d\nu} &= -\frac{(1-\mu)(\psi-\mu)A\left[\kappa^2+\mu(1-\nu)^2(\lambda-1)\right]}{D'^2} < 0, \quad \frac{d\eta^*}{d\psi} = \frac{(1-\nu)(1-\mu)\kappa A}{D'} > 0.
\end{aligned}$$

For the direct effect on the price, we use equation (31) to obtain:

$$\begin{aligned}
\frac{dp_U^*}{dA} &= \frac{\nu\lambda\mu}{D'} > 0, \quad \frac{dp_U^*}{d\mu} = \frac{\nu\lambda A\kappa}{D'^2} > 0, \quad \frac{dp_U^*}{d\psi} = 0,\\
\frac{dp_U^*}{d\lambda} &= \frac{\nu(1-\nu)\mu(1-\mu)A}{D'^2} > 0, \quad \frac{dp_U^*}{d\nu} = \frac{\lambda\mu(1-\mu)A}{D'^2} > 0.
\end{aligned}$$

Both the price p_U^* and the threshold η^* are independent of the distribution F.

Comparative statics: Fraction of high-cost lenders who insure Equation (36) defines m^* as a function of η^*. Therefore, the total effect of parameters $\alpha \in \{\nu, \lambda, \mu\}$ on m^* consists of a direct and indirect effect via screening, $\frac{dm^*}{d\alpha} = \frac{\partial m^*}{\partial\alpha} + \frac{dm^*}{d\eta^*}\frac{d\eta^*}{d\alpha}$:

$$\begin{aligned}
\frac{dm^*}{d\eta^*} &= -\frac{[\kappa(1-\mu)\psi-(1-\psi)\lambda\mu]f}{\mu(1-\mu)(\lambda-1)(1-\nu)(1-F)^2} < 0, \quad \frac{\partial m^*}{\partial\lambda} = \frac{(1-\mu)\psi-(1-\psi)\mu}{\mu(1-\mu)(1-\nu)(\lambda-1)^2(1-F)}F > 0,\\
\frac{\partial m^*}{\mu} &= \frac{\kappa\psi(1-\mu)^2+(1-\psi)\lambda\mu^2}{\mu^2(1-\mu)^2(\lambda-1)(1-\nu)(1-F(\eta^*))}F(\eta^*) > 0, \quad \frac{\partial m^*}{\partial A} = 0,\\
\frac{\partial m^*}{\partial\nu} &= -\frac{(1-\mu)\psi-(1-\psi)\mu}{\mu(1-\mu)(\lambda-1)(1-\nu)^2(1-F(\eta^*))}\lambda F(\eta^*) < 0,\\
\frac{\partial m^*}{\partial F} &= -\frac{\kappa(1-\mu)\psi-(1-\psi)\lambda\mu}{\mu(1-\mu)(\lambda-1)(1-\nu)(1-F)^2} < 0, \quad \frac{\partial m^*}{\partial\psi} = -\frac{\kappa(1-\mu)+\lambda\mu}{\mu(1-\mu)(\lambda-1)(1-\nu)(1-F)}F < 0.
\end{aligned}$$

The following total derivatives are unambiguous, $\frac{dm^*}{d\mu} > 0$, $\frac{dm^*}{dA} < 0$, $\frac{dm^*}{d\psi} < 0$, and the FOSD shift, $\frac{dm^*}{dF} < 0$. The total effect of ν on m^* can be ambiguous since the direct effect is negative and the indirect one is positive. A sufficient condition for non-monotonicity is the opposite sign of derivatives at both limits, $\nu \to \{0, 1\}$, where $\lim_{\nu\to 1} \frac{dm^*}{d\nu} = -\infty$ and

$$\lim_{\nu\to 0} \frac{dm^*}{d\nu} = -\frac{(1-\mu)\psi - (1-\psi)\mu}{\mu(1-\mu)(\lambda-1)(1-F)}\lambda F + A(1+(\lambda-1)\mu)\frac{\psi-\mu}{1-\mu}\frac{(1-\mu)\psi-(1-\psi)\lambda\mu}{\mu(1-\mu)(\lambda-1)(1-F)^2}f.$$

The sufficient condition for non-monotonicity is $\frac{F(1-F)}{f} |_{\nu\to 0} < A(1+(\lambda-1)\mu)\frac{\psi-\mu}{1-\mu}\frac{(1-\mu)\psi-(1-\psi)\lambda\mu}{(1-\mu)\lambda\psi-(1-\psi)\lambda\mu}$. Finally, to complete the proof of Proposition 9, we need to establish that $\frac{d\tilde{A}}{d\psi} < 0$, which is straightforward using equation (38) since F^{-1} is increasing.

B.4 Proof of Proposition 3

The illiquid equilibrium (or not liquid–NL) always exists. If the price of uninsured loans is zero, $p_U^* = 0$, only lemons are sold in this market, which justifies the zero price. When the liquid equilibrium does not exist, $\lambda < \min\{\underline{\lambda}_L, \tilde{\lambda}_L\}$, the illiquid equilibrium is unique. We have already shown that high-cost lenders always insure, $m^* = 1$, because of the gains from trade on the market for insured loans. The screening threshold is given by the indifference condition of the marginal lender who compares payoffs from screening, $\psi A - \eta$, and not screening but insuring, $\kappa\mu A$. Equating those yields the threshold $\widetilde{\eta^{NL}} = (\psi - \kappa\mu)A$, which is below the threshold in the illiquid equilibrium where insurance is unavailable, $\eta^{NL} = (\psi - \mu)A$.

When $(1-\kappa\mu)A \geq \bar{\eta}$, screening is full, $\eta^{NL} > \widetilde{\eta^{NL}} \geq \bar{\eta}$, irrespective of loan insurance. When $(1-\kappa\mu)A < \bar{\eta}$, screening is partial, $\widetilde{\eta^{NL}} < \bar{\eta}$, and there are three types of lenders. First, lenders of mass $1 - F(\eta^{NL})$ are high-cost irrespective of the availability of loan insurance. Those lenders strictly prefer the payoff in equilibrium with insurance, $\kappa\mu A$, to the payoff in equilibrium without insurance option, μA. Second, lenders of mass $F(\widetilde{\eta^{NL}})$ are low-cost irrespective of the availability of loan insurance. Those lenders are indifferent about insurance since their payoff is always $A - \eta_i$. Third, lenders of mass $F(\widetilde{\eta^{NL}}) - F(\eta^{NL})$ are high-cost when insurance is available and low-cost otherwise. Those lenders do not screen when insurance is available because $\kappa\mu A > A - \eta_i$. Since the payoff when screening is the same in both equilibria, $A - \eta_i$, they strictly prefer the equilibrium with insurance.

In sum, all high-cost lenders in the equilibrium with insurance are better off than in the equilibrium without the insurance option, and no lender is worse off. Therefore, the aggregate welfare is superior in the equilibrium with insurance option.

B.5 Proof of Proposition 4

Utilitarian welfare is the sum of the expected payoffs of lenders and of outside financiers. Given that outside financiers always break even in expectation, welfare is the expected value to lenders (up to a constant). Low-cost lenders of mass $F(\eta^*)$ have an expected payoff

$\nu\lambda p_U^* + (1-\nu)[\psi A + (1-\psi)p_U^*] - \eta_i$. Uninsured high-cost lenders of mass $(1-F(\eta^*))(1-m^*)$ have an expected payoff $\nu\lambda p_U^* + (1-\nu)[\mu A + (1-\mu)p_U^*]$, while insured high-cost lenders of mass $(1-F(\eta^*))m^*$ have an expected payoff κp_I^*. Integrating over all lenders i, welfare is

$$W(\psi) = \overbrace{\nu\lambda\Big\{ [F(\eta^*) + (1-F(\eta^*))(1-m^*)]\, p_U^* + m^*(1-F(\eta^*))p_I^* \Big\}}^{\text{Liquidity Shock}} + \overbrace{(1-\nu)m^*(1-F(\eta^*))p_I^*}^{\text{No shock insured}}$$

$$+ \overbrace{(1-\nu)[F(\eta^*)\left[\psi A + (1-\psi)p_U^*\right] + (1-m^*)(1-F(\eta^*))(\mu A + (1-\mu)p_U^*)]}^{\text{No shock uninsured}} - \overbrace{\int_0^{\eta^*} \eta dF(\eta)}^{\text{Screening costs}}. \tag{44}$$

Substituting the price of uninsured loans from (34) in the form $p_U\Big[\nu\big[F + (1-F)(1-m)\big] + (1-\nu)\big[(1-\psi)F + (1-\mu)\,(1-F)\,(1-m)\,\big]\Big] = \nu A\big(\psi F(\eta) + \mu\,(1-F(\eta))\,(1-m)\,\big)$ results in a simplified expression for welfare (12) for $\psi \to 1$.

Constrained-efficient allocation in liquid equilibrium We prove existence and the result on the intensive margin by showing that (i) welfare increases in m on the interval $m \in [0, m^*]$; and (ii) welfare decreases in m for $m \to 1$. Since the welfare function in equation (12) is continuous and defined everywhere in the interval $m \in (0,1)$, the planner's choice satisfies $m^P \in (m^*, 1)$, thus exceeding the competitive m^*.

The total derivative of welfare, $\frac{dW}{dm} = \frac{\partial W}{\partial m} + \frac{\partial W}{\partial p_U^*}\frac{dp_U^*}{dm} + \frac{\partial W}{\partial \eta^*}\frac{d\eta^*}{dm}$, is evaluated using (44):

$$\begin{aligned}
\frac{\partial W}{\partial m} &= (1-F)\Big[\kappa p_I^* - \nu\lambda p_U^* - (1-\nu)(\mu A + (1-\mu)p_U^*)\Big] = 0, \qquad (45)\\
\frac{\partial W}{\partial p_U^*} &= \nu\lambda(F + (1-F)(1-m)) + (1-\nu)\left[F(1-\psi) + (1-F)(1-m)(1-\mu)\right] > 0,\\
\frac{\partial W}{\partial \eta^*} &= f\Big[(1-\nu)(\psi-\mu)(A-p_U^*) - \eta^* + m^*\left[\nu\lambda p_U^* + (1-\nu)(\mu A + (1-\mu)p_U^*) - \kappa p_I^*\right]\Big] = 0.
\end{aligned}$$

Since $\frac{dp_U^*}{dm} > 0$ and $\frac{d\eta^*}{dm} < 0$, the total derivative is positive in the unregulated equilibrium due to the positive pecuniary externality, $\frac{dW}{dm}\mid_{m=m^*} = \frac{\partial W}{\partial p_U^*}\frac{dp_U^*}{dm} > 0$. The direct effect of insurance and screening on welfare is zero in the unregulated economy by envelope-type-argument (lenders choose insurance and screening privately optimally). The total derivative is also positive for any $\tilde{m} < m^*$ since $\frac{\partial W}{\partial m}\mid_{\tilde{m}} > 0$, $\frac{\partial W}{\partial p}\mid_{\tilde{m}} > 0$, $\frac{\partial W}{\partial \eta^*}\mid_{\tilde{m}} < 0$.

For the upper bound on m^P, we focus on $\psi \to 1$. (Only the result $m^P < 1$ does not necessarily generalize to all $\underline{\psi} < \psi < 1$.) In the limit of $m \to 1$, the price of uninsured loans equals payoff of high-quality loans and there is no screening, $\lim_{m\to 1} p_U = A$, $\lim_{m\to 1}\eta = 0$. Hence, the partial derivatives are $\lim_{m\to 1}\frac{\partial W}{\partial m} = -\kappa(1-\mu)A$, $\lim_{m\to 1}\frac{\partial W}{\partial p} = 0$ and $\lim_{m\to 1}\frac{\partial W}{\partial \eta^*} = f\kappa(1-\mu)A$. This implies that the total derivative is negative, $\lim_{m\to 1}\frac{dW}{dm} < 0$. From the proof of Proposition 1 (see equation 43), a higher m increases the price in secondary markets for uninsured loans and decreases screening. Thus $m^P > m^*$ implies $p_U^P > p_U^*$ and $\eta^P < \eta^*$.

To prove the result on the extensive margin, we compare the threshold of parameters at which insurance is zero in the unregulated equilibrium, $\{\tilde{A}, \tilde{\mu}_I, \tilde{\lambda}_I\}$, and in the constrained-efficient allocation, $\{A^P, \mu_I^P, \lambda_I^P\}$.

The parameter threshold in the unregulated equilibrium, $\{\tilde{A}, \tilde{\mu}_I, \tilde{\lambda}_I\}$, satisfies $m^* = 0$ and $\frac{\partial W}{\partial m} = (1-F)(\kappa\mu A - \nu\lambda p_U^* - (1-\nu)(\mu A + (1-\mu)p_U^*)) = 0$, which is the indifference condition for insurance. Substituting p_U^* from the break-even condition in $\frac{\partial W}{\partial m} = 0$ yields the condition (37) in Appendix B.3 satisfied with equality, and implies that insurance is positive when $\frac{\partial W}{\partial m} \mid_{m=0} > 0$, that is when $\frac{p_U}{A} \mid_{m=0} < \frac{\nu\lambda\mu}{\nu\lambda + (1-\nu)(1-\mu)}$. Appendix B.3 derives the bounds $\{\tilde{A}, \tilde{\mu}_I, \tilde{\lambda}_I\}$ and suggests that insurance is used for expensive enough screening costs $F(\cdot)$. We can rewrite $\frac{\partial W}{\partial m} = 0$ as

$$\frac{\partial W}{\partial m} = \left[-F\left(\kappa\psi + (1-\psi)\lambda\frac{\mu}{1-\mu}\right) + (\lambda-1)\mu(1-\nu)(1-F)\right]\frac{(1-\mu)(1-F)\nu A}{\nu + (1-\nu)\left((1-\psi)F + (1-\mu)(1-F)\right)} = 0, \tag{46}$$

which gives the lower bound $\tilde{\mu}_I$ in (40). Insurance is positive if $\frac{\partial W}{\partial m} \mid_{m=0} > 0$, i.e. if $\mu > \tilde{\mu}_I$.

The parameter threshold in the constrained efficient case, $\{A^P, \mu_I^P, \lambda_I^P\}$, satisfies $m^P = 0$ and $\frac{dW}{dm} = 0$. After substituting p_U^* we get:

$$\frac{p_U}{A} = \frac{\nu(\psi F + \mu(1-F))}{\nu + (1-\nu)\left((1-\psi)F + (1-\mu)(1-F)\right)} = \frac{\nu\lambda\mu}{\nu\lambda + (1-\nu)(1-\mu)} + \underbrace{\frac{\frac{\partial W}{\partial p}\frac{dp_U}{dm}}{(1-F)(\nu\lambda + (1-\nu)(1-\mu))A}}_{\text{New externality term (>0)}}. \tag{47}$$

Since the above externality term is positive, the LHS of (47) is higher than the LHS of (37). The LHS of (37) and (47) have the same functional form and are increasing in A and after a first-order stochastic dominance reduction in screening costs distribution $F(\cdot)$, and decreasing in λ. This implies that planner uses insurance for larger parameter space $A^P > \tilde{A}$, $\lambda_I^P < \tilde{\lambda}_I$, and relatively cheaper screening costs $F(\cdot)$. The sufficient condition for the existence of the bound on λ is $\nu \leq \frac{2\mu}{1+2\mu}$. We can rewrite $\frac{dW}{dm} = 0$ as

$$\begin{aligned}\frac{dW}{dm} \quad = \quad & \left[-F\left(\kappa\psi + (1-\psi)\lambda\frac{\mu}{1-\mu}\right) + (\lambda-1)\mu(1-\nu)(1-F) + \underbrace{\frac{\frac{\partial W}{\partial p}\frac{dp_U}{dm}\left(\nu + (1-\nu)\left((1-\psi)F + (1-\mu)(1-F)\right)\right)}{(1-\mu)(1-F)\nu A}}_{\text{New externality term (>0)}}\right] \\ & \cdot\frac{(1-\mu)(1-F)\nu A}{\nu + (1-\nu)\left((1-\psi)F + (1-\mu)(1-F)\right)} = 0,\end{aligned}$$

which implicitly gives a lower bound μ_I^P

$$\mu_I^P \equiv \frac{\left(\kappa\psi - (1-\psi)\lambda\frac{\mu}{1-\mu}\right)F}{(\lambda-1)(1-\nu)(1-F)} - \underbrace{\frac{\frac{\partial W}{\partial p}\frac{dp_U}{dm}\left(\nu + (1-\nu)\left((1-\psi)F + (1-\mu)(1-F)\right)\right)}{(\lambda-1)(1-\nu)(1-\mu)(1-F)^2\nu A}}_{\text{New externality term (>0)}}. \tag{48}$$

A direct comparison of (40) and (48) implies that $\mu_I^P < \tilde{\mu}_I$.

B.6 Proof of Proposition 5

Welfare for an illiquid market is $W^{NL}=\nu(\lambda-1)\mu A(1-F(\eta^{NL}))+\left[\psi F(\eta^{NL})+\mu(1-F(\eta^{NL}))\right]A-\int_0^{\eta^{NL}}dF(\tilde{\eta})$, where $\eta^{NL}=(\psi-\kappa\mu)A$. Welfare is $W^L=\nu(\lambda-1)\left(p_U+(\mu A-p_U)(1-F(\eta^L))m\right)+\left[\psi F(\eta^L)+\mu(1-F(\eta^L))\right]A-\int_0^{\eta^L}dF(\tilde{\eta})$ for a liquid market, subject to η^L is given by (22) , p_U by (34), and $p_U\lambda\geq A$. At some λ_L^P, the planner is indifferent between the illiquid equilibrium and equilibrium liquified with intervention, $W^{NL}=W^L$:

$$\overbrace{\nu(\lambda_L^P-1)\left(p_U+(\mu A-p_U)(1-F(\eta^L))m-\mu A(1-F(\eta^{NL}))\right)}^{\text{Higher gains from trade in liquid equilibrium}(>0)}$$
$$=\underbrace{(\psi-\mu)A\left(F(\eta^{NL})-F(\eta^L)\right)-\left(\int_0^{\eta^{NL}}dF(\tilde{\eta})-\int_0^{\eta^L}dF(\tilde{\eta})\right)}_{\text{Higher net benefits of screening in illiquid equilibrium}(>0)}. \tag{49}$$

Next, we show that the above equation implicitly and uniquely defines a $\lambda_L^P\in(1,\infty)$. For existence, the gains from trade term dominates for $\lambda\to\infty$, so $\lambda_L^P<\infty$, while this term vanishes for $\lambda\to 1$. The existence of λ_L^P follows. For uniqueness, we start with an intermediate result. At $\lambda=\lambda_L^P$, the liquid equilibrium can be sustained only with a level of insurance that exceeds the level in the unregulated equilibrium, because the unregulated market was illiquid. Thus, the first-order condition for the optimal insurance level m^P is

$$\frac{dW^L}{dm}+\gamma\frac{dp_U}{dm}=0, \tag{50}$$

where γ is the Lagrange multiplier for $p_U\lambda\geq A$. At $\lambda=\lambda_L^P$, the planner is indifferent between illiquid and liquid markets (via a subsidy $p_U=A/\lambda$), so $\gamma>0$. This together with $\frac{dp_U}{dm}>0$ and equation (50) implies $\frac{dW^L}{dm}<0$ at $\lambda=\lambda_L^P$. That is the planner would choose fewer insured lenders without the binding liquidity constraint. The total derivative of the welfare difference, $W^L\mid_{p_U=A/\lambda}-W^{NL}$, with respect to λ is:

$$\underbrace{\frac{dW^L}{dm}}_{<0}\underbrace{\frac{dm\mid_{p_U=A/\lambda}}{d\lambda}}_{<0}+\underbrace{\nu\left(\frac{A}{\lambda}+\left(\mu A-\frac{A}{\lambda}\right)(1-F(\eta^L))m-\mu A(1-F(\eta^{NL}))\right)}_{\text{higher gains from trade in liquid eq.}(>0)}>0. \tag{51}$$

Equation (49) implies that the gains from trade in the liquid equilibrium are higher. The sign, $\frac{dm|_{p_U=A/\lambda}}{d\lambda}<0$, is due to the positive effect of insurance on the price, $\frac{dp_U}{dm}>0$, (proven already) and that a higher λ reduces the price needed for liquifying the market. Hence, the welfare difference between a liquid and illiquid market increases monotonically in λ and, thus, (49) defines λ_L^P uniquely.

B.7 Proof of Proposition 6

This proof proceeds as follows. First, we condition on the liquid equilibrium and show that an insurance subsidy achieves constrained efficiency. We also derive the comparative statics for the size of this insurance subsidy. For a subset of parameters, where allocative efficiency is increased on the intensive margin, we make a distributional assumption. Second, we show that an uninsured loan sale subsidy can eliminate the illiquid equilibrium. However, the sale subsidy is welfare-dominated by the insurance subsidy in the liquid equilibrium.

B.7.1 Liquid equilibrium: insurance subsidy attains constrained efficiency

When subsidies for the sale of uninsured loans are not used, $b_U = 0$, the objective functions of the planner in (12) and the regulator in (18) are identical—except for the constant interim-date endowment term—and so are the indifference condition for screening and the break-even condition of outside financiers. To see this, we can rewrite (18) as

$$\begin{aligned} \max_{b_I} W^R &= \max_{b_I} \overbrace{\nu(\lambda-1)\left[p_U + m\left(1-F(\eta)\right)\left(p_I - p_U\right)\right]}^{\text{Gains from trade}} + \underbrace{\left(\psi F(\eta) + \mu(1-F(\eta))\right) A}_{\text{Fundamental value}} \\ & \quad - \underbrace{\int_0^{\eta} \tilde{\eta} dF(\tilde{\eta})}_{\text{Screening costs}} + \kappa(n + \underbrace{m(1-F(\eta))b_I - T}_{\text{Policy redistribution (=0)}}). \end{aligned}$$

Hence, a subsidy is set to achieve the constrained efficient price in the secondary market for uninsured loans, thus achieving constrained efficiency. Solving equation (15) and evaluating at $p(b_I) = p_U^P$ yields the optimal value of b_I^R stated in the proposition.

To solve for b_I^R, we proceed in two steps. First, we solve for the value of b_I^R when the condition (3) binds (so the respective Lagrange multiplier is positive, $\gamma > 0$) and the subsidy satisfies $p_U(b_I^R) = A/\lambda$. Since higher λ relaxes the condition (3), this case arises for $\lambda_L^P \leq \lambda < \lambda_B^P$, where λ_B^P is implicitly defined as $dW/dm = 0$ and $p_U = A/\lambda$. Substituting $p_U = \frac{A}{\lambda}$ condition in (15), we get

$$b_I^R = A \frac{\nu\lambda(1-\lambda\mu) + (1-\nu)(1-\mu)}{\kappa\lambda}. \tag{52}$$

Hence, b_I^R linearly increases in A but decreases in μ and λ and is independent of screening technology parameters ($F(\cdot)$, $\bar{\eta}$, and ψ).

Second, we find the optimal subsidy b_I^R when condition (3) does not bind, $\lambda \geq \lambda_B^P$. We focus on $\psi \to 1$ and rewrite parts of the total derivative of welfare (45), using the expression for the subsidy from the insurance indifference condition (15):

$$\begin{aligned} \frac{\partial W^R}{\partial m} &= -(1-F)\kappa b_I, \qquad \frac{\partial W^R}{\partial \eta^*} = fm\kappa b_I, && (53) \\ \frac{\partial W^R}{\partial p_U^*} &= \nu\lambda(F + (1-F)(1-m)) + (1-\nu)(1-F)(1-m)(1-\mu). && (54) \end{aligned}$$

The screening threshold is $\eta = \frac{(1-\nu)(1-\mu)A\kappa}{\nu\lambda + (1-\nu)(1-\mu)}\left(1 - \mu - \frac{b_I}{A}\right)$ and the fraction of insured high-cost lenders is

$$m = 1 - \frac{F\kappa(1-\mu-\frac{b_I}{A})}{(1-F)\left[\mu(\lambda-1)(1-\nu)(1-\mu)+\frac{\kappa b_I}{\nu A}\right]}. \tag{55}$$

Henceforth, we assume a uniform distribution of screening costs $\eta_i \sim \mathcal{U}(0,\bar{\eta})$. Using equations (53)–(55), it can be shown that the solution to the optimal insurance $\frac{dW}{dm}=0$ is the unique real root of the following cubic equation in $B \equiv \frac{b_I}{A}$:

$$\begin{aligned}
0 &= \Gamma_0+\Gamma_1 B+\Gamma_2 B^2+\Gamma_3 B^3, \quad \text{where} \qquad (56)\\
\Gamma_0 &= (\lambda-1)(1-\mu)^2\mu(1-\nu)\left((1-\nu(\lambda-1))^2-\mu(1-\nu)(1-\nu(\lambda-1)^2)\right),\\
\Gamma_1 &= (\lambda-1)(1-\mu)\mu(1-\nu)\big(\lambda(1-\mu)(2-\mu(1-\nu)-8\nu)(1-\nu)-(4-5\mu+\mu^2)(1-\nu)^2\\
&\quad +2\lambda^2\nu(1+2\mu(1-\nu)-2\nu)\big),\\
\Gamma_2 &= \frac{1}{\nu}\Big(-(1-\mu)(1-\nu)^3(2-\mu(4-5\nu)+2\mu^2(1-\nu))\\
&\quad +\lambda^2\nu^2(1+5\mu^2(1-\nu)^2-2\nu+\mu(-6+11\nu-5\nu^2))\\
&\quad -\lambda(1-\mu)(1-\nu)^2(1-6\nu+\mu(-3+17\nu-15\nu^2)\mu^2(2-6\nu+4\nu^2)\\
&\quad \lambda^2(1-\nu)\nu(2-2\mu^3(1-\nu)^2-6\nu-\mu^2(-10+27\nu-17\nu^2)-\mu(10-28\nu+15\nu^2))\Big),\\
\Gamma_3 &= (1-\mu(1+\nu))(1-\nu)(1-(\lambda-1)\nu)^2(1-\mu(1-\nu)-2(\lambda-1)\nu)\frac{1}{\nu^2}.
\end{aligned}$$

Since eq. (56) does not contain $\bar{\eta}$ or A, B^R is independent of these parameters, where $b_I^R \equiv AB^R$. Hence, b_I^R is independent of $\bar{\eta}$ and linearly increasing in A. We derive numerically that b_I^R increases in λ (Figure 12). One can also show that b_I^R is non-monotonic in μ and ν.

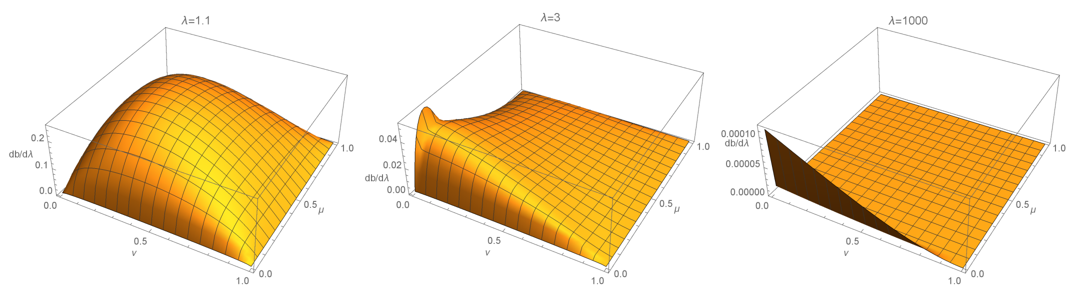

Figure 12: The derivative $\frac{db_I}{d\lambda}$ is always positive.

B.7.2 Subsidies for sales of uninsured loans used only off equilibrium

It is immediate that an illiquid equilibrium, $p_U^* = 0$, can be eliminated with a subsidy $b_U^R = A/\lambda$. It breaks the existence of an illiquid equilibrium, $p_U^* + b_U < A/\lambda$. Appendix B.6 defines λ_L^P for the dominance of the liquid equilibrium. Next, we compare the welfare of achieving the same target price, $p_U^T < A$, with an insurance subsidy, $p_U^T = p_U(b_I)$, and with an uninsured loan sale subsidy, $p_U^T = p_U + b_U$. Using the insurance indifference condition (15), the welfare with an insurance subsidy (18) can be expressed as

$$W^R(b_I) = \overbrace{\nu\lambda p_U^T + (1-\nu)\left[\psi F(\eta) + \mu\left(1-F(\eta)\right)\right]A + (1-\nu)\left[(1-\psi)F(\eta) + (1-\mu)\left(1-F(\eta)\right)\right]p_U^T + n}^{\text{Value to lenders}}$$
$$- \underbrace{\int_0^\eta \tilde{\eta}\, dF(\tilde{\eta})}_{\text{Screening costs}} - \underbrace{\kappa m\left(1-F(\eta)\right)b_I}_{\text{Policy costs}},$$

where $p_U = p_U^T$ and $b_I(p_U^T), \eta(p_U^T)$, and $m(\eta(p_U^T))$ are given by (15), (22), and (34), respectively. In contrast, welfare with effective subsidized sales of uninsured loans, $p_U^T > p_U^*$, is

$$W^R(b_U) = \overbrace{\nu\lambda p_U^T + (1-\nu)\left[\psi F(\eta) + \mu\left(1-F(\eta)\right)\right)A + (1-\nu)\left((1-\psi)F(\eta) + (1-\mu)\left(1-F(\eta)\right)\right]p_U^T + n}^{\text{Value to lenders}}$$
$$- \underbrace{\int_0^\eta \tilde{\eta}\, dF(\tilde{\eta})}_{\text{Screening costs}} - \underbrace{\kappa b_U \int q_i^U di}_{\text{Policy costs}},$$

where p_U is given by (24), $b_U = p_U^T - p_U$, $\eta = (1-\nu)(\psi-\mu)(A-p_U^T)$, and quantity of uninsured loans sold $\int q_i^U di = \nu + (1-\nu)\left[(1-\psi)F + (1-\mu)(1-F)\right]$.

Since the screening threshold is the same in both cases, these welfare expressions differ only in the policy cost term. Welfare under an insurance subsidy exceeds welfare under subsidized sales if

$$m\left(1-F\right)b_I < \left(p_U^T - p_U\right)\int q_i^U di, \tag{57}$$

which holds for $p_U^T < A$ (i.e. generically)Substituting for b_I from (15), for $m(1-F) = \frac{p_U^T(\nu+(1-\nu)((1-\psi)F+(1-\mu)(1-F)))-\nu(\psi F+\mu(1-F))A}{p_U^T(1-\mu+\nu\mu)-\nu\mu A}$ from (34), and for $p_U \int q_i^U di = \nu\left(\psi F + \mu(1-F)\right)A$ from (34), we can rewrite (57) as $\frac{1}{\kappa}\frac{[\nu\lambda+(1-\nu)(1-\mu)]p_U^T-\nu\mu A\lambda}{p_U^T(1-\mu+\nu\mu)-\nu\mu A} < 1$, which collapses to $p_U^T < A$.

B.8 Proof of Proposition 7

We focus on the equilibrium in which the market for loan insurance at $t=0$ is liquid. We omit the unstable equilibrium in which the secondary market for uninsured loans is illiquid, $p_U = 0$, but the loan insurance market is liquid, $p_I > 0$. This equilibrium requires that high-cost lenders are indifferent about insurance, $0 < m < 1$. But any deviation from the equilibrium level of m would lead to either the equilibrium in which all markets are illiquid or to the equilibrium in which all markets are liquid.

Positive analysis For the loan insurance market to be liquid, (some) high-cost lenders have to insure, $m > 0$, so the payoff from insuring has to (weakly) exceed the payoff from not insuring:

$$\kappa p_I \geq \nu\lambda p_U + (1-\nu)\left(\mu A + (1-\mu)p_U\right). \tag{58}$$

Note that $p_I \leq \mu A$ because high-cost lenders insure loans worth μA and low-cost lenders potentially insure lemons worth 0. Combining this with equation (58), we find that $p_I > p_U$. Hence, all low-cost lenders insure their lemons. Moreover, only some high-cost lenders insure in equilibrium, $m < 1$. Full insurance, $m = 1$, would imply that the price of uninsured loans is $p_U = A$, which is inconsistent with the supposed full insurance (similar to the main model).

The screening threshold equalizes the payoff from screening, $\psi(\nu\lambda p_U + (1-\nu)A) + (1-\psi)\kappa p_I - \eta$, and from not screening, $\nu\lambda p_U + (1-\nu)(\mu A + (1-\mu)p_U)$, which after substituting (58) with equality simplifies to

$$\eta = \psi(1-\nu)(1-\mu)(A - p_U). \tag{59}$$

The break-even conditions of outside financiers give the price expressions:

$$p_I = \frac{\mu A m(1-F)}{m(1-F) + F(1-\psi)} = \mu A - \overbrace{\mu A \frac{F(1-\psi)}{m(1-F) + F(1-\psi)}}^{\text{adverse selection discount}}, \tag{60}$$

$$p_U = \nu A \frac{\psi F + \mu(1-F)(1-m)}{\nu\left[\psi F + (1-F)(1-m)\right] + (1-\nu)(1-F)(1-m)(1-\mu)}. \tag{61}$$

Intuitively, the adverse selection discount vanishes for $\psi \to 1$.

Loan insurance by high-cost lenders m increases both prices p_U and p_I. As in the main text $dp_U/dm > 0$ as in (43). Moreover, $\frac{dp_I}{dm} = \frac{\partial p_I}{\partial m} + \frac{dp_I}{d\eta}\frac{d\eta}{dp_U}\frac{dp_U}{dm} > 0$, since $\frac{\partial p_I}{\partial m} > 0$, $\frac{dp_I}{d\eta} < 0$, $\frac{d\eta}{dp_U} < 0$, and $\frac{dp_U}{dm} > 0$.

Since $m < 1$, the expression (58) holds with equality, and thus

$$p_U = \mu A \frac{\nu\lambda - \overbrace{\frac{F(1-\psi)}{m(1-F) + F(1-\psi)}\kappa}^{\text{adverse selection discount}}}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{62}$$

Combining equations (61) and (62) gives

$$\nu \frac{\psi F + \mu(1-F)(1-m)}{\nu\left[\psi F + (1-F)(1-m)\right] + (1-\nu)(1-F)(1-m)(1-\mu)} = \mu \frac{\nu\lambda - \frac{F(1-\psi)}{m(1-F)+F(1-\psi)}\kappa}{\nu\lambda + (1-\nu)(1-\mu)}, \tag{63}$$

where η is given by (59). The RHS of (63) is the benefit of insurance and the LHS its opportunity costs. The loan payoff A enters only via the screening choice (59) and, thus, decreases the RHS and increases the LHS. In the limit of $A \to 0$, no lender screens, $F(\eta) = 0$, and the RHS collapses to $\frac{\mu\nu\lambda}{\nu\lambda + (1-\nu)(1-\mu)}$, while the LHS collapses to $\frac{\mu\nu}{\nu + (1-\nu)(1-\mu)}$. This implies that in this limit, insurance is strictly preferred. While for $A = \tilde{A}$ the benefit of insurance (RHS of 63) is strictly smaller than the cost of insurance (LHS of 63). The threshold of insurance usage in the main model, $\tilde{A}$, is given implicitly by (35). In contrast to (35), the LHS of (63) evaluated at $\tilde{A}$ is higher due to higher screening, and the absence of lemons sales by low-cost lenders in the uninsured market. At the same time, the RHS is lower due to the adverse selection discount. Taken together at $A = \tilde{A}$ high-cost lenders strictly prefer not to insure. By continuity, there exist a $\widehat{A} \in (0, \tilde{A})$ such that for $A < \widehat{A}$, insurance is used, $m > 0$. Note that since both RHS and LHS of (63) increase in m, there may exist multiple equilibria with liquid insurance. Figure 13 shows an example of this multiplicity.

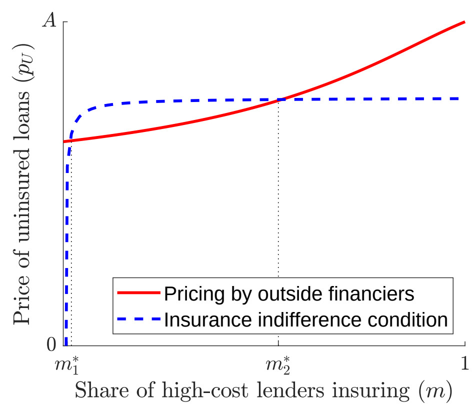

Figure 13: An example of multiple equilibria in the class of equilibria with a liquid insurance market. The red solid line plots the uninsured loans price p_U from the break-even condition of outside financiers (opportunity costs of insurance) and the blue dashed line shows the price p_U at which high-cost lenders are indifferent about insurance (insurance benefit). There are two equilibria with positive levels of m, but only the one with higher m is stable.

Higher λ increases p_U directly (eq. 62) and indirectly via lowering screening (eq. 59) and thus lowering also the amount of lemons insured by low-cost lenders. As a result, there is a threshold $\widehat{\lambda}_L$ such that for $\lambda \geq \widehat{\lambda}_L$ the liquid equilibrium exists (condition 3 holds) conditional on insurance being used. Due to the negative effect of adverse selection in insurance (see 62), liquid equilibrium with insurance is less likely, $\tilde{\lambda}_L < \widehat{\lambda}_L$.

Normative analysis

Regarding the extensive margin of insurance, we find that the equilibrium with insurance welfare-dominates the equilibria with an illiquid insurance market. The screening and insurance choice are privately optimal and all externalities are pecuniary. And since the equilibrium with insurance has higher prices in secondary markets, both p_I (by definition because the illiquid insurance equilibria have $p_I = 0$) and p_U (see above for dp_U/dm), welfare in the equilibrium with an additional option to insure welfare-dominates the equilibria without such an insurance option.

As in the main model, we can express the welfare in the equilibrium with a liquid insurance market as the sum of lender's payoffs:

$$\begin{aligned} W &= F\left[\psi(\nu\lambda p_U + (1-\nu)A) + (1-\psi)\kappa m^l p_I\right] - \int_0^\eta \eta_i dF \\ &\quad + (1-F)\left[m\kappa p_I + (1-m)\left[\nu\lambda p_U + (1-\nu)(\mu A + (1-\mu)p_U)\right]\right], \end{aligned}$$

where p_U and η are given by generalized (61) and (59):

$$
\begin{aligned}
p_U &= \nu A \frac{\psi F + \mu(1-F)(1-m)}{\nu\left[\psi(1-m^l)F + (1-F)(1-m)\right] + (1-\nu)(1-F)(1-m)(1-\mu)}. \\
\eta &= \psi(1-\nu)(1-\mu)(A-p_U) - (1-m^l)(p_I - p_U), \qquad (64)
\end{aligned}
$$

and m^l is the fraction of low-cost lenders with lemons insuring (in equilibrium, we have $m^l = 1$). As before, more insurance by high-cost lenders in the unregulated equilibrium increases welfare by improving prices in the uninsured loan sale market. Moreover, more insurance improves price in the loan insurance market:

$$
\frac{dW}{dm} = \underbrace{\frac{\partial W}{\partial m}}_{=0} + \underbrace{\frac{\partial W}{\partial p_U}}_{>0}\underbrace{\frac{dp_U}{dm}}_{>0} + \underbrace{\frac{\partial W}{\partial \eta}}_{=0}\frac{d\eta}{dm} + \underbrace{\frac{\partial W}{\partial p_I}}_{>0}\underbrace{\frac{dp_I}{dm}}_{>0} > 0.
$$

In order to evaluate the effect of insurance by low-cost lenders on welfare dW/dm^l, it is useful to rearrange the welfare condition using condition (60) to obtain

$$
\begin{aligned}
W &= F\left[\psi(\nu\lambda p_U + (1-\nu)A)\right] - \int_0^{\eta} \eta_i dF \\
&\quad + (1-F)\left[\kappa m\mu A + (1-m)\left[\nu\lambda p_U + (1-\nu)(\mu A + (1-\mu)p_U)\right]\right],
\end{aligned}
$$

where m^l does not affect welfare directly, but only by the price of uninsured loans, the screening threshold, and insurance by high-cost lenders. These are given (implicitly) by (64) and more generalized version of (62) and (63):

$$
p_U = \mu A \frac{\nu\lambda - \frac{m^l F(1-\psi)}{m(1-F)+m^l F(1-\psi)}\kappa}{\nu\lambda + (1-\nu)(1-\mu)},
$$

$$
\nu \frac{\psi F + \mu(1-F)(1-m)}{\nu\left[\psi F + (1-F)(1-m)\right] + (1-\nu)(1-F)(1-m)(1-\mu) + (1-m^l)F(1-\psi)} = \mu \frac{\nu\lambda - \frac{m^l F(1-\psi)}{m(1-F)+m^l F(1-\psi)}\kappa}{\nu\lambda + (1-\nu)(1-\mu)}.
$$

Therefore:

$$
\frac{dW}{dm^l} = \underbrace{\frac{\partial W}{\partial m^l}}_{=0} + \underbrace{\frac{\partial W}{\partial m}}_{>0}\underbrace{\frac{dm}{dm^l}}_{<0} + \underbrace{\frac{\partial W}{\partial \eta}}_{<0}\underbrace{\frac{d\eta}{dm^l}}_{>0} + \underbrace{\frac{\partial W}{\partial p_U}}_{>0}\underbrace{\frac{dp_U}{dm^l}}_{<0} < 0,
$$

$$
\begin{aligned}
\frac{\partial W}{\partial p_U} &= F\psi(\nu\lambda + (1-F)(1-m)\left[\nu\lambda + (1-\nu)(1-\mu)\right] > 0, \\
\frac{dp_U}{dm^l} &= \underbrace{\frac{dp_U}{dm}}_{>0}\underbrace{\frac{dm}{dm^l}}_{<0} - \frac{F(1-\psi)m(1-F)}{(m(1-F)+m^l F(1-\psi))^2\left[\nu\lambda + (1-\nu)(1-\mu)\right]}\mu A\kappa < 0, \\
\frac{\partial W}{\partial m} &= (1-F)\left[\kappa\mu A - \left[\nu\lambda p_U + (1-\nu)(\mu A + (1-\mu)p_U)\right]\right] > 0, \\
\frac{\partial W}{\partial \eta} &= f\Big[\psi(\nu\lambda p_U + (1-\nu)A) - \eta - \left[\kappa m\mu A + (1-m)\left[\nu\lambda p_U + (1-\nu)(\mu A + (1-\mu)p_U)\right]\right]\Big] \\
&= -f\kappa\left[m(\mu A - p_I) + (1-\psi)p_I\right] < 0, \\
\frac{d\eta}{dm^l} &= -\psi(1-\nu)(1-\mu)\underbrace{\frac{dp_U}{dm^l}}_{<0} + (p_I - p_U) > 0.
\end{aligned}
$$

Insurance by low-cost lenders lowers welfare. The direct effect of higher m^l is lower adverse selection in the secondary market for uninsured loans but higher adverse selection in insurance market. On the intensive margin, the former adverse selection redistributes resources from liquidity shocked lenders to lenders without liquidity shock and this reduces the social gains from trade. However, the adverse selection in insurance market redistributes resources from low-cost lenders to high-cost lenders but since both have the same expected marginal utility of consumption, there in no direct impact on social gains from trade.

The key negative effect of insurance by low-cost lenders is that it reduces the insurance by high-cost lenders. As a result, the insurance by low-cost lenders lowers allocative efficiency (lower p_I and p_U) and—despite an improvement in productive efficiency—the overall effect on welfare is negative.

B.9 Proof of Proposition 8

B.9.1 Relationship lending case

It is easy to show that early loan sales are equivalent to loan insurance in this case. Both choices result in the same payoffs for lenders–κp_0 for early sale and κp_I for insurance, where $p_0 = p_I = \mu A$ in equilibrium. Also both early sales and insurance serve as a commitment not to act on private information at $t = 1$. Therefore, both choices have the same positive pecuniary externality on the price of uninsured loans at $t = 1$, p_1, which is given by:

$$p_1 = \nu A \frac{F + (1-F)(1-m^I-m^S)\mu}{\nu\left[F + (1-F)(1-m^I-m^S)\right] + (1-nu)(1-\mu)(1-F)(1-m^I-m^S)}, \tag{65}$$

where m^I (m^S) is the fraction of high-cost lenders insuring (selling early). Therefore, all positive and normative result regarding loans insurance in the main text extend to early loan sales under relationship learning assumption.

B.9.2 Learning-by-holding case

Positive analysis There are two equilibria depending on whether high-quality loans are sold at $t = 1$ (liquid and illiquid). The illiquid equilibrium is equivalent to the illiquid equilibrium with insurance. All high-cost lenders sell early at price $p_0 = \mu A$, and low-cost lenders keep all loans till maturity. Comparing the respective payoffs gives a screening threshold $\eta^{NL} = (1-\kappa\mu)A$.

In the liquid equilibrium, all high-cost lenders sell early for a price $p_0 = \mu A + (1-\mu)p_1$ (outside financiers pass the payoff from keeping good loans until maturity and selling lemons). This is because the payoff from selling early, κp_0, strictly dominates the payoff of not selling early and not insuring, $\nu\lambda + (1-\nu)(\mu A + (1-\mu)p_1)$, as well as the payoff of insuring, $\kappa\mu A$. Hence, no lender insures.

The option to sell early increases adverse selection in the market at $t = 1$ because high-quality loans previously owned by high-cost lenders are never sold in this market:

$$p_1 = \frac{\nu F A}{\nu F + (1-F)(1-\mu)}. \tag{66}$$

The early loan sale has two opposing effects on the screening threshold. First, higher payoff for high-cost lenders from early sales lowers screening incentives. Second, lower price p_1 tends to increase them. The screening threshold equates the payoff from screening, $\nu\lambda p_1 + (1-\nu)A - \eta$, with the payoff from not screening, κp_0:

$$\eta = \max\left\{0, \left[(1-\nu)(1-\mu) - \nu\mu\lambda\right](A - p_1)\right\}. \tag{67}$$

The liquid equilibrium exists when $p_1 \geq A/\lambda$. A threshold that satisfies the liquidity condition has to satisfy

$$\frac{\nu F(\underline{\eta})}{\nu F(\underline{\eta}) + (1 - F(\underline{\eta}))(1-\mu)} A = \frac{A}{\lambda}, \tag{68}$$

where $\underline{\eta} = \max\left\{0, \left[(1-\nu)(1-\mu) - \nu\mu\lambda\right](A - A/\lambda)\right\}$. The RHS of equation (68) decreases in λ. The LHS of equation (68) is non-monotonic in λ, because F increases in $\underline{\eta}$, which is non-monotonic in λ. This implies that both for $\lambda = 1$ and for $\lambda \geq (1-\mu)(1-\nu)/(\mu\nu)$, the liquid equilibrium is not sustainable because of the implied variables $\eta = p_1 = 0$. Therefore, there may exist an interval $\lambda \in [\underline{\lambda}_L^S, \bar{\lambda}_L^S]$, where liquid equilibrium exists. Thresholds $\underline{\lambda}_L^S$ and $\bar{\lambda}_L^S$ are the two roots of (68) that lie on the interval $(1, (1-\mu)(1-\nu)/(\mu\nu))$. These thresholds exist for $A > A_L^S$, where A_L^S is implicitly given by

$$\frac{\nu F(A_L^S)}{\nu F(A_L^S) + (1 - F(A_L^S)(1-\mu)} = \frac{\mu\nu}{(1-\mu)(1-\nu)}.$$

Because of negative effects of early loan sales on the price p_1 (both direct and indirect through screening incentives), the lower threshold for the existence of liquid equilibrium is higher than in the benchmark model (with or without insurance), that is $\underline{\lambda}_L^S > \underline{\lambda}_L > \tilde{\lambda}_L$.

Normative analysis We show that a planner who controls early loan sales reduces the amount of these. Since it is not or efficient for low-cost lenders to sell early, this is equivalent to choosing the fraction of high-cost lenders who sell early, m^S.

First, we study the case where lower m^S could increase p_1 to sustain the liquid equilibrium. Using similar steps as in the Appendix B.6 we can show that the liquid equilibria is socially preferred for large enough $\lambda > \lambda_L^{PS}$, where $\lambda_L^{PS} < \underline{\lambda}_L^S$. Note that for $\lambda > \lambda_L^{PS}$ all lenders prefer the liquid equilibrium, because it gives an additional option to sell at positive price at $t = 1$. Even the lenders who are constrained not to sell early, (weakly) prefer the liquid equilibrium as they have an option to insure and achieve a higher payoff ($\max\{\kappa\mu A, \nu\lambda p_1 + (1-\nu)(\mu A + (1-\mu)p_1)\}$) than in the illiquid equilibrium ($\kappa\mu A$).

Second, we study whether the planner would like to reduce early sales in the liquid equilibrium, when $\lambda \in [\underline{\lambda}_L^S, \bar{\lambda}_L^S]$. We express the welfare as:

$$\begin{aligned} W &= (\nu\lambda p_1 + (1-\nu)A) - \int_0^\eta \tilde{\eta} dF(\tilde{\eta}) \\ &\quad + (1-F)\left\{m^S \kappa(\mu A + (1-\mu)p_1) + (1-m^S)\max\{\kappa\mu A, \nu\lambda p_1 + (1-\nu)(\mu A + (1-\mu)p_1)\}\right\}. \end{aligned}$$

It can be shown that a necessary condition for negative effects of early loan sales on welfare $dW/dm^S < 0$ collapses to:

$$(1-F)\left[\nu F(1-\nu)(\lambda - A) + \lambda(1-F)(1-\mu)\right] <$$
$$f\nu\lambda A\left[\nu\lambda\mu - (1-\nu)(1-\mu) + \frac{F}{1-F}\nu\lambda + (1-\mu)\frac{\nu F A\kappa}{\nu F + (1-F)(1-\mu)}\right]. \quad (69)$$

For $A \to \infty$ the above condition (69) is satisfied, while for $A \to 0$ it is not. By continuity there is a threshold $\bar{A}^S$ implicitly defined by (69) with equality, such that for $A > \bar{A}^S$ the planner wants to lower early loans sales in the liquid equilibrium.

A regulator can implement fewer loan sales m^S by imposing taxes T^S to sellers of loans at $t = 0$ and redistributing the proceeds to all lenders. Optimal tax makes high-cost lenders indifferent about early loan sale at the constrained-efficient price $p_1^P = p_U^P$, so:

$$T^S = (1-\mu)p_1^P - \frac{\max\left\{0, \nu\lambda(p_1^P - \mu A) + (1-\nu)(1-\mu)p_1^P\right\}}{\kappa}.$$

B.10 Proof of Proposition 10

We derive the privately optimal insurance coverage ω^*. The price in secondary markets for insured loans is $p_I^* = \frac{\nu+(1-\nu)(1-\mu)\omega}{\nu+(1-\nu)(1-\mu)}\mu A$, which implies that p_I^* monotonically increases in insurance coverage, $\frac{dp_I^*}{d\omega} > 0$. Lenders who insure do not screen, so their problem is

$$\max_{\omega} \nu\lambda p_I + (1-\nu)\left(\mu(A-k) + (1-\mu)p_I\right) = \frac{\nu\kappa + (1-\nu)(1-\mu)\left(\kappa + \nu(\omega-1)(\lambda-1)\right)}{\nu + (1-\nu)(1-\mu)}\mu A,$$

which increases in ω. Thus, the corner solution $\omega^* = 1$ is optimal.

Next, we consider the socially optimal choice of insurance coverage. The payoff of uninsured low-cost lenders ($\nu\lambda p_U + (1-\nu)A - \eta_i$) and high-cost lenders ($\nu\lambda p_U + (1-\nu)(\mu A + (1-\mu)p_U) - \eta_i$) also increases in ω due to the positive externality of insurance coverage on the price of uninsured loans $\frac{dp_U}{d\omega} = \frac{dp_U}{dp_I}\frac{dp_I}{d\omega} > 0$. Therefore, a planner who maximizes aggregate welfare also chooses full insurance coverage, $\omega^{SP} = 1$:

$$\begin{aligned}
\omega^{SP} &= \arg\max_{\omega} \overbrace{\nu\lambda p_U\left[F + (1-F)(1-m)\right] + (1-\nu)\left[FA + (1-F)(1-m)(\mu A + (1-\mu)p_U)\right]}^{\text{Value to uninsured lenders}} \\
&\quad + \underbrace{\left(\nu\lambda p_I + (1-\nu)\left(\mu(A-k) + (1-\mu)p_I\right)\right)m(1-F)}_{\text{Value to insured lenders}} - \underbrace{\int_0^\eta \tilde{\eta}\, dF(\tilde{\eta})}_{\text{Screening costs}} \\
&= \arg\max_{\omega} \nu\lambda p_U + (1-\nu)\left[FA + (1-F)(\mu A + (1-\mu)p_U)\right] - \int_0^\eta \tilde{\eta}\, dF(\tilde{\eta}), \qquad (70)
\end{aligned}$$

subject to (6) and (34), where equation (70) is obtained after substituting the indifference condition (10). The solution follows from $\frac{dW}{d\omega} = \left(\underbrace{\frac{\partial W}{\partial p_U^*}}_{>0} + \underbrace{\frac{\partial W}{\partial \eta^*}}_{=0}\underbrace{\frac{d\eta^*}{dp_U^*}}_{<0}\right)\underbrace{\frac{dp_U^*}{dp_I^*}}_{>0}\underbrace{\frac{dp^{I*}}{d\omega}}_{>0} > 0.$

B.11 Proof of Proposition 11

First, we study the equilibrium with insurance. Insurers' break-even condition, $k = (1-k)A(1-\mu)$, determines the insurance fee: $k = \frac{A(1-\mu)}{1+A(1-\mu)}$. The high-cost lenders are indifferent between the insurance payoff $(1-k)A(\nu\lambda+1-\nu) = \kappa A(\mu-\delta(1-\mu))$ and the non-insurance payoff $\nu\lambda p'_U + (1-\nu)(\mu A + (1-\mu)p'_U$. Equating those payoffs gives a condition for the price of uninsured loans:

$$p_U^{*\prime} = \frac{\nu\lambda\mu A - \kappa A\delta(1-\mu)}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{71}$$

Combining this equation with the break-even condition of outside financiers (34) gives

$$m^{*\prime} = 1 - \frac{\kappa F(1-\delta)}{(1-F)\left[\mu(1-\nu)(\lambda-1) - \frac{\kappa}{\nu}\delta(\nu+(1-\nu)(1-\mu)\right]}. \tag{72}$$

Finally, substituting p'_U from (71) into (6) gives:

$$\eta^{*\prime} = \frac{(1-\nu)(1-\mu)^2 A\kappa(1+\delta)}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{73}$$

The price in equation (71) must satisfy condition (3) to ensure a liquid equilibrium. Thus, a necessary condition for a liquid equilibrium when insurance is used is $\nu\left[\mu - \delta(1-\mu)\right]\lambda^2 - \left[\nu + \delta(1-\mu)(1-\nu)\right]\lambda - (1-\nu)(1-\mu) \geq 0$. Since only the larger root of this quadratic condition is positive, the condition collapses to $\lambda \geq \tilde{\lambda}'_L \equiv \frac{\nu+\delta(1-\mu)(1-\nu)}{2\nu[\mu-\delta(1-\mu)]} + \sqrt{\frac{[\nu+\delta(1-\mu)(1-\nu)]^2}{4\nu^2[\mu-\delta(1-\mu)]^2} + \frac{(1-\mu)(1-\nu)}{\nu[\mu-\delta(1-\mu)]}}$. For $\mu A > 1$, $\delta > 0$ and the threshold for the existence of liquid equilibrium is higher, $\tilde{\lambda}'_L > \tilde{\lambda}_L$. Insurance takes place on the subset $A < \tilde{A}'$, where the threshold $\tilde{A}'$ is implicitly defined by a combination of (71) and (34), where $m^{*\prime} = 0$:

$$\kappa(1-\delta)F(\eta) = (1-F(\eta))\left[\mu(1-\nu)(\lambda-1) - \frac{\kappa}{\nu}\delta(\nu+(1-\nu)(1-\mu))\right], \tag{74}$$

where $\eta(\tilde{A}')$ and $\delta(\tilde{A}')$. For $\mu A > 1$, $\delta > 0$, which implies $\tilde{A}' < \tilde{A}$. $\tilde{A}'$ is unique since $\frac{dm}{dA} < 0$. To prove this, we define Ξ as the difference of the two expressions for price, (71) and (34):

$$\Xi \equiv \frac{F + (1-F)(1-m)\mu}{\nu F + (1-F)(1-m)(\nu + (1-\nu)(1-\mu))} - \frac{\lambda\mu - \frac{\kappa}{\nu}(1-\mu)\delta}{\nu\lambda + (1-\nu)(1-\mu)} = 0,$$

and then show that $\frac{dm}{dA} = -\frac{\partial\Xi/\partial A}{\partial\Xi/\partial m} < 0$. If $\mu A > 1$ and $A < \tilde{A}'$, then $\delta > 0$ and $\eta^{*\prime}$ given by (73) exceeds η^* given by (32), and $p_U^{*\prime}$ given by (71) is smaller than price p_U^* given by (31), which together with dp_U/dm implies that $m^{*\prime} < m^*$.

Second, regarding normative implications, it is straightforward to show that the constrained efficient level of insurance exceeds the unregulated level at both the intensive margin and the extensive margin by following the same steps as in the proof for case with insurance fee charged at $t = 2$ in Appendix B.5.

B.12 Proof of Proposition 12

Definition 4. *A perfect Bayesian equilibrium with loan insurance comprises choices of screening $\{s_i\}$, insurance $\{\ell_i\}$, loan sales $\left\{q_i^I, q_i^U\right\}$, financier beliefs about loan quality $\phi_{i,t}$, secondary market prices p_I and p_U, and an insurance fee k such that:*

1. *At $t = 1$, each lender i optimally chooses sales of insured and uninsured loans for each realized liquidity shock $\lambda_i \in \{1, \lambda\}$, denoted by $q_i^I(s_i, \lambda_i, \ell_i)$ and $q_i^U(s_i, \lambda_i, \ell_i)$, given the prices p_I and p_U and choices of screening s_i and insurance ℓ_i.*
2. *At $t = 1$, outside financiers use Bayes' rule to update their beliefs $\phi_{i,1}(q_i^U, q_i^I, \ell_i)$ on the equilibrium path, and prices p_I and p_U are set for outside financiers to expect to break even, given screening $\{s_i\}$ and insurance $\{\ell_i\}$ choices, the fee k, and sales $\{q_i^I, q_i^U\}$.*
3. *At $t = 0$, each lender i chooses its screening s_i and loan insurance ℓ_i to maximize the expected utility, given prices p_I and p_U, the fee k, and sales q_i^I and q_i^U:*

$$\max_{s_i,\ell_i,c_{i1},c_{i2}} \mathbb{E}\left[\lambda_i c_{i1} + c_{i2} - \eta_i s_i\right] \qquad \textit{subject to}$$
$$c_{i1} = q_i^U(s_i, \lambda_i, \ell_i)\, p_U + q_i^I(s_i, \lambda_i, \ell_i)\, p_I,$$
$$c_{i2} = \left[\ell_i - q_i^I\right](A-k) + \left[1 - \ell_i - q_i^U\right] \times \begin{cases} A & w.\ p. \quad s_i + \mu(1-s_i) \\ 0 & \qquad\quad (1-\mu)(1-s_i). \end{cases}$$

4. *At $t = 0$, outside financiers use Bayes' rule to update their beliefs $\phi_0(\ell)$ on the equilibrium path, and the fee k is set for financiers to break even in expectation, given screening $\{s_i\}$ and insurance $\{\ell_i\}$ choices.*

Risk retention as signal of loan type. In a separating equilibrium with both high-cost and low-cost lenders, sellers of high-quality loans choose $q^U \in (0, 1]$ (since $\ell_i \in \{0, 1\}$), and sellers of low-quality loans choose $q^{U\prime} \neq q^U$, such that $p_U(q^U) = A$ and $p_U(q^{U\prime}) = 0$. Since lenders cannot commit to negative consumption, high-cost lenders with lemons will always want to mimic sellers with high-quality loans since $q^U p_U(q^U) = q^U A > q^{U\prime} p_U(q^{U\prime}) = 0$. Hence, there exists no separating equilibrium with partial screening, $\eta^* < \bar{\eta}$.

However, there could exist an equilibrium with $q^U < 1$, where all lenders screen and, therefore, loan quality becomes public information. We derive the threshold screening cost by equating the payoff from screening, $\nu[\lambda p_U q^U + (1 - q^U)A] + (1 - \nu)A - \eta$, and payoff when not screening, $\nu[\lambda p_U q^U + (1 - q^U)\mu A] + (1 - \nu)(\mu A + (1 - \mu)p_U q^U)$:

$$\begin{aligned} \eta &= (1-\mu)[\nu(1-q^U)A + (1-\nu)(A - p_U q^U)] && (75) \\ &= (1-\mu)(1-q^U)A, && (76) \end{aligned}$$

where the second equality comes from $p_U = A$. Equation (76) implies that there are no high-cost lenders, $\eta \geq \bar{\eta}$, if retention is large enough, $(1 - q^U) \geq \frac{\bar{\eta}}{(1-\mu)A}$. Thus, a sufficient condition for ruling out this equilibrium is $\bar{\eta} \geq (1 - \mu)A$.

Pooling equilibria with partial sales. The rest of the proof focuses on the pooling equilibria with partial sales and shows that our main results are qualitatively unchanged.

Define $\bar{q}^U \equiv \min\left\{0, 1 - \frac{\bar{\eta}}{(1-\mu)A}\right\}$ as the maximum loan sales consistent with full screening, $\eta^* \geq \bar{\eta}$. Then there exist a continuum of pooling perfect Bayesian equilibria with $q^U \in (\bar{q}^U, 1]$ in the appropriately generalized liquid equilibrium, $\lambda > \tilde{\lambda}_I(q^U)$, where the out-of-equilibrium beliefs of financiers that a sold loan is of high quality is $\phi_{i,1} = 0$ if $q_i^U \neq q^U$.

If insurance is used in this equilibrium, high-cost lenders have to be indifferent between payoff when not insuring, $\nu\lambda p_U q^U + \nu(1-q^U)\mu A + (1-\nu)(\mu A + (1-\mu)p_U q^U)$, and insurance when insuring, $\kappa\mu A$. Equating those payoffs determines the price of uninsured loans:

$$p_U^* = \frac{\nu\mu A\left[\lambda + \frac{(\lambda-1)(1-q^U)}{q^U}\right]}{\nu\lambda + (1-\nu)(1-\mu)}, \tag{77}$$

which is a generalization of (31). This price decreases in q^U, $dp_U/dq^U < 0$, because higher uninsured loan sales make insurance relatively less attractive, and a lower price of uninsured loans satisfies the insurance indifference equation. Using (75), the screening threshold is

$$\eta^* = \frac{(1-\mu)\kappa A}{\nu\lambda + (1-\nu)(1-\mu)}\left[(1-\nu)(1-\mu) + \nu(1-q^U)\right], \tag{78}$$

which is a generalization of (32). The screening threshold decreases with q^U, $d\eta/dq^U < 0$, since a higher q^U lowers the net benefits of screening from loans held to maturity in case of liquidity shock, term $(1-\mu)\nu(1-q^U)A$, and increases the payoff from the sale of lemons when not screening, term $(1-\nu)p_U q^U$, where $dp_U q^U/dq^U > 0$. Combining (77) with the break-even condition of outside financiers (34), the fraction of insured high-cost lenders is

$$m^* = 1 - \frac{F(\eta^*)\left[\kappa q^U(1-\mu) - \mu(\lambda-1)(1-q^U)\right]}{\mu(1-F(\eta^*))(\lambda-1)[(1-\nu)(1-\mu) + (1-q^U)\nu]}, \tag{79}$$

which is a generalization of (36). Hence, $m^* > 0$ whenever

$$A < \tilde{A}(q^U) \equiv \frac{\nu\lambda + (1-\nu)(1-\mu)}{(1-\mu)\kappa[(1-\nu)(1-\mu) + (1-q^U)\nu]} F^{-1}\left(\frac{\mu(\lambda-1)[(1-\nu)(1-\mu) + (1-q^U)\nu]}{\kappa(1-\mu)q^U + \mu(\lambda-1)(1-\nu)(q^U-\mu)}\right). \tag{80}$$

It is straightforward to show that the constrained efficient level of insurance exceeds the unregulated level at both the intensive margin and the extensive margin by following the same steps as in the proof for case $q^U = 1$ in Appendix B.5.

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